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Probabilistic condition assessment of structures by multiple FE model identification considering measured data uncertainty

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Abstract. A new procedure is proposed for assessing probabilistic condition of structures considering effect of measured data uncertainty. In this procedure, multiple Finite Element (FE) models are identified by using weighting vectors that represent the uncertainty conditions of measured data. The distribution of structural parameters is analysed using a Principal Component Analysis (PCA) in relation to uncertainty conditions, and the identified models are classified into groups according to their similarity by using a *K*-means method. The condition of a structure is then assessed probabilistically using FE models in the classified groups, each of which represents specific uncertainty condition of measured data. Yeondae bridge, a steel-box girder expressway bridge in Korea, is used as an illustrative example. Probabilistic condition of the bridge is evaluated by the distribution of load rating factors obtained using multiple FE models. The numerical example shows that the proposed method can quantify uncertainty of measured data and subsequently evaluate efficiently the probabilistic condition of bridges.

Keywords: condition assessment; FE Model update; measurement uncertainty; principal component analysis; *K*-means clustering; load rating

1. Introduction

The purpose of condition assessment is to provide a diagnosis of its current state, predict upcoming performance degradation, and ultimately prevent gradual or sudden failure. The accuracy of an assessment can be assured by a reliable Finite Element analysis. However, FE models frequently lack accuracy in terms of the actual behavior of a structure. The discrepancies between predicted and observed behavior are mainly accounted by modeling uncertainty, which is a combination of various factors including assumptions in the design process, deterioration of the structures, variability of material properties, and the limited resolution of the FE model and numerical errors in discretization using finite number of elements.

The modeling uncertainty can be reduced by updating the FE model using measured data, of which procedure mostly involves solving an optimization problem (Park *et al*. 2012). The conventional updating procedure usually assumes that the measured data is accurate and involves deterministic values, thus a single optimal FE model is identified. However, measured data is also

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deterministic values, thus a single optimal FE model is identified. However, measured data is also known to exhibit considerable variability due to non-stationary behavior of structure, noise and bias during data acquisition, numerical error in data processing, and influence of adverse environmental conditions such as temperature and humidity variation. The risk of an error in the identification of single updated model under the existence of data uncertainties has been reported (Robert-Nicoud *et al*. 2000, Schuëller *et al*. 2008).

Variability of FE model caused by data uncertainty can be dealt by using a probabilistic approach such as inverse Stochastic FEM (Mares *et al*. 2006, Soize *et al*. 2008, Govers and Link 2009). Bayesian model updating has also been proposed to find the most plausible posterior PDF of structural parameters (Katafygiotis *et al*. 1998, Beck and Au 2002, Park *et al*. 2010). Uncertainty of measured data can also be represented by variation of weighting factors for optimization problem (Friswell and Mottershead 1999, Steenackers and Guillaume 2006). The non-uniqueness of optimal model due to uncertainties has also been dealt by identification of multiple models (Zarate and Caicedo 2008, Groulet *et al*. 2010).

This paper proposes a procedure for a probabilistic condition assessment of structures considering measured data uncertainty. In this procedure, multiple Finite Element (FE) models are identified through successive optimizations by using sets of weighting vectors. The successive optimizations can overcome the limitation of conventional single model update to deal with the uncertainty of measured data, and estimate probabilistic information about the structural condition. The distribution of multiple FE models is analysed using the Principal Component Analysis (PCA) in relation to uncertainty conditions of measured data. The PCA is utilized for dimension reduction and feature extraction of the multiple models, and improving the clustering accuracy. The multiple FE models can be classified into groups according to their similarity by using a *K*-means clustering. Combined use of PCA and *K*-means clustering can provide probabilistic information about state of the structure, and reduce subjectivity in the interpretations and assessments. Finally, the condition of a structure can be expressed probabilistically by the distribution of performance indices such as load rating factor using FE models in the classified groups, each of which represents specific uncertainty condition of measured data. Furthermore, probability of failure is also estimated using multivariate normal distribution function describing the distribution of structural parameters. Yeondae bridge, a steel-box girder expressway bridge in Korea, is used as an illustrative example. In the example, the distribution of load rating factors and the probability of a bridge failure are evaluated for probabilistic assessment of bridge condition. Numerical example shows that the proposed method can quantify uncertainty of measured data and subsequently evaluate efficiently the probabilistic condition of bridges.

2. Updating multiple finite element models

2.1 Formulation of successive optimization problems

A procedure for updating a FE model involves different types of measured data, e.g., natural frequencies, mode-shapes, damping ratios, displacement, strain, and so on, in order to prevent the occurrence of under-fitted solutions. This represents the multi-objectivity of a FE model update problem. However, it was pointed out that the multi-objective optimization of more than three objectives would be computationally too expensive, and may converge to the wrong solution (Jung *et al.* 2010). Especially in case of the large-scale structure examples, the problem of convergence and the credibility of a solution may be worsened because sufficient number of structural parameters and target responses should be considered. As an alternative, the problem of finding an optimal FE model satisfying various types of responses can be formulated as an aggregated single-objective function by assigning a weighting factor for each residual. Typically, a single FE model is updated by optimization of the objective function. The single updated model can reflect specific structural condition and is not suitable to represent probabilistic condition of the structure considering uncertainty of the measured data.

In this paper, successive optimization is proposed to deal with the uncertainty of measured data, and to estimate probabilistic condition of structure. For the purpose, a set of optimization problems with each member in the set corresponding to a different choice of the weights are constructed. Varying weighting factors represent different levels of measurement uncertainties. Accordingly, multiple FE models are updated from the optimization problems incorporating measurement uncertainties. An optimization problem corresponding to a specific choice of the weight factors is expressed as follows

$$\min\ J_i(\boldsymbol{\theta}_i) = \mathbf{W}_i^T \mathbf{r}(\boldsymbol{\theta}_i) = \sum_{j=1}^{M} w_{ji} r_j(\boldsymbol{\theta}_i) \quad \text{subjected to} \quad \boldsymbol{\theta}_{lb} \le \boldsymbol{\theta}_i \le \boldsymbol{\theta}_{ub} \tag{1}$$

The objective function J_i, denoted by the index i, is a component in the sets of optimization problems corresponding to a different choice of the weights $\mathbf{W}_i$, by which specific uncertainty condition is represented. The subscript j denotes different type of measurements. The $\boldsymbol{\theta}$ is a vector of updated structural parameters that are normalized to the initial values. Structural parameters of different units and magnitudes can be considered effectively by the normalization. The vector of updated structural parameters for the i-th optimization problem is expressed as $\boldsymbol{\theta}_i = [\theta_{1i}, \theta_{2i}, \ldots, \theta_{Pi}]^T$ where P is the number of parameters. The optimal value of $\boldsymbol{\theta}_i$ that can minimize the objective function is identified within a reasonable range defined by $\boldsymbol{\theta}_{lb}$ and $\boldsymbol{\theta}_{ub}$, a lower and upper bound vector, respectively. Weighting factor w_{ji} in Eq. (1) represents weight for the residual of j-th response. An weighting vector $\mathbf{W}_i = [w_{1i}, w_{2i}, \ldots, w_{Mi}]^T$, composed by M weighting factors, represents specific uncertainty condition of measured data in the i-th optimization. The residual r_j represents discrepancy between the analytical model and a real structure with respect to the j-th type of measurement.

$$r_j(\boldsymbol{\theta}_i) = \sqrt{\frac{1}{N_u} \sum_{n=1}^{N_u} \left(\frac{u_{a,n}(\boldsymbol{\theta}_i) - u_{m,n}}{u_{m,n}} \right)^2} \tag{2}$$

where u is the concerned responses such as displacement and natural frequency, subscript n represents the n-the measurement within the N_u total measurements, subscripts a and m represent the analyzed and measured values, respectively. When calculating mode-shape residual $r_{i\phi}$, Modal Assurance Criterion (MAC) is generally adopted.

$$r_\phi = 1 - \mathrm{MAC}(\theta_i) = 1 - \frac{[\phi_m^T \phi_a(\theta_i)]^2}{[\phi_m^T(\theta_i)\phi_m(\theta_i)] \cdot [\phi_a^T \phi_a]} \tag{3}$$

where ϕ_m and ϕ_a are mode-shape of real structure and analysis model, respectively.

The Sequential Quadratic Programming (SQP) is utilized to find optimal FE models for the constrained non-linear optimization problems, each of which is characterized by a weighting vector $\mathbf{W}_i$ representing specific uncertainty condition. The SQP method approximates the objective function to a quadratic model, and solves a sequence of optimization sub-problems iteratively until convergence is achieved. Karush-Kuhn-Tucker (KKT) conditions and the rate of change of the $\boldsymbol{\theta}$ are considered to check convergence.

2.2 Identification of multiple FE models from sets of weighting vectors

Weighting factors can be determined by the reciprocals of the variance of the corresponding measurements (Friswell and Mottershead 1999). In this paper, various levels of measurement uncertainty are represented by variation of the weighting vector $\mathbf{W}_i$. Varying relative magnitude of weighting factors for different responses account for different level of uncertainty. For example, smaller weight factor can be assigned to the measured data that is expected to contain large uncertainty. On the other hand, large weighting factor of a residual implies that the related data contain small uncertainty. In this way, a weighting vector $\mathbf{W}_i$ can represent specific condition of measurement uncertainties. Accordingly, various conditions of data uncertainty can be reflected by sets of weighting vectors as $\mathbf{W} = [\mathbf{W}_1, \mathbf{W}_2, \ldots, \mathbf{W}_N]$.

To include all possible conditions of measurement uncertainties, the generated weighting vectors are uniformly distributed in feasible space. The directional angles of the weighting vector are taken as variables. For example, if we deal with FE model update associated with three types of responses, a weighting vector $\mathbf{W}_i$ include three weighting factors. The $\mathbf{W}_i$ can be expressed by the three directional angles as Eq. (4).

$$\mathrm{W}_i = [w_{i1}, w_{i2}, w_{i3}]^T = \|\mathrm{W}_i\| \cdot \left[\frac{w_{i1}}{\|\mathrm{W}_i\|}, \frac{w_{i2}}{\|\mathrm{W}_i\|}, \frac{w_{i3}}{\|\mathrm{W}_i\|} \right]^T = \|\mathrm{W}_i\| \cdot [\cos\alpha_i, \cos\beta_i, \cos\gamma_i]^T \tag{4}$$

where α_i, β_i and γ_i are directional angles between the $\mathbf{W}_i$ and three axes for the responses. By setting the norm of the $\mathbf{W}_i$ as one and considering dependency between directional angles, the Eq. (4) can be expressed with two variables as

$$\mathbf{W}_i = \left[\cos\alpha_i, \cos\beta_i, \sqrt{1-\cos^2\alpha_i - \cos^2\beta_i} \right]^T \tag{5}$$

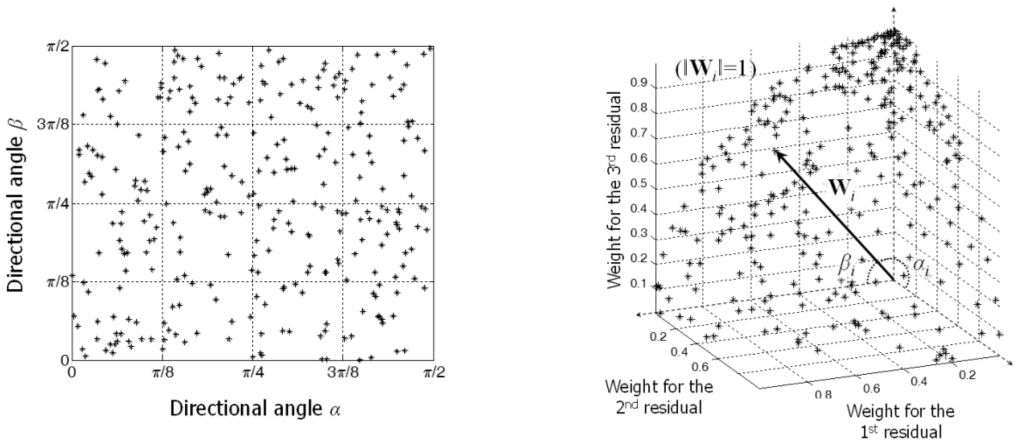
Therefore, generating weighting vectors for three types of responses is equivalent to sampling two directional angles α and β uniformly in 2-D Euclidean space (Fig. 1(a)). The direction angles are acute, i.e., $0 \le \alpha \le \pi/2$ and $0 \le \beta \le \pi/2$ because the weighting factors should be larger than or equal to zero. Fig. 1(b) shows the consequent weighting vectors that compose surface of 1/8 sphere in the first octant.

In case N weighting vectors are generated as $\mathbf{W} = [\mathbf{W}_1, \mathbf{W}_2, \ldots, \mathbf{W}_N]$, N optimal FE models are finally identified by N successive optimizations and compose a matrix of optimal structural parameters $\boldsymbol{\Theta} = [\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \ldots, \boldsymbol{\theta}_N]$. The i-th column vector $\boldsymbol{\theta}_i$ indicates the properties of the i-th updated model, and each of the p rows corresponds to considered structural parameter. Thus, the matrix $\boldsymbol{\Theta}$ expresses the variability of multiple FE models by the distribution of structural parameter values. Fig. 2 shows an example of updated multiple FE models distributed in the space of structural parameters. Each point represents probable status of structure under the employed uncertainty condition, which is represented by a corresponding weighting vector. Accordingly, distribution of the updated FE models provides probabilistic information about the structural condition due to the measurement uncertainties.

3. Probabilistic structural condition assessment using multiple FE models

3.1 Analysis of structural parameters distribution of multiple FE models

The Principal Component Analysis (PCA) is utilized for the purposes of dimension reduction and feature extraction of the multiple models. Applying the PCA is also beneficial to improve the clustering accuracy (Ding and He 2004). In addition, the PCA enables to visualize multiple FE models using a couple of principal components only, thus analysis on the grouped models becomes easier.



(a) Uniform sampling of two directional angles (b) Generating sets of weighting vectors

Fig. 1 Example of weighting vector generation for three types of responses

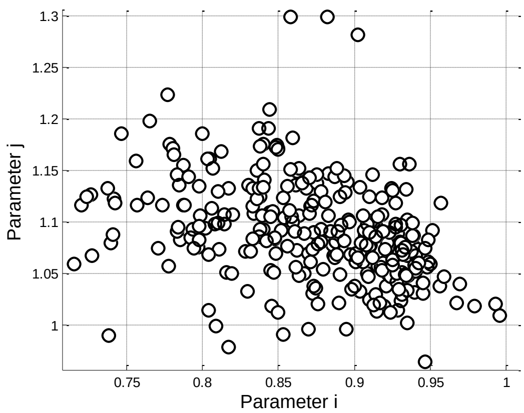


Fig. 2 Distribution of multiple FE models in the space of structural parameters

As a first step to find principal components, the covariance of structural parameters for the N optimal FE models are computed.

$$\mathbf{S} = E[(\mathbf{\Theta} - E[\mathbf{\Theta}])(\mathbf{\Theta} - E[\mathbf{\Theta}])^T] \tag{6}$$

Each entry of the covariance matrix **S** is calculated as

$$S_{pq} = \frac{1}{N}\sum_{i=1}^{N}\left(\theta_{pi} - \bar{\theta}_p\right)\left(\theta_{qi} - \bar{\theta}_q\right) \tag{7}$$

where θ_{pi} is p-th component of optimal structural parameter vector $\boldsymbol{\theta}_i$, and $\bar{\theta}_p$ is a mean value given by Eq. (8).

$$\bar{\theta}_p = \frac{1}{N}\sum_{i=1}^{N}\theta_{pi} \tag{8}$$

By applying eigenvector decomposition on **S**, eigenvector matrix **u** and eigenvalue matrix $\mathbf{\Lambda}$ are obtained.

$$\mathbf{S} = \mathbf{u}\mathbf{\Lambda}\mathbf{u}^T \tag{9}$$

The eigenvector matrix **u** contains principal components in its column vectors, which are bases of transformed space. The contribution of principal components to explain the variance of the distribution of structural parameters is calculated from diagonal components of $\mathbf{\Lambda}$.

Finally, the coordinate of each model in the transformed space is calculated as

$$\hat{\boldsymbol{\theta}}_i = \mathbf{u}^T\left(\boldsymbol{\theta}_i - \bar{\boldsymbol{\theta}}\right) \tag{10}$$

where $\bar{\boldsymbol{\theta}} = \left[\bar{\theta}_1, \bar{\theta}_2, \ldots, \bar{\theta}_P\right]^T$.

The relationship between the distribution of structural parameters and uncertainty conditions of the measured data can be characterized by the principal components on the transformed space.

3.2 Grouping of multiple FE models according to similarity

The K-means method is further implemented in order to classify the multiple FE models into several groups according to the similarity, which is now revealed by the application of PCA. Combined use of PCA and K-means clustering can group the multiple models, and reduce subjectivity in the interpretations and assessments. The objective function for the clustering is given by

$$J = \sum_{k=1}^{K}\sum_{i=1}^{N} r_{ik}\left\|\hat{\boldsymbol{\theta}}_i - \boldsymbol{\mu}_k\right\|^2 \tag{11}$$

where $\boldsymbol{\mu}_k$ is the centroid of the k-th cluster and r_{ik} indicates membership of the i-th model in the k-th cluster. If the i-th model is assigned to cluster k then $r_{ik} = 1$, otherwise $r_{ik} = 0$. Term inside Σ measures the sum of the inner-distances of data points that are assigned to the k-th cluster. The algorithm iteratively finds centroids of clusters $\boldsymbol{\mu}_k$ and r_{ik} for each of N updated models. The number of clusters K is determined by calculating score function which evaluates the quality of clustering by two quantities: the Between Cluster Distance (*BCD*) and the Within Cluster Distance (*WCD*). The former one indicates the degree of separation of clusters from each other, whereas the second term indicates the degree of compactness of each cluster. The score function is thus defined as

$$S.F. = BCD/WCD \tag{12}$$

where

$$BCD = \frac{1}{NK}\sum_{k=1}^{K}\left\|\overline{\boldsymbol{\mu}} - \boldsymbol{\mu}_k\right\|^2 n_k \tag{13}$$

$$WCD = \frac{1}{K}\sum_{k=1}^{K}\sqrt{\frac{1}{n_k}\sum_{i=1}^{N} r_{ik}\left\|\hat{\boldsymbol{\theta}}_i - \boldsymbol{\mu}_k\right\|^2} \tag{14}$$

where n_k is the number of FE models in the k-th cluster, and $\overline{\boldsymbol{\mu}}$ is the mean of centroids of all K clusters. Score function values are calculated for different number of clusters, and optimal number of clusters for the maximum score is chosen.

The FE models in each identified group may be characterized by similar property of weighting vectors each of which represents specific uncertainty condition of measured data. Thus, depending on the features of the groups and characteristic of concerned assessment, a specific group of FE models is selected to assess the condition of structures probabilistically.

3.3 Probabilistic assessment of condition using the multiple FE models

A set of FE models provides the probabilistic information about the structural condition considering measurement uncertainties. Thus, the variability of multiple models indicates a likelihood of status that the structure may exhibit under the employed uncertainty of measured data, rather than modeling uncertainty. Accordingly, the updated models can be used for probabilistic assessment of structural condition.

The condition of structure can be expressed as distribution of any types of structural performance index, such as load rating factor, seismic fragility and safety index. In this paper, distribution of Load Rating Factors (RF) is calculated by numerical simulations with the multiple FE models. The distribution of RFs provides statistical information about vehicle load carrying capacity of bridge. When all models are used, the estimated distribution can consider overall variability of multiple models caused by all possible conditions of data uncertainty. A group of similar models can also be selected to consider specific status of structure according to the classified conditions of data uncertainty.

When the statistical information is not necessary, a model representing each cluster can also be selected from the centroids of clusters by Eq. (15).

$$\boldsymbol{\theta}_c = \left[\mathbf{u}^T\right]^{-1}\boldsymbol{\mu}_c + \overline{\boldsymbol{\theta}} \tag{15}$$

where $\boldsymbol{\mu}_c$ is the coordinate of centroid of selected group in the principal component space, and $\boldsymbol{\theta}_c$ the corresponding structural parameters. Structural assessment using the representative model yields the average condition of the structure.

Probability of bridge failure is also evaluated for rigorous probabilistic assessment. A probability model for multiple FE models is obtained by fitting a multivariate normal distribution to the matrix of transformed structural parameters $\hat{\mathbf{\Theta}}$. Once the mean vector $\boldsymbol{\mu}_{\hat{\theta}}$ and covariance matrix $\boldsymbol{\Sigma}_{\hat{\theta}}$ are identified, a set of offspring models are sampled from the PDF $p(\hat{\boldsymbol{\theta}}) = N(\boldsymbol{\mu}_{\hat{\theta}}, \boldsymbol{\Sigma}_{\hat{\theta}})$ and expressed by a matrix $\hat{\mathbf{\Theta}}_{offspring}$. The offspring models in principal component space is inversely transformed to the space of structural parameters as $\mathbf{\Theta}_{offspring} = \mathbf{u}^{-T}\hat{\mathbf{\Theta}}_{offspring} + \bar{\boldsymbol{\theta}}\vec{\mathbf{1}}$, where $\vec{\mathbf{1}}$ is a row vector of which components are 1, and $\mathbf{\Theta}_{offspring}$ indicates structural parameters of offspring models. Through the numerical simulation using the offspring models, probabilistic assessment of structural condition can be conducted rigorously. In this paper, probability that the bridge does not meet the safety criteria with respect to the rating factor (RF) is calculated by using the offspring models. The RF equals to 1 defines a safety criteria indicating necessity of bridge reinforcement or vehicle regulation due to insufficient load carrying capacity, rather than actual collapse of the bridge.

The overall procedure for multiple FE model identification, grouping and probabilistic condition assessment is shown in the flow chart (Fig. 3).

4. Application to the assessment of bridge condition

4.1 Application to Yeondae bridge

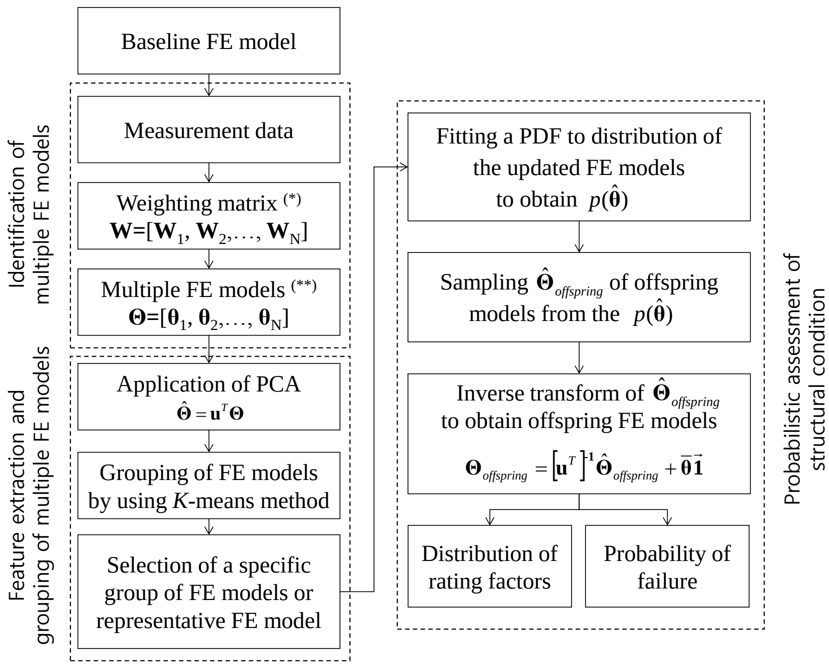
Yeondae bridge is comprised of a composite steel box girder with two cells as shown in Fig. 4. The bridge is located in a test road section of the expressway 45 in Korea.

Static loading tests were carried out under various load cases using two test trucks, and dynamic loading tests were conducted by running test trucks on the bridge with a varying speed from 5 km/h to 100 km/h. Natural frequencies, mode-shapes and dynamic amplification factor were identified from the instrumented accelerations and dynamic displacements (Kim *et al*. 2013). In this study, three types of measured data are employed: the first 3 natural frequencies and the associated mode-shapes, and vertical displacements of box girders obtained by 3 static load cases (Figs. 5 and 6).

4.2 Identification of multiple FE models

A baseline FE model has been developed based on the design documents. The box girders, cross frames and bracings are modeled using three-dimensional frame elements, and elastic spring support elements are used to represent the bearings. The equivalent sectional properties of each box girder are calculated considering the composite concrete deck. The structural behavior computed by using this FE model was proven to be in good accordance with the actual one measured at the completion of construction (Kim *et al*. 2013).

The baseline FE model is updated by the proposed procedure in order to evaluate condition of the bridge. In the optimization process, 37 structural parameters are considered including the coefficient of spring support elements, the mass of structural and substructural members, Young's modulus of the composite box-girder, cross frame and slab, moment of inertia of girder and cross frame. Table 1 summarizes the structural parameters and their allowable bounds considered in the optimization.



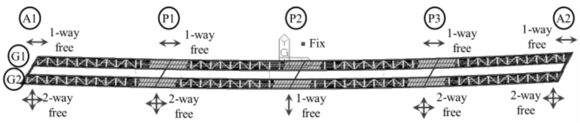
(*) Consideration of data uncertainty

(**) Successive optimizations

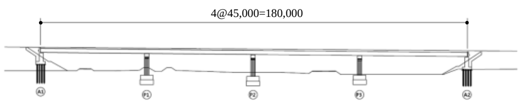
Fig. 3 Overall schema of multiple FE model identifications, feature extraction, and probabilistic assessment of structural condition considering data uncertainty

(a) Composite twin box-girders

(b) Side view

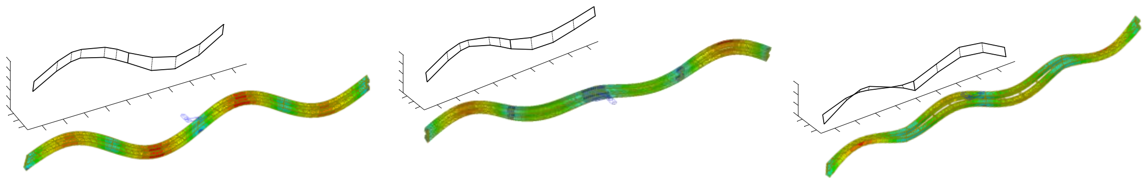


(c) Plan view



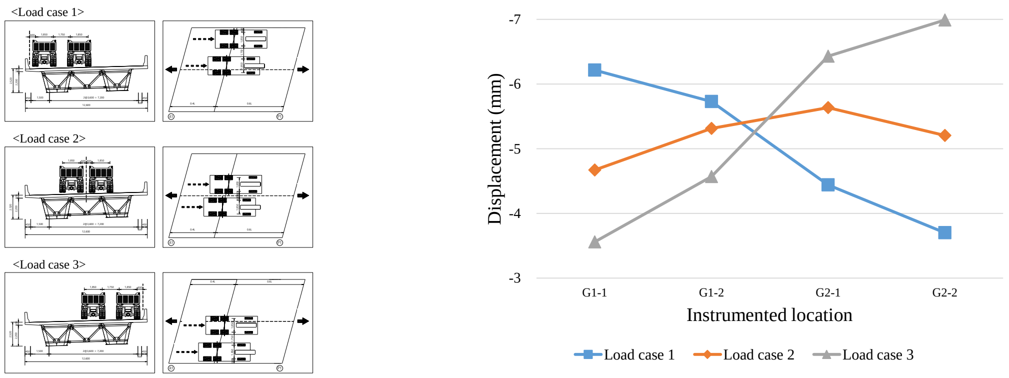
(d) Elevation

Fig. 4 Yeondae bridge



(a) The 1st mode (2.32Hz) (b) The 2nd mode (2.61Hz) (c) The 3rd mode (3.42Hz)

Fig. 5 Measured modal properties



(a) Three load cases (b) The measured displacements of box girders (mm)

Fig. 6 Measured static displacements

The objective function for the *i*-th optimal FE model is formulated as Eq. (16) considering three types of responses.

$$\min\ J_i(\boldsymbol{\theta}) = \mathbf{W}_i^T \mathbf{r}_i(\boldsymbol{\theta}) = w_{if} r_{if} + w_{i\phi} r_{i\phi} + w_{i\delta} r_{i\delta} \tag{16}$$

where r_{if} is the natural frequency residual, $r_{i\phi}$ the mode-shape residual, and $r_{i\delta}$ the displacement residual. The 300 sets of weighting vectors are sampled to represent uncertainty of the responses and, accordingly, a matrix of structural parameters $\boldsymbol{\Theta} = [\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \ldots, \boldsymbol{\theta}_{300}]$ for the 300 updated FE models are identified by successive optimizations.

4.3 Analysis of structural parameters distribution of multiple FE models

A matrix of transformed coordinate of 300 FE models $\hat{\boldsymbol{\Theta}}$ is obtained along with eigenvector matrix u and eigenvalue matrix Λ by applying PCA to the by applying PCA to the $\boldsymbol{\Theta}$. The Fig. 7 shows the contribution of a few dominant principal components for the variance of the FE models in the case of Yeondae bridge, which indicates that more than 90% of the variance of the multiple FE models can be explained by the first 9 principal components.

Table 1 Structural parameters and their allowable bounds considered in the optimization

Structural parameters	Allowable bounds (%)
Coefficients of spring support elements in translation and rotational direction	±30
Mass of girders (span 1, 2, 3, 4), cross frame	±10
Young's modulus of girders (span 1, 2, 3, 4), cross frame, and slab	±20
Torsional stiffness of girders (span 1, 2, 3, 4), cross frame, and slab	±25
Moment of inertia (I_{yy}) of girders (span 1, 2, 3, 4), cross frame, and slab	±10
Moment of inertia (I_{zz}) of girders (span 1, 2, 3, 4), cross frame, and slab	±10
Area of transverse slab	±30
Mass of substructural member (vertical, rotational)	±30

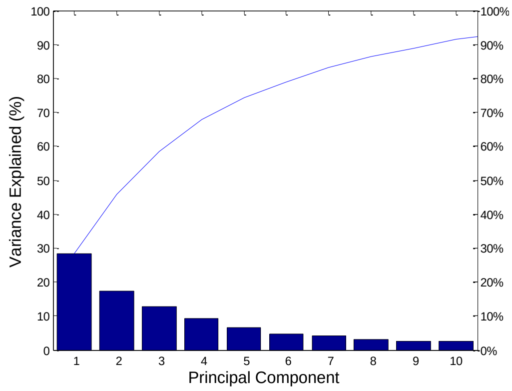
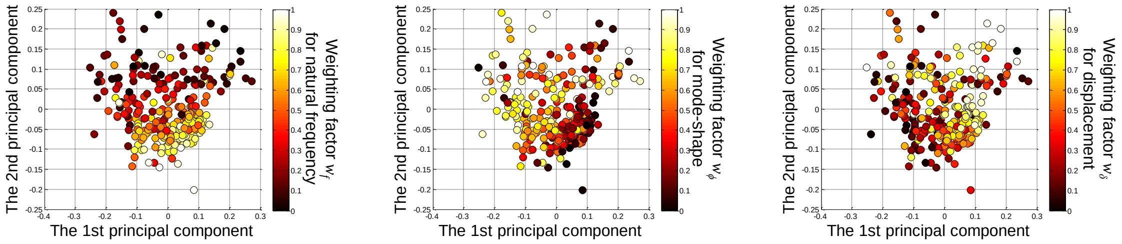


Fig. 7 Contribution of each principal component in the variance of multiple FE models

The relationship between the distribution of structural parameters and uncertainty conditions of the measured data can be characterized sufficiently by the two largest principal components in this example, as represented by gradual variation of weighting factors along the principal axes in Fig. 8. Although not presented here, we found that the relationship between the distribution of structural parameters and uncertainties does not reveal any specific characteristics in the spaces defined by other remaining higher-order principal components. Each point represents each of multiple FE models in the orthogonal space. The 2nd principal axis obviously explains the relationship between multiple FE models and the weighting factor for natural frequency residual (Fig. 8(a)). Variation of the weighting factor for mode-shape residual is also well represented by combination of the 1st and 2nd principal axes in Fig. 8(b). In the meantime, wide dispersion of higher weightage of mode-shape may suggest the insignificance of MAC based residual due to different orders of identified modal parameter and the structural matrices as discussed in Mukhopadhyay *et al*. (2012). The variation of weighting factor for displacement residual is less apparent than others (Fig. 8(c)).



(a) Natural frequency (b) Mode-shape (c) Static displacement

Fig. 8 Variation of the weighting factors of each measured data for updated FE models

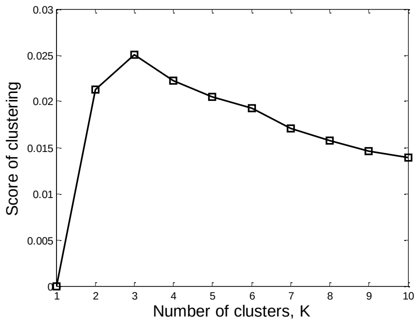


Fig. 9 Scores of clustering for different number of clusters

K-means method is then applied to the $\hat{\mathbf{\Theta}}$ to classify models according to similarity. The value of score function is evaluated to assure the quality of cluster results while varying the number of clusters from 1 to 10, where the optimal number of clusters is found to be three (Fig. 9). According to the estimated scores, the multiple FE models are classified into three groups as shown in Fig. 10.

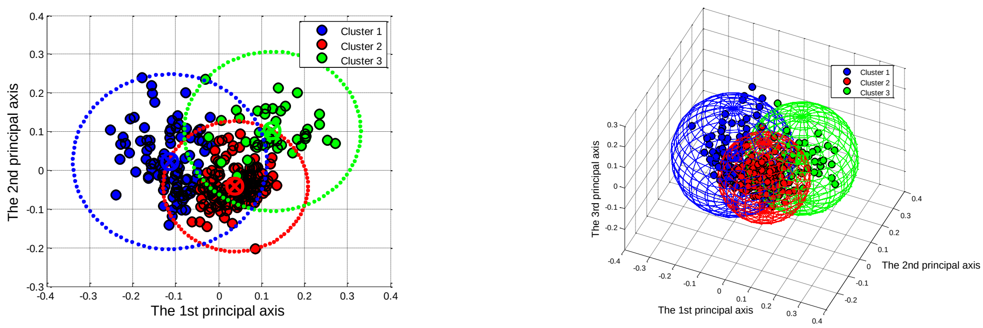


Fig. 10 Classification of multiple FE models into 3 groups

Table 2 Number of FE models in each cluster and characteristics of clusters

	Cluster 1	Cluster 2	Cluster 3
Number of FE models	99	160	41
Characteristic of cluster	Higher weighting factor for mode-shape residual	Higher weighting factor for natural frequency residual	Higher weighting factor for displacement residual

Each group, or cluster, can be characterized by the variation of weighting factors. For example, FE models characterized by higher weighting factor for natural frequency residual, shown in more bright yellow color in Fig. 8(a), are grouped together in the cluster 2. The cluster 2 also includes the models for which all the measurement types are weighted equally. Thus, the FE models in the cluster are expected to be suitable for the evaluations requiring good prediction on the natural frequency, such as seismic fragility analysis. Similarly, the cluster 1 and cluster 3 are characterized by higher weighting for mode-shape and displacement residuals, respectively (Figs. 8(b) and 8(c)). The multiple FE models can be classified into distinct groups without subjectivity by the combined use of PCA and *K*-means. Table 2 shows the number of FE models in three clusters, and their characteristics with respect to weighting factor.

4.4 Probabilistic assessment of bridge condition using load rating factor

Condition assessment of bridges evaluates the remaining resistance capacity of major structural components of bridge against external loads. Especially, the vehicle load carrying capacity is quantified by evaluating load rating factor. The Rating Factor (RF) is usually computed by using a single updated FE model by which bridge condition is reflected restrictively. On the contrary, the proposed procedure utilizes multiple updated FE models that can represent possible states of structural conditions, and the estimated rating factors provides statistical information on bridge condition.

The rating factors are evaluated with respect to the maximum positive bending moment for the strength limit state I (MLTM 2012). Table 3 shows the results obtained by using different sets of FE models: all the identified FE models and FE models in each of the three clusters. Since rating factors are highly dependent on the stiffness of the bridge, it can be concluded that the mean value of 2.16, computed by using FE models in the cluster 3 that is characterized by higher weighting factors for displacement residual, represents properly the current condition of the bridge. This is also confirmed by more rigorous analysis where highly refined updated FE model based on measured data produced a rating factor of 2.15 (Kim *et al.* 2013). It is noteworthy that, when 50% of the FE models closest to the centroid of each cluster are used, all the standard deviations have decreased considerably while the mean values have not changed much. Fig. 11 also shows that using the FE models of cluster 3 results in narrower and shifted distribution of rating factor compared to the case of using all the identified FE models.

Table 3 Rating factors using different set of FE models

	All identified FE models	Cluster 1	Cluster 2	Cluster 3
Mean value	2.06	2.06 (1.98)*	2.03 (2.03)*	2.16 (2.18)*
Standard deviation	0.129	0.153 (0.104)*	0.089 (0.064)*	0.128 (0.081)*

( )* : values estimated by using 50% closest FE models to the centroid of each cluster

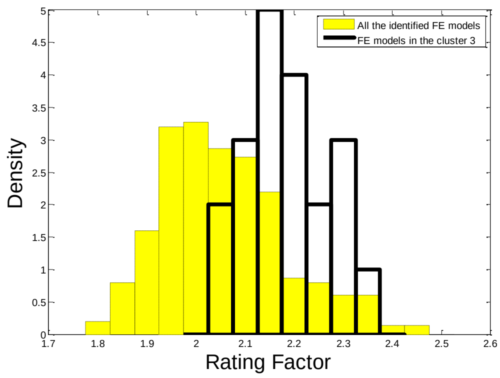


Fig. 11 Distribution of rating factors estimated by using all identified FE models and FE models in the cluster 3

When probabilistic information is not necessary, the proposed procedure can simply use one representative FE model located at the centroid of each cluster. In this case, the rating factors are evaluated as 2.01, 2.01 and 2.11 for cluster 1, 2 and 3, respectively, which are close to the mean values for each cluster. Thus, the evaluation of bridge condition using the representative FE model can also reflect each cluster's characteristics in this procedure.

The probability of bridge failure is computed as $p_f = P[RF \leq 1]$. Here, the limit state function indicates insufficient load carrying capacity. To evaluate the probability, a distribution of rating factors is estimated by numerical simulations with 10,000 offspring FE models, which are generated from a multivariate normal distribution for the updated FE models. Then, the probability of insufficient load carrying capacity is evaluated by using a Gaussian PDF that is fitted to the distribution of rating factors. The evaluations are repeated by using FE models of each cluster, respectively. Table 4 lists statistical properties of the rating factors estimated by using the offspring models, and the probability of bridge failure for the identified clusters. The probability computed by using FE models in each cluster is significantly small compared to the one using all the identified FE models because data uncertainties are well taken care by clustering. The probabilistic assessment of the current structural condition by the proposed procedure can provide useful and efficient information for optimizing repair and maintenance plan in terms of lifetime cost and safety.

Table 4 Statistical parameters of rating factors obtained by using 10,000 offspring models and probability of insufficient load carrying capacity

	All identified FE models	Cluster 1	Cluster 2	Cluster 3
Mean value of RF	2.01	2.09	2.06	2.15
Standard deviation of RF	0.39	0.26	0.24	0.26
p_f	2.54×10^{-3}	1.57×10^{-5}	4.38×10^{-6}	4.18×10^{-6}

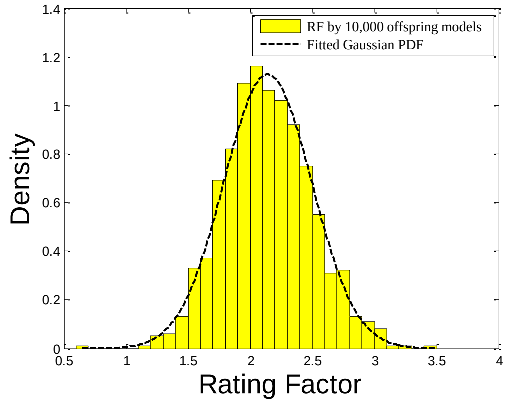


Fig. 12 Fitting of a Gaussian distribution (μ = 2.01, σ = 0.39) to the distribution of rating factors estimated by considering all identified FE models

5. Conclusions

In this paper, a new procedure for assessing probabilistic condition of bridge considering measured data uncertainty has been proposed. The uncertainty of measured data is represented by the variation of weighting vectors. Multiple updated FE models are identified by successive optimizations using varying weighting vectors for specific measured data, and are classified into groups according to their similarity by using Principal Component Analysis (PCA) and *K*-means clustering. Probabilistic condition of a structure is properly assessed by using a selected group of updated FE models depending on the degree of data uncertainties. It is shown in the application example of Yeondae bridge that the proposed procedure can effectively evaluate probabilistic distribution of load rating factors for representing the current remaining resistance capacity of the bridge. The probabilistic assessment of the current structural condition by the proposed procedure can provide useful and efficient information for optimizing repair and maintenance plan in terms of lifetime cost and safety.

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