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Locating and identifying model-free structural nonlinearities and systems using incomplete measured structural responses

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Abstract. Structural nonlinearity is a common phenomenon encountered in engineering structures under severe dynamic loading. It is necessary to localize and identify structural nonlinearities using structural dynamic measurements for damage detection and performance evaluation of structures. However, identification of nonlinear structural systems is a difficult task, especially when proper mathematical models for structural nonlinear behaviors are not available. In prior studies on nonparametric identification of nonlinear structures, the locations of structural nonlinearities are usually assumed known and all structural responses are measured. In this paper, an identification algorithm is proposed for locating and identifying model-free structural nonlinearities and systems using incomplete measurements of structural responses. First, equivalent linear structural systems are established and identified by the extended Kalman filter (EKF). The locations of structural nonlinearities are identified. Then, the model-free structural nonlinear restoring forces are approximated by power series polynomial models. The unscented Kalman filter (UKF) is utilized to identify structural nonlinear restoring forces and structural systems. Both numerical simulation examples and experimental test of a multi-story shear building with a MR damper are used to validate the proposed algorithm.

Keywords: nonlinear structural systems; nonparametric identification; nonlinear restoring force; extended Kalman filter; unscented Kalman filter, incomplete measurements

1. Introduction

Structural nonlinearity is a common phenomenon encountered in engineering structures under severe dynamic loading (Kerschena *et al*. 2006, Feng *et al*. 2009). Also, structural nonlinearity can be used as the indicator of the development of structural damage under dynamic excitation (Yan *et al*. 2008, Lee *et al*. 2010, Yi *et al*. 2013). Consequently, it is necessary to detect, localize and identify structural nonlinearities using structural dynamic measurements for damage detection, performance evaluation and remaining service life forecasting of engineering structures (Lu *et al*. 2013, Xia *et al*. 2014). However, identification of nonlinear dynamics structures is much less established than linear structures. For some complex nonlinear structural systems, the

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mathematical forms of models are not clear due to the lack of the knowledge of the patterns of nonlinear performances (Masri *et al*.1982, 2006)

Nonparametric identification approaches for model-free structural nonlinear systems have been proposed. The most popular nonparametric procedure for nonlinear systems is the restoring force surface (RFS) approach (Masri and Caughey 1979, Masri *et al*. 1982), or called the force-state mapping technique (Crawley and Aubert 1986). Further extensions and implementation strategies of RFS based methodologies were also investigated (Al-Hadid and Wright 1992, Masri *et al*. 2005, 2006, Hernandez-Garcia *et al*. 2010). In the RFS based approaches, a nonlinear restoring force is represented by polynomial series expansion. The polynomial-basis model is a non-parametric model in which structural nonlinear restoring force is approximated by a linear combination of the power-series expansion of the system displacement and velocity. In recent years, Xu *et al*. (2012) and He *et al*. (2012) presented data-based identification of nonlinear restoring force using power series polynomial model, in which a power series polynomial modeling approach involving the instantaneous values of system displacement and velocity was proposed to approximate structural nonlinear restoring force behaviors. The performances of the above RFS based approaches have been validated by both numerical simulation and experimental studies. However, in prior studies on nonparametric identification of nonlinear structures, it is prerequisite that the locations of structural nonlinearities are known and complete measurements of structural responses are available. If the locations of structural nonlinearities are not clear, the expansions of all structural nonlinear restoring forces lead to tremendous undermined coefficients in the polynomial series expansions.

In practice, it is impossible to deploy so many sensors to measure all response outputs of a structural system (Yi *et al*. 2012, Lei *et al*. 2013). The extended Kalman filter (EKF) has been widely used for structural identification with incomplete structural responses (Hoshiya and Saito 1984, Yang *et al*. 2006, Lei *et al*. 2013, Su *et al*. 2014). Although EKF can be extended for the identification of nonlinear structures (Yang *et al*. 2006, Lei *et al*. 2012, Lei and He, 2013), the degree of accuracy of the EKF relies on the validity of the linear approximation and is not effective to identify strong nonlinear structural systems (Wu and Smyth 2007). The unscented Kalman filter (UKF) (Julier *et al*. 2004, Mariani and Ghisi 2007, Omrani *et al*. 2013) is superior to the EKF in the identification of strong nonlinear structural systems. The UKF is based on the unscented transform of a deterministic sampling scheme consisting of the so-called “sigma points”. In each step, the UKF generates a set of sigma points and updates the prediction formulas based on these sigma points. Therefore, if the UKF is directly applied to the identification of a model-free nonlinear structure in which the locations of structural nonlinearities are not prerequisite, it is difficult or even impossible to identify so many undetermined coefficients in the polynomial series expansions.

In this paper, an identification algorithm is proposed for locating and identifying model-free structural nonlinearities and nonlinear structural systems using incomplete measurements of structural responses. First, an equivalent linear structural system is established for an original nonlinear system. The equivalent linear system is identified by extended Kalman filter (EKF) and the locations of structural nonlinearities can be identified. Then, the model-free structural restoring forces at the locations of identified structural nonlinearities are assumed to be approximated by power series polynomials. The unscented Kalman filter (UKF) is utilized to estimate the polynomials and the nonlinear structural system. Several numerical simulation examples with different nonlinear restoring force models and locations are used to validate the effectiveness of the proposed algorithm. Also, the identification of a multi-story shear building equipped with MR damper which emulate nonlinear story restoring force (Kim *et al*. 2011, Woo *et al*. 2013) is used to illustrate the application of the proposed algorithm.

2. The Identification problem

The equation of motion for a nonlinear structure can be written as

$$\boldsymbol{M}\ddot{\boldsymbol{x}}(\mathrm{t})+\boldsymbol{C}\dot{\boldsymbol{x}}(\mathrm{t})+\boldsymbol{K}\boldsymbol{x}(\mathrm{t})+\boldsymbol{F}_n[\boldsymbol{x}(\mathrm{t}),\dot{\boldsymbol{x}}(\mathrm{t})]=\boldsymbol{B}\boldsymbol{f}(\mathrm{t}) \tag{1}$$

where ***M, C, K*** are the linear mass, damping and stiffness matrices of the structure, respectively; $\boldsymbol{x}(\mathrm{t})$, $\dot{\boldsymbol{x}}(\mathrm{t})$ and $\ddot{\boldsymbol{x}}(\mathrm{t})$ are the vectors of the displacement, velocity and acceleration responses, respectively; $\boldsymbol{F}_n[\boldsymbol{x}(\mathrm{t}),\dot{\boldsymbol{x}}(\mathrm{t})]$ is a vector of the nonlinear restoring force, $\boldsymbol{B}$ and $\boldsymbol{f}(\mathrm{t})$ are the external force influence matrix and external force vector, respectively. Usually, the mass of a structure can be estimated accurately, so ***M*** is assumed known in this paper.

For general complex nonlinear systems, it is hard to establish proper mathematical form of the nonlinear restoring force models due to the lack of the knowledge of the pattern of nonlinear performances. The power series polynomials involving the instantaneous values of the relative displacement and velocity responses of the structural system are utilized to approximate the total nonlinear restoring forces in the identification problem (Masri *et al*. 2005, 2006, 2010, Xu *et al*. 2012), i.e.

$$F_{n,i,i-1}[\dot{\boldsymbol{x}}(\mathrm{t}),\boldsymbol{x}(\mathrm{t})]\approx F_{n,i,i-1}[s_{i,i-1},v_{i,i-1},c^{i,i-1}]\approx\sum_{k=0}^{p}\sum_{j=0}^{q}c_{j,k}^{i,i-1}s_{i,i-1}^{j}v_{i,i-1}^{k}\quad(k+j\neq 0) \tag{2}$$

in which $F_{n,i,i-1}[\dot{\boldsymbol{x}}(\mathrm{t}),\boldsymbol{x}(\mathrm{t})]$ denotes the nonlinear restoring force between ***i***-th and (***i***-1)-th degrees of freedom; $c_{j,k}^{i,i-1}$ are the coefficients of power series polynomials; *p* and *q* are the highest power of polynomials, which are depended on the level of nonlinearity; $s_{i,i-1}$ and $v_{i,i-1}$ represents the relative displacement and relative velocity between ***i***-th and (***i***-1)-th degrees of freedom, respectively.

When the external excitation is weak, the nonlinear restoring forces can be neglected and the equation of motion of the structural system reduces to

$$\boldsymbol{M}\ddot{\boldsymbol{x}}_l(\mathrm{t})+\boldsymbol{C}\dot{\boldsymbol{x}}_l(\mathrm{t})+\boldsymbol{K}\boldsymbol{x}_l(\mathrm{t})=\boldsymbol{B}\boldsymbol{f}(\mathrm{t}) \tag{3}$$

where $\boldsymbol{x}_l(\mathrm{t})$, $\dot{\boldsymbol{x}}_l(\mathrm{t})$ and $\ddot{\boldsymbol{x}}_l(\mathrm{t})$ are the vectors of the displacement, velocity and acceleration responses of the linear structure, respectively.

The stiffness and damping parameters of the above linear structural system can be identified by the extended Kalman filter (EKF) using only partial measurements of the structural responses under weak excitation.

The extended state vector of the above linear structure is defined as $\boldsymbol{X}_l=\{\boldsymbol{x}_l^T,\dot{\boldsymbol{x}}_l^T,\boldsymbol{\theta}_l^T\}^T$ with $\boldsymbol{\theta}_l$ being a vector of linear stiffness and damping parameters. Then, the equation for the extended state vector of the equivalent linear structure can be derived as

$$\dot{\boldsymbol{X}}_l=\begin{Bmatrix}\dot{\boldsymbol{x}}_l\\ \boldsymbol{M}^{-1}[\boldsymbol{B}\boldsymbol{f}(t)-\boldsymbol{K}\boldsymbol{x}_l-\boldsymbol{C}\dot{\boldsymbol{x}}_l]\\ 0\end{Bmatrix}=\boldsymbol{g}(\boldsymbol{X}_l,\boldsymbol{f}) \tag{4}$$

Usually, it is easy to deploy some accelerometers on structures to measure structural partial acceleration responses. Therefore, the discrete form of equation for measured acceleration vector can be expressed as

$$\boldsymbol{y}_{l,k+1} = -\boldsymbol{D}\boldsymbol{M}^{-1}\left(\boldsymbol{C}\dot{\boldsymbol{x}}_{l,k+1} + \boldsymbol{K}\boldsymbol{x}_{l,k+1} + \boldsymbol{B}\boldsymbol{f}_{k+1}\right) + \boldsymbol{v}_{k+1} = \boldsymbol{h}\left(\boldsymbol{X}_{l,k+1}, \boldsymbol{f}_{k+1}\right) + \boldsymbol{v}_{k+1} \tag{5}$$

where $\boldsymbol{y}_{l,k+1}$ is the measured linear acceleration response vector at time $t = (k+1)\times\Delta t$ (Δt is the sampling time step), $\boldsymbol{D}$ denotes the location of accelerometers, $\boldsymbol{v}_{k+1}$ is the measurement noise assumed as Gaussian white noise.

Based on the EKF, the extended state vector can be identified as

$$\hat{\boldsymbol{X}}_{l,k+1|k+1} = \tilde{\boldsymbol{X}}_{l,k+1|k} + \boldsymbol{K}_{k+1}[\boldsymbol{y}_{l,k+1} - \boldsymbol{h}(\tilde{\boldsymbol{X}}_{l,k+1|k}, \boldsymbol{f}_{k+1})] \tag{6}$$

$$\tilde{\boldsymbol{X}}_{l,k+1|k} = \hat{\boldsymbol{X}}_{l,k|k} + \int_{t_k}^{t_{k+1}} \boldsymbol{g}(\hat{\boldsymbol{X}}_l, \boldsymbol{f})\, dt \tag{7}$$

where $\hat{\boldsymbol{X}}_{l,k+1|k+1}$ is the estimated state vector $\boldsymbol{X}_l$ at time $t = (k+1)\times\Delta t$, $\boldsymbol{K}_{k+1}$ is the Kalman Gain matrix at the time instant (Hoshiya and Saito 1984, Lei *et al*. 2013).

Therefore, the linear structural parameters can be identified using the EKF with the partial measurements of structural responses under weak excitation.

Under strong external excitation, structural nonlinear effect becomes significant. For the identification of the model-free nonlinear system described by Eq. (1), some algorithms such as the unscented Kalman filter (UKF) can be utilized. However, when the locations of structural nonlinearities are unknown, it is difficult or even impossible to identify so many undermined coefficients $c_{j,k}^{i,i-1}$ if the UKF is directly applied to the whole nonlinear system. Since structural nonlinearities only exist in local areas of a whole structure, it is better to firstly identify the locations of structural nonlinearities. A novel identification approach is proposed for this purpose in this paper.

2.1 Locating the nonlinearity in nonlinear structural systems

For the above nonlinear structural system under strong external excitation, an equivalent linear structural system is introduced with the following equation of motion

$$\boldsymbol{M}_e\ddot{\boldsymbol{x}}_e(\mathrm{t}) + \boldsymbol{C}_e\dot{\boldsymbol{x}}_e(\mathrm{t}) + \boldsymbol{K}_e\boldsymbol{x}_e(\mathrm{t}) = \boldsymbol{B}\boldsymbol{f}(\mathrm{t}) \tag{8}$$

in which $\boldsymbol{x}_e(\mathrm{t})$, $\dot{\boldsymbol{x}}_e(\mathrm{t})$ and $\ddot{\boldsymbol{x}}_e(\mathrm{t})$ are the vectors of the displacement, velocity and acceleration responses of the equivalent linear system, respectively; $\boldsymbol{M}_e$,$\boldsymbol{C}_e$ and $\boldsymbol{K}_e$ are the equivalent linear mass, damping and stiffness matrices, respectively; Usually, the mass matrix is hardly changed when nonlinear behavior appears, so $\boldsymbol{M}_e$=$\boldsymbol{M}$.

Analogously, the state vector and the equivalent linear stiffness and damping parameters can be identified by the extended Kalman filter (EKF) using the limited measurements of the nonlinear structural responses.

$$\hat{\boldsymbol{X}}_{e,k+1|k+1} = \tilde{\boldsymbol{X}}_{e,k+1|k} + \boldsymbol{K}_{e,k+1}[\boldsymbol{y}_{k+1} - \boldsymbol{h}(\tilde{\boldsymbol{X}}_{e,k+1|k}, \boldsymbol{f}_{k+1})] \tag{9}$$

$$\tilde{\boldsymbol{X}}_{e,k+1|k} = \hat{\boldsymbol{X}}_{e,k|k} + \int_{t_k}^{t_{k+1}} \boldsymbol{g}(\hat{\boldsymbol{X}}_e, \boldsymbol{f})\, dt \tag{10}$$

where $\hat{\boldsymbol{X}}_{e,k+1|k+1}$ is the estimated state vector $\boldsymbol{X}_e$ at time $t = (k+1)\times\Delta t$ and $\boldsymbol{X}_e = \left\{ \boldsymbol{x}_e^T, \dot{\boldsymbol{x}}_e^T, \boldsymbol{\theta}_e^T \right\}^T$ with $\boldsymbol{\theta}_e$ being the parametric vector of the equivalent linear structural system.

Then, the locations of structural nonlinearity can be identified by comparing the differences between the identified structural parameters of equivalent linear system and those of the linear structure. Also, the extents of differences indicate the levels of structural nonlinearities.

2.2 Identification of the nonlinear restoring forces and system

After the identification of the locations of structural nonlinearities, the unscented Kalman filter (UKF), which is suitable for the identification of strong nonlinearities, is adopted herein for the identification of the model-free nonlinear system described by Eqs. (1) and (2); however, only the nonlinear forces at the locations of identified structural nonlinearities are approximated by the power series polynomials in which the orders of polynomials p and q are selected based on the levels of identified structural nonlinearities indicated by the extents of differences between the identified structural parameters of the equivalent linear system and those of the linear one in the above section.

In the recursive identification process by UKF, the selection of proper initial values of power series polynomials is important for the identification convergence. This can be accomplished based on the identified equivalent linear structural system by the EKF in the above section.

Based on the identified acceleration, velocity and displacement responses of the equivalent linear structure, the algebraic coefficients used to approximate the total nonlinear restoring forces can be identified by means of least-square techniques. These estimated coefficients are used as the proper initial values of power series polynomials.

Then, the UKF is utilized for the identification of nonlinear system described by Eqs. (1) and (2). The extended state equation and observation equations are nonlinear equations expressed as:

$$\dot{\boldsymbol{X}} = \boldsymbol{g}(\boldsymbol{X}, \boldsymbol{f}) \tag{11}$$

$$\boldsymbol{y}_{k+1} = \boldsymbol{h}(\boldsymbol{X}_{k+1}, \boldsymbol{f}_{k+1}) + \boldsymbol{v}_{k+1} \tag{12}$$

in which $\boldsymbol{X} = \left\{ \boldsymbol{x}^T, \dot{\boldsymbol{x}}^T, \boldsymbol{\theta}^T \right\}^T$ with $\theta_i = c_{j,k}^{i,i-1}$

Based on the unscented transform, 2N+1 deterministic sample points $\chi_{i,k|k}$ can be computed as (Julier *et al*. 2004)

$$\boldsymbol{\chi}_{i,k|k} = \begin{cases} \hat{\mathrm{X}}_{k|k} & , \quad i = 0 \\ \hat{\mathrm{X}}_{k|k} + \left(\sqrt{(N+\lambda)\hat{\boldsymbol{P}}_{\boldsymbol{X},k|k}} \right)_i & , \quad i = 1, \cdots N \\ \hat{\mathrm{X}}_{k|k} - \left(\sqrt{(N+\lambda)\hat{\boldsymbol{P}}_{\boldsymbol{X},k|k}} \right)_i & , \quad i = \mathrm{N}+1, \cdots 2N \end{cases} \tag{13}$$

where N is the dimension of $\boldsymbol{X}$, $\hat{\mathrm{X}}_{k|k} = E\{\boldsymbol{X}_k\}$, $\hat{\boldsymbol{P}}_{\boldsymbol{X},k|k} = E\left\{\left(\boldsymbol{X}_k - \hat{\boldsymbol{X}}_{k|k}\right)\left(\boldsymbol{X}_k - \hat{\boldsymbol{X}}_{k|k}\right)^T\right\}$, λ is a scaling parameter, and $(\bullet)_i$ is the i-th column of the matrix.

First, the sample points and extended state vector are predicated as

$$\boldsymbol{\chi}_{i,k+1|k} = \boldsymbol{\chi}_{i,k|k} + \int_{k\Delta t}^{(k+1)\Delta t} g\left(\boldsymbol{X}_{t,k}, \boldsymbol{f}\right) dt \; ; \quad \tilde{\boldsymbol{X}}_{k+1|k} = \sum_{i=0}^{2N} W_i^m \boldsymbol{\chi}_{i,k+1|k} \tag{14}$$

with the predicted error covariance expressed as

$$\tilde{\boldsymbol{P}}_{\boldsymbol{X},k+1|k} = \sum_{i=0}^{2N} W_i^c (\boldsymbol{\chi}_{i,k+1|k} - \tilde{\boldsymbol{X}}_{k+1|k})(\boldsymbol{\chi}_{i,k+1|k} - \tilde{\boldsymbol{X}}_{k+1|k})^T + \boldsymbol{Q}_{k+1} \tag{15}$$

where W_i^m and W_i^c are the weights for the mean and covariance, respectively (Julier *et al*. 2004)

Then, the measurement update steps are as follows

$$\hat{\boldsymbol{X}}_{k+1|k+1} = \tilde{\boldsymbol{X}}_{k+1|k} + \boldsymbol{K}_{k+1}(\boldsymbol{y}_{k+1} - \hat{\boldsymbol{y}}_{k+1|k+1}) \tag{16}$$

$$\tilde{\boldsymbol{y}}_{i,k+1|k+1} = \boldsymbol{h}\left(\boldsymbol{\chi}_{i,k+1|k}, \boldsymbol{f}_{k+1}\right) \quad \hat{\boldsymbol{y}}_{k+1|k+1} = \sum_{i=0}^{2N} W_i^m \tilde{\boldsymbol{y}}_{i,k+1|k} \tag{17}$$

$$\hat{\boldsymbol{P}}_{\boldsymbol{y},k+1} = \sum_{i=0}^{2N} W_i^c (\hat{\boldsymbol{y}}_{k+1|k+1} - \tilde{\boldsymbol{y}}_{i,k+1|k+1})(\hat{\boldsymbol{y}}_{k+1|k+1} - \tilde{\boldsymbol{y}}_{i,k+1|k+1})^T + \boldsymbol{R}_{k+1} \tag{18}$$

$$\hat{\boldsymbol{P}}_{\boldsymbol{Xy},k+1} = \sum_{i=0}^{2n} W_i^c \left\{\boldsymbol{\chi}_{i,k+1|k} - \tilde{\boldsymbol{X}}_{k+1|k}\right\}\left\{\hat{\boldsymbol{y}}_{k+1|k} - \tilde{\boldsymbol{y}}_{i,k+1|k}\right\}^T \tag{19}$$

$$\boldsymbol{K}_{k+1} = \hat{\boldsymbol{P}}_{\boldsymbol{Xy},k+1}(\hat{\boldsymbol{P}}_{\boldsymbol{y},k+1})^{-1}; \quad \hat{\boldsymbol{P}}_{\boldsymbol{X},k+1|k+1} = \tilde{\boldsymbol{P}}_{\boldsymbol{X},k+1|k} - \boldsymbol{K}_{k+1}\boldsymbol{P}_{y,k+1}\boldsymbol{K}_{k+1}^T \tag{20}$$

After the identification of the state vector and the coefficients of the power series polynomials used to represent the total nonlinear restoring forces, the behaviors of the model-free of nonlinear restoring forces are identified.

3. Numerical validations

To validate the proposed identification algorithm, numerical identification examples of an 8-story nonlinear shear building with different nonlinear restoring force models and locations are used. The building is subject a white noise excitation at the top floor of the building. The numerical calculated acceleration responses are treated as “measured responses” for the identification problem. To consider partial response measurements, only the acceleration responses at the 1st, 2nd, 4th, 5th and 7th floor levels are used. Also, the calculated acceleration responses are added with the corresponding white noise with 5% noise-to-signal ratio in root mean square (RMS) to consider the influences of measurement noises.

The linear structural parametric values are selected as: mass of floor m_i=500kg, story stiffness

k_i=200 kN/m, story damping c_i=0.20 kN·s/m (i=1,2,…,8). These parameters can be identified using the EKF with the partial acceleration responses of the building under weak external excitation. The identified linear stiffness parameters $k_{l,i}$ and linear damping values $c_{l,i}$ are listed in Table 1.

3.1 Identification of nonlinear restoring forces with Dahl model

In this study a modified Dahl model (Dahl 1976), which can capture many commonly observed types of nonlinear behavior of MR dampers, is employed. The modified Dahl model is described by the following equation for nonlinear force as

$$F_{ni}(t) = k_{0i}u_i(t) + c_{0i}\dot{u}_i(t) + f_{di}\mathbf{z}_i(t) + f_{0i} \tag{21}$$

in which, k_{0i}=the stiffness coefficient, c_{0i} = the viscous damping coefficient, f_{di}=the adjustable coulomb friction, f_{0i}=the initial force, u_i=the displacement of the i-th damper, and $\mathbf{z}_i$=a dimensionless parameter which describes the coulomb friction by the equation as:

$$\dot{\mathbf{z}}_i = \sigma_i \dot{u}_i \cdot (1 - \mathbf{z}_i \cdot \mathrm{sgn}(\dot{u}_i)) \tag{22}$$

and σ_i= the coefficient used to control the shape of the nonlinear restoring curve.

3.1.1 Identification of nonlinear restoring forces with Dahl model in the 1st story

When the 8-story building is subject to a strong white noise excitation, structural nonlinear effect needs to be considered. In this example, it is assumed that story nonlinear restoring force with Dahl model occurs in the 1st floor. The Dahl model parameters are selected as: k_{01} = 0.05 kN/m; c_{01}=0.30 kN/m; f_{d1}=0.20 kN; f_{01}=0.0 kN; σ_1=50000. However, the location of the nonlinear restoring force and the nonlinear force parameters are only used in generating the nonlinear structural responses.

Table 1 Parameters of the equivalent linear and linear building (Dahl model in the 1st story)

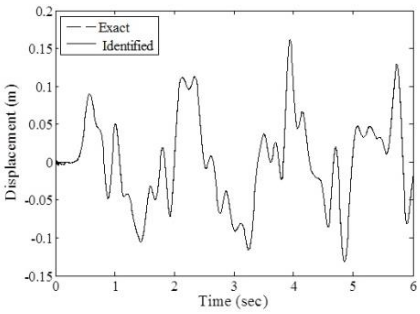
Story No.	$k_{e,i}$ (kN/m)	$k_{l,i}$ (kN/m)	$c_{e,i}$ (kN·s/m)	$c_{l,i}$ (kN·s/m)
1st	200.15	200.73	**0.73**	**0.21**
2nd	199.36	199.96	0.18	0.20
3rd	199.59	200.09	0.19	0.20
4th	199.88	199.79	0.21	0.21
5th	200.64	200.16	0.19	0.18
6th	199.86	199.76	0.19	0.19
7th	200.05	200.25	0.21	0.21
8th	200.66	200.69	0.21	0.19

An equivalent linear building structure is established and its parameters can be identified by EKF with the measured incomplete nonlinear structural responses. The identification results of equivalent linear stiffness $k_{e,i}$ and equivalent linear damping $c_{e,i}$ parameters are listed and compared with those of the linear building in Table 1.

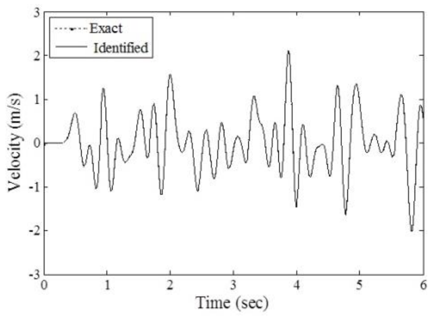
From the comparison values in Table 1, it is noted that there is significant differences between the identified equivalent linear damping and the identified linear damping in the 1st floor as marked by values in bold. Therefore, the location of structural nonlinearity is identified.

Table 2 Identification parameters of the nonlinear building (Dahl model in the 1st story)

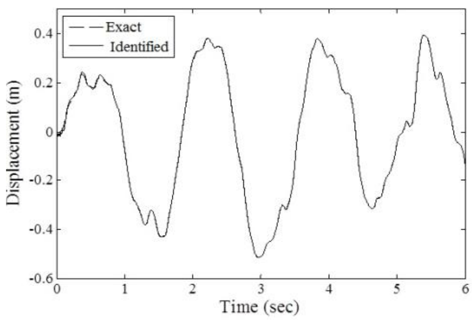
Story No.	1st	2nd	3rd	4th	5th	6th	7th	8th	
k_i (kN/m)	200.2	200.0	200.1	199.8	200.2	199.6	200.2	199.3	
c_i(kN·s/m)	0.24	0.22	0.19	0.20	0.20	0.20	0.20	0.17	
Coefficients of polynomial in the 1st story									
	$c_{1,0}$	$c_{0,1}$	$c_{2,0}$	$c_{1,1}$	$c_{0,2}$	$c_{3,0}$	$c_{2,1}$	$c_{1,2}$	$c_{0,3}$
Initial	121.5	662.6	-1102.9	144.3	-3.1	5703.8	-395.7	169.0	-88.8
Identified	106.3	570.6	-937.5	125	-24.3	4848.2	-336.3	194.7	-69.2



(a) Displacements of the 1st story



(b) Velocity of the 1st story



(c) Displacements of the 8th story

Fig. 1 Comparisons of the structural displacement and velocity responses

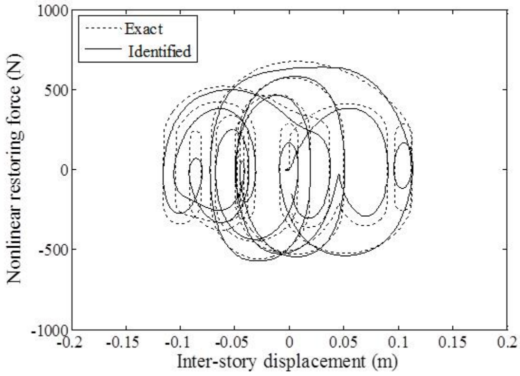


Fig. 2 Restoring force at the 1st story (Dahl model)

Based on the levels of structural nonlinearities indicated by the extents of differences of parameters in Table 1, the orders of power series polynomial for the approximation of nonlinear restoring force at the 1st floor are selected as p=q=3, so there are 9 undermined coefficients in the power series polynomial. Then, the unscented Kalman filter (UKF) is utilized to identify the nonlinear structure. The initial values of the coefficients in the power series polynomial are estimated by the least-square technique with the identification results of the corresponding equivalent linear structure. The estimated results are shown in Table 2. In Table 2, the identified structural parameters and coefficients of polynomial are also shown.

Figs. 1(a)-1(c) show the comparisons of the identified structural displacement and velocity responses with the exact structural responses. In Fig. 2, the identified nonlinear restoring force at the 1st floor of the building is compared with the exact force. It is demonstrated that the identification results are in good agreement with the exact ones.

3.1.2 Identification of nonlinear forces with Dahl model in the 1st and the 5th stories

To consider the identification of multiple nonlinearities in the building, it is assumed that story nonlinear restoring forces with Dahl model occur in both the 1st and the 5th floors with the parameters as: k_{01}= 0.05 kN/m; c_{01}=0.30 kN/m; f_{d1}=0.20 kN; f_{01}=0.0 kN; σ_1=50000; k_{05} = 0.05 kN/m; c_{05}=0.30 kN/m; f_{d5}=0.15 kN; f_{05}=0.0 kN; σ_5 =40000

Again, an equivalent linear building structure is established and its parameters are identified by the EKF using the nonlinear structural responses. The identification results are shown and compared with those of the linear building in Table 3. From the comparisons, it is noted that there are large differences between the identified equivalent linear parameters and the linear structure parameters in both the 1st floor and the 5th floors as marked by values in bold, which also indicates the locations of the two structural nonlinearities.

Then, the unscented Kalman filter (UKF) is utilized to identify the nonlinear building. The initial values of the coefficients in the power series polynomials for approximation of the two nonlinear restoring forces at the 1st and the 5th floors are again estimated by the least-square technique based on the above identification results of the equivalent linear structure. In Table 4, all the identified structural parameters and coefficients of polynomial are also shown.

Table 3 Parameters of the equivalent linear & linear building (Dahl model in the 1st & 5th stories)

Story No.	$k_{e,i}$ (kN/m)	$k_{l,i}$ (kN/m)	$c_{e,i}$ (kN·s/m)	$c_{l,i}$ (kN·s/m)
1st	198.87	200.73	**0.70**	**0.21**
2nd	201.38	199.96	0.17	0.20
3rd	199.18	200.09	0.21	0.20
4th	200.41	199.79	0.20	0.21
5th	201.23	200.16	**0.63**	**0.18**
6th	200.32	199.76	0.20	0.19
7th	199.58	200.25	0.22	0.21
8th	200.93	200.69	0.20	0.19

Table 4 Identification parameters of the nonlinear building (Dahl models in the 1st & 5th stories)

Story No.	1st	2nd	3rd	4th	5th	6th	7th	8th
k_i (kN/m)	199.3	201.0	200.0	200.5	200.5	199.8	200.0	199.4
c_i(kN·s/m)	0.23	0.19	0.21	0.20	0.22	0.22	0.20	0.19

Coefficients of polynomial in the 1st story									
	$c_{1,0}$	$c_{0,1}$	$c_{2,0}$	$c_{1,1}$	$c_{0,2}$	$c_{3,0}$	$c_{2,1}$	$c_{1,2}$	$c_{0,3}$
Initial	379.1	623.4	147.3	-102.7	3.2	-5917.9	43.6	-147.7	-81.6
Identified	303.7	500.9	117.9	-49.9	-10.1	-4734.3	35.0	-119.2	-21.0
Coefficients of polynomial in the 5th story									
	$c_{1,0}$	$c_{0,1}$	$c_{2,0}$	$c_{1,1}$	$c_{0,2}$	$c_{3,0}$	$c_{2,1}$	$c_{1,2}$	$c_{0,3}$
Initial	273.6	531.2	702.0	-0.4	3.7	-7789.5	-481.7	-161.7	-48.7
Identified	219.3	427.0	561.6	11.6	20.8	-6231.7	-384.8	-118.3	-26.3

Figs. 3(a) and 3(b) show the comparisons of the identified and exact displacement responses at the 1st and the 5th floors of the building. In Figs. 4(a) and 4(b), the identified two nonlinear restoring forces at the 1st and the 5th stories are compared with the exact forces.

3.2 Identification of the nonlinear restoring force with Bingham model in the 1st story

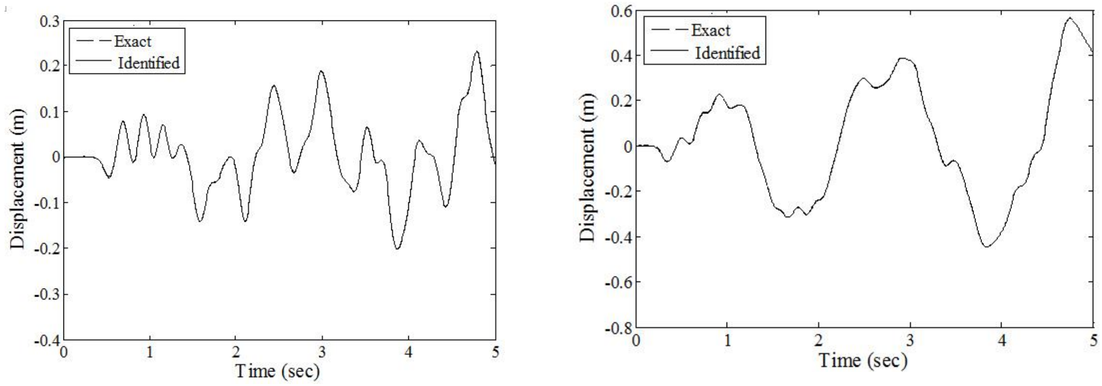
Bingham model is another simple and useful model to describe the nonlinear behavior of MR damper (Woo *et al.* 213). The nonlinear restoring force with Bingham model is described by:

$$F_{ni} = f_{ci} \cdot \mathrm{sgn}(\dot{u}_i) + c_{0i}\dot{u}_i + f_{0i} \tag{23}$$

in which f_{ci}, c_{0i} and f_{0i} are model parameters, $\dot{u}_i$ is the velocity of the i-th damper, respectively. In this numerical example, it is assumed that story nonlinear restoring force with Bingham model in the 1st story of the building and the Bingham model parameters are selected as: $f_{ci} = 0.02\,\mathrm{k}N$, $c_{01} = 0.15\,kNs/m$, $f_{01} = 0$.

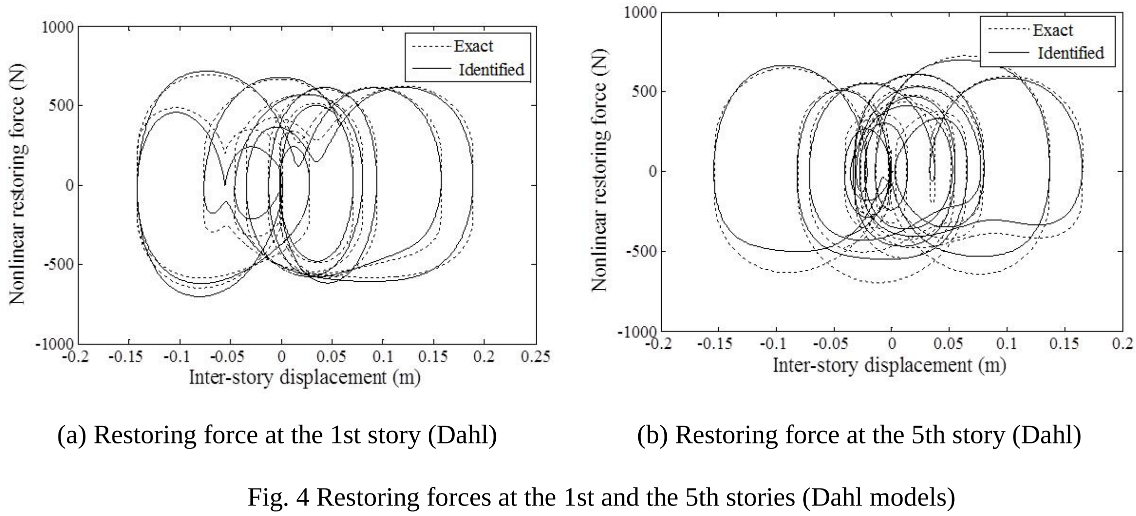
Analogously, an equivalent linear structure is established and identified by EKF using the partial measurements of structural nonlinear responses. The identification results of equivalent linear parameters are shown and compared with linear ones in Table 5.

From the comparisons of the identified equivalent linear parameters with the linear ones in Table 5, the location of the structural nonlinearity can be identified. Then, the unscented Kalman filter (UKF) is utilized to identify the nonlinear building. Table 6 summaries the identified structural parameters and coefficients of the power series polynomial.



(a) Displacements of the 1st story (b) Displacements of the 5th story

Fig. 3 Comparisons of the displacement responses at the 1st and 5th stories (Dahl models)



(a) Restoring force at the 1st story (Dahl) (b) Restoring force at the 5th story (Dahl)

Fig. 4 Restoring forces at the 1st and the 5th stories (Dahl models)

Figs. 5 and 6 show the identified displacement responses and the nonlinear restoring force of the 1st story. The identification results are in good agreements with the exact responses and force.

Table 5 Parameters of the equivalent linear and linear building (Bingham model the 1st story)

Story No.	k_{ei} (kN/m)	k_{li} (kN/m)	c_{ei} (kN·s/m)	c_{li} /(kN·s/m)
1st	199.47	200.73	**0.60**	**0.21**
2nd	200.16	199.96	0.20	0.20
3rd	199.92	200.09	0.21	0.20
4th	200.25	199.79	0.20	0.21
5th	200.34	200.16	0.19	0.18
6th	199.78	199.76	0.19	0.19
7th	200.11	200.25	0.21	0.21
8th	200.71	200.69	0.21	0.19

Table 6 Identification parameters of the nonlinear building (Bingham model in the 1st story)

Story No.	1st	2nd	3rd	4th	5th	6th	7th	8th	
k_i (kN/m)	199.8	200.2	200.1	200.1	200.0	200.0	199.9	199.6	
c_i(kN·s/m)	0.24	0.20	0.20	0.20	0.20	0.20	0.20	0.18	
Coefficients of polynomial in the 1st story									
	$c_{1,0}$	$c_{0,1}$	$c_{2,0}$	$c_{1,1}$	$c_{0,2}$	$c_{3,0}$	$c_{2,1}$	$c_{1,2}$	$c_{0,3}$
Initial	-295.7	559.5	464.5	5.2	-4.5	26675	3330.0	287.5	-177.1
Identified	-248.4	482.3	394.8	11.3	-11.9	22674	2830.5	240.8	-151.0

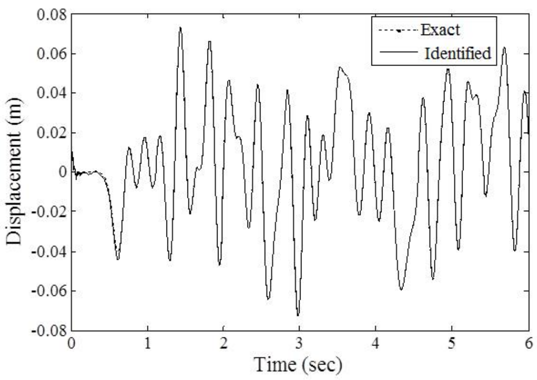


Fig. 5 Displacement of the 1st story

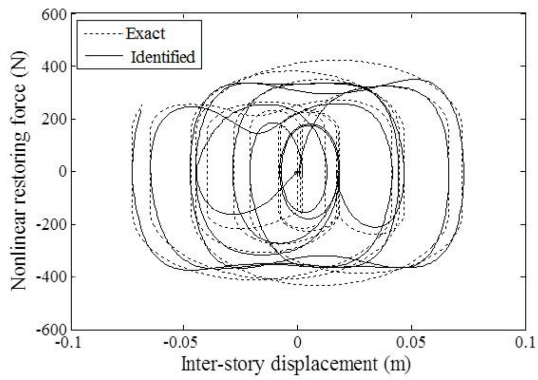


Fig. 6 Restoring force at the 1st story (Bingham)

4. Experimental validation

To further validate the feasibility of the proposed algorithm, the application of the proposed algorithm in conjunction with a 5-story shear building equipped with a MR damper shown in Fig. 7(a) is employed. The 5-story lab experimental building model behaves as a lumped mass shear-type building model as the beam elements in the model are much stronger than the flexible columns. The model is excited by a MB Dynamics Modal 50A vibration shaker. The shaker induces random white noise excitation to the building at the 3rd story which is installed with a force transducer to record the excitation. An MR damper RD-1097-01 from the Load Company is installed in the 1st story of the building as shown in Fig. 7(b). A force transducer is attached to one end of the MR damper to measure the nonlinear force of the MR damper.

Fig. 7(a) Lab experiment of a 5-story building

Fig.7(b) MR damper installed in the 1st story

Table 7 Parameters of the equivalent linear & linear building (Lab experiment)

Story No.	$k_{e,i}$ (kN/m)	$k_{l,i}$ (kN/m)	$c_{e,i}$ (N·s/m)	$c_{l,i}$ (N·s/m)
1st	40.72	40.16	**10.51**	**1.22**
2nd	117.78	121.38	1.45	1.33
3rd	118.09	120.25	1.88	2.03
4th	128.48	129.58	1.59	1.42
5th	127.73	131.66	2.32	2.15

Table 8 Identification parameters of the nonlinear building (Lab experiment)

Story No.	1st	2nd	3rd	4th	5th
k_i (kN/m)	39.73	116.85	118.66	125.63	128.94
c_i(kN·s/m)	1.35	1.47	2.26	1.29	2.48

Coefficients of polynomial in the 1st story

	$c_{1,0}$	$c_{0,1}$	$c_{2,0}$	$c_{1,1}$	$c_{0,2}$	$c_{3,0}$	$c_{2,1}$	$c_{1,2}$	$c_{0,3}$
Identified	201.1	0.81	-0.94	0.12	-0.02	4.85	-0.33	0.19	-0.07

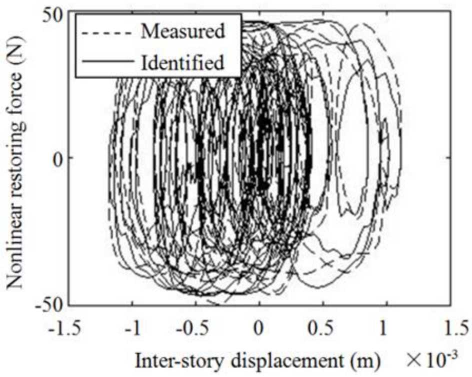


Fig. 8 Restoring force of the MR damper (Lab experiment)

Only 3 light PCB accelerometers are deployed on the 1st, 3rd and 5th floors to measure structural acceleration responses at the corresponding floor levels, respectively. Both the excitation and corresponding model responses are measured and used for the identification of the building model equipped with a MR damper by the proposed algorithm.

The structural parameters of the linear building model can be identified by EKF with the measured structural acceleration responses without the MR damper. The identification results are shown in Table 7. Then, an equivalent linear building is established and its equivalent structural parameters are identified by the EKF using the measured structural acceleration responses with the installed MR damper. The identification results of equivalent structural parameters are shown and compared with those of the linear structural values in Table 7. From the comparison, it is noted that there is a large difference between the identified equivalent linear damping in the 1st story as marked by values in bold. Thus, the location of the MR damper is identified.

Then, the UKF is utilized to identify the nonlinear building with model-free MR damper. In Table 8, The identified structural parameters and coefficients of polynomial are shown.

Finally, the identified nonlinear restoring force of the MR damper is compared with the measured force in Fig. 8. These two forces are close to each other.

5. Conclusions

In this paper, an identification algorithm is proposed for locating and identifying model-free nonlinear restoring forces and systems using only partial measurements of structural responses. First, an equivalent linear structural system is established for a nonlinear structure. The equivalent linear structure is identified by the extended Kalman filter (EKF). The locations of structural nonlinearities can be identified by comparing the differences between the identified the equivalent linear structural parameters and those of the structure under weak excitation. Also, these differences indicate the levels of structural nonlinearities. Then, model-free nonlinear restoring forces at the locations of identified structural nonlinearities are approximated by power series polynomials with the orders depended on the level of structural nonlinearities. The unscented Kalman filter (UKF) is utilized to identify the nonlinear structural restoring forces and system with the initial values of power series polynomials estimated by least-square techniques with the identified values of equivalent linear structure.

Compared with prior approaches of direct identification of the nonlinear structures, the proposed algorithm greatly simplifies the identification difficulties. Several numerical simulation examples with different nonlinear models and locations have validated the effectiveness of the proposed algorithm. Also the application of the proposed algorithm for the identification of a multi-story shear building equipped with MR damper has further validated the proposed algorithm. It is shown that the proposed algorithm is feasible for the identification of model-free nonlinear restoring forces and nonlinear structural systems using only partial measurements of structural responses. Although only shear-type structures are used in the numerical examples of the manuscript, the proposed algorithm can also be used for the identification of other types of nonlinear structures.

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References

Al-Hadid, M.A. and Wright, J.R. (1992), “Estimation of mass and modal mass in the identification of non-linear single and multi degree of freedom systems using the force-state mapping approach”, *Mech. Syst. Signal Pr.*, **6**(4), 383-401.

Crawley, E.F. and Aubert, A.C. (1986), “Identification of nonlinear structural elements by force-state mapping”, *AIAA J.*, **24**(1), 155-162.

Dahl, P.R. (1976), “Solid friction damping of mechanical vibrations”, *AIAA J.*, **14**(12), 1675-1682.

Feng, M.Q. (2009), “Application of structural health monitoring in civil infrastructure”, *Smart Struct. Syst.*, **5**(4), 469-482.

He, J., Xu, B. and Masri, S.F. (2012), “Restoring force and dynamic loadings identification for a nonlinear chain-like structure with partially unknown excitations”, *Nonlinear Dynam.*, **69**(1-2), 231-245.

Hernandez-Garcia, M.R., Masri, S.F. and Ghanem, R. (2010), “A structural decomposition approach for detecting, locating, and quantifying nonlinearities in chain-like systems”, *Struct. Control Health Monit.*, **17**(7) ,761-777.

Hoshiya, M. and Saito, E. (1984), “Structural identification by extended Kalman filter”, *J. Eng. Mech. - ASCE*, **110(**12), 1757-1771.

Julier, S.J. and Uhlmann, J.K. (2004), “Unscented filtering and nonlinear estimation”, *P. IEEE*, **92**(3), 401-422.

Kerschena, G., Wordenb, K., Vakakis, A.F. and Golinvala, J.C. (2006), “Past, present and future of nonlinear system identification in structural dynamics”, *Mech. Syst. Signal Pr.*, **20**(3),505-592.

Kim, I.H., Jung, H.J. and Kim, J.T. (2011), “Numerical investigation of an MR damper-based smart passive control system for mitigating vibration of stay cables”, *Struct. Eng. Mech*., **37**(4), 443-458.

Lee, Y.S., Vakakis, A.F., McFarland, D.M. and Bergman, L.A. (2010), “A global-local approach to Nonlinear system identification: a review”, *Struct. Control Health Monit.*, **17**(7),742-760.

Lei, Y., Wu, Y. and Li, T. (2012), “Identification of nonlinear structural parameters under limited input and output measurements”, *Int. J. Nonlinear Mech.*, **47**(10), 1141-1146.

Lei, Y., Liu, C., Jiang, Y.Q. and Mao, Y.K. (2013), “Substructure based structural damage detection with limited input and output measurements”, *Smart Struct. Syst.*, **12**(6), 619-640.

Lei, Y. and He, M.Y. (2013), “Identification of the nonlinear properties of rubber-bearings in base-isolated buildings with limited seismic response data”, *Sci. China-Technol. Sci.*, **56**(5), 1224-1231.

Lu, W., Teng, J. and Xu, Y.L. (2013), “Identification of damage in dome-like structures using hybrid sensor measurements and artificial neural networks”, *Smart Mater. Struct.*, **22**(10), doi:10.1088/0964-1726/22/10/105014

Mariani, S. and Ghisi, A. (2007), “Unscented Kalman filtering for nonlinear structural dynamics”, *Nonlinear Dynam.*, **49**(1-2), 131-150.

Masri, S.F. and Caughey, T.K. (1979), “A nonparametric identification technique for nonlinear dynamic problems”, *J. Appl. Mech. - ASME*, **46**(2), 433-447.

Masri, S.F., Sassi, H. and Caughey, T.K. (1982), “A nonparametric identification of nearly arbitrary nonlinear systems”, *J. Appl. Mech. – ASME*, **49**(3), 619-628.

Masri, S.F., Miller, R.K., Saud, A.F. *et al*. (1987), “Identification of nonlinear vibrating structures: part II -application”, *J. Appl. Mech. – ASME*, **109**, 923-929.

Masri, S.F., Caffery, J.P., Caughey, T.K., Smyth, A.W. and Chassiakos, A.G. (2005), “A general data-based approach for developing reduced-order models of non-linear MDOF systems”, *Nonlinear Dynam.*, **39**(1-2), 95-112.

Masri, S.F., Tasbihgoo, F., Caffery, J.P., Smyth, A.W. and Chassiakos, A.G. (2006), “Data-based model-free representation of complex hysteretic MDOF systems”, *Struct. Control Health Monit.*, **13**(1), 365-387.

Omrani, R., Hudson, R.E. and Taciroglu, E. (2013), “Parametric identification of nondegrading hysteresis in a laterally and torsionally coupled building using an unscented Kalman filter”, *J. Eng. Mech. - ASCE*, **139**(4), 452-468.

Su, J.Z., Xia Y. and Xu Y.L. (2014), “Settlement monitoring of a supertall building using the Kalman filtering technique and forward construction stage analysis”, *Adv. Struct. Eng.*, **17**(6), 881-893

Woo, S.S. and Lee, S.H. (2013), “Damping updating of a building structure installed with an MR damper”, *Smart Struct. Syst.*, **12**(6), 695-705.

Wu, M. and Smyth, A.W. (2007), “Application of the unscented Kalman filter for real-time nonlinear structural system identification”, *Struct. Control Health Monit.*, **14** (7),971-990.

Xia, Y., Chen, B. and Zhou, X.Q. (2013), “Field monitoring and numerical analysis of Tsing Ma Suspension Bridge temperature behavior”, *Struct. Control Health Monit.*, **20**(4), 560-575.

Xu, B., He, J. and Masri, S. F. (2012), “Data-based Identification of nonlinear restoring force under spatially incomplete excitations with power series polynomial model”, *Nonlinear Dynam.*, **67**(3), 2063-2080.

Yan, A.Z., Teng, J. and Lu, Z.X. (2008), “Effect of geometrical nonlinearity on the SAVIS device”, *Adv. Struct. Eng.*, **11**(1), 91-105.

Yang, J.N., Lin, S., Huang, H.W. and Zhou, L. (2006), “An adaptive extended Kalman filter for structural damage identification”, *Struct. Control Health Monit.*,**13**(4), 849-867.

Yi, T.H., Li, H.N. and Gu, M. (2012), “Sensor placement for structural health monitoring of Canton Tower”. *Smart Struct. Syst.*, **10**(4-5), 313-329.

Yi, T.H., Li, H.N. and Sun, H.M. (2013), “Multi-stage structural damage diagnosis method based on ‘energy-damage’ theory”. *Smart Struct. Syst.*, **12**(3-4), 345-361.