Smart Structures and Systems, *Vol. 15, No. 1 (2015) 227-244*
DOI: http://dx.doi.org/10.12989/sss.2015.15.1.227

Structural damage identification based on genetically trained ANNs in beams

Li Peng-hui, Zhu Hong-ping, Luo Hui* and Weng Shun

Hubei Key Laboratory of Control Structure, School of Civil Engineering and Mechanics, Huazhong University of Science and Technology, Wuhan 430074, China

(Received January 9, 2014, Revised May 17, 2014, Accepted May 25, 2014)

Abstract. This study develops a two stage procedure to identify the structural damage based on the optimized artificial neural networks. Initially, the modal strain energy index (MSEI) is established to extract the damaged elements and to reduce the computational time. Then the genetic algorithm (GA) and artificial neural networks (ANNs) are combined to detect the damage severity. The input of the network is modal strain energy index and the output is the flexural stiffness of the beam elements. The principal component analysis (PCA) is utilized to reduce the input variants of the neural network. By using the genetic algorithm to optimize the parameters, the ANNs can significantly improve the accuracy and convergence of the damage identification. The influence of noise on damage identification results is also studied. The simulation and experiment on beam structures shows that the adaptive parameter selection neural network can identify the damage location and severity of beam structures with high accuracy.

Keywords: damage identification; modal strain energy index; artificial neural network; genetic algorithm; principal component analysis

1. Introduction

Damage identification in an early stage is a critical issue to ensure the structural integrity and safety. Identifying damage can extend the service life and reduce the maintenance cost of the structure. The damage identification has the key importance to promote the structure safety. Structural damage identification has two problems: damage index selection and the accuracy of damage identification. Modal strain energy change (MSEC) is sensitive to the local damage. Meanwhile the artificial neural network can identify the damage exactly. The combination of the modal strain energy (MSE) and artificial neural networks has the potential to solve the two problems of damage identification.

Shi and Law (2002) utilized the MSEC to detect the damage location and to derive the sensitivity of the modal strain energy. Only incomplete measured mode shapes are required to detect the damage location and severity. Choi and Samali (2008, 2010) extracted the MSEC from the intact and damaged structure to establish the damage index, from which the damage location and severity are identified. This method is effective in severe damage and ineffective in slight

*Corresponding author, Associate professor, E-mail: autumn_luoh@163.com

http://www.techno-press.org/?journal=sss&subpage=8 ISSN: 1738-1584 (Print), 1738-1991 (Online)

damage. Hsu (2008) implemented the MSEC to detect the damage in a frame structure, by improving the iterative process and considering the modal error, noise and incomplete measurement. Yan and Ren (2010, 2011) investigated the modal strain energy sensitivity to identify the damage location and extent. The first order sensitivity formulae of element for a real symmetric undamped structure are derived based on the algebraic eigensensitivity method. Dixit (2011) proposed the damage detection method based on the modal strain energy. The mass and stiffness formula are derived to establish the damage index.

The MSE has the good ability to locate the damage area accurately and ANNs can adaptively achieve the pattern recognition to obtain the damage severity. Many researchers have combined the MSE and ANNs to locate and qualify damage. Xu (2006) introduced the MSE to locate the damage and ANNs to qualify the damage. Kim (2008) developed the acceleration signals and ANNs to monitor the occurrence and location of damage in structures. Bakhary and Hao (2007, 2010) proposed the damage detection method based on the vibrational parameter and ANNs. The neural network model is established by the statistical method. The modal parameters are input of the ANNs, and the damage location and severity are obtained. González (2008) proposed the damage detection based on ANNs and statistical method. The inputs of ANNs are frequencies and mode shapes, and outputs are mass and stiffness. Park (2009) proposed the sequential approaches for damage detection in beams using time-modal features and ANNs. Firstly an acceleration-based neural network is designed to detect the occurrence of damage. Then a modal feature-based neural network is designed to estimate the location and extent. Dackermann (2010) developed a method to identity defects combined the principal component analysis (PCA) and neural networks ensembles. PCA-compressed damage index values are used as inputs to evaluate location and severity. González-Pérez (2011) introduced the ANNs for structural damage identification in vehicular bridge. The method can predict the location and severity with high accuracy in the bridge.

The present research has shown that the MSE combined with ANNs can identify the damage extents when the scale of neural networks is small. Nevertheless the practical structure is complex and has many degrees of freedom (DOFs) involved. The ANNs have many limitations for training a reliable network model such as the high computational cost, low accuracy and convergence problem. The genetic algorithm (GA) optimized neural network can improve the limitation of neural network. In this study, two step procedures are proposed. Firstly, the modal strain energy index is used to locate the damage. Secondly, the genetically trained ANNs is employed to qualify the damage extents. The PCA reduces the input variables of neural network and the computational cost. The genetic algorithm is implemented to optimize the parameters of the ANNs and achieve the automate parameter selection in different damage cases. The accuracy and convergence of damage identification are improved obviously by the genetically trained ANNs.

2. Methodology

2.1 Modal strain energy

The modal strain energy of Bernoulli-Euler beam is defined as Eq. (1) (Shi 2002)

$$U = \frac{1}{2} EI \int_0^L \left(\frac{d^2 y}{dx^2} \right)^2 dx \tag{1}$$

where EI is the flexural stiffness of the cross section, x is the longitude along the axis of beam, y is the vertical deformation of beam.

The modal strain energy of beam in the *i*-th mode shape and the *j*-th element is denoted as

$$U_{ij} = \frac{1}{2}(EI)_j \int_j \left(\frac{\partial^2 \phi_i(x)}{\partial x^2} \right)^2 dx \tag{2}$$

where $\phi_i(x)$ is the *i*-th mode shape of the beam.

The stiffness change of beam in the *i*-th mode shape and the *j*-th element is expressed as Eq. (3) (Dackermann 2010)

$$\frac{EI_j}{EI_j^d} = \frac{\int_j \left[\phi_i^{"d}(x) \right]^2 dx \int_0^L \left[\phi_i^{"}(x) \right]^2 dx}{\int_j \left[\phi_i^{"}(x) \right]^2 dx \int_0^L \left[\phi_i^{"d}(x) \right]^2 dx} = \frac{f_{ij}^d}{f_{ij}} \tag{3}$$

where superscript d denotes the damaged status, $\phi_i^{"}(x)$ is the *i*-th curvature mode shape of beam.

Considering the whole available m measurement modes, the damage index is given by

$$\beta_j = \frac{\sum_{i=1}^{m} f_{ij}^d}{\sum_{i=1}^{m} f_{ij}} \tag{4}$$

To enhance the performance of damage identification, the damage index β_j is transformed into the standard normal space and the normalized modal strain energy index (MSEI) is established as

$$MSEI = \frac{\beta_j - \mu_{\beta_j}}{\sigma_{\beta_j}} \tag{5}$$

where μ_{β_j} and σ_{β_j} are the mean and standard deviation value of β_j respectively.

2.2 Principal Component Analysis (PCA)

PCA is a statistical method to reduce the dimensionality. It linearly transforms an original set to a smaller set. The compressed variants are the linear components of original variants, and the components are uncorrelated. The PCA can be used to reduce the input of neural networks effectively. The four steps of data compression by PCA are shown as follows. Step 1, the data are normalized by

$$\bar{x}_j = \frac{1}{m}\sum_{i=1}^{m} x_{ij}\ ,\ s_j = \sqrt{\frac{\sum_{i=1}^{m}(x_{ij} - \bar{x}_j)^2}{m}}\ ,\ \tilde{x}_{ij} = \frac{x_{ij} - \bar{x}_j}{s_j} \tag{6a, 6b, 6c}$$

where, x_{ij} is the samples, m is the total sample numbers, $\bar{x}_j$ and s_j are the mean and standard values of samples respectively, $\tilde{x}_{ij}$ is the scandalized samples.

Step 2, the covariant matrix $[C]$ is established as

$$[C] = \frac{[\tilde{x}]^T[\tilde{x}]}{m-1} \tag{7}$$

Step 3, the eigenvalues λ_i and eigenvectors $\{P_i\}$ of covariant matrix $[C]$ are calculated by Eq. (8) (Dackermann 2010)

$$[C]\{P_i\} = \lambda_i\{P_i\} \tag{8}$$

Step 4, the accumulated contribution rate (ACR) of the first r principal components is defined as

$$\text{ACR} = \sum_{i=1}^{r} \lambda_i \Big/ \sum_{i=1}^{m} \lambda_i \, (r \le m) \tag{9}$$

The first principal component (PC) represents the most significant contribution of the data set, and it has the largest eigenvalue and associated eigenvector. The second PC is orthogonal to the first PC and has the second significant contribution of the data set. When the accumulated contribution rate of the first r components is big enough, it is considered that the first r components conclude the most information of all m original variants. The first r components are chosen to reduce the dimension for damage identification.

2.3 GA Trained ANNs

The ANNs are regarded as nonlinear mathematical functions that map a set of input variables to the output variables. The backpropagation neural network (BPNN) is a multiple layer forward network and it consists of an input layer, an output layer and several hidden layers. The neural network with two hidden layers is utilized in this study. The input of the ANNs is modal strain energy deviations of element and the output is elemental flexural stiffness. The steepest descent learning algorithm is chosen to obtain high learning accuracy. We set learning rate 0.3, momentum constant 0.9 and error tolerance 0.0001 for training parameters. The Sigmoid function is chosen as the transfer function to train the neural networks.

The parameters are chosen by a trial procedure in the most proposed research. In this study, the parameters of neural networks such as learning coefficients, momentum coefficients and numbers of neurons in each hidden layer are optimized by genetic algorithm. The flow chart of GA optimized ANNs is shown in Fig. 1.

The GA is a global searching process based on the Darwin's principal of natural selection and evolution. It consists of three main operations: selection, mutation and crossover. The GA starts with an initial population, and the initial population is generated randomly with real numbers between 0 and 1. Each chromosome in the population is real coded and contains the number of neurons in two hidden layers, learning and momentum coefficients of neural network. The population size of 30 is selected in this study. The objective function is defined as

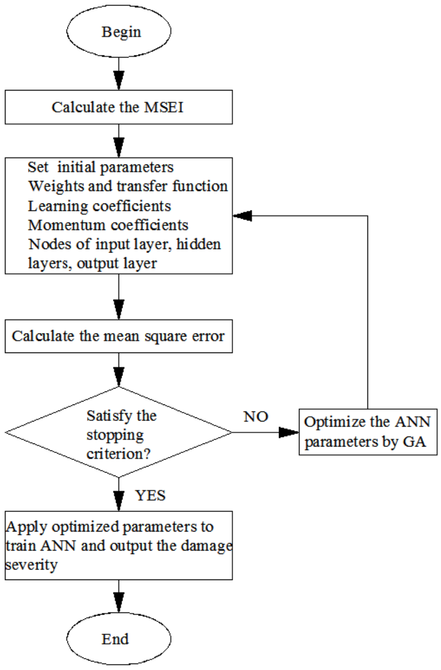

Fig. 1 Flow chart of GA optimized ANNs

$$E_f = \sum_{i=1}^{n} abs(y_i - o_i) \tag{10}$$

where, n is the output number of neural network, y_i is the true output of BPNN of *i*-th node, o_i is the predicted output of BPNN of *i*-th node.

The fitness values are calculated by fitness function for the selection operator. The fitness value F is defined as

$$F = \frac{1}{1+E_f} \tag{11}$$

The chromosomes are selected for reproduction of the future populations based on their fitness with the selection operation. The crossover operator takes the chromosomes of two parents, randomly selected and then exchanges part of the genes, which results in two new chromosomes for the child generation. The simulated binary crossover (SBX) is chosen in this paper. The probability of crossover defines the ratio of the number of offspring produced in each generation to the population size. The mutation operator introduces a change in one or more of chromosome genes. The probability of mutation is defined as the ratio of the number of mutated genes to the total number of genes. The process is stopped when the number of generations is completed or the mean square error is less than the predefined value.

Table 1 Damage cases of simply supported beam

Case no.	Damaged elements	Severity
1	6	10%
2	6	50%
3	6,10	10%,20%
4	6,10	20%,50%

Table 2 Natural frequencies of intact and damaged beam

Case no.	Mode1	Mode2	Mode3	Mode4	Mode5
Intact	22.88	91.37	205.04	363.12	564.49
1	22.81	90.89	204.72	362.85	562.04
2	22.24	87.45	202.52	360.82	545.52
3	22.53	90.85	202.43	362.26	556.70
4	21.68	90.18	196.00	360.38	540.77

3. Numerical example

3.1 Numerical Model

The dimension of simply supported beam is 2.0 m× 0.02 m× 0.04 m, with the density of 7850 kg/m^3, poison ratio of 0.3, elastic modular of 1.96×10^{11} N/m^2. The damage of beam is modeled by reduction of flexural stiffness. The stiffness reduction are 10%, 20% and 50%, and the corresponding damage ratios are 0.1, 0.2 and 0.5 respectively. The ANSYS software package is used for finite element simulation, and the beam is divided into 20 elements along the length. The first five natural frequencies and mode shapes are extracted to calculate the modal strain energy and the MSEI are established by Eq. (5). In order to investigate the noise effect on the performance of the proposed method, two noise levels are considered. The 0.1% and 3% are considered for frequencies and mode shapes respectively as noise level A. The 0.1% and 10% are considered for frequencies and mode shapes respectively as noise level B. Two damaged elements are 6 and 10 respectively. Four damage cases are listed in Table 1.

The first five natural frequencies of the intact and four damaged cases in beam are listed in Table 2. It can be observed that the presence of damage in beam causes a small decrease in the natural frequencies in all damage cases. Thus the natural frequency change cannot identify the damage location clearly.

The first five mode shapes of intact beam and damaged beam in the case 4 are shown in Fig. 2.

It also can be observed that the damage causes a small change of mode shapes. It is difficult to identify the damage location from the change of mode shapes.

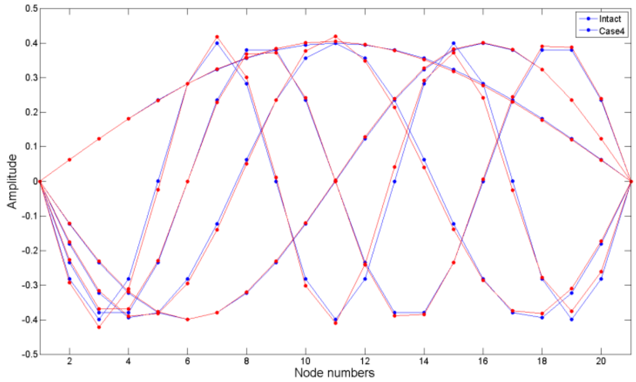

Fig. 2 The first five mode shapes of intact and damaged beam

3.2 Localization

The MSEI are calculated by the first five natural frequencies and mode shapes via Eq. (5). Then the damage locations in four damage cases are determined. Fig. 3 shows the MSEI result without noise and the Figs. 4 and 5 show the MSEI results with two noise levels.

It can be observed in Fig. 3 that the MSEI can identify the location without noise exactly in the single and multiple damage cases. Figs. 4 and 5 show the MSEI values of four damage cases when the frequencies and mode shapes are contaminated by noises. In order to enhance the identification effect, those elements whose MSEI exceed 0.4 are selected as suspected damage elements. As shown in the Figs. 4 and 5, the suspected elements are damaged elements in four damage cases. Thus, the locating ability of MSEI is demonstrated when the modal data are contaminated by noise.

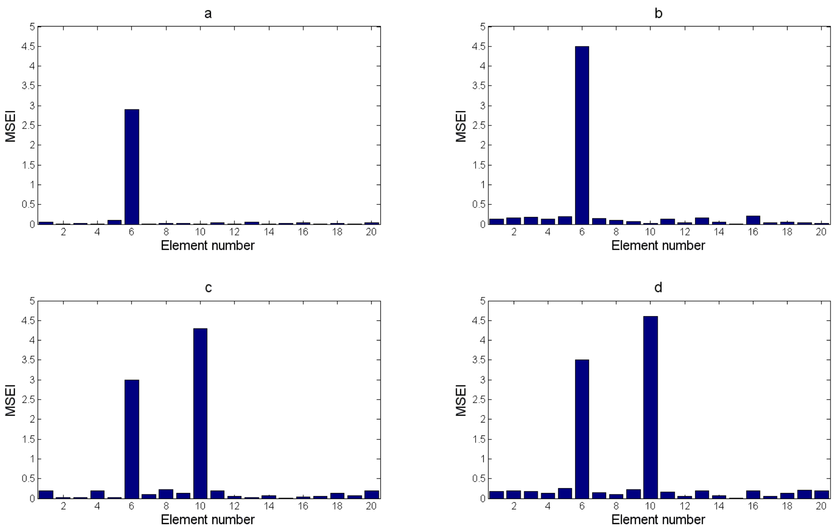

Fig. 3 MSEI values in Case 1- Case 4 without noise

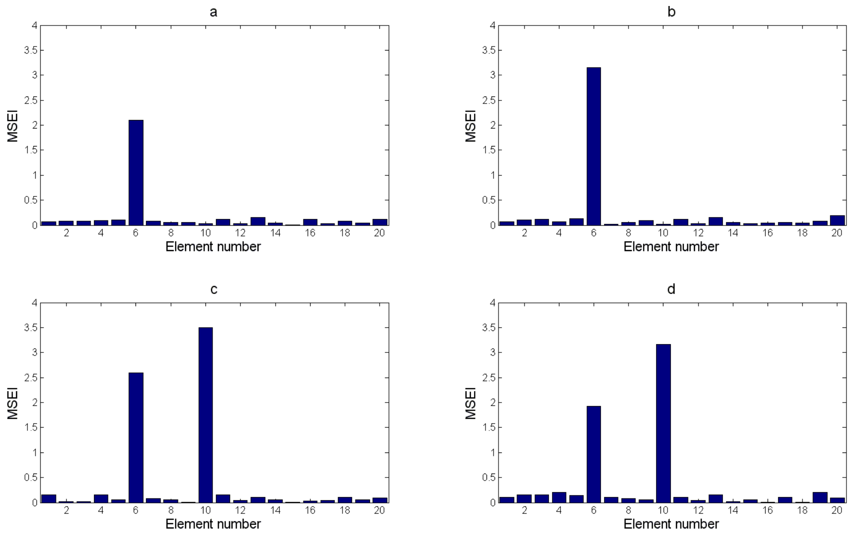

Fig. 4 MSEI values in Case 1- Case 4 with 3% noise

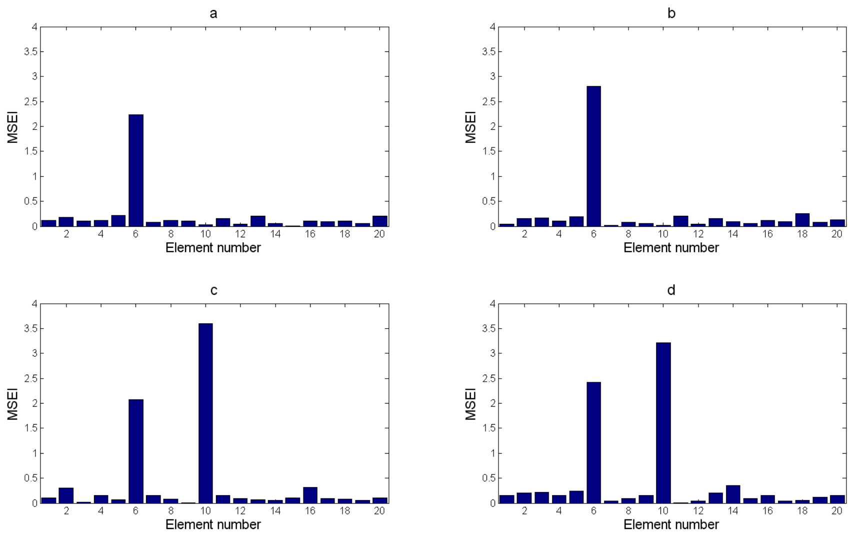

Fig. 5 MSEI values in Case 1- Case 4 with 10% noise

3.3 Qualification

3.3.1 Principal component selection

The principal component analysis is utilized to compress the input nodes of the neural networks. The beam is divided into twenty elements and the first five mode shapes are used. Thus the number of samples is 20. The ‘princomp’ function in MATLAB is utilized to transfer MSEI to the principal component space. According to the Eq. (11), the contributions of twenty components are calculated in Fig. 6.

From Fig. 6, it can be observed the first component accounts for 26.2% of the original data, the first and second components contribute 50.1% of the data and the first three components contribute to 62.5%. The cumulative contribution of last four components is less than 2.2%. In this study, the components are chosen as the input of neural network when the accumulated contribution rate of the first *r* components is 97%. Therefore the first sixteen components which represent 97.8% of original data are regarded as the most significant components. The first sixteen components are used as input for the neural networks.

3.3.2 GA trained BPNN

The backpropagation neural network has one input layer, one output layer and two hidden layers. The PCA compressed MSEI is the input of network and the elemental flexural stiffness is the output of network. The number of neurons in the hidden layers, learning coefficients and momentum coefficients are optimized by the genetic algorithm. The beam has twenty elements and the first five mode shapes are considered. The total 600 samples are generated, out of which 540 are used for training and the rest of 60 are used for testing.

The genetic algorithm has three procedures by selection, crossover and mutation. The population size of 30 is selected in this study. A crossover probability of 0.9 is adopted as it suits many engineering problems. A mutation probability of 0.01 is selected. The process is stopped when 30 generations are completed or the mean square error is less than the given value of 0.0001. The parameters of GA trained BPNN are listed in Tables 3-5.

The optimized parameters of neural network are utilized to obtain the damage extents. The qualification results are shown in the next section.

3.4 Results

The identified results of four damage cases without noise are shown in Fig. 7. And the results with two noise levels are shown in Figs. 8 and 9 respectively.

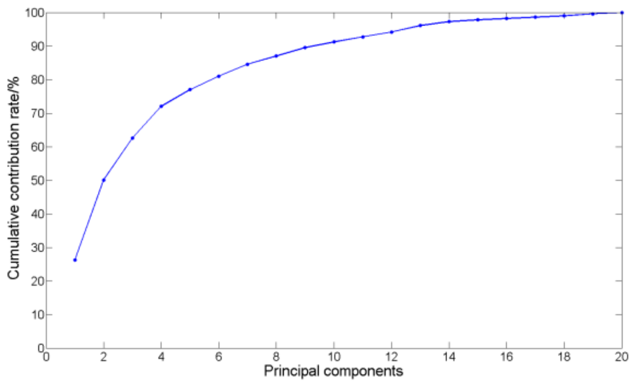

Fig. 6 Cumulative contribution of principal components

Table 3 GA optimized BPNN parameters without noise

Case no.	Nodes of 1st layer	Nodes of 2nd layer	Learning coefficients	Momentum coefficients	Mean square error	Iterations
1	16	13	0.7353	0.4468	0.0000969	116
2	15	12	0.7251	0.4231	0.0000926	194
3	10	7	0.7521	0.4325	0.0000922	136
4	15	10	0.7361	0.4136	0.0000969	159

Table 4 GA optimized BPNN parameters with 3% noise

Case no.	Nodes of 1st layer	Nodes of 2nd layer	Learning coefficients	Momentum coefficients	Mean square error	Iterations
1	17	11	0.7265	0.4326	0.0000928	176
2	12	10	0.7163	0.4569	0.0000961	231
3	11	9	0.7952	0.4422	0.0000914	165
4	13	7	0.7225	0.4551	0.0000923	241

Table 5 GA optimized BPNN parameters with 10% noise

Case no.	Nodes of 1st layer	Nodes of 2nd layer	Learning coefficients	Momentum coefficients	Mean square error	Iterations
1	15	11	0.7145	0.4628	0.0000935	374
2	15	9	0.7233	0.4447	0.0000938	221
3	10	6	0.7624	0.4644	0.0000954	259
4	14	8	0.7065	0.4982	0.0000940	336

The proposed algorithms identify the damage extents exactly without noise in all the single and multiple damage cases according to Fig. 7. Damage quantification is successful with a maximum of 7% error under 3% measurement noise as shown in Fig. 8. Damage quantification is successful with a maximum of 12% error under 10% measurement noise as Fig. 9.

3.5 Discussion

The training error curve of the original BPNN and genetic trained BPNN in case 4 with 10% noise is shown in Fig. 10. The original BPNN has low convergence speed and trends to the local extreme value, but the neural network can search the most suitable value via the genetic algorithm.

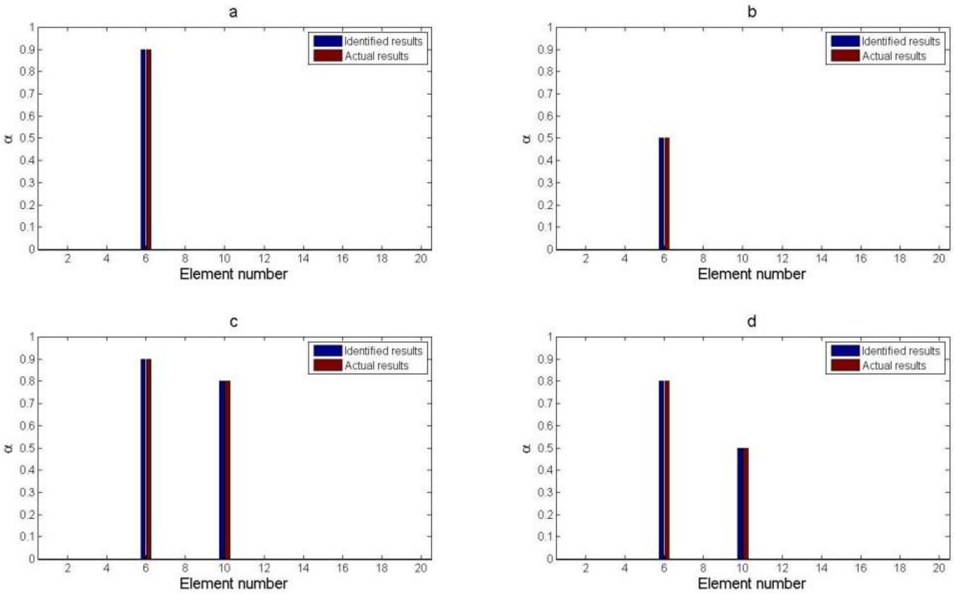

Fig. 7 Damage extents of Case 1- Case 4 without noise

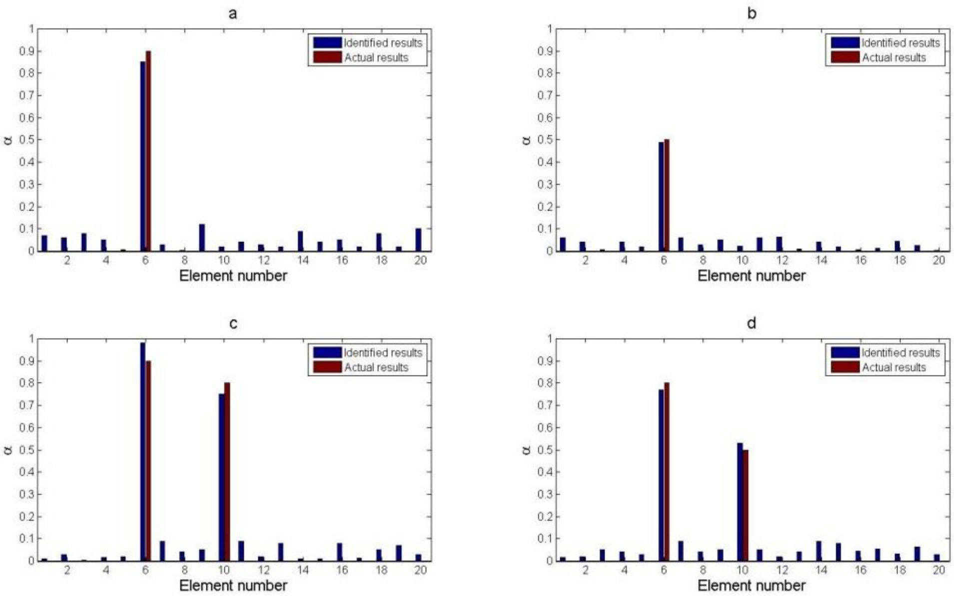

Fig. 8 Damage extents of Case 1- Case 4 with 3% noise

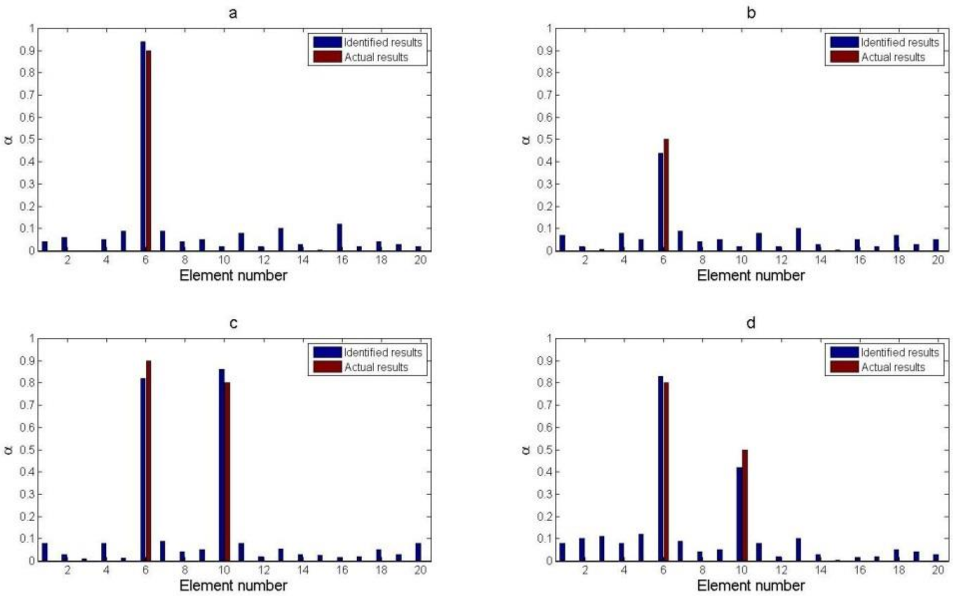

Fig. 9 Damage extents of Case 1- Case 4 with 10% noise

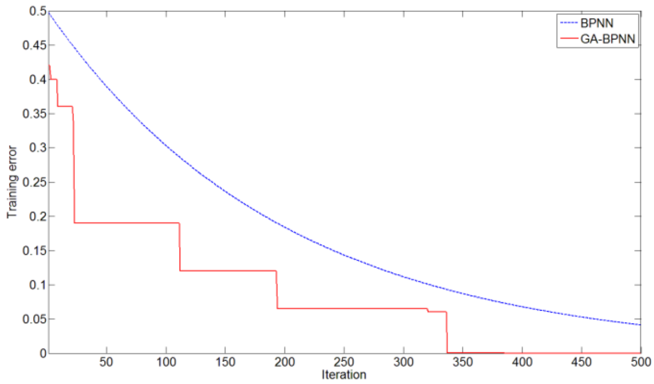

Fig. 10 Mean square error of BP and GA trained NN with 10% noise

The relationship between damage ratio and generation of element 6 without noise are shown in Figs. 11 and 12. From Figs. 11 and 12, the GA-BPNN generates 13 times to obtain the stable identified results and the original BPNN needs at least 24 times in case 4. The relationship between damage ratio and generation of element 6 with 10% noise are shown in Figs. 13 and 14. From Figs. 13 and 14, the GA-BPNN generates 18 times to obtain the stable identified results and the original BPNN needs at least 28 times in case 4. The convergence speed to achieve the correct identified results are promoted by the GA trained BPNN. It can be observed that the genetically trained neural network performance better than the original neural network even when the modal data contaminated by the noise.

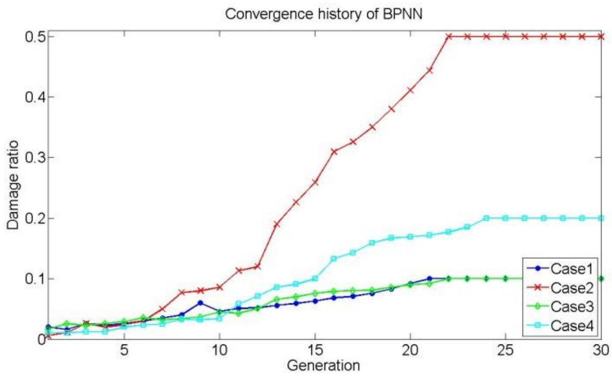

Fig. 11 Convergence history of BPNN for four damage cases without noise

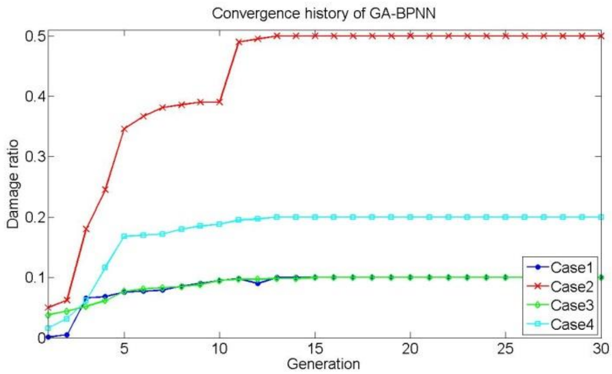

Fig. 12 Convergence history of GA-BPNN for four damage cases without noise

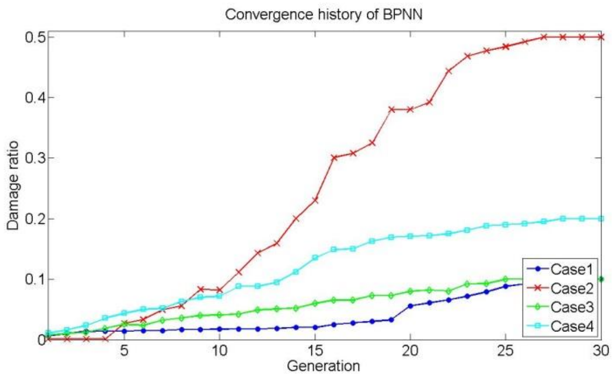

Fig. 13 Convergence history of BPNN for four damage cases with 10% noise

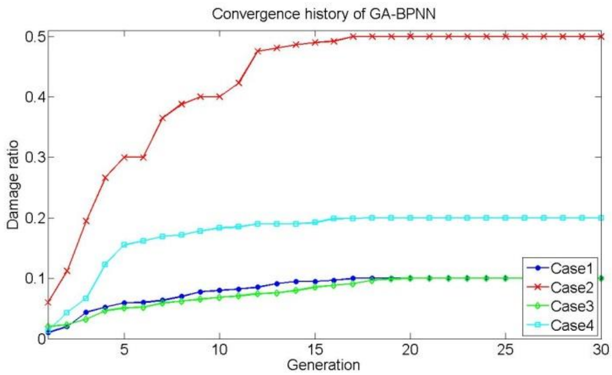

Fig. 14 Convergence history of GA-BPNN for four damage cases with 10% noise

The GA-BPNN method can identify the damage extents in all the single and multiple damages with high accuracy. The genetically trained neural works has higher convergence speed and achieve the results more accurately than the neural networks.

4. Experimental example

The laboratory example is a steel cantilever beam as shown in Fig. 15. The beam is 50.1 mm wide, 3.0 mm high and 750 mm long as illustrated in Fig. 16. The mass density was measured as 8.026×10^3 kg/m^3. To assure that the boundary condition was not altered in each testing, two thick blocks were welded on both sides of the clamped end as shown in Fig. 16.

The structure is tested in the intact state and four damage cases respectively which are given in Table 6. The beam is first tested without damage as 'Case 0'. Afterwards, the beam is cut at Location 1 as shown in Fig. 16 with depth of d = 5 mm, 10 mm, and 15 mm gradually, corresponding to 'Case 1', 'Case 2', 'Case 3' respectively. In 'Case 4', the beam is additionally cut at Location 2 with depth of d = 10 mm. The width of the cuts is b =5 mm in all the damage cases.

Fig. 15 Experimental beam specimen

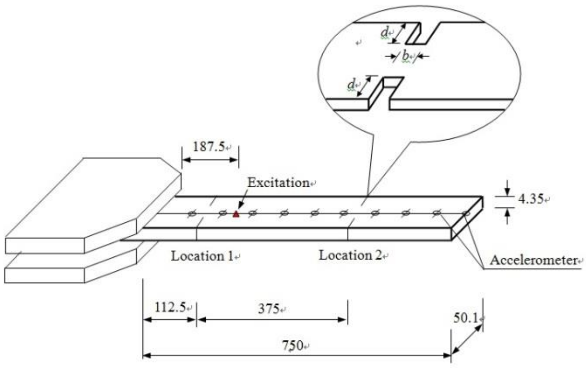

Fig. 16 Beam specimen setup (Unit: mm)

Table 6 Damage cases of cantilever beam

Case 1	Case 2	Case 3	Case 4
Location 1 d = 5 mm	Location 1 d = 10 mm	Location 1 d = 15 mm	Location 1 d = 15 mm Location 2 d = 10 mm

The frequency response function is shown in Fig. 17. The measured frequencies in the different states are shown in Table 7.

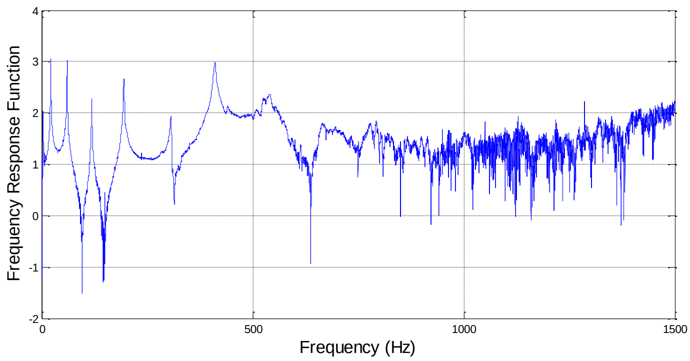

Fig. 17 Frequency response function

Table 7 Measured frequencies in the different cases

Modes	Case 0	Case 1		Case 2		Case 3		Case 4	
	Freq. (Hz)	Freq. (Hz)	Diff. (%)	Freq. (Hz)	Diff. (%)	Freq. (Hz)	Diff. (%)	Freq. (Hz)	Diff. (%)
1	3.499	3.438	-1.75%	3.500	0.02%	3.426	-2.09%	3.422	-2.22%
2	21.848	21.851	0.01%	21.518	-1.51%	21.497	-1.61%	21.201	-2.96%
3	60.290	60.280	-0.02%	59.580	-1.18%	59.668	-1.03%	59.003	-2.13%
4	118.819	118.685	-0.11%	117.399	-1.20%	116.817	-1.69%	116.611	-1.86%
5	194.708	193.715	-0.51%	190.254	-2.29%	188.426	-3.23%	187.289	-3.81%

The damage identification results of case 1-4 in cantilever beam are shown in Fig. 18. From Fig. 18, the MSEI can identify the damage location with high accuracy under measurement noise. Damage quantification is successful with a maximum of 9.5% error due to the measurement noise. The proposed algorithm is verified in the steel cantilever experiment with acceptable accuracy.

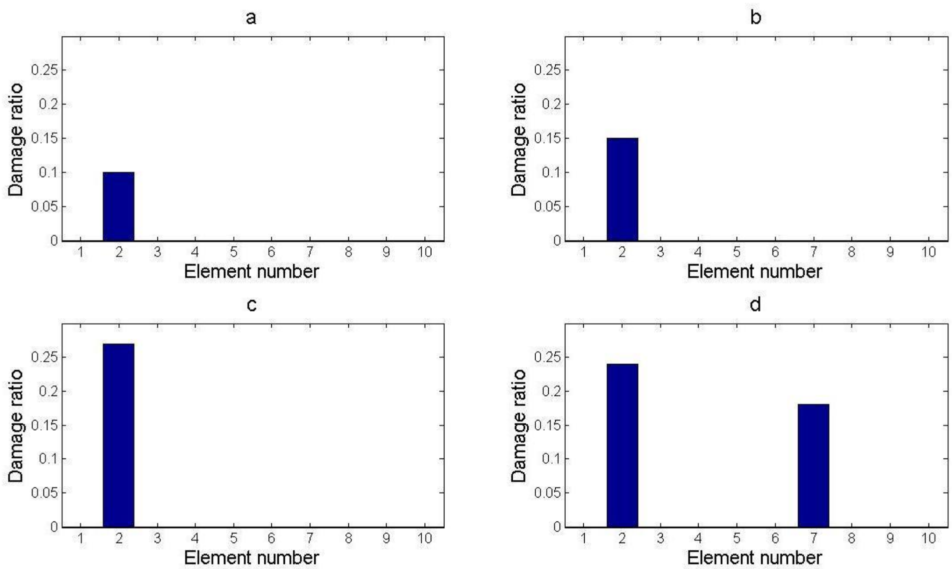

Fig. 18 Damage identification results of Case 1- Case 4 in cantilever beam experiment

5. Conclusions

The two stage procedures are proposed to identify the damage of the beam structures. Firstly, the modal strain energy index is established to identify the damage location. Secondly, the principal component analysis is utilized to reduce the inputs of the neural networks and the genetic algorithm trained neural networks are utilized to identify the damage severity. The PCA is an effective method to reduce the inputs of neural network and save the computing time. A numerical example of the simply supported beam and experimental example of cantilever beam are studied to verify the method. The identification results of the numerical and experimental examples show that the genetically trained neural networks have higher identification accuracy and convergence speed than neural networks even in the presence of measurement noise in the modal data.

Acknowledgments

The research is supported by the 973 Project (2011CB013800) and National Natural Science Fund of China (51178203).

References

Bakhary, N., Hao H. and Deeks, A.J. (2007), "Damage detection using artificial neural network with consideration of uncertainties", *Eng. Struct.*, **29**(11), 2806-2815.

Bakhary, N., Hao, H. and Deeks, A.J. (2010), "Substructuring technique for damage detection using statistical multi-stage artificial neural network", *Adv. Struct. Eng.*, **13**(4), 619-639.

Choi, F.C., Li, J. and Samali, B. (2008), "Application of the modified damage index method to timber beams", *Eng. Struct.*, **30**(4), 1124-1145.

Dackermann, U., Li, J.C. and Samali, B. (2010), "Dynamic-based damage identification using neural network ensembles and damage index method", *Adv. Struct. Eng.*, **13**(6), 1001-1016.

Dixit, A. and Hanagud, S. (2011), "Single beam analysis of damaged beams verified using a strain energy based damage measure", *Int. J. Solids. Struct.*, **48**(3-4), 592-602.

González, M.P. and Zapico, J.L. (2008), "Seismic damage identification in buildings using neural networks and modal data", *Comput. Struct.*, **86**(3-5), 416-426.

González-Pérez, C. and Valdés-González, J. (2011), "Identification of structural damage in a vehicular bridge using artificial neural networks", *Struct. Health. Monit.*, **10**(1), 33-48.

Hsu, T.Y. and Loh, C.H. (2008), "Damage diagnosis of frame structures using modified modal strain energy change method", *J. Eng. Mech. -ASCE*, **134**(11), 1000-1012.

Kim, J.T. (2008), "Acceleration-based neural networks algorithm for damage detection in structures", *Smart. Struct. Syst.*, **4**(5), 583-604.

Park, J.H., Kim, J.T. and Hong, D.S. (2009), "Sequential damage detection approaches for beams using time-modal features and artificial neural networks", *J. Sound. Vib.*, **323**(1-2), 451-474.

Samali, B., Li, J. and Choi, F.C. (2010), "Application of the damage index method for plate-like structures to timber bridges", *Struct. Control. Health.*, **17**(8), 849-871.

Shi, Z.Y., Law, S.S. and Zhang, L.M. (2002), "Improved damage quantification from elemental modal strain energy change", *J. Eng. Mech. -ASCE*, **128**(5), 521-529.

Xu, H.P. and Humar, J.M. (2006), "Damage detection in a girder bridge by artificial neural network technique", *Comput-Aided. Civil. Infrastruct. Eng.*, **21**(6), 450-464.

Yan, W.J., Huang, T.L. and Ren, W.X. (2010), “Damage detection method based on element modal strain energy sensitivity”, *Adv. Struct. Eng.*, **13**(6), 1075-1088.

Yan, W.J. and Ren, W.X. (2011), “A direct algebraic method to calculate the sensitivity of element modal strain energy”, *Int. J. Numer. Meth. Bio.*, **27**(5), 694-710.