Smart Structures and Systems, *Vol. 14, No. 5 (2014) 831-845*
DOI: http://dx.doi.org/10.12989/sss.2014.14.5.831

Time-varying physical parameter identification of shear type structures based on discrete wavelet transform

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(Received May 3, 2013, Revised November 11, 2013, Accepted November 13, 2014)

Abstract. This paper proposed a discrete wavelet transform based method for time-varying physical parameter identification of shear type structures. The time-varying physical parameters are dispersed and expanded at multi-scale as profile and detail signal using discrete wavelet basis. To reduce the number of unknown quantity, the wavelet coefficients that reflect the detail signal are ignored by setting as zero value. Consequently, the time-varying parameter can be approximately estimated only using the scale coefficients that reflect the profile signal, and the identification task is transformed to an equivalent time-invariant scale coefficient estimation. The time-invariant scale coefficients can be simply estimated using regular least-squares methods, and then the original time-varying physical parameters can be reconstructed by using the identified time-invariant scale coefficients. To reduce the influence of the ill-posed problem of equation resolving caused by noise, the Tikhonov regularization method instead of regular least-squares method is used in the paper to estimate the scale coefficients. A two-story shear type frame structure with time-varying stiffness and damping are simulated to validate the effectiveness and accuracy of the proposed method. It is demonstrated that the identified time-varying stiffness is with a good accuracy, while the identified damping is sensitive to noise.

Keywords: physical parameter identification; time-varying parameter; discrete wavelet transform; shear type structure; multi-scale analysis

1. Introduction

The identification of structural parameters is an inverse problem in structural dynamics. Normally, the structural parameters to be identified are classified as two categories: modal parameters and physical parameters. The modal parameters such as natural frequencies, mode shapes and damping ratios characterize the dynamic properties of a structure, while the physical parameters such as stiffness and damping are the direct parameters to characterize a structure. Therefore, to identify the structural parameters means to know the structure.

At present, most research work on the identification of structural parameters is focused on the positive and negative problem of linear time-invariant system. However, the structural parameters of many practical civil engineering structures often vary over time during their operational process

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http://www.techno-press.org/?journal=sss&subpage=8 ISSN: 1738-1584 (Print), 1738-1991 (Online)

due to the environmental erosion, structural damage, material aging and load effect etc. Consequently, identifying these time-varying structural parameters is more beneficial to monitoring operational condition and diagnosing damage of the structure. The time-varying structural parameters cause the structure exhibit time-varying dynamic characteristics, so that the identification of time-varying structural parameters is more difficult than the identification of time-invariant structural parameters.

Many algorithms for modal parameter identification of time-varying structures have been proposed. Xu *et al*. (2003a) proposed a time-varying modal parameter identification method through building time-varying autoregressive model of the structure using non-stationary time serial. Xu *et al*. (2003b) also proposed a time-frequency based analysis method to identify time-varying modal parameter of civil engineering structures. Liu (1997) put forward a subspace-based identification technique and used pseudo-modal parameters to characterize the dynamic properties of time-varying system. The presented algorithm was verified by setting up an axially moving cantilever beam experiment (Liu and Deng 2004). Pang *et al*. (2005) suggested an improved subspace method using ensemble response sequence to identify the modal parameters of linear time-varying structures.

The wavelet transform as an advanced time-frequency analysis technique has been designed for non-stationary time signals in the past two decades. The wavelet analysis reveals the detail and approximation of a time signal at multiple levels and retains the transient characteristics of the data series. Therefore, wavelet transform method is widely applied to the dynamic signal analysis of time-varying structures. A continuous wavelet-based technique has been advised by Hera *et al*. (2005) and Hou *et al*. (2006) for identification of instantaneous modal parameters of a time-varying structure. Recently, Wang and Ren (2013) presented the instantaneous frequency identification of time-varying structures by continuous wavelet transform where the singular value decomposition (SVD) and dynamic optimization technique are implemented to extract the wavelet ridges. To improve the quality of extracted wavelet ridges, a synchrosqueezed wavelet transform enhanced by extended analytical mode decomposition method is proposed to identify the modal parameters of time-varying structures (Wang *et al*. 2013).

Since the physical parameters directly reflect the dynamic characteristics of a structure, the physical parameter identification of time-varying structures is more meaningful to civil engineering structures. Instead of modal parameter identification, however, the physical parameter identification is even difficult. Only a few researches are focused on the physical parameter identification of a time-vary structure. Shi and Law (2007) proposed a Hilbert transform and empirical mode decomposition (EMD) method to identify the stiffness and damping of time-varying multi-degrees of freedom dynamical systems. Wang and Chen (2012) further proposed an improved recursive Hilbert transform based method for shear type structures with time varying parameters. Ghanem and Romeo (2000) presented a discrete wavelet identification approach for a time-varying structure analysis. Their algorithm associated with a differential equation model related to the input and output responses using the wavelet Galerkin approach. Cooper and Worden (2000) proposed an on-line adaptive tracking technique to track time-varying parameter by using a forgetting factor in the standard formulation. Yang and Lin (2004, 2005) proposed an on-line adaptive tracking technique based on the least-squares estimation to identify the time-varying parameter. Li and Shi (2007) proposed a sub-space identifying method for the physical parameter identification of time-varying system based on free response data. Due to the complexity of the time-varying structural problem, these proposed algorithms have respective advantages and disadvantages. More related research work need to be carried out.

This paper is aimed at presenting a discrete wavelet transform method to identify the physical parameter of time-varying shear type structures. The proposed method expands the time-varying parameter into multi-scale approximation and detail signal by discrete wavelet basis. By ignoring the detail signal, the time-varying parameter is evaluated only using profile signal, and the scale coefficients that reflect the low frequency signal can be identified using least-squares method. In such a way, the original time-varying parameter can be reconstructed by using the identified time-invariant scale coefficients. A two-story shear type frame structure with time-varying stiffness and damping are simulated to validate the effectiveness and accuracy of the proposed method.

2. Wavelet expansion and reconstruction

2.1 Multiresolution analysis

The space $L^2(R)$ of measurable function R can be decomposed as a sequence of closed subspaces defined as follow

$$\cdots, V_0 = V_1 \oplus W_1, V_1 = V_2 \oplus W_2, V_j = V_{j+1} \oplus W_{j+1}, \cdots \tag{1}$$

where $\oplus$ is a direct sum of space. V_j is the scale space and W_j is the wavelet space of discrete orthogonal wavelet transform.

For any square integrable function $x(t) \in L^2(R)$, its multi-resolution analysis can be carried out by projecting the function onto the scale and wavelet space of different scales. Assuming that the space $L^2(R)$ is decomposed into J^{th} scale as follow

$$L^2(R) = \sum_{j=-\infty}^{J} W_j \oplus V_j \tag{2}$$

the function $x(t)$ can be orthogonally expanded as follow

$$x(t) = f_s^j(t) + f_s^j(t) = \sum_{k=-\infty}^{\infty} c_{J,k}\phi_{J,k}(t) + \sum_{j=-\infty}^{\infty}\sum_{k=-\infty}^{\infty} d_{j,k}\psi_{j,k}(t) \tag{3}$$

in which $f_s^j(t)$ is the profile signal projected to the scale space V_j, $f_s^j(t)$ is the detail signal projected to the wavelet space W_j, $c_{J,k}$ is the scale coefficient of discrete wavelet transform under J^{th} scale, $\phi_{J,k}(t)$ is the scale function of wavelet transform, $d_{j,k}$ is the wavelet coefficient under J^{th} scale, and $\psi_{j,k}(t)$ is the wavelet function.

The multiresolution analysis can be calculated through the filter banks. Assuming that h_0 and h_1 are respectively the impulse responses of low-pass and high-pass filters corresponding to the wavelet decomposition, g_0 and g_1 are respectively the impulse responses of low-pass and high-pass filters corresponding to the wavelet reconstruction, the scale coefficients and wavelet coefficients can be recursively calculated by the Mallat's pyramid algorithm as follows

$$\begin{cases} c_{j+1,k} = \sum_m h_0(m-2k)c_{j,m} \\ d_{j+1,k} = \sum_m h_1(m-2k)c_{j,m} \end{cases} \quad (4)$$

where k is the length of wavelet coefficients and it is related to the length of signal and the number of scale. The corresponding reconstructed algorithm of coefficients is expressed as

$$c_{j-1,m} = \sum_k c_{j,k} g_0(m-2k) + \sum_k d_{j,k} g_1(m-2k) \quad (5)$$

When the sampling frequency of a signal is higher than Nyquist frequency, the sample sequence $x(n)$ of signal $x(t)$ can be approximately considered as the scale coefficients $c_{0,k}$ decomposing on zero scale. If the sequence $x(n)$ is decomposed to scale $j=1$, the scale and wavelet coefficients are expressed as $c_{1,k}$ and $d_{1,k}$ respectively. So the original signal can be reconstructed by Mallat's pyramid algorithm as follows

$$x(n) = \sum_k c_{1,k} g_0^1(n-2k) + \sum_k d_{1,k} g_1^1(n-2k) \quad (6)$$

The sequence $x(n)$ can be decomposed continuously to the higher scale, and the original signal can be reconstructed by using Eqs. (5) and (6). The decomposition and reconstruction procedures can be implemented using the filter banks as shown in Fig. 1 ($j=3$).
where $H_0(Z)$ and $H_1(Z)$ are the low-pass and high-pass decomposition filter respectively, while $G_0(Z)$ and $G_1(Z)$ are the low-pass and high-pass reconstruction filter respectively.

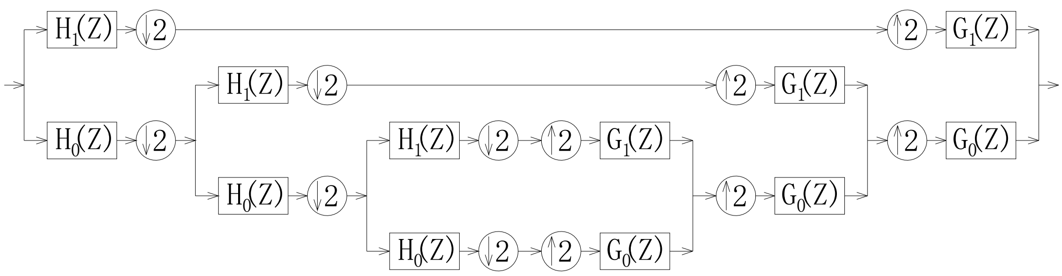

Fig. 1 Multiresolution analysis on scale *j*=3

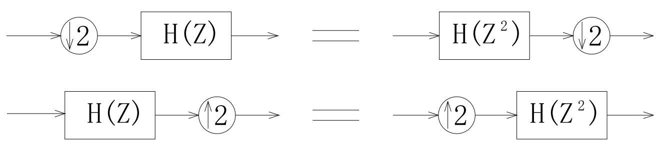

Fig. 2 The equivalent relation of position-swapping between up (down) sampler and filter

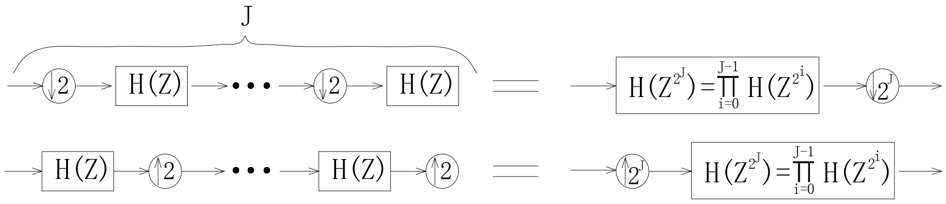

Fig. 3 The equivalent relation of position-swapping with J filter modules connected in series

In order to make the notation more compact, the equivalent relation of position-swapping between up(down) sampler and filter in the multi-rate filter banks is used in this study (Vaidyanathan 1990, Vetterli and Herley 1992) as shown in Fig. 2.

It can be seen from Fig. 2 that the cascade of a sub-sample by 2, followed by a filter $H(Z)$, is equivalent to the filter $H(Z^2)$ followed by the same sub-sample. Moreover, when such multiple filter modules are connected in series, the similar equivalent relation of position-swapping exists as presented in Fig. 3.

Thus, the multiresolution analysis structure as shown in Fig. 2 can be transformed into the equivalent structure as shown in Fig. 4.

In the general case of multiresolution analysis on scale J, the equivalent structure contains a filter banks with J+1 branch. For the decomposition of a signal, the low-pass branch involves in an undersampling by 2^J and the transform function of the equivalent filter is expressed as

$$H_0^J(Z) = H_0(Z)H_0(Z^2)\cdots H_0(Z^{2^{J-1}}) \tag{7}$$

The branch corresponding to the detail signal involves in a sampling by 2^J and the transform function of the equivalent filter can be calculated as

$$\begin{cases} H_1^j(Z) = H_1(Z) & j=1 \\ H_1^j(Z) = H_1(Z)H_1(Z^2)\cdots H_1(Z^{2^{j-1}}) & j=2,3,\cdots J \end{cases} \tag{8}$$

The same algorithm is suitable to the signal reconstruction. The corresponding transform function of the equivalent filter can be obtained by substituting reconstruction filter G with decomposition filter H .

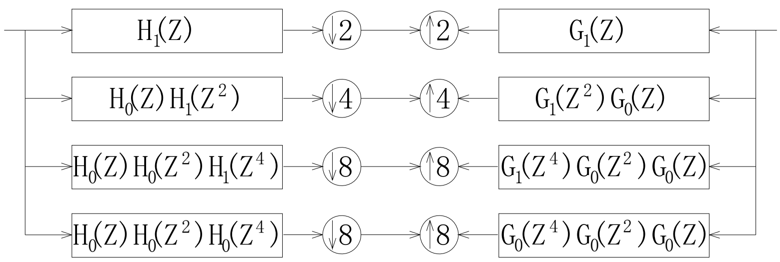

Fig. 4 Equivalent multiresolution analysis structure

If a discrete signal $x(n)$ is decomposed on scale J using multiresolution analysis where the scale and wavelet coefficients are $c_{J,k}$ and $d_{j,k}$ respectively, the original signal can then be reconstructed as follow

$$x(n)=\sum_{k} c_{J,k} g_0^J(n-2^J k)+\sum_{j=1}^{J}\sum_{k} d_{j,k} g_1^j(n-2^j k) \tag{9}$$

2.2 Wavelet scale coefficient reconstruction of time varying signal

A slow time-varying signal expressed as follows is considered

$$x(t)=\begin{cases} 1+0.5\times t & 0\le t<2 \\ 1 & 2\le t<4 \\ 0.5+0.5\times\cos(0.5\pi\times(t-4)) & 4\le t<8 \\ 0.5 & 8\le t\le 10 \end{cases} \tag{10}$$

The db3 wavelet is used to decompose the signal into scale $J=4$, and the signal is reconstructed using the scale and wavelet coefficients respectively. In such a way, the low frequency profile signal and high frequency detail signal can be obtained. The reconstructed low frequency components on different scales are shown in Fig. 5, and the reconstructed high frequency signal is shown in Fig. 6.

It can be observed from Fig. 5 and Fig. 6 that the energy of the signal mainly concentrates on the low frequency components. In other words, the energy on the high frequency part is very small, which only exists in the position of signal break. Therefore, for a slow time-varying signal, through muiltresolution analysis and ignoring the detail component by setting the wavelet coefficients as zero, the original signal can be approximately reconstructed only using scale coefficients. So Eq. (9) is simplified as

$$x(n)\approx\sum_{k} c_{J,k} g_0^J(n-2^J k) \tag{11}$$

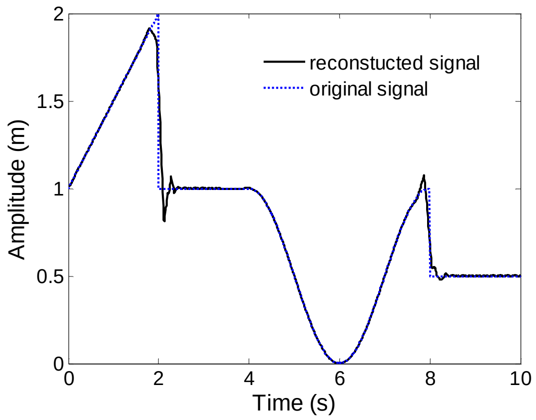

Fig. 5 Reconstructed signal with scale coefficients reflecting low frequency components

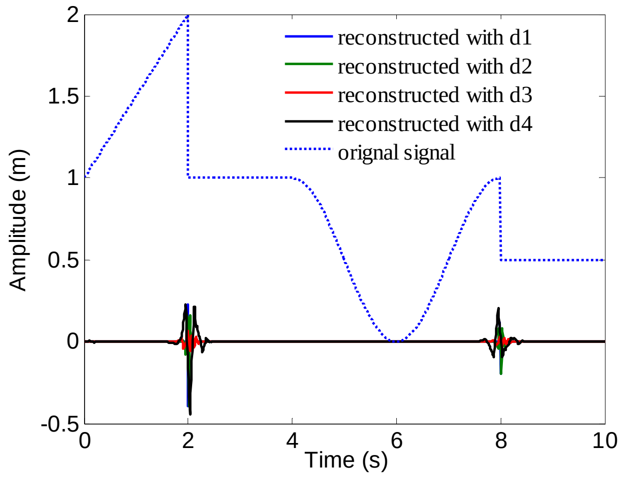

Fig. 6 Reconstructed signal with different wavelet coefficients ($\mathrm{d}_i, i = 1,2,3,4$ indicates the decomposed wavelet coefficients corresponding to scale *i*)

3. Physical parameter identification

For the physical parameter identification of time-varying systems, Tsatsanis and Giannakis (1993) expanded the time-varying parameter onto a finite set of wavelet basis sequences and transformed the time-varying problem into a time invariant model. They estimated the time-varying spectrum using their proposed method. In this study, the time-varying physical parameter will be similarly expanded at the multi-scale as the profile and detail signal using multiresolution analysis, and then the physical parameters are identified by solving linear equations using least-squares method. The detail algorithm is presented as follows.

3.1 Single-degree-of-freedom time-varying system

Considering a single-degree-of-freedom system with time-varying stiffness and damping, the equation of motion can be expressed as

$$m\ddot{x}(t) + c(t)\dot{x}(t) + k(t)x(t) = f(t) \tag{12}$$

The discrete format of Eq. (12) can be represented by

$$m\ddot{x}(n) + c(n)\dot{x}(n) + k(n)x(n) = f(n) \tag{13}$$

All stiffness at different time point constitutes a discrete serial signal, so it can be decomposed at different scales using multiresolution analysis. Assuming that the wavelet and scale coefficients are known, the original stiffness can be approximately reconstructed according to Eq. (11)

$$k(n) \approx \sum_i k_{J,i} g_0^J (n - 2^J i) \tag{14}$$

where $k_{J,i}$ are the decomposed scale coefficients.

The same algorithm can be used to deal with the damping coefficients, and the result is expressed as

$$c(n) \approx \sum_i c_{J,i} g_0^J (n - 2^J i) \tag{15}$$

Substituting Eqs. (14) and (15) into Eq. (13), one can obtain the transformed motion equation as follows

$$\sum_i g_0^J (n - 2^J i)\dot{x}(n)c_{J,i} + \sum_i g_0^J (n - 2^J i)x(n)k_{J,i} = f(n) - m\ddot{x}(n) \tag{16}$$

If the input load and structural response are measured in a discrete time point $n = 1 \sim N$, substituting into Eq. (16), one can obtain the following equation

$$G \times CK = R \tag{17}$$

where $[G] = [G(c) \quad G(k)]$,

$$[G(c)] = \begin{bmatrix} g_0^J (1-2^J)\dot{x}(1) & \cdots & g_0^J (1-2^J i)\dot{x}(1) \\ g_0^J (2-2^J)\dot{x}(2) & \cdots & g_0^J (2-2^J i)\dot{x}(2) \\ \vdots & \vdots & \vdots \\ g_0^J (N-2^J)\dot{x}(N) & \cdots & g_0^J (N-2^J i)\dot{x}(N) \end{bmatrix} \tag{18}$$

$$[G(k)] = \begin{bmatrix} g_0^J (1-2^J)x(1) & \cdots & g_0^J (1-2^J i)x(1) \\ g_0^J (2-2^J)x(2) & \cdots & g_0^J (2-2^J i)x(2) \\ \vdots & \vdots & \vdots \\ g_0^J (N-2^J)x(N) & \cdots & g_0^J (N-2^J i)x(N) \end{bmatrix} \tag{19}$$

$$[CK] = [c_{J,1} \cdots c_{J,i} \ k_{J,1} \cdots k_{J,i}]' \tag{20}$$

$$[R] = [f(1) - m\ddot{x}(1) \cdots f(n) - m\ddot{x}(n)]' \tag{21}$$

Now, the scale coefficients of $c_{J,i}$ and $k_{J,i}$ are the unknown time invariant variables, so the identification task of time-varying system is transformed into estimating time-invariant scale coefficients. It can be resolved using least-squares method as follow

$$[CK] = [G^T G]^{-1} G^T R \tag{22}$$

where superscription ^{T} and $^{-1}$ indicate the transpose and inverse of the matrix respectively.

Substituting the resolved results into Eqs. (14) and (15), the time-varying stiffness and damping of a structure can be identified.

3.2 Muiltiple- degree-of-freedom time-varying shear type structure

For a muiltiple-degree-of-freedom time-varying shear type structure, the physical parameter identification procedure is similar to that of a single-degree-of-freedom system. On the sake for convenience, we consider a two-story shear type frame structure with time-varying stiffness and damping coefficient as shown in Fig. 7. The motion equation of such a two-story shear type frame can be expressed by

$$\begin{cases} m_1\ddot{x}_1 + (c_1 + c_2)\dot{x}_1 - c_2\dot{x}_2 + (k_1 + k_2)x_1 - k_2x_2 = f_1 \\ m_2\ddot{x}_2 - c_2\dot{x}_1 + c_2\dot{x}_2 - k_2x_1 + k_2x_2 = f_2 \end{cases} \tag{23}$$

Considering all time-varying stiffness and damping as the unknown variables, one can transform Eqs. (23) into (24)

$$\begin{cases} \dot{x}_1c_1 + (\dot{x}_1 - \dot{x}_2)c_2 + x_1k_1 + (x_1 - x_2)k_2 = f_1 - m_1\ddot{x}_1 \\ (\dot{x}_1 - \dot{x}_2)c_2 + (x_2 - x_1)k_2 = f_2 - m_2\ddot{x}_2 \end{cases} \tag{24}$$

In Eq. (24), each equation is similar with Eq. (13). Again, decomposing and reconstructing every variable using above proposed procedure, one can obtain the following equation

$$G_M \times CK_M = R_M \tag{25}$$

where

$$G_M = \begin{bmatrix} G_1(c_1) & G_1(c_2) & K_1(k_1) & K_1(k_2) \\ G_2(c_1) & G_2(c_2) & K_2(k_1) & K_2(k_2) \end{bmatrix} \tag{26}$$

in which the subscription 1 of $G_1(c_2)$ indicates that it corresponds to the first equation in Eq. (24). c_2 indicates the corresponding unknown damping coefficient. Each matrix in G_M can be deduced using similar method with that of Eq. (18). Limited by length, only the matrices $G_1(c_2)$ and $K_1(k_2)$ are listed as follows

$$[G_1(c_2)] = \begin{bmatrix} g_0^J(1-2^J)(\dot{x}_1(1)-\dot{x}_2(1)) & \cdots & g_0^J(1-2^J i)(\dot{x}_1(1)-\dot{x}_2(1)) \\ g_0^J(2-2^J)(\dot{x}_1(2)-\dot{x}_2(2)) & \cdots & g_0^J(2-2^J i)(\dot{x}_1(1)-\dot{x}_2(1)) \\ \vdots & \vdots & \vdots \\ g_0^J(N-2^J)(\dot{x}_1(N)-\dot{x}_2(N)) & \cdots & g_0^J(N-2^J i)(\dot{x}_1(N)-\dot{x}_2(N)) \end{bmatrix} \tag{27}$$

$$[K_1(k_2)] = \begin{bmatrix} g_0^J(1-2^J)(x_1(1)-x_2(1)) & \cdots & g_0^J(1-2^J i)(x_1(1)-x_2(1)) \\ g_0^J(2-2^J)(x_1(2)-x_2(2)) & \cdots & g_0^J(2-2^J i)(x_1(1)-x_2(1)) \\ \vdots & \vdots & \vdots \\ g_0^J(N-2^J)(x_1(N)-x_2(N)) & \cdots & g_0^J(N-2^J i)(x_1(N)-x_2(N)) \end{bmatrix} \tag{28}$$

In Eq. (25), the matrix CK_M can be represented by

$$[CK_M] = [c_{J,1}^1 \cdots c_{J,i}^1 \; c_{J,1}^2 \cdots c_{J,i}^2 \; k_{J,1}^1 \cdots k_{J,i}^1 \; k_{J,1}^2 \cdots k_{J,i}^2]' \tag{29}$$

in which the superscript 2 of $c_{J,1}^2$ indicates that it corresponds to the unknown damping coefficient c_2.

The matrix R_M of Eq. (25) can be described by

$$[R_M] = \begin{cases} [f_1(1) - m_1\ddot{x}_1(1) \cdots f_1(N) - m_1\ddot{x}_1(n)]' \\ [f_2(1) - m_2\ddot{x}_2(1) \cdots f_2(N) - m_2\ddot{x}_2(n)]' \end{cases} \tag{30}$$

Now, the matrix CK_M can be also resolved using least-squares method and the time-varying physical parameter can then be identified.

If noise is present in the measured signal, the equations to be solved are often ill-posed. The Tikhonov regularization technique is implemented in the paper to reduce the solution error.

4. Numerical verification

A two-story shear type frame structure with time-varying stiffness and damping is simulated to validate the aforementioned method. The frame is shown in Fig. 7, and the equation of motion can be expressed by Eq. (23).

The masses of frame are assumed to be a constant value of $m_1 = m_2 = 2500$ kg. The 40 second El-Centro earthquake load is applied to the frame base, and corresponding dynamic responses are calculated by using the Fourth-order Runge-Kutta method. The sample frequency of 50 Hz is used in the dynamic response calculation. To take into account for noise effect, the simulated Gauss white noise is added to the calculated time-history responses.

In this study, two kinds of time-varying cases are simulated. Case one is both time-varying stiffness and time-varying damping coefficient. Case two is the time-varying stiffness but constant damping coefficient.

4.1 Case one with time-vary stiffness and damping

In this case, the changes in stiffness k_1 and damping c_1 with time are assumed to be mutational, while the changes in stiffness k_2 and damping c_2 with time are assumed to be linear. The unit of stiffness and damping is kN/m and kN.s/m respectively. The changes in assumed physical parameters are specified as follows

$$k_1 = \begin{cases} 250 & t \le 8s \\ 175 & t > 8s \end{cases}, \text{ and } k_2 = 200 - 3 \times t \tag{31}$$

$$c_1 = \begin{cases} 2.5 & t \le 15s \\ 3.75 & t > 15s \end{cases}, \text{ and } c_2 = \begin{cases} 2.5 & t \le 25s \\ 2.5 + 0.175 \times (t - 25) & t > 25s \end{cases} \tag{32}$$

As mentioned above, the displacement, velocity and acceleration responses of two lumped mass can be calculated by using the Fourth-order Runge-Kutta method. Fig. 8 illustrates such time-history responses of lumped mass m_1

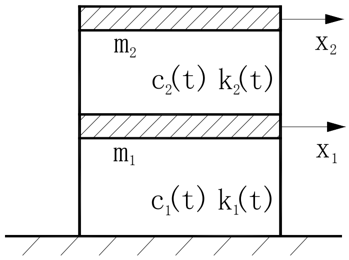

Fig. 7 Two stories time-varying shearing frame model

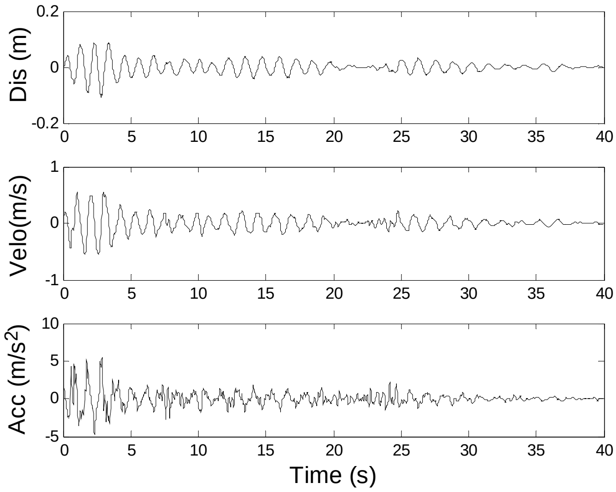

Fig. 8 Calculated responses of mass m_1 (Dis: displacement, Velo: velocity, Acc: acceleration)

To consider the influence of noise, 3% simulated Gauss white noise is added to the calculated response signals. By expanding the time-varying physical parameter to scale $J = 5$ using db3 wavelet and reconstructing the parameter using the decomposition scale coefficient, the physical parameters can be identified according. The Tikhonov regularization method is used to deal with the ill-posed equation. Figs. 9 to 12 illustrate the identified time-varying stiffness and damping coefficients with and without noise.

It can be seen from Figs. 9-12 that the identified time-varying stiffness k_1 and k_2 in case of without noise are in good agreement with the theoretical values (Eq. (31)). When noise is present in the signal, the identified error is slightly larger than that of without noise, but it still can effectively track the change of time-varying parameters. The identified result of stiffness k_2 that varies linearly is better than the identified result of stiffness k_1 that varies abruptly. With regards to the identification of time-varying damping, it can be observed that the identified error of damping coefficients c_1 and c_2 with noise is larger than the stiffness identification. It means that the identification of time-varying damping parameter is more sensitive to noise. In addition, the identification error is larger at the end of time due to the end effect of wavelet transform.

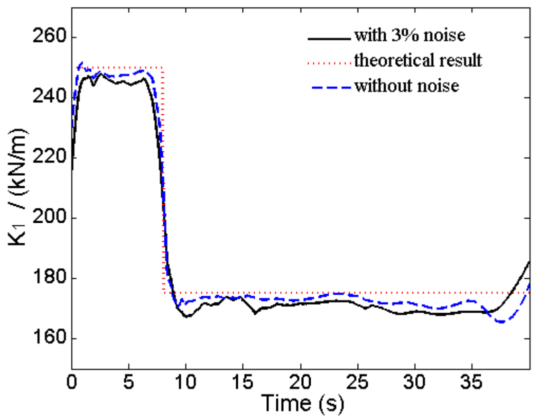

Fig. 9 Identified time-varying stiffness k_1

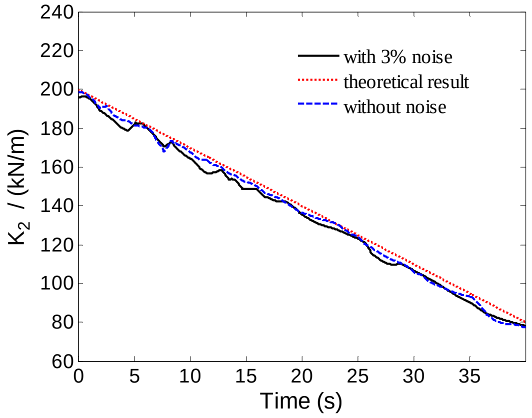

Fig. 10 Identified time-varying stiffness k_2

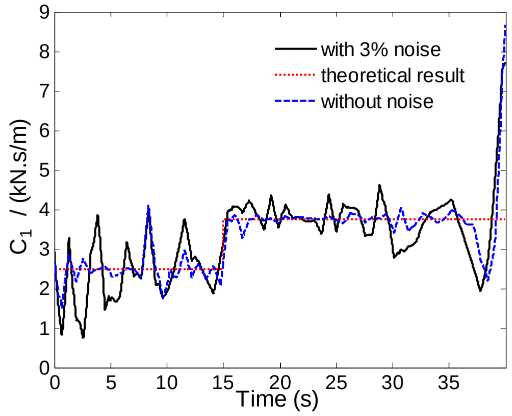

Fig. 11 Identified time-varying damping c_1

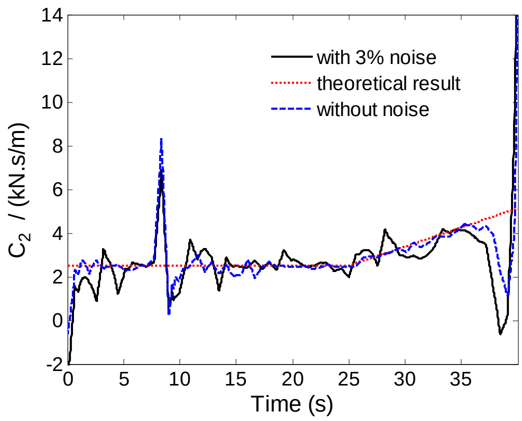

Fig. 12 Identified time-varying damping c_2

4.2 Case two with time-vary stiffness but constant damping

In this case, the stiffness parameters are time-varying but the damping parameters remain unchanged. The constant damping coefficients are assumed to be $c_1 = c_2 = 1$ kN.s/m. Let the stiffness k_1 vary quadratically and k_2 vary periodically according to the following equations

$$k_1 = 500 - 0.1 \times t^2 \text{, and } k_2 = 200 + 40 \sin(\frac{2}{35} \pi \times t) \tag{33}$$

As the same as previous example, the dynamic responses of frame structure excited by the seismic input can be obtained through the numerical calculation. The 5% Gauss white noise is added to the calculated responses to simulate the real measurement noised signal. By expanding the time-varying physical parameter to scale using db3 wavelet and reconstructing the parameter using the decomposition scale coefficient, the physical parameters can be identified accordingly. The Tikhonov regularization method is used to deal with the ill-posed equation. Figs. 13 to 14 illustrate the identified time-varying stiffness with and without noise.

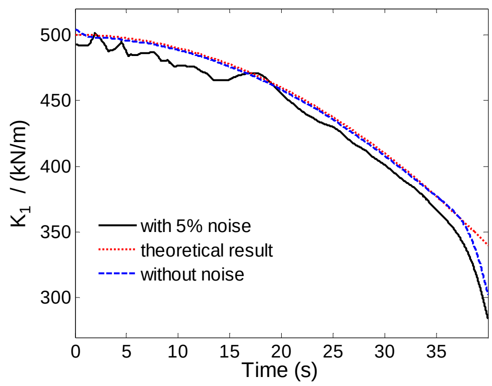

Fig. 13 Identified time-varying stiffness k_1

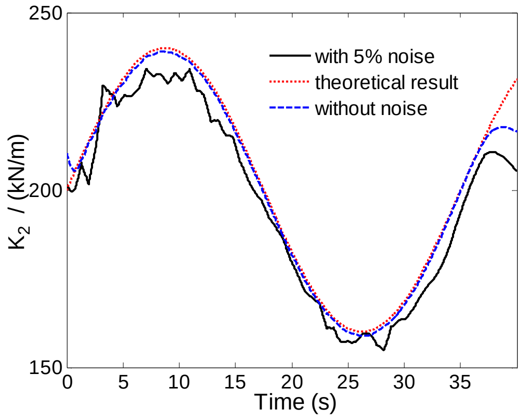

Fig. 14 Identified time-varying stiffness k_2

It is again demonstrated that the proposed method can effectively identify the time-varying physical parameters of a structure. When 5% noise is present in the response signals, the identified result is still satisfactory. It can been found that the identification error increases at both ends. It is clearly due to the end effect of wavelet transform.

5. Conclusions

A physical parameter identification method for time-varying shear type structures is proposed in the paper based on discrete wavelet transform. The time-varying physical structural parameters are first expanded on a multi-scale as the profile and detail signals using wavelet multiresolution analysis. By ignoring the detail signal, the time-varying parameter is reconstructed only using the profile signal. The identification of time-varying problem is therefore transformed to the identification of equivalent time-invariant problem. The Tikhonov regularization method is implemented to reduce the influence of ill-posed problem.

The results of a numerical verification indicate that the proposed algorithm is capable of identifying the time-varying stiffness and damping coefficient. It is demonstrated that when structural stiffness and damping vary together with time, the time-varying physical parameters can be effectively identified even if noise is present. The identification result of time-varying stiffness is better than that of time-varying damping coefficients, which means that the time-varying damping identification is more sensitive to noise. When the structural stiffness varies smoothly, for example linear or periodic variation, the identification precision is higher than that of stiffness that varies abruptly. In addition, the proposed wavelet-based method is adaptive so that there is no need to assume the variation form of time-varying physical parameters beforehand.

Acknowledgments

This study is supported by the National Natural Science Foundation of China (Grant No. 51078357) and the National Outstanding Young Scholar Fund of China (Grant No. 50925828)

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