

An evolutionary fuzzy modelling approach and comparison of different methods for shear strength prediction of high-strength concrete beams without stirrups

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Abstract. In this paper, an Adaptive nerou-based inference system (ANFIS) is being used for the prediction of shear strength of high strength concrete (HSC) beams without stirrups. The input parameters comprise of tensile reinforcement ratio, concrete compressive strength and shear span to depth ratio. Additionally, 122 experimental datasets were extracted from the literature review on the HSC beams with some comparable cross sectional dimensions and loading conditions. A comparative analysis has been carried out on the predicted shear strength of HSC beams without stirrups via the ANFIS method with those from the CEB-FIP Model Code (1990), AASHTO LRFD 1994 and CSA A23.3 – 94 codes of design. The shear strength prediction with ANFIS is discovered to be superior to CEB-FIP Model Code (1990), AASHTO LRFD 1994 and CSA A23.3 – 94. The predictions obtained from the ANFIS are harmonious with the test results not accounting for the shear span to depth ratio, tensile reinforcement ratio and concrete compressive strength; the data of the average, variance, correlation coefficient and coefficient of variation (CV) of the ratio between the shear strength predicted using the ANFIS method and the real shear strength are 0.995, 0.014, 0.969 and 11.97%, respectively. Taking a look at the CV index, the shear strength prediction shows better in nonlinear iterations such as the ANFIS for shear strength prediction of HSC beams without stirrups.

Keywords: ANFIS; shear strength; HSC beams; tensile reinforcement ratio; shear span to depth ratio; concrete compressive strength

1. Introduction

The latest progress taking place in mechanical properties and economical evaluation explains the use of HSC in construction endeavors. However, HSC demonstrates its brittle nature; in

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comparison with normal strength concrete (NSC) that necessitates some concerns to be raised over HSC applications.

The shear strength prediction of HSC beams does not have an easy path as it is complex. Despite the multifarious studies and investigations on the shear strength of HSC beams (Khan *et al.* 2000, Mohammadhassani *et al.* 2011, Zeidan *et al.* 2011, Voo *et al.* 2011), it is still in need of further examination and investigation due to the HSC's non-homogeneous, nonlinear and non-isotropic nature under a combined shear and bending state of stresses. In reinforced concrete (RC) beams; the shear resistance is a combination of various mechanisms equipped with shear reinforcement, aggregate interlocking (V_a), dowel action of the longitudinal reinforcement (V_d) and the intact un-cracked concrete in the compression zone (V_{cc}). Fig. 1 shows that in the case of RC beams without stirrups, the beams fail as the diagonal cracks seem to have built up in the web of beams.

The major parameters that leave an impact on the shear strength of RC beams without stirrups are shear span to effective depth ratio (a/d), tensile reinforcement ratio (ρ), strength of concrete, beam size, support conditions, *etc.* a/d is the most significant parameter that affects the shear strength of RC beams. Berg (1962) and Ahmad and Lue (1987) have found that the shear strength of RC beams increases with the decreased a/d ratio. Kani (1964) concludes that as far as the beams with $a/d < 2.50$ (deep beams) are concerned; the shear strength is affected with the a/d variation because of the load transferring mechanism from the load point to the support point via the compression strut trajectory. Fig. 2 shows the RC beam behavior with the variation of (a/d).

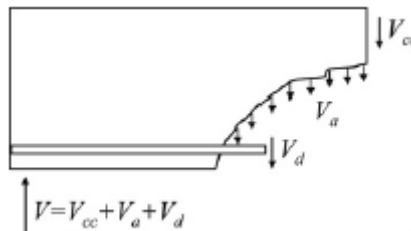


Fig. 1 Shear resistance components of RC beams without stirrups

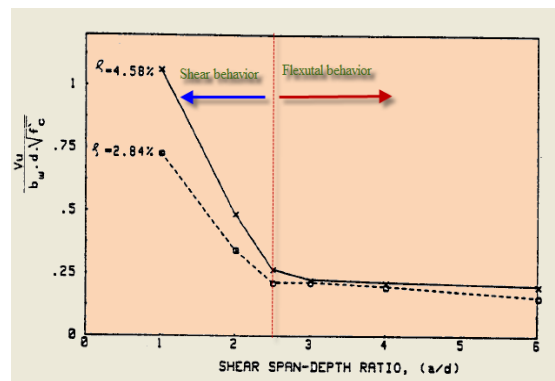


Fig. 2 Behavior prediction of RC beams with variation of shear span-depth ratio (Wafa and AShour (1994))

As shown in Fig. 2 and based on the Maggregor (1997) findings, RC beam behavior is classified with its attributed a/d ratio. With close reference to the ACI code approach; deep beams are those with $a/d < 2.5$ while normal beams contain a/d more than 2.50.

Fig. 1 confirms that the tensile reinforcement ratio (ρ) affects the shear strength of RC beams without stirrups. With shear deformation occurring in the splintered concrete beam, the tension reinforcement is vulnerable to a degree of shear stress (Choi *et al.* 2009). Lee and Kim (2008) demonstrate that the shear strength of RC beams comes in a range, according to the diversity of tensile reinforcement ratio. They also conclude that the shear strength of normal beams follows the strength of concrete being used.

There are many national design codes which foretell the shear strength of HSC beams. However, the research done over the years has not made uniform these codes in shear strength prediction and instead, has under or over-estimated HSC's shear strength. Yang *et al.* (2007) conclude that there is an absence of confirmed rational procedure that can predict the shear strength of RC beams. They have further demonstrated that most code provisions and approaches for shear design of deep beam are generally unable to capture the effect of varying parameters on shear strength prediction. Ahmed *et al.* (1986) have concluded, for instance, that the ACI code could be un-conservative in shear strength prediction for under-reinforced HSC beams. However, for beams with values of ρ exceeding 1%, the experimental shear strength was found to have been conforming better with the shear design criteria of the ACI code (2008).

With this in mind, the design codes are not fully able to make some kind of predictions on the actual behaviour and the precise shear strength of HSC beams.

The high price of process such as the casting, curing and testing and the intricate behavior of HSC beams motivate the pursuit for easy, precise and effective method in the shear strength prediction of these elements. Thus, a clear understanding of the shear behavior of HSC beams with continuous new technologies in software and computer science is very much required.

To date, fuzzy inference systems (FIS) and neuro-fuzzy / fuzzy-neural systems have both been used effectively for modelling in many engineering applications like concerning the stability and serviceability of structures (Hakim *et al.* 2011, Mohammadhassani *et al.* 2013a, b, c) analysis and vibration control (Li *et al.* 2013); Airport Pavement Structural Analysis (Gopalakrishnan and Ceylan 2009); Prediction of Short-Term Operational Water Levels (Shiri *et al.* 2011), Crack Performance Prediction (Sun and Qiu 2011); Prediction of Metallic Water Pipelines Clair and Sinha (2011) and Forecast Modeling of Monthly Runoff (Ren *et al.* 2011).

Mohammadhassani *et al.* (2013) revealed that the ANFIS's results are highly accurate, precise and satisfactory in comparison with other applied methods in deep beam deflection prediction.

Fuzzy logic systems are very much compatible with the concept of modelling the ill-defined or very complex relationship between variables in environments which at the same time, constitutes an alternative that is more precise.

The qualitative variables and mathematical relationships in this technique bring about a more spot-on decision-making process. Fuzzy logic, a self-learning technique, was pioneered by Zadeh (1965), which provides a mathematical framework based on expert knowledge for the conversion of vagueness evaluation variables into an automatic evaluation strategy.

Fuzzy-neural systems are a component of an intelligent system that integrate some prominent characteristics of artificial neural networks (ANNs) and FIS for the construction of power computing tools. ANFIS adopts the ANN theory for the determination of the properties (fuzzy membership functions and fuzzy rules) of data samples in the learning of a fuzzy inference system.

The ANFIS is an improved version from the Takagi-Sugeno fuzzy model (Takagi 1985), where a FIS is implemented through a feed-forward network and a hybrid learning method that include the back propagation theory from ANNs, recursive least square (RLS) method and clustering techniques which are used in integration to build up the FIS appropriately according to data. In other words, the ANFIS marries both the fuzzy logic and ANNs, by making use of the ANNs' mathematical properties in tuning rule based on the FIS that resembles humans' information-processing method through their brains. ANFIS has shown a very promising record in modeling nonlinear systems where learning features of the data set and adjusting the system characteristics accordingly to a given error criterion (Jang 1993) are two of its abilities. Like the ANN, the ANFIS does not have any problem mapping unseen inputs to their outputs by learning the rules derived from earlier data.

The determined values of physical parameters (input) and the real values of shear strength prediction of HSC beams without stirrups (output) have played their part in training the fuzzy neural network.

In this study; an alternative approach using ANFIS is employed to predict the shear strength of HSC beams without stirrups, and then the results are placed in comparison with the design codes such as CEB-FIP Model Code (1990), AASHTO LRFD 1994, CSA A23.3 – 94 and ANFIS method.

1.1 Research significant

The shear strength of RC beams without stirrups has been in the minds and interests of various researchers (Wafa and Ashour 1994, Choi *et al.* 2009, Bukhari and Ahmad 2007, Sudheer *et al.* 2010). However, there has been proof that a good understanding of shear behavior of such beams is a rarity, but then again, this is explained by the complex nature of the affecting parameters governing the shear strength of concrete beams without stirrups. In this study, the ANFIS seeks to predict the ultimate shear strength of HSC beams without stirrups. Also a comparative prediction of shear strength of 122 HSC beams without web reinforcement is carried out using the ANFIS method and CEB-FIP Model Code (1990), AASHTO LRFD 1994 and **CSA A23.3 – 94**. The outcomes of this paper should be able to pave the way for researchers to predict the shear strength of HSC beams without stirrups more flawlessly, and the results will then be very useful for upcoming research and the amendments which are to be done in existing shear design codes.

2. Material and method

2.1 Data collection and experimental study

The used data choose some experimental studies by (Bukhari and Ahmad 2007, Ali 2001, Yaqub 2002, Elahi 2003) on HSC beams without web reinforcement. Each specimen was tested as a simply-supported RC beam under three-point loading (Fig. 3). A vertical load was imposed on the failure by a hydraulic jack in a load frame.

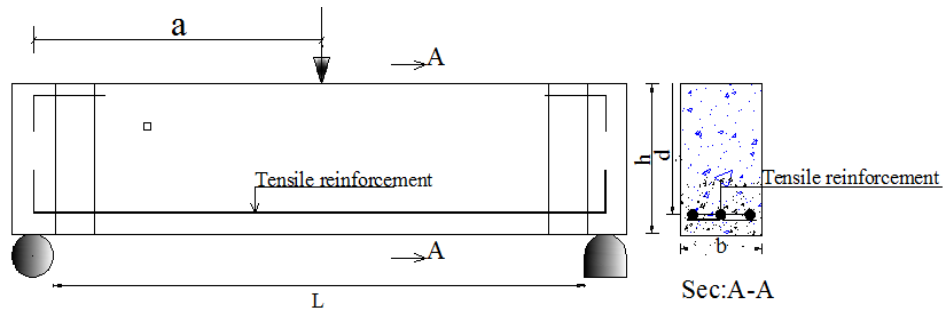


Fig. 3 Beam cross-section and reinforcement details

2.2 Numerical study

2.2.1 Shear strength database

The complete database for HSC beams without stirrups, as established in the literature review, is used for developing the ANFIS model. These data are divided into two sets: a training set containing 80 percent random data and testing set comprising 20 percent random data.

The input parameters used are concrete compressive strength (f_{cu}), shear span to depth ratio ($\frac{a}{d}$) and tensile reinforcement ratio (ρ) and only one neuron in the output layer as shear strength of beams (V). In the database, f_{cu} falls in the range from 6.47 to 8.59, ρ is between 0.0035 and 0.0194 and $\frac{a}{d}$ ranges between 1 and 6.

2.2.2 Fuzzy expert system

Human logic will not find it too troubling to process uncertainties and vague concepts in appropriate situations. Fuzzy logic, similarly, allows the modelling of uncertainties and imitates the human brain's thinking, reasoning and perception (Abraham 2005). Based on the Boolean logic, two concepts either 'True' or 'False', by 1 and 0 respectively, are applied, so a proposition can only be true or false. Fuzzy logic opens doors for intermediate values between these two values where the classical theory of the binary membership in a set expands to incorporate memberships between 0 and 1. This allows each proposition to be either True or False to a certain degree. With X as the space of objects and x as an element of X , a classical set $A, A \subseteq X$, is defined as a collection of elements $x \in X$, where x can either belong or not belong to set A . In other words, set A is described in Eq. (1)

$$A = \{x | x \in X\} \quad (1)$$

where as, a fuzzy set A in X is defined by Eq. (2)

$$A = \{(x, \mu_A(x)) | x \in X\} \quad (2)$$

where $\mu_A(x)$ is the membership function for the fuzzy set A . Here, A is a linguistic term (label) that is determined by the fuzzy set. The membership function maps each element of x to a membership

grade between zero and one ($\mu_A(x) \in [0,1]$). For example, this set can present x as ‘Medium’, which is a linguistic term used to describe by a fuzzy set with soft boundaries. Fig. 4 shows two sets, one based on the Boolean logic and the other on the fuzzy logic.

2.2.3 Fuzzy Inference System (FIS) (Mohammadhassani 2013)

Fuzzy systems provide the means of speaking on behalf of the expert knowledge of the human about the process in light of the fuzzy (IF–THEN) rules, denoting as the basic unit for capturing knowledge in a fuzzy system. In the same vein with a conventional rule in artificial intelligence, a fuzzy rule carries with it two components: an ‘IF’ part and a ‘THEN’ part which are also labelled familiarly as antecedent and consequent, respectively. The main structure of the fuzzy rule is shown in Eq. (3)

$$IF \text{ <antecedent> } THEN \text{ <consequent>} \quad (3)$$

The antecedent of a fuzzy rule can conditionally be satisfied up to a certain extent. Similar to the typical rules, the antecedent of a fuzzy rule may bring together multiple simple conditions into a complex string with the use of AND, OR and NOT logic operators. The consequence of a fuzzy rule can be categorized further into:

- Fuzzy consequent (Eq. (4)) where C is a fuzzy set.
- Functional consequent (Eq. (5)) where p, q and r are constant.

$$IF \text{ } x \text{ is } A \text{ and } y \text{ is } B \text{ THEN } f \text{ is } C \quad (4)$$

$$IF \text{ } x \text{ is } A \text{ and } y \text{ is } B \text{ THEN } f = px + qy + r \quad (5)$$

Basically, FIS manipulates the experiences gained by an expert into the system design which are composed of 4 blocks (Fig. 5). A FIS has a ‘fuzzifier’ that transforms the ‘crisp’ inputs into fuzzy inputs by membership functions that signify fuzzy sets of input vectors. It also contains a knowledge-base that considers the information provided by the expert with regards to linguistic fuzzy rules. An inference-system (Engine) uses them together through reasoning and a ‘defuzzifier’ that transforms the fuzzy results of the inference into a crisp output assisted by the ‘defuzzification’ method (Herrera and Lozano 2003).

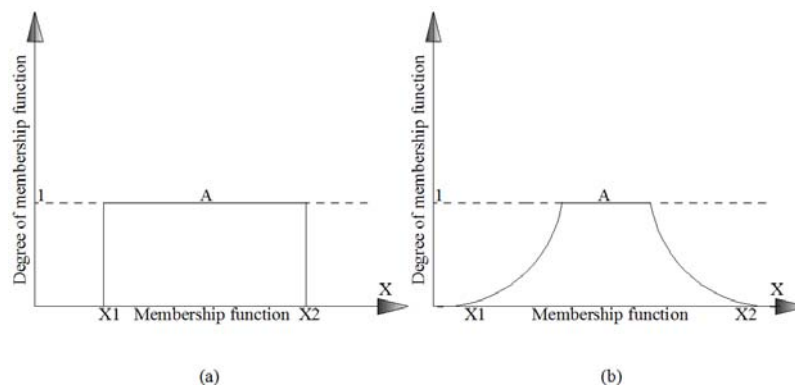


Fig. 4 An example of: (a) Classical Boolean set, and (b) Fuzzy Logic set

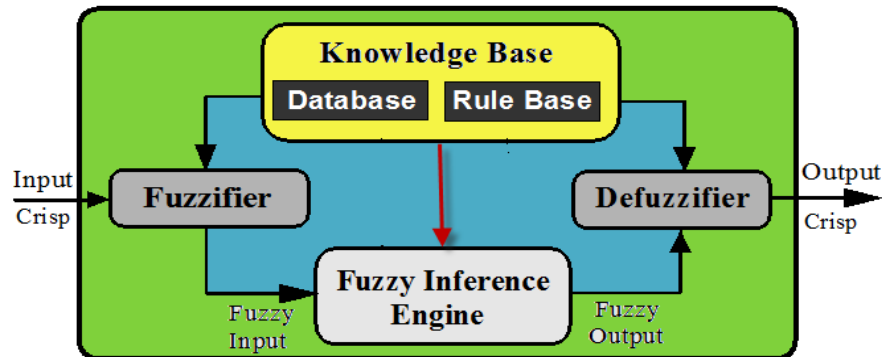


Fig. 5 A flow diagram of a fuzzy inference system (FIS) (Mohammadhassni 2013b)

The knowledge-base has two components: a data-base, which is the membership functions of the fuzzy sets used in the context of fuzzy rules, and a rule-base boasting off a compiled set of linguistic rules that are combined by a specific operator. The generic structure of a FIS is shown in Fig. 5. The two common types of the FIS are dissimilar, according to the differences between the specifications of the consequent part of fuzzy rules (Eqs. (4) and (5)). The first fuzzy system adopts the inference method proposed by Mamdani who establishes that the rule consequent is defined by fuzzy sets and has the structure of Eq. (5) (Mamdani and Assilian 1975).

The second fuzzy system proposed by Takagi, Sugeno and Kang (TSK) contains an inference engine where the fuzzy rule's conclusion is made up of a weighted linear combination of the crisp inputs rather than a fuzzy set (Takagi and Sugeno 1985). The TSK system contains a structure shown in Eq. (5). The TSK models are seen to be suitable for resembling large non-linear systems.

The knowledge-base containing the database and rule-base of a FIS can be gathered from the knowledge of an expert. He will normally pick out the membership functions and rules. In this way, fuzzy models help extract expert knowledge at a level deemed appropriate. Fuzzy systems can also be constructed from the data and the problem of knowledge acquisition can then be alleviated. Various techniques have been tried and tested for the data analysis with the best possible accuracy. There are two approaches commonplace to construct the FIS using available data. The first approach is where the fuzzy system rules are often designated a priori and the parameters of the membership functions are tailored during the learning process from input to output data through an evolutionary algorithm (e.g., genetic algorithm). In the second approach, the fuzzy system can be produced with the hybrid neural nets. The neural net defines the membership functions' shape; this is what is termed as the adaptive network-based fuzzy inference system (Jang 1993).

2.2.4 ANFIS model

The fuzzy neural network was the brainchild of Zadeh (1965) as a mathematical framework that is used to deal with vagueness variables and problems. The fuzzy inference system is the real process in which a given input is mapped to the output using fuzzy logic criteria.

Fuzzy logic works well to develop the systems used by experts, which models the human brain by means of capturing human knowledge. A fuzzy logic set is possibly implied by a membership function that points to the degree of membership within the set and maps the elements of universe on the numerical values during the interval $[0,1]$. (Mashrei *et al.* 2010)

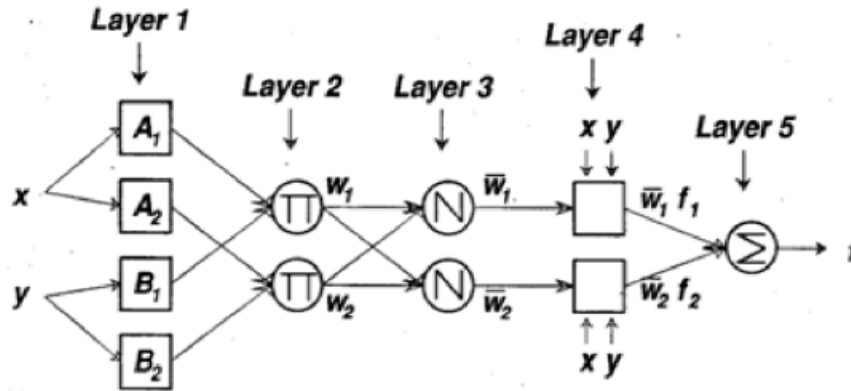


Fig. 6 The common architecture of ANFIS (Mammadhassani 2013)

ANFIS is a technology combined with the FIS with back propagation algorithms. This boasts off neural network and neuro-fuzzy systems advantages while it manipulates the learning capability of the NNs and the ability to integrate easily which serves as the reason for the substantial shortening of the learning process. The neuro-fuzzy systems are based on linguistic rules. The ANFIS as the first –order Sugeno fuzzy model is used to adapt both the linear and nonlinear parameters of an FIS. Fig. 6 shows the common architecture of the ANFIS model with two inputs of x and y and one output of f for the first-order sugeno fuzzy model.

As can be seen; the ANFIS has five layers including, the fuzzify layer, product layer, normalized layer, defuzzify layer, and total output layer. With the assumption of only two membership functions for each of the input data x and y , the general form of a first-order TSK type of fuzzy if–then rule has been provided by Eq. (6). Here, we re-write the rule I of the ANFIS as

Let us assume that- two inputs X and Y and one output Z

$$\text{Rule } i: \quad \text{If } x \text{ is } A_i \text{ and } y \text{ is } B_i, \quad \text{then} \quad \tilde{f}_i = p_i x + q_i y + r_i, \quad i=1,2,\dots,n \quad (6)$$

In which; n is the number of rule and p_i , q_i and r_i represent the parameters ascertained during the training process.

Layer one shows the first stage of the loading process; and where the membership functions (u) of the linguistic labels A_i and y is B_i are assessed as demonstrated in Eqs. (7) and (8)

$$O_i^1 = u_{A_i}(x), \quad i = 1,2,\dots,n \quad (7)$$

$$O_i^1 = u_{B_i}(y), \quad i = 1,2,\dots,n \quad (8)$$

Here, i is the membership grade of a fuzzy set and it makes specific the degree to which the given input x or y meets the quantifies. Usually, the membership function for a fuzzy set can come in any parameterized membership function, such as the triangle, trapezoidal, Gaussian, or generalized Bell function. Parameters in this layer are labelled the Antecedence Parameters.

In layer two; which is the product layer, the previously calculated membership degrees of linguistic variables are multiplied as expressed in Eq. (9). Each node output speaks for the firing strength of a rule.

$$O_i^2 = w_i = u_{Ai}(x)u_{Bi}(y), \quad i = 1, 2, \dots, n \quad (9)$$

Layer three is normalized layer in which for the normalization process, it calculates the ratio of the i th rule's firing strength to the sum of all rules' firing strengths

$$O_i^3 = \overline{w_i} = \frac{w_i}{\sum_{i=1}^n w_i} \quad i = 1, 2, \dots, n \quad (10)$$

In layer four; every node i stands as an adaptive node with a node function. The relationship for these nodes is described as Eq. (11)

$$O_i^4 = \overline{w_i} f_i = \overline{w_i} (p_i x + q_i y + r_i) \quad i = 1, 2, \dots, n \quad (11)$$

This layer is the defuzzification layer. Their outputs in this layer are dependent on the parameter(s) which relate to the adaptive nodes, and the learning rule clarifies how these parameters are altered to diminish the measure of the prescribed error (Jang 1993). Finally the fifth layer computes the overall output as the summation of all incoming signals

$$O_i^5 = \sum_{i=1}^n \overline{w_i} f_i \quad i = 1, 2, \dots, n \quad (12)$$

2.2.5 System modeling and evaluation (Mohammadhassani 2013)

System modeling changes the parameters of an adaptive intelligent system to suit unidentified actual engineering system transfer function. Fig. 7 shows a schematic modeling problem's system with the use of an adaptive intelligent system. As shown in this figure, the parameters of the estimated intelligent system are set with the proper learning methods to make sure that there is an accurate estimation of the actual system. In other words, the performance function, typically the Mean Squared Error (MSE) between the intelligent system's output and the actual response is minimized.

The MSE is used for monitoring the network performance. The objective of the function in the system's modeling problems is expressed below

$$MSE = \frac{1}{L} \sum_{k=1}^L (\hat{y}(k) - y(k))^2 \quad (13)$$

where $y(k)$ is the noisy output of the actual system (measured or observed output), $\hat{y}(k)$ is the adaptive intelligent system output and L is the number of instances. Some cases are noise-free where $y(k)$ is equal to $d(k)$ which is the output wanted. When noise is present, $\hat{y}(k)$ becomes the estimation of the desired output or semi desired output.

The MSE and Correlation Coefficient / Pearson Coefficient (R) values are used in this research to evaluate the methods being compared. The MSE is a risk function which corresponds to the anticipated value of the squared error loss. The larger the MSE, the more distant the estimation is from the true data points.

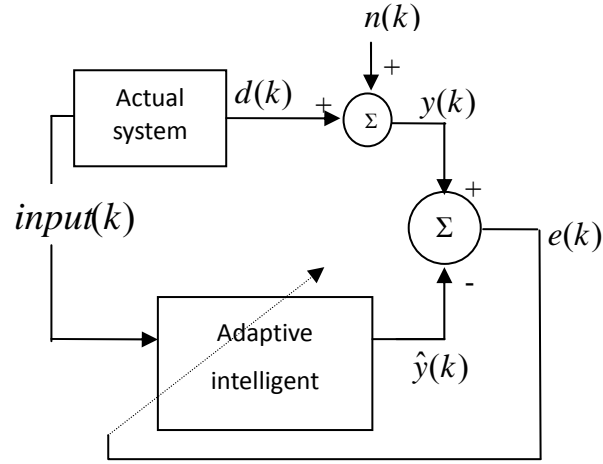


Fig. 7 System modeling using adaptive intelligent system

R^2 is the degree of success in reducing the standard deviation (SD) and very widely used in the sciences as a measure of the strength of linear dependence between two variables. Eq. (14) presents the R value as follows.

$$R^2 = 1 - \frac{\sum_{k=1}^L (y(k) - \hat{y}(k))^2}{\sum_{k=1}^L (y(k) - y_{ave})^2} \quad (14)$$

where $\hat{y}(k)$ is the output predicted by ANN, $y(k)$ is the actual (observed) output, y_{ave} is the averaged actual output and L is the total number of training/testing instances.

2.2.6 Training and testing of neural networks

Training implies presenting the network with the experimental data and learning its weights, or modifying it if deemed necessary, so that it predicts the target correctly. However, training the network with success necessitates the presentation of many choices and training experiences.

The master unit of the network is a complex network of neurons that acts in parallel and works as a numerical processing unit. The consequence of the connection between neurons is referred to as the weight of the internal connection. In the generation process, the network gets random weight amount to find optimum correlation between the experimental data.

3. Results and discussion

3.1 ANFIS architecture for shear strength prediction

Firstly, Using the MATLAB version 7.11.0, inputs and the target in the database were normalized in reference to Eq. (15) below for implementing the ANFIS model. The normalized data fall in the interval $[-1, +1]$. ANFIS has better efficiency with the original data normalization.

$$(p_i)_n = \frac{2(p_i - (p_{\min}))}{(p)_{\max} - (p)_{\min}} - 1 \quad (15)$$

which; (p_i) is normalized value of data set.

$(p)_{\min}$ is minimum value of the parameter under normalization and

$(p)_{\max}$ is maximum value of the parameter under normalization .

Eq. (16) is used for obtaining the outputs with the same units, similar to the original databases after the training and input simulation.

$$p'_i = \frac{[(p_i)_n + 1][(p)_{\max} - (p_{\min})]}{2} + (p_{\min}) \quad (16)$$

in where p'_i shows the original value of the data set.

With the application of this process; the ANFIS model was built and trained with a 3-input, 790 nodes, 392 linear parameters, 588 of nonlinear parameters and 98 rules as shown in Fig. 8.

The evaluations of both the processes of training and testing data are presented in Table 1, carrying the MSE and R values using MATLAB in the range of used datasets.

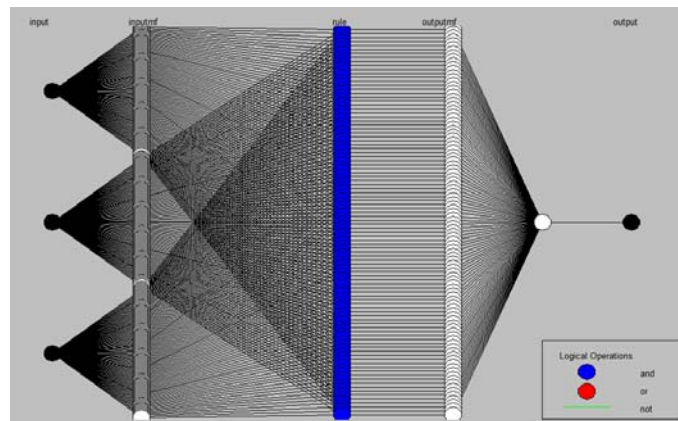


Fig. 8 architecture of ANFIS model

Table 1 MSE and R values from ANFIS in training and testing of datasets

Methods	Training Set			Testing set		
	Instances	MSE	R	Instances	MSE	R
ANFIS	97	0.1818	0.9066	25	0.1730	0.9321

As noted, the R values from the ANFIS for test data is 0.9321 which offers a degree of fascination to a scientist due to value close to one, which implies that there is a very high confidence level involved.

3.2 Shear strength prediction using the Canadian Standards for the Design of Concrete Structure (CSA A23.3 – 94)

The CSA Code applied to concrete of compressive strength up to 80MPa is based on the Modified Compression Field Theory (MCFT) for shear and torsion design in flexural regions. Concrete contribution to shear is supplied in Eq. (17).

$$V_{cg} = 1.3\lambda\phi_c\beta\sqrt{f'_c}b_wd_v \quad (\text{N.mm}) \quad (17)$$

where f'_c = 28 days Cylindrical compressive strength of concrete,

d = Effective beam depth and

V_{cg} = shear strength provided by concrete

ϕ_c = Concrete resistance factor

The predicted values of shear strength within the used input datasets that apply the CSA code approach are presented in Appendix 1.

3.3 Shear strength prediction using AASHTO LRFD Bridge design specifications (1994)

Based on the AASHTO LRFD code approach, the shear strength of the HSC beams without stirrups is calculated using following equation

AASHTO LRFD suggests the MCFT as the design method for shear in RC members. The concrete contribution to shear strength is

$$V_c = \frac{\beta'}{12} \sqrt{f'_c} b_v d_v \quad (\text{N.mm}) \quad (18)$$

in which

d_v is the effective shear depth.

also $\beta \text{ AASHTO} = 12 \beta \text{ CSA}$ (19)

The predicted values of shear strength in the range of input datasets apply the AASHTO LRFD code approach is presented in Appendix 1.

3.4 Shear strength prediction using European CEB-FIP Model Code (1990)

CEB-FIP Model Code (1990) uses equation 20 for Shear design of RC members.

$$V_u = [t_R K(1.2 + 40\rho) + 0.9\rho_v f_{yv}] b_w d \quad \text{SI units} \quad (20)$$

where

$$t_R = 0.0525(f'_c)^{\frac{2}{3}} \quad (21)$$

$$K = 1.6 - d > 1.0 \text{ (d in m);} \quad (22)$$

$$\rho = A_s/bwd < 0.02 \quad (23)$$

The predicted values of shear strength in the range of input datasets apply the CEB-FIP Model Code (1990) approach are presented in Appendix 1.

3.5 Comparison of different methods in the shear strength prediction of HSC beams without stirrups

The ratio of experimental to predicted values of the shear strength of HSC beams without stirrups using the CEB-FIP Model Code (1990), AASHTO LRFD Bridge Design Specifications (1994), Canadian Standards for the Design of Concrete Structure (CSA A23.3 – 94) and ANFIS method are presented in Appendix 1.

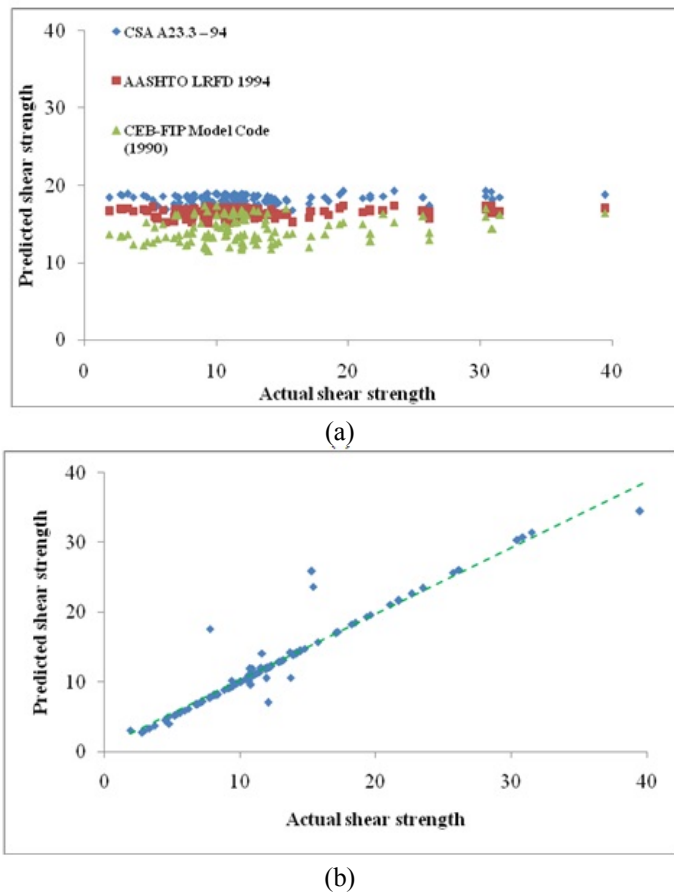
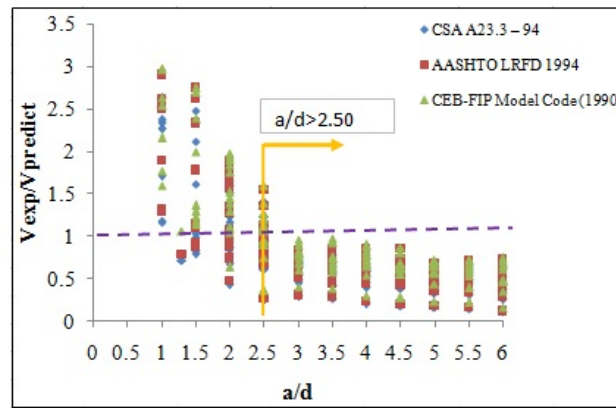


Fig. 9 Shear strength prediction performance from (a) CEB-FIP Model Code (1990), AASHTO LRFD 1994, CSA A23.3 – 94 (b) ANFIS

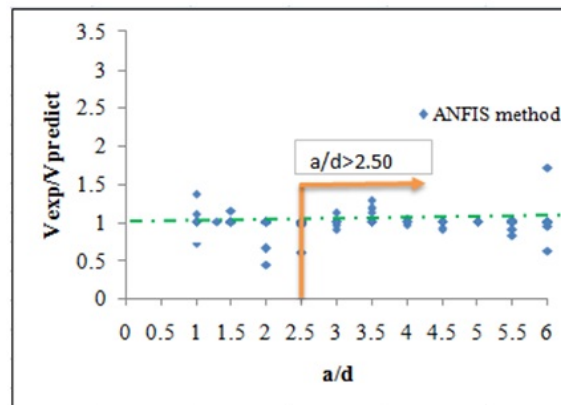
To place in comparison the performance of a variety of codes and methods aforementioned, the graphs between the actual and predicted shear strength are plotted. The best method or code prediction will emerge in a way to show the scatter around a line of perfect agreement (i.e. a line at 45). Thus, referring to the performance of the CEB-FIP Model Code (1990), AASHTO LRFD 1994, CSA A23.3 – 94 and ANFIS method are shown in Figs. 9(a) and 9(b) between the actual and predicted shear strength values.

Fig. 9(a) shows that the CEB-FIP Model Code (1990), AASHTO LRFD 1994 and CSA A23.3 – 94 predict the shear strength of HSC beams in a constant amount for this dataset. As noted from this figure; the CSA prediction has a higher value than the CEB-FIP Model Code (1990) and AASHTO LRFD 1994 predictions.

Fig. 9(b) reveals that the proposed ANFIS method is highly accurate as compared to the CEB-FIP Model Code (1990), AASHTO LRFD 1994, CSA A23.3 – 94 with this dataset for the shear strength prediction of HSC beams without stirrups.



(a)



(b)

Fig. 10 Variation of $\frac{V_{exp}}{V_{predict}}$ with a/d (a) CEB-FIP Model Code (1990), AASHTO LRFD 1994, CSA A23.3 – 94 (b) ANFIS

In relation to this, the relationships of $\frac{V_{exp}}{V_{predict}}$ are presented against input parameters (a/d , f'_c and ρ) separately for all the codes and methods which are used in this study for shear strength predictions.

Figs. 10(a) and 10(b) presents the variation of $\frac{V_{exp}}{V_{predict}}$ of HSC beams without stirrups with the a/d .

Fig. 10: Variation of $\frac{V_{exp}}{V_{predict}}$ with a/d (a) CEB-FIP Model Code (1990), AASHTO LRFD 1994, CSA A23.3 – 94 (b) ANFIS

Based on Fig. 10(a), the codes of design used in this study are able to foresee the shear strength of HSC beams without stirrups in lower values than actual values for $a/d < 2.5$. Meanwhile, it is predicted in higher values for a/d more than 2.50. These findings provide the rational for the significance of a/d on shear strength and behavior of RC beams as allocated to deep and normal beams.

The comparison between Fig. 10(a) and 10(b) shows that a large variation in the ratio of actual to predicted shear strength is obtained with the CEB-FIP Model Code (1990), AASHTO LRFD 1994 and CSA A23.3 – 94 in comparison to the ANFIS method. It shows an improved performance by the ANFIS method in all ranges of the a/d ratios, where most values are nearing 1.

However all the applied methods have overrated the shear strength of HSC beams without stirrups in a diverse range of a/d more than 2.50 but the ANFIS prediction is considerably close to the real shear strength. In a much more realistic context, it can be explained by the shear strength of normal concrete beams with $a/d > 2.50$ that is not impacted with the a/d ratio, wrapped up in contemporary design codes (Choi *et al.* 2009 and ACI 2008).

In sequel, $\frac{V_{exp}}{V_{predict}}$ was plotted against the compressive strength of the concrete (f'_c) and this is shown in Fig. 11 concerning the applied method and code provisions.

Fig. 11(a) highlights the fact that the CEB-FIP Model Code (1990), AASHTO LRFD 1994 and CSA A23.3 – 94 have gravely over-estimated the shear strength of HSC beams without stirrups.

Fig. 11(b) shows that the shear strength predication by ANFIS are mostly undeterred by the variation in the compressive strength of concrete and demonstrate a better performance than the CEB-FIP Model Code (1990), AASHTO LRFD 1994 and CSA A23.3 – 94.

In continuation, the effect of the tensile reinforcement ratio (ρ) is investigated on the shear strength of HSC beams without stirrups and $\frac{V_{exp}}{V_{predict}}$ was plotted against ρ for used methods and codes of design.

Fig. 12 shows a similar effect of f'_c , whereby the ANFIS is not marked in any way by the tensile reinforcement ratio variation.

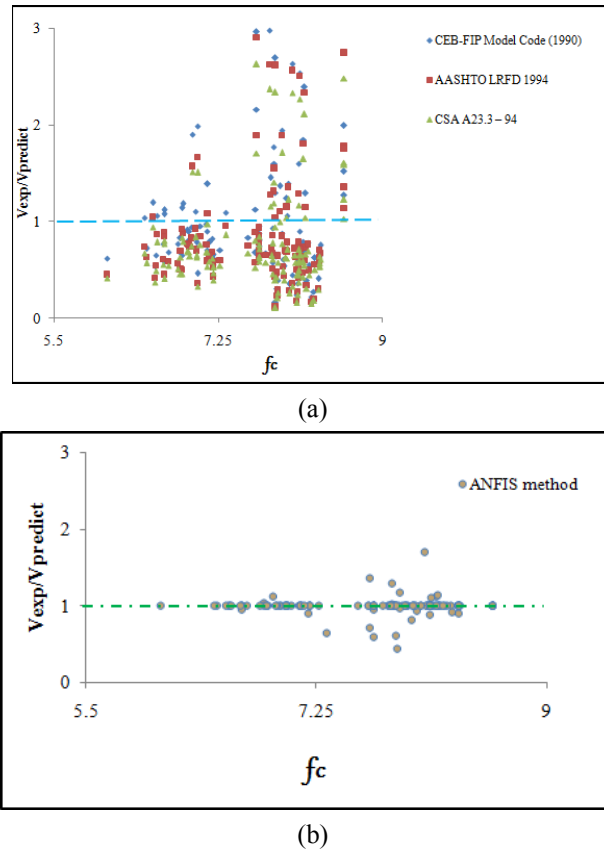


Fig. 11 Variation of $\frac{V_{exp}}{V_{predict}}$ against f'_c (a) CEB-FIP Model Code (1990), AASHTO LRFD 1994, CSA A23.3 – 94 (b) ANFIS

3.6 Parametric analysis

The influence of input parameters on the shear strength of HSC beams without stirrups is studied using the ANFIS, Canadian Standards for the Design of Concrete Structure (CSA A23.3 – 94), AASHTO LRFD Bridge Design Specifications (1994) and European CEB-FIP Model Code (1990) and the comparison of all methods is tabulated in Table 2. This table shows the value of average (AVG), variance (VAR), correlation coefficient (CORR), Mean Square Error (MSE) and coefficient of variation (CV) of the ratio of actual to predicted shear strength for all specimens inside the dataset.

CV makes a useful statistical index for making comparison of the variation degree from one data series to another, even if the means are radically dissimilar to one another.

The average ratio of actual to predicted shear strength of all specimens is 0.995, 0.773, 0.854 and 0.968 with ANFIS, CSA A23.3 – 94, AASHTO 1994 and CEB-FIP Model Code (1990),

respectively.

It has been observed that the best average ratio of $V_{exp}/V_{predict}$ is 0.995, with the least coefficient of variation of 11.97 for the ANFIS prediction. Hence, the ANFIS prediction supplies the best answer to the datasets.

As presented, the MSE values from ANFIS are approximately more than 20 times smaller than the values derived from other methods. Furthermore, the R^2 values from ANN is 0.969 simultaneously signing off as a great value to a scientist due to its close value to 1 meaning very high confidence.

Table 2 shows that the ANFIS is able to generate the best and reliable outputs in comparison with other mentioned codes.

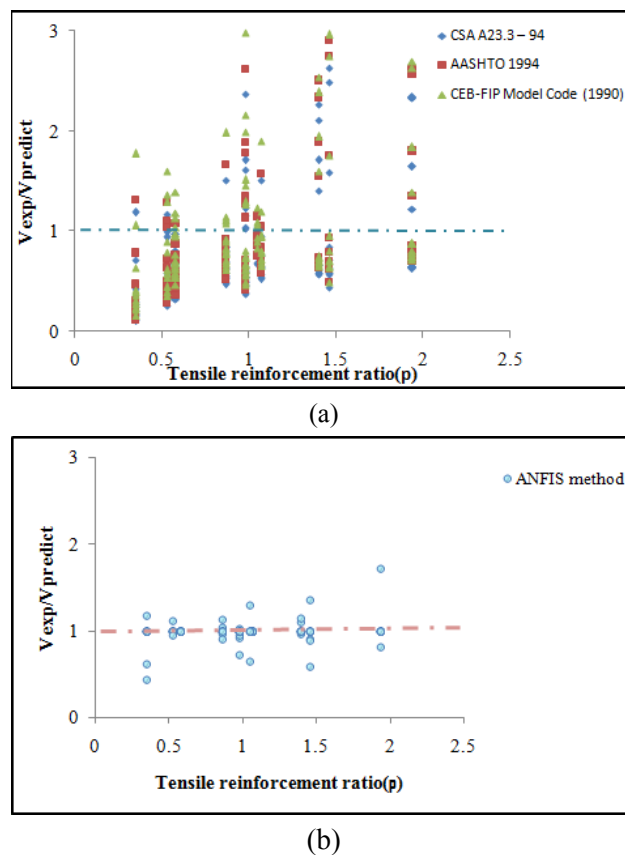


Fig. 12 Variation of $\frac{V_{exp}}{V_{predict}}$ against ρ (a) CEB-FIP Model Code (1990), AASHTO LRFD 1994, CSA A23.3 – 94 (b) ANFIS

4. Conclusions

In this paper; a comparative study has been done on the shear strength prediction of HSC beams without stirrups using the ANFIS method, CEB-FIP Model Code (1990), AASHTO LRFD 1994 and CSA A23.3 – 94. For this purpose, data had been used and taken from the reported studies consisting of 122 HSC beams without stirrups. Based on this study; the following conclusions had been drawn:

This study shows the potential of the ANFIS as an optional modelling technique for shear strength predication of HSC beams without stirrups in place of other empirical or code approaches. Results have uncovered that the ANFIS method is very accurate and precise as compared to the other used codes of design with this dataset for the shear strength prediction of HSC beams without stirrups. The mean and variance of the ratio between both the predicted and measured shear strength of HSC beams without stirrups are 0.995 and 0.014, respectively for the ANFIS prediction.

The result shows that the shear strength predications by the ANFIS are mostly undeterred by the variation in the compressive strength of concrete and tensile reinforcement ratio (ρ) and performs better than the CEB-FIP Model Code (1990), AASHTO LRFD 1994 and CSA A23.3 – 94.

The parametric study in the analyzed datasets shows that the ANFIS has represented the best answer to the predicted amount. Based on the CV index, the shear strength prediction is gained better in the ANFIS method due to its nonlinearity iterations talent.

The CV index is 11.97%, 66.23%, 66.18% and 60.69% for the ANFIS method, CSA A23.3 – 94, AASHTO 1994 and CEB-FIP Model Code (1990) whereby the best answer to these datasets has been attained from the ANFIS method.

Table 2 Average, Variance, Correlation Coefficient, MSE and Coefficient of Variation for different method prediction within datasets

	ANFIS	CSA A23.3 – 94	AASHTO 1994	CEB-FIP Model Code (1990)
AVG	0.995	0.773	0.854	0.968
VAR	0.014	0.262	0.320	0.345
CORR	0.969	0.196	0.204	0.443
MSE	0.014	0.312	0.338	0.343
CV	11.97	66.23	66.18	60.69

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Appendix

Appendix 1: the predicted shear strength of HSC beams using CEB-FIP Model Code (1990), AASHTO LRFD 1994, CSA A23.3 – 94 and ANFIS method

Title	f'_c (ksi)	ρ %	a/d	V_{exp}	$V_{exp}/V_{predict}$			
					ANFIS	CAN	ASHTO	EUR
55 High strength Concrete beam data from [Elahi,2003]								
BF1	7.84	0.35	1	21.71	1	1.183751	1.309409	1.770799
BF2	7.99	0.35	1.3	13.08	1	0.706263	0.781362	1.05314
BF3	7.87	0.35	2	7.79	0.441728	0.424061	0.468995	0.633849
BF4	8.1	0.35	2.5	4.49	1	0.240751	0.266311	0.35834
BF5	8.32	0.35	3	5.18	1	0.287778	0.303279	0.405956
BF6	7.89	0.35	3.5	4.78	1.176673	0.259783	0.28726	0.388302
BF7	7.88	0.35	4	3.72	1	0.202174	0.223692	0.302193
BF8	8.27	0.35	4.5	3.33	1	0.176752	0.195423	0.272727
BF9	8.09	0.35	5	2.92	1	0.156652	0.173294	0.233041
BF10	8.24	0.35	5.5	2.74	1	0.145745	0.161176	0.216259
BF11	7.86	0.35	6	1.91	0.623287	0.103974	0.11506	0.155411
BG1	8.11	0.53	1	21.69	1	1.163003	1.285714	1.593681
BG2	7.91	0.53	1.5	18.26	1	0.990776	1.096038	1.363704
BG3	7.86	0.53	2	17.22	1	0.937908	1.036725	1.291823
BG4	8.12	0.53	2.5	12.14	1	0.650938	0.719621	0.891336
BG5	8.16	0.53	3	10.8	1.12418	0.575386	0.638298	0.788897
BG6	8.15	0.53	3.5	7.82	1	0.418182	0.462448	0.572894
BG7	7.95	0.53	4	7.19	1	0.38928	0.430539	0.535369
BG8	8.09	0.53	4.5	8.3	1	0.445279	0.492582	0.610743
BG9	8.04	0.53	5	5.95	1	0.320237	0.354167	0.439763
BG10	8.19	0.53	5.5	5.34	1	0.303927	0.335849	0.389497
BG11	8.01	0.53	6	4.72	0.944926	0.254585	0.281623	0.34963
BH1	7.8	0.98	1	43.29	1	2.365574	2.617291	2.977304

BH2	8.18	0.98	1.5	19.34	1	1.032017	1.141677	1.289333
BH3	7.81	0.98	2	21.1	1	1.152376	1.274924	1.450172
BH4	8.34	0.98	2.5	11.41	1	0.603066	0.667251	0.750658
BH5	7.94	0.98	3	8.85	1	0.479675	0.530576	0.602041
BH6	8.17	0.98	3.5	10.1	1	0.539242	0.596574	0.673333
BH7	8.12	0.98	4	10.86	1	0.581682	0.643365	0.726908
BH8	8.28	0.98	4.5	9.38	0.92127	0.497613	0.550147	0.61996
BH9	8.22	0.98	5	8.33	1	0.443557	0.490288	0.553121
BH10	7.83	0.98	5.5	8.366	1.022963	0.455913	0.503976	0.573014
BH11	7.86	0.98	6	7.1	1	0.3865	0.427711	0.485636
B11	8.12	1.4	1	42.26	1.107042	2.263524	2.503555	2.535093
B12	8.17	1.4	1.5	39.48	1.144235	2.108974	2.331955	2.386941
B13	7.93	1.4	2	31.51	1	1.707859	1.889089	1.942663
B14	7.85	1.4	2.5	25.7	1	1.399782	1.548193	1.594293
B15	8.18	1.4	3	12.32	1	0.657417	0.727273	0.743961
B16	8.08	1.4	3.5	10.62	1	0.570354	0.630641	0.646379
B17	7.89	1.4	4	10.65	0.970927	0.569519	0.640024	0.658627
B18	7.76	1.4	4.5	10.68	1	0.572654	0.647273	0.667917
B19	7.83	1.4	5	11.14	1	0.601187	0.67068	0.691067
BII0	8.13	1.4	5.5	12.05	1	0.649596	0.715134	0.730746
BII1	7.98	1.4	6	11.43	1	0.616838	0.683204	0.702088
BJ1	8.04	1.94	1	42.99	1	2.330081	2.558929	2.627751
BJ2	7.86	1.94	1.5	43.38	1	2.332258	2.613253	2.691067
BJ3	8.16	1.94	2	30.43	1	1.642202	1.798463	1.839782
BJ4	8	1.94	2.5	22.67	1	1.222102	1.352625	1.389093
BJ5	8.11	1.94	3	13.18	1	0.706702	0.780806	0.800243
BJ6	7.82	1.94	3.5	11.91	1	0.640323	0.718769	0.741133
BJ7	7.69	1.94	4	13.9	1	0.751351	0.845499	0.874214
BJ8	7.93	1.94	4.5	14.15	1	0.766938	0.847305	0.870769
BJ9	8.21	1.94	3	12.88	1	0.68877	0.758539	0.775436

BJ10	7.97	1.94	5.5	11.62	0.821045	0.623391	0.694146	0.713321
BJ11	8.07	1.94	6	12.08	1.713354	0.645299	0.717766	0.735688
27 High strength Concrete beam data from [Bukhari and Ahmad,2007]								
rcb10	7.14	0.58	2	17.03	1	0.969818	1.075805	1.385679
rcb11	7.65	0.58	2.5	14.46	1	0.798013	0.882784	1.123543
rcb12	7.13	0.58	3	9.3	1	0.531732	0.588235	0.699774
rcb13	7.12	0.58	3.5	11.93	1	0.682494	0.755063	0.971498
rcb14	6.87	0.58	4	7.74	1	0.450786	0.498711	0.645538
rcb15	7.2	0.58	4.5	6.8	1	0.386803	0.427673	0.549273
rcb16	6.68	0.58	5	6.83	1	0.403188	0.446114	0.580289
rcb17	6.58	0.58	5.5	6.21	1	0.369423	0.408553	0.533047
rcb18	7.03	0.58	6	5.59	1	0.321819	0.355824	0.459326
rcb20	7.03	0.87	2	26.13	1	1.504318	1.663272	1.981046
rcb21	7.01	0.87	2.5	14.46	1	0.83391	0.922194	1.101295
rcb22	7.13	0.87	3	11.9	1	0.680778	0.752688	0.896084
rcb23	6.92	0.87	3.5	11.93	1.132276	0.692397	0.765725	0.916283
rcb24	6.85	0.87	4	10.67	1.0453	0.62252	0.688387	0.825213
rcb25	7.65	0.87	4.5	9.4	1	0.518764	0.57352	0.674803
rcb26	7.27	0.87	5	9.43	1	0.535795	0.590482	0.701115
rcb27	6.59	0.87	5.5	8.16	1	0.485425	0.536489	0.647619
rcb28	7.2	0.87	6	8.19	1	0.466135	0.515094	0.612565
rcb30	6.98	1.07	2	26.13	1	1.509532	1.569369	1.894851
rcb31	6.56	1.07	2.5	15.76	1	0.939213	1.040264	1.191232
rcb32	7.06	1.07	3	13.2	1	0.758185	0.838628	0.94964
rcb33	6.97	1.07	3.5	12.9	1	0.745665	0.824808	0.936139
rcb34	7.02	1.07	4	10.83	1	0.624207	0.690249	0.78308
rcb35	6.95	1.07	4.5	11.02	1	0.638101	0.733688	0.801455
rcb36	6.49	1.07	5	9.43	1	0.565009	0.624503	0.717656
rcb37	6.72	1.07	5.5	9.14	1	0.53828	0.581425	0.679554
rcb38	7.2	1.07	6	9.17	1	0.521615	0.57673	0.651741

18 High strength Concrete beam data from [Ali, 2001]								
BSA1	6.88	0.58	1.5	14.16	1	0.789298	0.872996	1.18
BSB1	6.47	0.58	2	11.96	1	0.670028	0.736	1.038194
BSC1	7.02	0.58	2.5	11.43	1	0.642135	0.705556	0.939967
BSD1	6.85	0.58	3	8.11	1	0.453073	0.501236	0.678094
BSE1	6.07	0.58	3.5	7.23	1	0.407324	0.445746	0.614796
BSF1	6.82	0.58	4	9.11	1	0.510364	0.558896	0.762982
BSA2	6.87	0.87	1.5	14.75	1	0.833333	0.916149	1.138117
BSB2	6.68	0.87	2	14.19	1	0.794958	0.878638	1.115566
BSC2	6.68	0.87	2.5	13.66	0.963003	0.763128	0.84321	1.073899
BSD2	7.19	0.87	3	10.9	0.908243	0.617564	0.677019	0.815868
BSE2	6.67	0.87	3.5	9.74	1	0.547191	0.600493	0.766326
BSF2	7.16	0.87	4	10.4	1	0.579387	0.64	0.780781
BSA3	7.98	1.05	1.5	18.52	1	1.034637	1.146749	1.234667
BSB3	7.33	1.05	2	15.4	0.650912	0.858417	0.950617	1.086037
BSC3	6.6	1.05	2.5	13.93	1	0.780392	0.865217	1.053707
BSD3	7.57	1.05	3	12.02	1	0.669638	0.744272	0.829538
BSE3	7.83	1.05	3.5	13.73	1.29366	0.771348	0.852795	0.927076
BSF3	6.94	1.05	4	12.22	1	0.682682	0.754321	0.894583
22 High strength Concrete beam data from [Yaqub, 2002]								
BSA1	7.66	0.98	1	30.94	0.721668	1.705623	1.886585	2.154596
BSB1	8.59	0.98	1.5	30.84	1	1.608764	1.775475	1.988395
BSC1	8.59	0.98	2	23.48	1	1.222917	1.351756	1.513862
BSD1	8.59	0.98	2.5	19.63	1	1.022396	1.130109	1.265635
BSE1	7.69	0.98	3	11.57	0.955283	0.636764	0.7042	0.803472
BSF1	7.69	0.98	3.5	10.63	1	0.58503	0.639206	0.738194
BSG1	8.33	0.98	4	10.01	1	0.52963	0.585723	0.658986
BSH1	8.33	0.98	4.5	10.04	1	0.531217	0.587478	0.660961
BSI1	8.33	0.98	5	10.07	1	0.532804	0.589233	0.662936
BSJ1	8.11	0.98	5.5	9.14	1	0.489818	0.541469	0.61219

BSK1	8.11	0.98	6	6.89	1	0.369239	0.408175	0.461487
BSA2	7.66	1.46	1	47.61	1.364219	2.624587	2.903049	2.958981
BSB2	8.59	1.46	1.5	47.64	1	2.48125	2.74266	2.74424
BSC2	8.59	1.46	2	30.4	1	1.583333	1.750144	1.751152
BSD2	7.69	1.46	2.5	15.3	0.591407	0.842047	0.931223	0.949132
BSE2	7.69	1.46	3	12.87	1	0.70831	0.783323	0.798387
BSF2	7.69	1.46	3.5	11.28	1	0.620804	0.686549	0.699752
BSG2	8.33	1.46	4	11.96	1	0.632804	0.699824	0.703529
BSH2	8.33	1.46	4.5	10.7	0.900135	0.566138	0.626097	0.629412
BSI2	8.11	1.46	5	10.73	1	0.575027	0.635664	0.64213
BSJ2	8.11	1.46	5.5	10.77	0.897201	0.57717	0.638033	0.644524
BSK2	8.11	1.46	6	8.19	1	0.438907	0.48519	0.490126