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Comprehensive piezo-thermo-elastic analysis of a thick hollow spherical shell

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Abstract. The present paper develops piezo-thermo-elastic analysis of a thick spherical shell for generalized functionally graded piezoelectric material. The assumed structure is loaded under thermal, electrical and mechanical loads. The mechanical, thermal and electrical properties are graded along the radial direction based on a power function with three different non homogenous indexes. Primarily, the non homogenous heat transfer equation is solved by applying the general boundary conditions, individually. Substitution of stress, strain, electrical displacement and material properties in equilibrium and Maxwell equations present two non homogenous differential equation of order two. The main objective of the present study is to improve the relations between mechanical and electrical loads in hollow spherical shells especially for functionally graded piezoelectric materials. The obtained results can evaluate the effect of every non homogenous parameter on the mechanical and electrical components.

Keywords: piezoelectric; thick hollow spherical shell; functionally graded piezoelectric material; non homogenous

1. Introduction

Indian men have found new group of materials named “Ceylon Magnet”. These materials have tendency to absorb the tiny particles when those are heated. Quartz has been known as a first piezoelectric material. Piezoelectric materials have different applications especially in electro mechanical systems such as systems including the sensors and actuators. The piezoelectric materials can exchange the input electric potential to mechanical deformation and conversely mechanical deformation to electric potential. The first case can be applied in micro positioning as actuator and the second case can be applied in sensing and measuring applications as sensors. The piezoelectric effect is scientifically discovered by Pierre and Jacques Curie in 1880. Piezoelectric structures are very applicable in the industrial systems. Disk, hollow cylindrical and spherical shells are three famous structures that can be used as sensor or actuator.

For many applicable conditions, it is appropriate to investigate the relation between the applied loads and displacement or electric potential in a piezoelectric structure such as hollow spherical

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shell. In order to control the distribution of the displacement or electric potential in a piezoelectric structure, functionally graded piezoelectric material (FGPM) can be used. The properties of this material vary continuously along the thickness direction.

The present study develops the piezo-thermo-elastic relations for a functionally graded piezoelectric hollow spherical shell under mechanical, thermal and electrical loads. All material properties are graded along the thickness direction based on a power function. Three compositions of material properties are considered based on three selected non homogenous parameters. The effect of different types of non homogeneity is investigated on the responses of the hollow spherical shell, comprehensively.

Functionally graded materials were created for the first time in laboratory by a Japanese group of material scientist. For many advantageous properties, these materials can be used in the vigorous environments such as nuclear reactors and chemical laboratory with abruptly gradient of pressure and temperature.

A brief literature can justify studying about functionally graded piezoelectric structures under mechanical, thermal and electrical loads as a novel subject in scope of electro elastic analysis.

Mindlin presented the primary researches about piezoelectric structures. He also presented the other studies about forced and high frequency vibration of piezoelectric plates (Mindlin 1952, Mindlin 1972, Mindlin 1984). Crawley and De Luis (1987) developed the use of piezoelectric materials in intelligent systems. This material can be used as a sensor or an actuator in electro mechanical systems. Many researchers analyzed the structures with bonded piezoelectric layers (Im and Atluri 1989, Lee 1990, Crawley and Anderson 1990).

Yamanouchi and Koizumi presented the concept of functionally graded materials for the first time in Japan (Yamanouchi *et al*. 1990, Koizumi 1993). New theories for analysis of the structure including the piezoelectric layers are developed based on the shear deformation and layerwise theories (Wang and Rogers 1991, Huang and Wu 1996, Jonnalagadda *et al*. 1994, Mitchell and Reddy 1995). Three-dimensional thermo-electro-elastic response of multilayered hybrid composite plates was developed by Xu *et al*. (1995). Each of the fundamental unknowns (three displacement, electric potential …) was expressed in terms of a double Fourier series in the Cartesian surface coordinates. Piezo-thermo-elastic solution of a finite transversely isotropic piezoelectric cylindrical shell under thermal, mechanical and electrical loads has been addressed by Kapuria *et al*. (1996). Solution of the governing equations was obtained in terms of potential functions which satisfy the boundary conditions at the ends. The axisymmetric loadings were expanded as a Fourier series in the axial coordinate.

Dube *et al*. (1996) presented piezo-thermo-elastic solution of infinitely long, simply-supported, orthotropic, piezoelectric, flat panel in cylindrical bending under pressure, thermal and electrostatic excitation. Fourier series have been employed for extension of the fields of displacements, electric potential and temperature. Tang *et al*. (1996) presented thermo-electro-elastic response of multilayered hybrid composite plates based on two-dimensional plate theories. The modeling approaches have been contained shear deformation theories and predictor-corrector procedures. Kapuria *et al*. (1997) employed first-order shear deformation and classical lamination theories for thermo-electro-mechanical solution of hybrid rectangular plates.

Senthil and Batra (2001) analyzed the elastic plates with distributed or segmented piezoelectric layers using the classical laminated plate and the first order shear deformation theories. The obtained results were compared with an analytical solution. Benjeddou *et al*. (2002) proposed an exact two-dimensional analytical solution for the free-vibration analysis of simply supported piezoelectric adaptive plates.

As observed, researchers have not been considered any non homogeneity for studied piezoelectric structures before 2003. In this year Chen *et al*. (2002) studied a FG piezoceramic hollow sphere by using 3D electro elastic formulation. Transient electro elastic analysis of a non homogenous sphere based on a power function was analytically investigated by Ding *et al*. (2003a). Dynamic thermo elastic analysis of a functionally graded pyroelectric sphere under mechanical and electrical loads under a uniform temperature rising was performed by Ding *et al*. (2003b) based on definition of a dependent variable.

The piezo thermo elastic solution of a functionally graded hollow cylinder was presented by Ying and Zhi-Fei (2005). They assumed that only piezoelectric coefficient vary quadratically in radial direction while the other material parameters were assumed to be constant. Thermo-electro-elastic transient analysis of functionally graded piezoelectric hollow structures was analytically investigated under thermal, mechanical and electrical loads by Dai and Wang (2005). Liew *et al*. (2005) investigated thermo-piezo-electrical analysis of multilayered composite plates. The analysis was performed using the three-dimensional equations of thermo-piezo-elasticity and the differential quadrature (DQ) numerical technique.

Magneto thermo elastic analysis for solution of stress and perturbation of magnetic field in FGP hollow structures are analytically investigated using the elasticity theory by Dai and Fu (2007). Two previous studies investigated on the sphere structure that made of piezoelectric materials. The non homogeneity is considered identical for all mechanical and electrical properties. Due to this incompleteness and simplified manner for variable material properties, the present study proposes a comprehensive investigation on the piezo-thermo-elastic analysis of thick hollow spherical shell that made of general piezoelectric non homogenous material. For presentation of the general solution, three types of material with three non homogenous variables are considered and the effects of every non homogenous index are investigated individually on the different mechanical and electrical components. Exact solution of cylindrical shell made of functionally graded piezoelectric materials under cylindrical bending was carried out by Wu and Syu (2007). Transient piezothermoelastic analysis of a hollow sphere made of functionally graded piezoelectric material was studied by Ootao and Tanigawa (2007). An overview of various three-dimensional (3D) analytical approaches for the analysis of multilayered and functionally graded (FG) piezoelectric plates and shells has been developed by Wu *et al*. (2008). A review was contained four different approaches.

Khoshgoftar *et al*. (2009) presented the comprehensive thermo elastic analysis of a functionally graded piezoelectric cylindrical shell under electrical and mechanical loads. They supposed all mechanical and electrical properties to be variable along the thickness direction. They considered three different materials for those analyses. Wu and Huang (2009) developed three-dimensional analysis of doubly curved functionally graded (FG) piezo-thermo-elastic shells under thermal loads. Exponent-law dependency has been considered for variation of material properties. Wu and Jiang (2011) developed three dimensional coupled analysis of simply supported FGP circular hollow sandwich cylinders under thermal loads. A parametric study of the influence of the geometric values, material-property gradient index and boundary conditions has been performed. Three-dimensional (3D) coupled analysis of simply-supported; doubly curved FGP shells using a meshless collocation method were developed by Wu *et al*. (2012). The effect of gradation has been evaluated on the field variables in the FG shells and plates under thermal loads.

Arefi and Rahimi (2010, 2012) studied the effect of nonhomogeneity on the thermal and electrical behavior of functionally graded cylinder. A functionally graded piezoelectric rotating cylinder as mechanical sensor under pressure and thermal loads is analytically investigated by

Rahimi *et al*. (2011) for evaluation of angular velocity of rotary devices. Arefi and Rahimi (2011, 2012) investigated on the general formulation, linear and nonlinear analyses of piezoelectric structures by using the energy method.

Studying the previous published works indicates that have not been reported a comprehensive analysis with evaluation of all non homogenous indexes. All previous papers considered one non homogenous index for all variable properties. Due to this incompleteness, the authors focus on a FGP hollow spherical shell with three non homogenous indexes. Effect of every non homogenous index is studied on the all mechanical and electrical components, individually. These analyses direct designers and engineers for selection of best and optimized material property distributions.

2. Solution of the heat conduction equation

The present section deals with the thermal analysis of a functionally graded hollow spherical shell (Fig. 1) based on some simplified assumptions. The reduced governing differential heat transfer equation for symmetric and steady state one dimensional distribution of temperature can be expressed as (Frank 1996, Lienhard 2008)

$$\frac{1}{r^2}\frac{\partial}{\partial r}(k_T(r)r^2\frac{\partial T}{\partial r})=0 \qquad r_a \le r \le r_b \tag{1}$$

where, $\mathrm{r_a}$ and $\mathrm{r_b}$ are the inner and outer radii, $k_T(r)$ is the thermal conductivity which is assumed as a function of the radius r. As mentioned previously, the modified differential equation is of order two and consequently must be had two boundary conditions. In the general state, the boundary conditions may be defined as a linear combination of temperature distribution and differentiation of that as follows (Khoshgoftar *et al*. 2009)

$$C_{11}^T T(r_a)+C_{12}^T T'(r_a)=f_1,\quad C_{21}^T T(r_b)+C_{22}^T T'(r_b)=f_2 \tag{2}$$

where, $C_{ij}{}^T\ (i=1,2; j=1,2)$ are constants which depend on the thermal conductivity and the thermal convection, f_1 and f_2 are constants which are evaluated at the inner and outer radii, respectively. The solution procedure of temperature distribution can be completed by devoting the appropriate function to $k_T(r)$ as a function of r (Eq. (3))

$$k_T(r)=k_0 r^k \tag{3}$$

The general solution of temperature distribution is obtained as

$$T(r)=A_1 r^{-(k+1)}+A_2 \qquad k\neq 0 \tag{4}$$

where, A_1, A_2 are constants of integration which are given by

$$\begin{aligned}A_1&=\frac{r_a^2 r_b^2(C_{11}^T f_2-C_{21}^T f_1)}{C_{21}^T r_b^2(-C_{11}^T r_a^{(-k+1)}+C_{12}^T r_a^{-k}(k+1))+C_{11}^T r_a^2(C_{21}^T r_b^{-k+1}-kC_{22}^T r_b^{-k})},\\ A_2&=\frac{f_1 r_a^{2+k}(C_{21}^T r_b-C_{22}^T(k+1))+f_2 r_b^{2+k}(-C_{11}^T r_a+C_{12}^T(k+1))}{C_{21}^T r_b^{2+k}(C_{11}^T r_a-C_{12}^T(k+1))+C_{11}^T r_a^{k+2}(-C_{21}^T r_b+C_{22}^T(k+1))}\end{aligned} \tag{5}$$

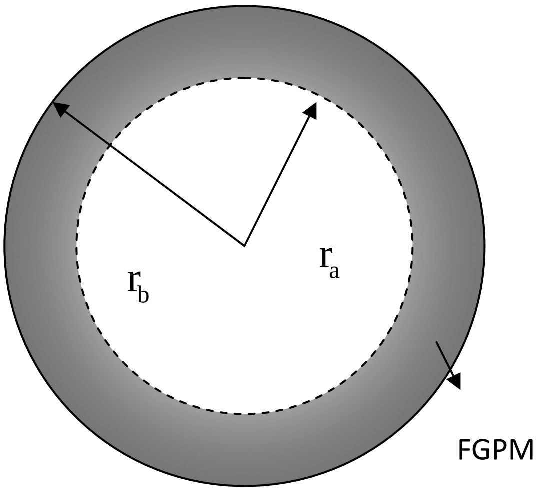

Fig. 1 The schematic figure of a functionally graded hollow spherical shell

3. Piezothermoelastic problem

In this section, general equations of a thick-walled hollow spherical shell with general non homogeneity are presented. In the general case, the stress includes the thermal, mechanical and piezoelectric terms and can be obtained from Eq. (6) (Ding *et al*. 2003a, Ding *et al*. 2003b, Dai and Wang 2005, Dai and Fu 2007).

$$\sigma_{rr} = C_{rr}\varepsilon_{rr} + 2C_{r\theta}\varepsilon_{\theta\theta} - e_{rr}E_r - \beta_r T, \quad \sigma_{\theta\theta} = C_{r\theta}\varepsilon_{rr} + (C_{\theta\theta} + C_{\phi\phi})\varepsilon_{\theta\theta} - e_{r\theta}E_r - \beta_\theta T \tag{6}$$

where, σ_{ij} and ε_{ij} (i,j=r,θ) are the components of stress and strain tensor, $T(r)$ is the temperature distribution, $E(r)$ is the electric field, $C_{ij}\,(i=r,\theta,\phi;\, j=r,\theta,\phi)$ is the elastic stiffness and $e_{ij}\,(i,j=r,\theta)$ is the piezoelectric coefficient. $\sigma_{r\theta}$ is zero due to symmetry. β can be obtained from (Khoshgoftar *et al*. 2009)

$$\beta_r = C_{rr}\alpha_r + C_{r\theta}\alpha_\theta, \quad \beta_\theta = C_{r\theta}\alpha_r + C_{\theta\theta}\alpha_\theta \tag{7}$$

where, $\alpha_i\,(i=r,\theta)$ is the coefficient of thermal expansion.

After presentation of above equations, the appropriate relations for strain components and electric field can be derived. For a symmetric spherical structure, only nonzero displacement component is radial displacement $u(r)$. Therefore by consideration of above assumption, the radial and circumferential components of strain can be derived as (Ding *et al*. 2003a, Ding *et al*. 2003b, Khoshgoftar *et al*. 2009, Lai *et al*. 1999)

$$\varepsilon_{rr} = \frac{\partial u}{\partial r}, \quad \varepsilon_{\theta\theta} = \varepsilon_{\phi\phi} = \frac{u}{r} \tag{8}$$

Due to imposing an electric potential on a piezoelectric structure, the electric field can be derived by using the minus divergence of electric potential φ as (Khoshgoftar *et al*. 2009)

$$E_r = -\frac{\partial \varphi}{\partial r} \tag{9}$$

The electrical displacement D_r which includes the strain, the electrical field and temperature, can be derived as

$$D_r = e_{rr}\varepsilon_{rr} + 2e_{r\theta}\varepsilon_{\theta\theta} + \eta E_r - pT \tag{10}$$

Due to symmetric distribution of all mechanical and electrical components, the circumferential component of the electrical displacement, D_θ is zero. In Eq. (10), η is the dielectric constant and p is the pyroelectric constant.

One of the main relations for analysis of the problem can be expressed in this stage. The equilibrium equation of stress components can be expressed in the spherical coordinate system as follows (Lai *et al*. 1999, Boresi 1993)

$$\frac{\partial \sigma_{rr}}{\partial r} + 2\frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0 \tag{11}$$

The equilibrium equation of electric displacement which is known as Maxwell's equation expresses that the divergence of the electrical displacement vanishes at any point within the media. By applying the Del operator in spherical coordinate system and employing a symmetric condition, we'll have (Dai and Wang 2005, Dai and Fu 2007)

$$div\,(D) = \frac{\partial (r^2 D_r)}{\partial r} = 0 \rightarrow \frac{\partial D_r}{\partial r} + \frac{2D_r}{r} = 0 \tag{12}$$

Eqs. (11) and (12) are adequate for obtaining unknown function u, φ. Before substituting the component of the electric displacement in the Maxwell's equation, the appropriate functions for all properties are assumed as (Wu and Syu 2007)

$$C_{ij} = C_{ij0}r^l \quad e_{ri} = e_{ri0}r^l \quad p = p_0 r^{b+l} \quad \beta = \beta_0 r^{b+l} \quad \alpha = \alpha_0 r^b \quad \eta = \eta_0 r^l \tag{13}$$

where, l and b are the non homogenous parameters of the material properties. Substitution of the above properties into Eq. (13), yields two components of the stress and electrical displacement

$$\begin{aligned} \sigma_{rr} &= C_{rr0}r^l\frac{\partial u}{\partial r} + 2C_{r\theta 0}r^l\frac{u}{r} + e_{rr0}r^l\frac{\partial \varphi}{\partial r} - \beta_{r0}r^b +{}^l T \\ \sigma_{\theta\theta} &= C_{r\theta 0}r^l\frac{\partial u}{\partial r} + (C_{\theta\theta 0} + C_{\phi\phi 0})r^l\frac{u}{r} + e_{r\theta 0}r^l\frac{\partial \varphi}{\partial r} - \beta_{\theta 0}r^b +{}^l T \\ D_r &= e_{rr0}r^l\frac{\partial u}{\partial r} + 2e_{r\theta 0}r^l\frac{u}{r} - \eta_\circ r^l\frac{\partial \varphi}{\partial r} - p_\circ r^b +{}^l T \end{aligned} \tag{14}$$

After substitution of Eq. (14) into two equilibrium equations of mechanical and electrical components (Eqs. (11) and (12)), we'll have two non homogenous ordinary differential equations

$$\left[C_{rr0}\right]r^2\frac{\partial^2 u}{\partial r^2}+\left[C_{rr0}(l+2)\right]r\frac{\partial u}{\partial r}+2\left[(C_{r\theta 0}(l+1)-(C_{\theta\theta 0}+C_{\theta\phi 0})\right]u+\left[e_{rr0}\right]r^2\frac{\partial^2\varphi}{\partial r^2}+\left[e_{rr0}(l+2)-e_{r\theta 0}\right]r\frac{\partial\varphi}{\partial r}$$
$$=\left[\beta_{r0}(b+l-1)-2\beta_{\theta 0}\right]r^{b+1}T(r)+\beta_{r0}r^{b+2}\frac{\partial T(r)}{\partial r} \tag{15a}$$

$$\left[e_{rr0}\right]r^2\frac{\partial^2 u}{\partial r^2}+\left[e_{rr0}(l+2)+2e_{r\theta 0}\right]r\frac{\partial u}{\partial r}+2\left[(e_{r\theta 0}(l+1)\right]u-\left[\eta_\circ\right]r^2\frac{\partial^2\varphi}{\partial r^2}-\left[\eta_\circ(l+2)\right]r\frac{\partial\varphi}{\partial r}$$
$$=p_\circ(b+l+2)r^{b+1}T(r)+p_\circ r^{b+2}\frac{\partial T(r)}{\partial r} \tag{15b}$$

The above ordinary differential equations can be analytically solved for studying the behaviors of a non homogenous hollow spherical shell under thermal, electrical and mechanical loads.

4. Solution of the problem

Solution of the obtained ordinary differential equations in the previous section can be expressed as summation of homogenous and particular solutions as follows

$$u=u_h+u_p,\quad \varphi=\varphi_h+\varphi_p \tag{16}$$

where the first term in the right-hand side of both equations is the homogenous solution (u , ϕ_h)and the second term is the particular solution (u_p , ϕ_p).

A simple change of variable $r=e^S$ can be employed for transformation of obtained Cauchy-Euler, non homogenous, uncoupled differential equations to solvable differential equations as follows.

$$\left[C_{rr0}\lambda^2+C_{rr0}(l+1)\lambda+2C_{r\theta 0}(l+1)-2(C_{\theta\theta 0}+C_{\theta\phi 0})\right]u+\left[e_{rr0}\lambda^2+(e_{rr0}(l+1)-e_{r\theta 0})\lambda\right]\varphi=0 \tag{17a}$$

$$\left[e_{rr0}\lambda^2+e_{rr0}(l+1)+2e_{r\theta 0})\lambda+2(l+1)e_{r\theta 0}\right]u+\left[-\eta_0\lambda^2-\eta_0(l+1)\lambda\right]\varphi=0 \tag{17b}$$

where, λ is the derivative of unknown functions u,ϕ with respect to s: $(\lambda=\dfrac{d}{ds})$

Determinant of above simplified differential equations can yield the fourth order characteristic equation as follows

$$[\lambda^4+2(l+1-\frac{e_{rr0}e_{r\theta 0}}{C_{rr0}\eta_0+e_{rr0}{}^2})\lambda^3+[(l+1)^2+\frac{4(l+1)e_{rr0}e_{r\theta 0}+2\eta_0(l+1)-2\eta_0(C_{\theta\theta 0}+C_{\theta\phi 0})}{C_{rr0}\eta_0+e_{rr0}{}^2})\lambda^2$$
$$+2\frac{(l+1)\eta_0(C_{\theta\theta 0}+C_{\theta\phi 0})}{C_{rr0}\eta_0+e_{rr0}{}^2}]u=0 \tag{18}$$

Due to non zero assumption of radial displacement in the general state; the coefficient of u in Eq. (18) must be zero.

$$\lambda^4+2(l+1-\frac{e_{rr}e_{r\theta 0}}{C_{rr0}\eta_0+e_{rr0}{}^2})\lambda^3+((l+1)^2+\frac{4(l+1)e_{rr0}e_{r\theta 0}+2\eta_0(l+1)-2\eta_0(C_{\theta\theta 0}+C_{\theta\phi 0})}{C_{rr0}\eta_0+e_{rr0}{}^2})\lambda^2$$
$$+2\frac{(l+1)\eta_0(C_{\theta\theta 0}+C_{\theta\phi 0})}{C_{rr0}\eta_0+e_{rr0}{}^2}\lambda=0 \tag{19}$$

Eq. (19) is known as the characteristic equation. The characteristic equation is of fourth order and therefore, in general state, has four roots. The solution has different cases which correspond to the classification of the values of roots in Eq. (19). The same procedure has been developed in the previous published paper of authors and therefore can be eliminated from this paper (Khoshgoftar *et al*. 2009).

Solving Eqs. (15(a)) and (15(b)) together with the appropriate boundary conditions yields the distribution of stress, strain, electric field and electric displacement.

5. Numerical results and discussion

In order to investigate the effect of non homogeneity on the responses of structure, it is appropriate to select an applied material for this purpose. Cadmium Selenide is selected as the material that can be presented as a FGPM (Ootao and Tanigawa 2007, Khoshgoftar *et al*. 2009). The properties of Cadmium Selenide are given as

$$\begin{gathered}\alpha_{r0}=2.458\times10^{-6}1/K,\alpha_{\theta 0}=4.396\times10^{-6}1/K,C_{rr0}=83.6Gpa,C_{\theta\theta 0}=74.1Gpa,C_{\phi\phi 0}=74.1Gpa,C_{r\theta 0}=39.3Gpa\\ e_{rr0}=0.347C/m^2,e_{r\theta 0}=0.16C/m^2,\eta_0=9.03\times10^{-11}C^2/Nm^2,p_0=2.94\times10^{-6}C/m^2k\end{gathered} \tag{20}$$

After substitution of the material properties into derived equations, the different types of non homogeneity may be investigated on the responses of the problem. As mentioned in the previous section, three different non homogenous parameters are selected for analysis of the problem. b, l, k are three types of non homogenous indexes. Different numerical values may be devoted for non homogenous indexes. For tending to these aims, a classic manner may be proposed for selection of non homogenous parameters. Three compositions are proposed for the present study. For every composition, two parameters is fixed and one remained parameter can be varied between -2, 2 with unit increment.

Three compositions of material properties are named as Material 1, 2 and 3 which are presented in Table 1.

The solution procedure can be completed by determination of boundary conditions. Six thermal, electrical and mechanical boundary conditions can be written as

$$\begin{aligned} &T(r=r_\alpha)=200 && T(r=r_b)=0\\ &\sigma_{rr}(r=r_\alpha)=-80Mpa && \sigma_{rr}(r=r_b)=0\\ &\varphi(r=r_\alpha)=0 && \varphi(r=r_b)=0\end{aligned} \tag{21}$$

where, r_a, r_b are the inner and outer radii and equal to 0.6, 1, respectively.

All mechanical and electrical components can be derived for different values of non homogenous index. The obtained results can be classified for three defined materials named as Material 1, 2 and 3 in Table 1.

5.1 Material 1: investigation on the effect of first non homogenous index k

The effect of first non homogenous index that introduced as Material 1 can be studied in the present section on the mechanical and electrical components.

Shown in Fig. 2 is the radial distribution of temperature along the thickness direction of hollow spherical shell for different values of non homogenous index. This figure indicates that the temperature decreases with increasing the non homogenous index. The distribution has linear behavior for *l=-2*. This behavior can be understood directly from Equation of heat transfer (Eq. (1)). Figs. 3 and 4 show the radial distribution of radial displacement and electric potential for five values of non homogenous index, respectively.

Table 1 Non homogeneous parameters

	L	b	k
	0	2	2
	0	2	1
Material 1	0	2	0
	0	2	-1
	0	2	-2
	0	2	0
	0	1	0
Material 2	0	0	0
	0	-1	0
	0	-2	0
	2	2	1
	1	2	1
Material 3	0	2	1
	-1	2	1
	-2	2	1

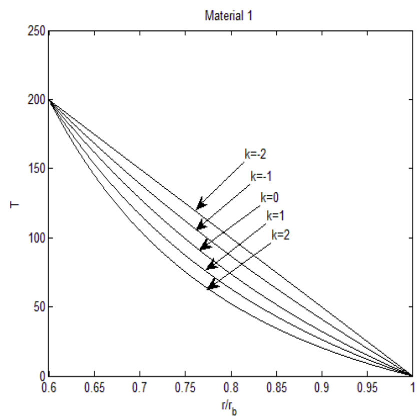

Fig. 2 The radial distribution of temperature for changing the non homogeneity of Material 1

The value of the radial displacement decreases with increasing the value of non homogenous index. Studying the gradient of change for different values of non homogenous index through a determined radial location for Material 1, 2 and 3 can present the important results. As depicted in Fig. 3, this gradient is not significant for $k \leq 0$. It seems that this behavior is because of approximately linear distribution of temperature for $k \leq 0$. Therefore, it can be concluded that the temperature gradient has significant effect on the radial displacement for Material 1.

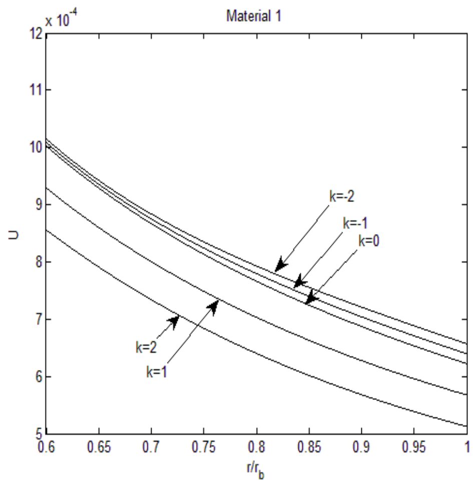

Fig. 3 The radial distribution of radial displacement for changing the non homogeneity of Material 1

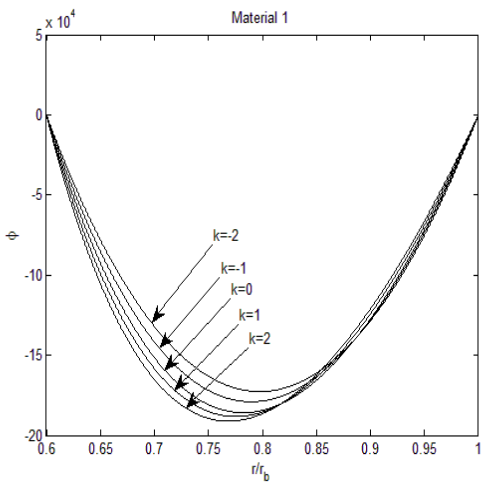

Fig. 4 The radial distribution of electric potential for changing the non homogeneity of Material 1

Fig. 4 indicates that the distribution of electric potential is approximately independent of non homogeneity for $r/r_b \geq 0.85$. This behavior can be under effect of high temperature at inner radius rather than outer radius. Therefore, the local temperature rising has significant effect on the electric potential. This behavior can be because of implicit effect of thermal non homogenous index, k, on the Maxwell and equilibrium equations. It is predictable that the distribution of electric potential must be significantly under effect of the non homogenous index of Material 2 and 3.

Shown in Figs. 5 and 6 are the radial distribution of the radial and circumferential stresses along the thickness direction for changing the non homogeneity of Material 1. It is observed that the distribution of the radial stress is approximately independent of non homogeneity of Material 1.

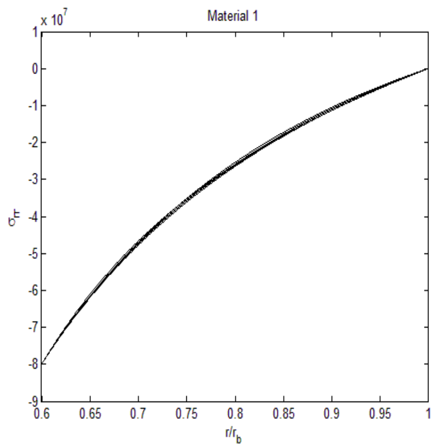

Fig. 5 The radial distribution of radial stress for changing the non homogeneity of Material 1

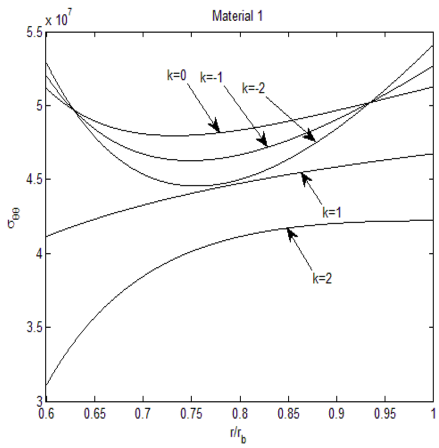

Fig. 6 The radial distribution of circumferential stress for changing the non homogeneity of Material 1

For $-2 \le k \le 0$, the circumferential stress has different behavior from $k \ge 1$. For the first case, the distribution has 2 point of intersection along the wall sphere. For these points, we have the identical values of circumferential stress for different non homogeneity of Material 1. For the second case, the behavior is uniformly along the thickness and there is no intersection point. In case 2, the value of circumferential stress decreases with increasing the non homogeneity of Material 1.

Shown in Figs. 7 and 8 are the radial distribution of electric field and electric displacement along the thickness direction for changing the non homogeneity of Material 1.

Fig. 7 indicates that the radial distribution of electric field for five values of non homogenous index has three different regions along the wall of the hollow spherical shell. Two intersection points are depicted in Fig. 7 that the values of electric field are identical for different non homogenous index.

The electric displacement is only component of electrical and mechanical components that has uniformly distribution along the wall of hollow spherical shell.

5.2 Material 2: Investigation on the effect of second non homogenous index b

Shown in Fig. 9 is the radial distribution of temperature along the thickness direction for changing non homogeneity of Material 2.

Due to independent solution of heat transfer equation by considering the one non homogenous parameter, *k,* it is obvious that the temperature distribution is unchangeable for changing the non homogeneity of Material 2. This behavior may be observed in temperature distribution of Material 3.

Shown in Figs. 10 and 11 are the radial distribution of the radial displacement and electric potential along the thickness direction for changing the non homogeneity of Material 2, respectively.

These figures indicate that the obtained results can be classified for different ranges of non homogeneity. It is observed that two distributions are significantly under effect of non homogenous index for $b \le -1$. For $b \ge 0$, it isn't observed such significant dependency.

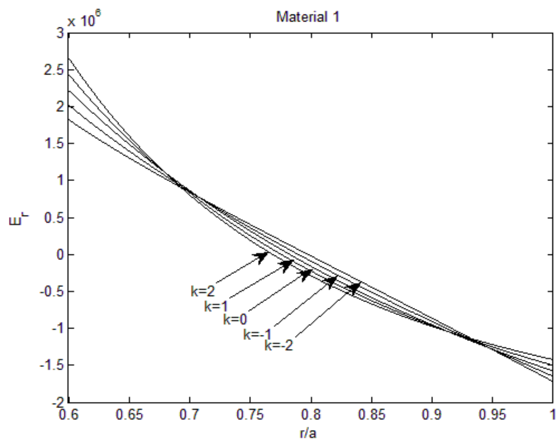

Fig. 7 The radial distribution of electric field for changing the non homogeneity of Material 1

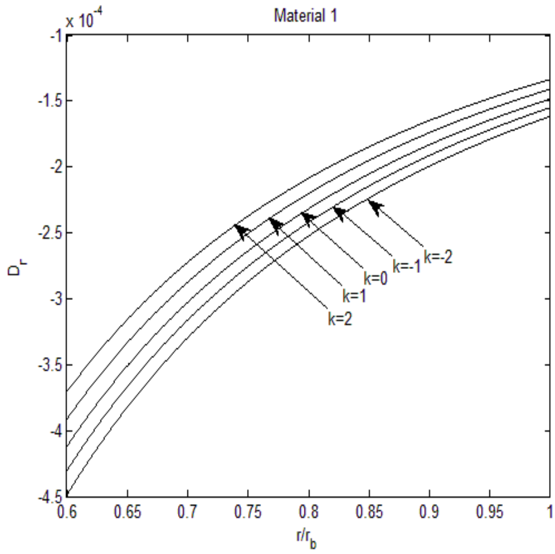

Fig. 8 The radial distribution of electric displacement for changing the non homogeneity of Material 1

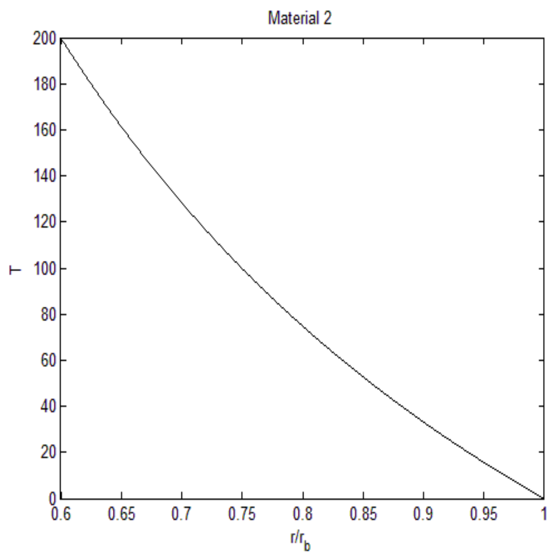

Fig. 9 The radial distribution of temperature for changing the non homogeneity of Material 2

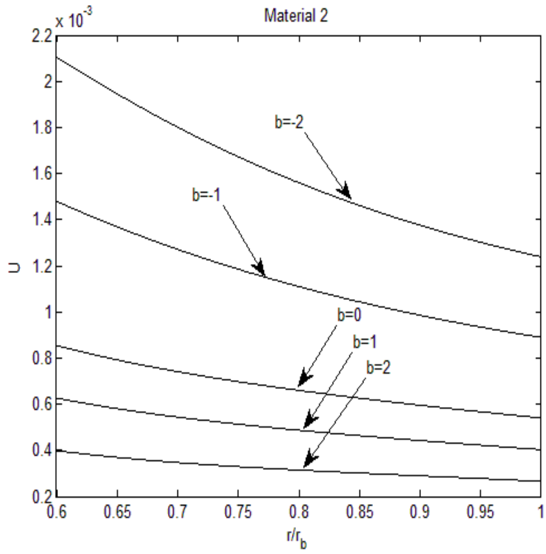

Fig. 10 The radial distribution of radial displacement for changing the non homogeneity of Material 2

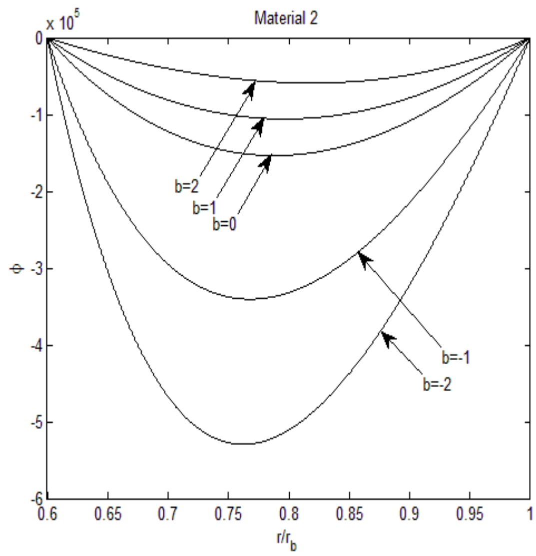

Fig. 11 The radial distribution of electric potential for changing the non homogeneity of Material 2

Figs. 12 and 13 show the radial distribution of the radial and circumferential stresses for different non homogenous index of Material 2. The radial distribution of electric field and electric displacement can be presented in Figs. 14 and 15.

The previous observation in Figures.10, 11 can be observed again in Figs. 12-15 for radial and circumferential distribution of stresses, electric field and electric displacement. The significant effect of non homogenous index of Material 2 on the radial and circumferential stresses, electric field and electric displacement can be understood for $b \le -1$.

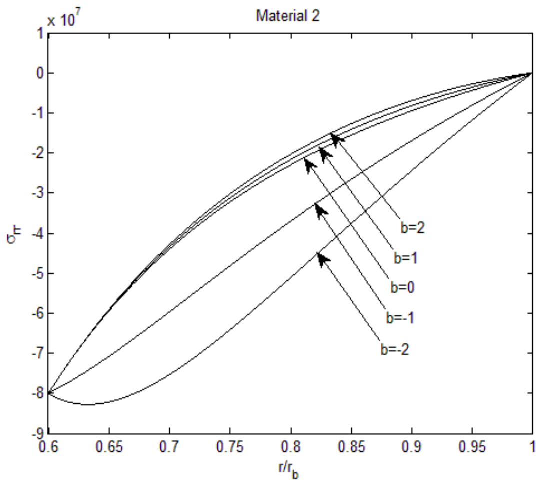

Fig. 12 The radial distribution of radial stress for changing the non homogeneity of Material 2

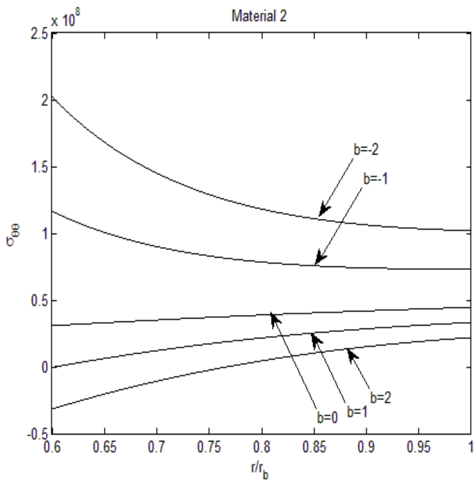

Fig. 13 The radial distribution of circumferential stress for changing the non homogeneity of Material 2

5.3 Material 3: Investigation on the effect of third non homogenous index l

Due to presented reason in section 5.2, the temperature distribution is independent of non homogenous index of Material 2, 3 and therefore can be withdrawn in this section.

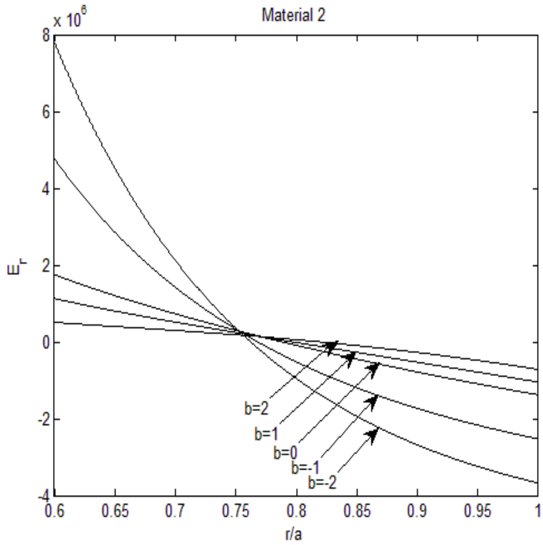

Fig. 14 The radial distribution of electric field for changing the non homogeneity of Material 2

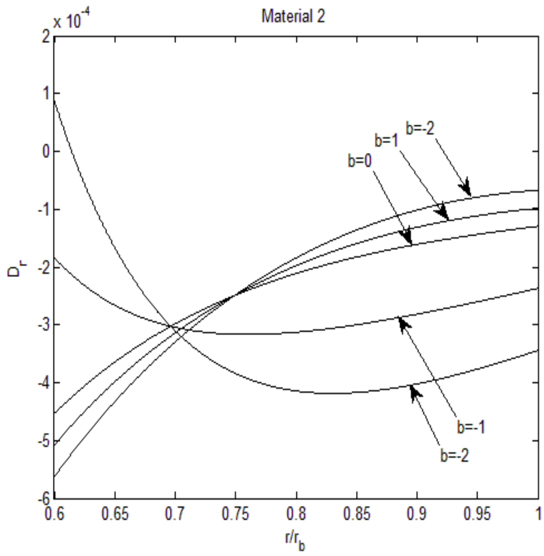

Fig. 15 The radial distribution of electric displacement for changing the non homogeneity of Material 2

Figs. 16 and 17 show the radial distribution of radial displacement and electric potential for different values of non homogenous index of Material 3. Only uniformly distribution of mechanical and electrical components can be observed for Material 3 while *l* is only variable non homogenous index (*b, k* are constant according to Table 1). The values of the radial displacement and electric potential decrease with increasing the value of non homogenous index.

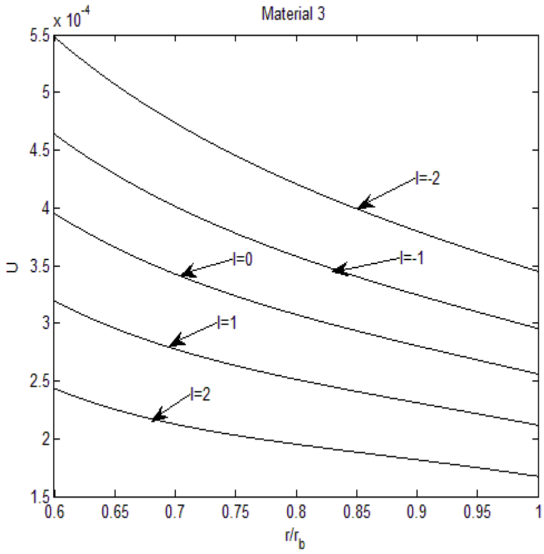

Fig. 16 The radial distribution of radial displacement for changing the non homogeneity of Material 3

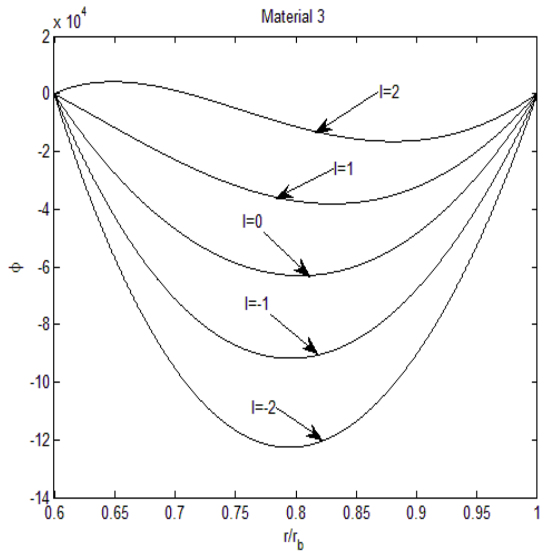

Fig. 17 The radial distribution of electric potential for changing the non homogeneity of Material 3

Shown in Figs. 18 and 19 are the radial distribution of the radial and circumferential stresses along the thickness direction for Material 3, respectively. It is observed in Fig. 18 that the absolute value of the radial stress decreases with increasing the non homogenous index.

The distribution of circumferential stress has considerable behavior. The value of this stress is negative at inner radius. This value tends to a positive value at outer radius. From the observed trend in Fig. 19, it can be predicted a total negative distribution of circumferential stress for large positive values of *l*. the most thickness of hollow spherical shell can be under positive circumferential stress for large negative values of the non homogenous index. Shown in Figs. 20 and 21 are the radial distribution of the electric field and electric displacement along the thickness direction for Material 3, respectively.

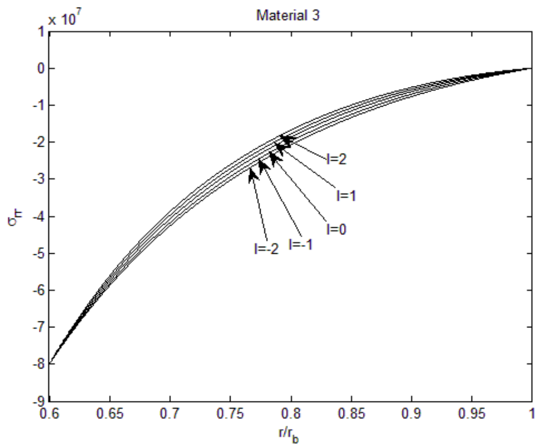

Fig. 18 The radial distribution of radial stress for changing the non homogeneity of Material 3

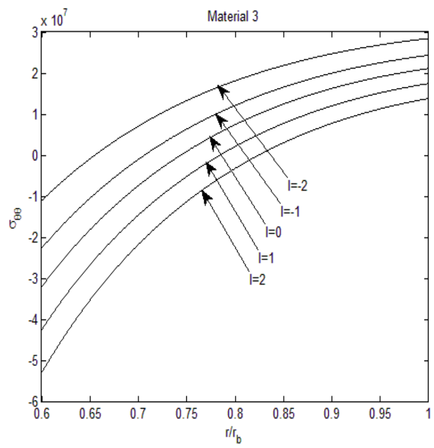

Fig. 19 The radial distribution of circumferential stress for changing the non homogeneity of Material 3

Two figures have an intersection point at the middle surface of hollow spherical shell. At regions that the radius is greater than the middle surface, the absolute value of two components decreases with increasing the non homogenous index.

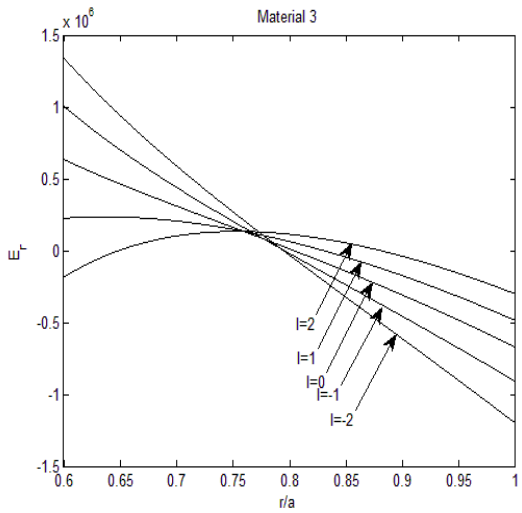

Fig. 20 The radial distribution of electric field for changing of the non homogeneity of Material 3

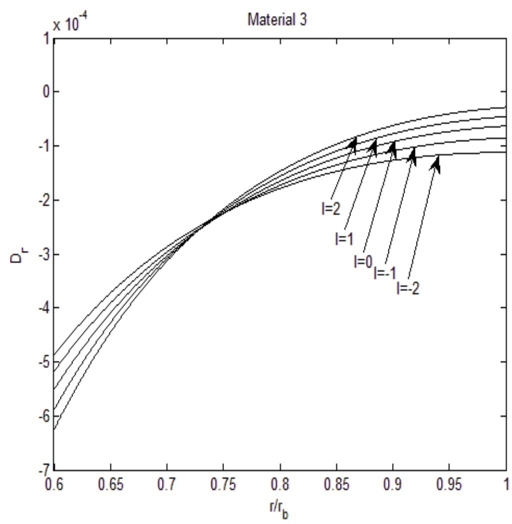

Fig. 21 The radial distribution of electric displacement for changing of the non homogeneity of Material 3

6. Conclusions

The present paper developed the equilibrium equations of mechanical and electrical components for a spherical structure made of functionally graded piezoelectric materials. Three different types of material are selected which for every material, one non homogenous index is considered variable and the other variables are considered constant. Solution of the defined problem for every material presents a comprehensive investigation about the effect of the different non homogenous indexes on the piezo-thermo-elastic responses of a hollow spherical shell. After solution of heat transfer equation in the spherical coordinate system with some simplified assumptions, the obtained temperature distribution is entered into Maxwell and equilibrium equation. Characteristic equation is derived in the general condition (Eq. (19)). By substitution of different numerical values into characteristic equation, the different responses can be obtained for different roots of characteristic equation. The main conclusions can be classified as follows:

1. As mentioned in abstract and introduction, the main purpose of the present study is to develop and improve the relation between mechanical and electrical components in a hollow spherical shell using the functionally graded materials in order to best controlling of systems including the thermal and pressure sensors or actuators. This comprehensive solution can present vast and valid options for designer and manufactures for attaining to best design.
2. The obtained results indicate that the absolute values of the radial displacement and electric potential decreases with increasing the values of non homogenous index. Because of applying the inner pressure, the radial displacement is positive for all values of non homogenous index. The effect of different non homogenous index on the radial displacement for Material 1, 2 and 3 indicates that this effect is considerable, significant and uniformly for Material 3. The minor and irregular effect of non homogenous material on the radial displacement and electric potential can be investigated for Material 1, 2.
3. Investigation on the Material 2 indicates that for this material, the distribution of different mechanical and electrical components has two separate behavior for $b \leq -1$ and $b \geq 0$. The gradient of change of mechanical and electrical components at a defined location of the wall of the hollow spherical shell indicates that this gradient is significant for $b \leq -1$. The mentioned gradient decreases significantly for $b \geq 0$.
4. The uniformly and regular distribution for different values of non homogenous index can be investigated for Material 3 where non homogenous index l changes gradually and continuously.
5. The achieved results justify application of functionally graded materials for attaining to optimization and best design of most applicable structures such as hollow spherical shell.

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