Smart Structures and Systems, *Vol. 14, No. 1 (2014) 55-70*
DOI: http://dx.doi.org/10.12989/sss.2014.14.1.055

Multi-type, multi-sensor placement optimization for structural health monitoring of long span bridges

Rohan N. Soman*, Toula Onoufriou[a], Marios A. Kyriakides[b], Renos A. Votsis[c] and Christis Z. Chrysostomou[d]

Department of Civil Engineering and Geomatics, Cyprus University of Technology, 31, Archbishop Kyprianou Street, Limassol, Cyprus

(Received February 28, 2014, Revised May 28, 2014, Accepted June 25, 2014)

Abstract. The paper presents a multi-objective optimization strategy for a multi-type sensor placement for Structural Health Monitoring (SHM) of long span bridges. The problem is formulated for simultaneous placement of strain sensors and accelerometers (heterogeneous network) based on application demands for SHM system. Modal Identification (MI) and Accurate Mode Shape Expansion (AMSE) were chosen as the application demands for SHM. The optimization problem is solved through the use of integer Genetic Algorithm (GA) to maximize a common metric to ensure adequate MI and AMSE. The performance of the joint optimization problem solved by GA is compared with other established methods for homogenous sensor placement. The results indicate that the use of a multi-type sensor system can improve the quality of SHM. It has also been demonstrated that use of GA improves the overall quality of the sensor placement compared to other methods for optimization of sensor placement.

Keywords: long span bridge; sensor placement optimization; mode shape expansion; modal identification; modal clarity index; genetic algorithm

1. Introduction

The significant increase in the demands of the built environment observed over the last three decades, together with the limiting financial and natural resources have led towards the development of innovative techniques for monitoring the performance of structures. The methodology to monitor a structure through the evaluation of its in-service performance is known as Structural Health Monitoring (SHM).

A proper SHM system can trigger alarms of structural deterioration early enough so as to schedule maintenance actions well in advance, thus reducing maintenance costs, and more importantly avoid severe structural deterioration that can lead to collapse. This realization has led to partial acceptance of the SHM systems for deployment on important infrastructure. The major

*Corresponding author, Ph. D. Student, E-mail: rnsoman@gmail.com
[a] Professor,
[b] Dr.
[c] Dr.
[d] Associate Professor

http://www.techno-press.org/?journal=sss&subpage=8 ISSN: 1738-1584 (Print), 1738-1991 (Online)

hindrance in the widespread acceptance of SHM systems is the high cost of the SHM equipment (Uhl 2009). The aim of the bridge owners is to minimize the cost of the SHM system thus putting a restriction on the number of sensors used. This restriction makes the optimization of sensors a necessity for maximum information quality.

Damage detection is a significant component of the function of the SHM. The traditional SHM systems mainly focus on monitoring the vibration characteristics of the structure like mode shapes and frequencies through the use of accelerometers for damage detection. Due to the global nature of these properties, these systems are sensitive to damages of large extent only (Doebling *et al*. 1998). In order to improve the sensitivity of the SHM system to small scale, local damage the use of local level sensors is necessary. Strain sensors are inexpensive and robust for damage detection at the local level but the range of detection is limited (Chakraborty and DeWolf 2006). If the damage occurs outside the immediate vicinity of the strain sensors, it might be missed. Thus multi-type sensing making use of accelerometers for global damage diagnosis and strain sensors for local damage detection is required to ensure detection of damage in the structure (Law *et al*. 2005, Sim *et al*. 2011).

The optimized sensor placement (OSP) problem has been investigated by many researchers in the field of mechanical, aerospace and civil engineering (Heo *et al*. 1997, Fedorov and Hackl 1994, Talebjinad *et al*. 2011). The work in the area of OSP has been largely restricted to the use of one type of sensors (homogenous network). So when the network consists of multi-type sensors these approaches yield a sub-optimal solution. Hence in order to maximize the use of sensors, the problem of multi-type sensor placement should be treated as one optimization problem.

The optimization problem has been solved through many approaches. Papadimitriou (2004) uses the Backward and Forward Sequential Sensor Placement where the sensors are removed/ added in order to improve the information content sequentially. Kammer (1991) tries to improve the information content, through the use of Effective Independence Method (EFI) by removing the sensor which contributes the least to a norm of Fisher Information Matrix (FIM). Both of these approaches are difficult to be implemented to large structures where there is a large number of possible sensor locations, and deleting of sensor positions one at a time is computationally expensive. When more than one sensor is removed at a time, the solution is sub-optimal. Worden and Burrows (2001) proposed the Simulated Annealing and Genetic Algorithm (GA) technique for OSP for damage detection of plate structures. These methods are known to give near-optimal solutions and work well even when the problem size is large e.g., on bridge structures.

In SHM the commonly used principles for sensor placement are the Kinetic Energy Methods (Heo *et al*. 1997), the FIM based methods (Kammer 1991) and many other derivatives of these norms. The primary function of these SHM systems is Modal Identification (MI), and in turn accurate damage detection (Worden and Burrows 2001). The performance of the damage detection methodology depends on the location of the sensors on the structure. In order to improve the damage detection resolution, Mode Shape Expansion (MSE) can be used (Levine *et al*. 1994). In addition, the MSE allows the estimation of the stresses and the displacements which occur at degrees of freedom which are not instrumented. MSE is essentially an interpolation process based on the collected data. This interpolated data should be as close to the real values as possible. Hence Accurate Mode Shape Expansion (AMSE) becomes a valid principle to optimize the sensor placement in order to achieve higher resolution of damage localization.

The present study aims at optimizing the sensor placement for joint optimization of multi-type sensor network for MI and AMSE. Due to the large size of the optimization problem when applied to a real structure, the integer Genetic Algorithm (Haupt and Haupt 2004) has been employed for optimizing the selected principles. In addition, the results obtained from the optimization are compared with homogenous sensor placements and sensor placement using established methods for OSP.

The rest of the paper is organised as follows. Section 2 provides the theoretical formulation of the problem. It elaborates the different application demands and their mathematical treatment. Section 3 covers the numerical modelling aspects of the long span bridge which was used for the validation of the methodology. Section 4 presents the sensitivity studies and the simulated results for the selected application demands. Finally, Section 5 discusses the key conclusions and the envisaged future work in the area.

2. Optimization of sensor placement

A schematic of the optimization process is given in Fig. 1.

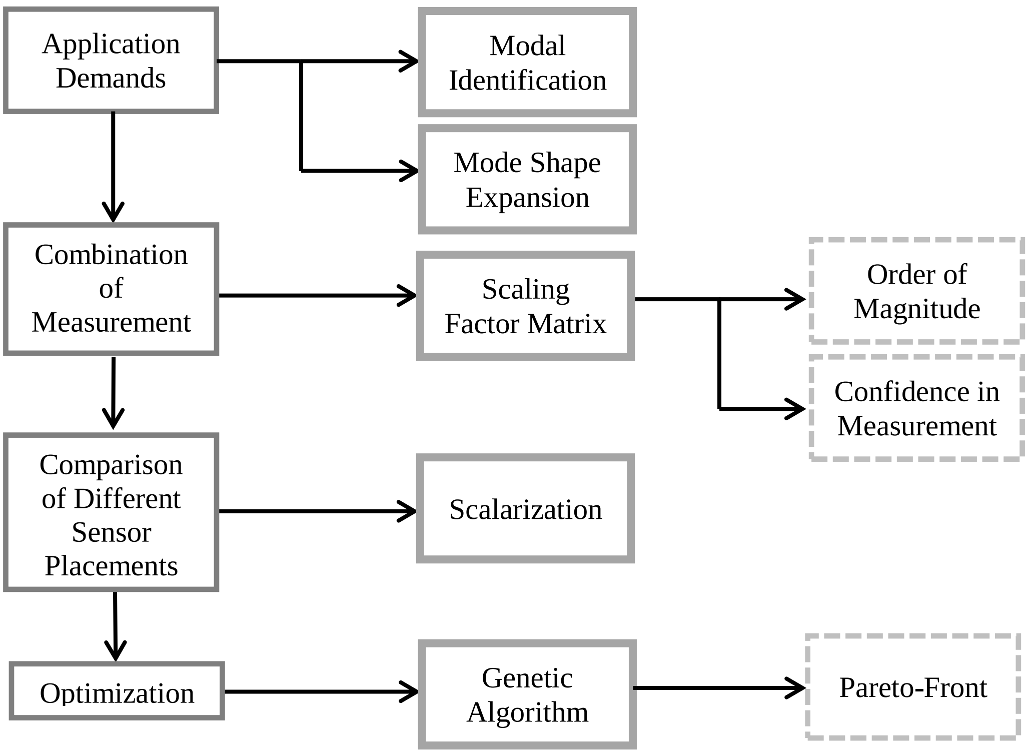

Fig. 1 Schematic of Optimization Process

The multi-objective optimization process starts, with the selection of the optimization principles, based on the application at hand. In order to simplify the optimization algorithm, the application demands are combined into a single metric through scalarization. Different optimized results are obtained by varying the scalarizing factor. These results may then be presented in a concise form through the use of a Pareto-front. The optimization process can be achieved through any optimization algorithm like the GA (Haupt and Haupt 2004), Simulated Annealing (Worden and Burrows 2001) etc. For the present case keeping in mind the large problem size and need for multi-objective optimization, GA was selected as the optimization algorithm. In order to overcome the problems associated with heterogeneous sensor networks, slight customization of the optimization algorithm for the combination of different sensor measurements is necessary. Each of the steps of the optimization process are explained in detail in the following sections.

2.1 Application demands

The optimization of sensor placement needs to be carried out with a particular application in mind. The application for which the optimization is carried out then can be expressed in the form of a qualitative metric for comparison. The application demand commonly used for placement of accelerometers is modal identification. But in the present scenario, as the methodology tries to improve the resolution of damage detection, accurate mode shape expansion is also a key demand for the sensor placement optimization. The numerical formulation of each of these demands is explained here.

2.1.1Modal identification

In order to perform accurate modal analysis it is important to identify the mode shapes and distinguish them from each other. This process of identification and distinction is known as Modal Identification. A widely accepted method for distinction of the mode shapes is the use of Modal Assurance Criterion (MAC) (Ewins 2000). MAC makes use of the orthogonality of the mode shapes with respect to the system mass matrix and allows us to qualitatively assess the distinction between different modes. However, in the case of heterogeneous network consisting of strain sensors and accelerometers, the orthogonality of the mode shapes to each other is lost. Thus, MAC cannot be used and therefore Modal Clarity Index (MCI)is employed (Natarajan *et al*. 2006).

The MCI is based on the least squares method. The best-fit amplitude matrix λ is constructed making use of Eq. (1)

$$\lambda_{p,q} = \frac{\sum_{i=1}^{n} \alpha_{i,p}\alpha_{i,q}}{\sum_{i=1}^{n} \alpha_{i,q}{}^{2}} \tag{1}$$

where p and q are the modes being compared, n is the total number of sensors deployed, α is the scaled modal matrix comprising of strain measurements and displacement measurements.

The Modal Clarity Index can then be obtained as the difference between the excited mode p and the best fit mode q (Natarajan *et al*. 2006).

$$MCI_{p,q} = [\alpha_p - (\lambda_{p,q} \cdot \alpha_p)]^T \cdot [\alpha_p - (\lambda_{p,q} \cdot \alpha_p)] \tag{2}$$

where p, q,α have the same definitions to the corresponding parameters used in Eq. (1).

The MCI matrix is a square matrix with dimensions equal to the modes of interest. Ideally, the matrix should have zeros along diagonal elements and high values at off-diagonal locations. A higher value at an off-diagonal location indicates good distinction between the mode shapes. In order to maximize the identification of different mode shapes the sum of off-diagonal elements should be as high as possible. A threshold for the lower values of the off-diagonal elements can be incorporated in the optimizing algorithm to ensure good identification of all mode shapes.

2.1.2 Accurate mode shape expansion

SHM is deployed primarily for Level 2 damage detection (determining existence and location of damage) (Rytter 1993). The resolution of the isolation of damage is often restricted by the density or spread of the sensor on the system and hence in most cases is not used by bridge owners. There is a need to improve this resolution of damage location determination. One way of achieving this is by performing MSE. MSE is used to estimate the response of the structure at the degrees of freedom (dofs) which are not equipped with a sensor, based on the measurements at few dofs. The MSE needs to be undertaken in order to improve the effectiveness of the damage detection methods. Many MSE methodologies have been proposed in the literature based on expansion methods like the Guyan Reduction (Guyan 1965), the Dynamic Reduction (Kidder 1973), and the System Equivalent Reduction Expansion Process (SEREP) (O'Callahan *et al*. 1989). The SEREP method is based on mode shapes as opposed to the other methods which are based on system stiffness and system mass matrices. SEREP method does not make use of orthogonality of the mode shapes and hence can be applied to the integrated modal matrix formed by the combination of strain sensors and accelerometers.

The SEREP method can be applied using the Eq. (3)

$$y_{estimate} = \Gamma \cdot \left[\left({\Gamma_m}^T \Gamma_m \right)^{-1} \cdot {\Gamma_m}^T \right] \cdot y_m \tag{3}$$

where, $y_{estimate}$ is the estimated response, Γ is the system modal matrix, Γ_m is the modal matrix for the measured dofs, and y_m is the measured response of the structure.

A full scale expansion is possible from a limited number of measured dofs using this method. The accuracy of the expansion depends on the number of the sensors used, as well as on their location. The accuracy of the expansion is assessed by taking the mean of the absolute relative error between the predicted responses and the actual response obtained from finite element simulations. The equation for the mean relative error is given in Eq. (4).

$$Relative\ Error = \frac{|y_{estimate} - y_{actual}|}{|y_{actual}|} \tag{4}$$

A low value of mean relative error (MRE) signifies an accurate prediction, and hence it should be as low as possible.

2.2 Combination of measurements

The combined modal matrix consists of strain and displacement data for the elements and nodes respectively. The combined modal matrix is highly ill-conditioned due to the large difference in the order of magnitudes and any operation on this will lead to wrong results. Thus, to overcome the problem of the ill-conditioning, scaling of the quantities is required. Mathematically, the scaling matrix should be the covariance of the measurement noise of individual sensors. The measurement noise can be assumed as a zero-mean stationary Gaussian noise (Kammer 1991). The noise is uncorrelated, giving a diagonal matrix. But it is difficult to estimate the measurement noise before deployment. Hence, a scaling matrix, which is independent of the measurement noise, is employed.

The scaling matrix should have similar characteristics to the covariance of the measurement noise; it should reflect the order of magnitude of the measurement and also the confidence level in the quality of sensors. The order of magnitude can be captured through the normalization for each degree of freedom, with all the mode shapes considered.

Thus, given the combined modal matrix $\tilde{\alpha}$, Eq. (5)

$$[\tilde{\alpha}] = \begin{bmatrix} A_{1,1} & . & . & A_{1,m} \\ . & . & . & . \\ . & . & . & . \\ A_{n,1} & . & . & A_{n,m} \\ B_{1,1} & . & . & B_{1,m} \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ B_{s,1} & . & . & B_{s,m} \end{bmatrix}_{(n+s)\times m} \tag{5}$$

where, $\tilde{\alpha}$ is the combined modal matrix, $A_{i,j}$=modal strain for the j^{th} mode and i^{th} dof, $B_{i,j}$=modal displacement for the j^{th} mode and i^{th} dof, n is the number of strain sensors, s is the number of accelerometers, and m is the measured mode numbers, we can compute the scaling matrix, Eq. (6)

$$S = \begin{bmatrix} \max(A_1)\times MC & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & \max(A_n)\times MC & . & . & . \\ . & . & . & \max(B_1) & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & \max(B_s) \end{bmatrix}_{(n+s)\times(n+s)} \tag{6}$$

where, *MC*, is the relative confidence in the measurement of strain to that of displacement mode shape based on accelerometer data.

For the present study, the *MC* is taken to be equal to1 as the confidence in the measurements of FE-based values is 100% for both types of measurements. In case of actual experimental data, the relative confidence in measurements can be incorporated. In the study undertaken by Unger *et al.* (2005) the error in measurement of natural frequency, strain and mode shapes were found to be 1:4:5. This trend is also reflected in (Ortel *et al.* 2012). This relative ratio can be found from the data sheet of the sensor manufacturer and or some basic studies prior to full scale deployment on the structure.

2.3 Comparison of different sensor placements

In a multi- objective optimization problem, it is likely to have more than one optimum solution. There are also cases where the objectives of optimization are contradictory to each other, and improving one objective might lead to relaxing the other. So there is a need to select the proper trade-off before the decision for an OSP can be taken. One way to achieve this trade-off is to integrate the two objectives into one objective through the use of linear scaling factor. In this study, the two objectives are Accurate Mode Shape Expansion, and Modal Identification with the MRE and the MCI serving as the corresponding metrics respectively. The MCI should be as high as possible, while MRE should be as low as possible. These two contradictory principles can be scalarized and combined linearly through the use of Eq. (7).

$$g = [\beta\times(\mathrm{MCI}) - (1-\beta)\ \mathrm{MRE}] \tag{7}$$

Thus, the cost function for each of the sensor placements can be computed easily and compared in a subjective way. The results can be presented in the form of a Pareto-front for different values of β ranging between 0 and 1 based on the relative weights assigned to each of the optimization principles.

2.4 Optimization

The optimization problem can be treated as a minimization problem where, the sensor placement with a combined metric of AMSE and MI is combined using a suitable weighing factor. This factor is decided on the basis of the application demands specific to the case at hand. Also, for a long span bridge structure, the possible locations of sensor placement and the available number of sensors makes the problem size large, and in order to reduce the computational load, a meta-heuristic approach of optimization is necessary. Meta-heuristic approaches allow a better search of the sample space in order to find near optimal solutions and hence cannot guarantee the absolute optimum solution. This tradeoff is especially important in case of large problem size and non-linearity in the cost function, which make the computations very expensive.

The Integer GA was chosen as the optimization tool due to the simplicity it brings to the problem formulation and ability to combine different metrics for multi-objective optimization (Haupt and Haupt 2004).

3. Numerical modelling of a long span bridge

For the validation of the proposed methodology, the Great Belt East Bridge was used. The Great Belt Bridge, shown in Fig. 2 is a suspension bridge in service since 1997.

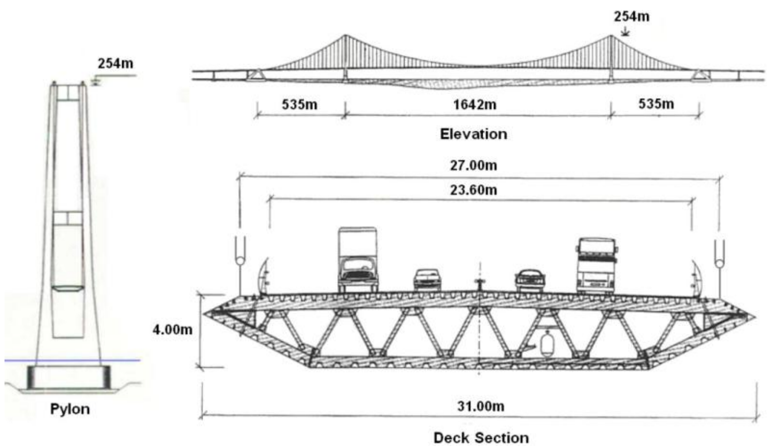

Fig. 2 Main Dimensions of Great Belt East Bridge

The bridge spans the navigation channel connecting the Danish islands of Zealand and Funen. It has a central span of 1624m and two side spans of 535m each. The entire length of the girder is suspended using two massive pylons which by that scale a height of 254m. The bridge uses a continuous bridge girder without supports at the pylons (Weight, 2009).

The bridge was modeled in the commercial software ABAQUS v.6.11. A fish-bone model (Chan *et al*. 2006) was employed for the modeling, which is simple to construct and fulfills the accuracy demands for our application of validation. The bridge deck mass was concentrated in three locations, two equal masses were assumed at the locations where the hangers are attached, and one mass below the neutral axis to preserve the location of the neutral axis and the center of gravity.452 beam elements (Element type: B31) with equivalent properties to the bridge girder were used to model the spine of the girder. The main cables and the hangers were modeled as beam elements with very low stiffness in compression and bending and suitable mass density as indicated in the design drawings.

The pylons were modeled using 30 beam elements for each tower. Fig. 3 shows the finite model of the bridge along with the specific support conditions. The end conditions were based on the design drawings and the model updating carried out based on experimental data available in literature (Larsen 1993). A static analysis was initially carried out in to determine the pretension in the cable. Non- linear analysis was performed to incorporate the geometric non- linearity of the main cable due to its sag. The natural frequencies of the first two lateral, two longitudinal and two torsional modes of vibration obtained through FE analysis were compared to the corresponding experimentally obtained natural frequencies, found in literature (COWI internal report 2000) for model validation. From Table 1 it can be observed that the corresponding natural frequencies are in good agreement as their difference is less than 3.3%.

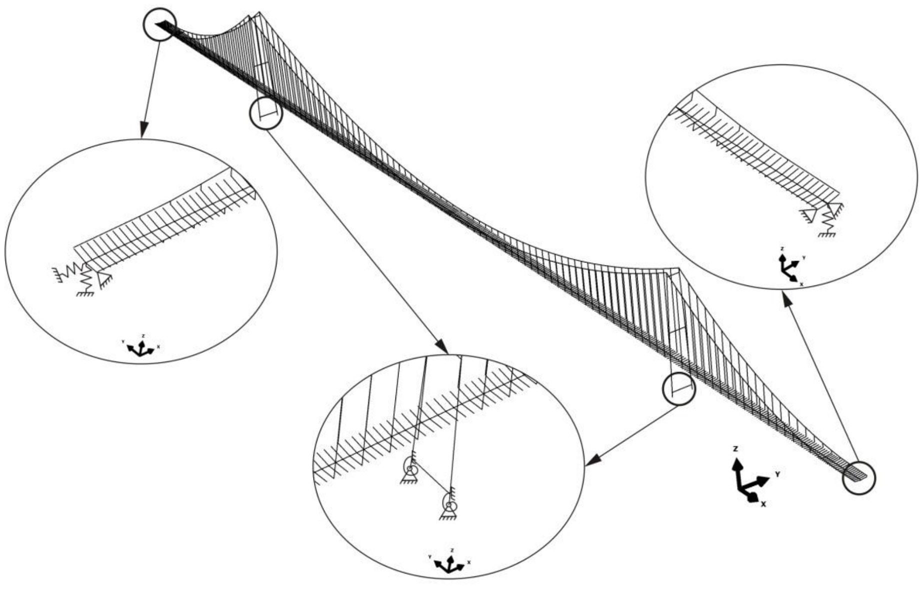

Fig. 3 FE model of Great Belt East Bridge

Table 1 Dynamic Validation of FE model

FEM Predicted (Hz)	Target Frequency (Hz)*	Difference (%)	Mode Specification
0.0537	0.052	3.269	Lateral Sway
0.103	0.100	3.000	Longitudinal Bending
0.115	0.113	1.770	Longitudinal Bending
0.123	0.121	1.653	Lateral Sway
0.278	0.278	0.000	Torsion
0.382	0.383	-0.261	Torsion

*Larsen, A. (1993)

Table 2 Performance of optimization principles with change in number of sensors (accelerometers and strain sensors)

Number of Sensors	Distribution	Sum of MCI	Mean Error
905	Every DOF	1.9302×10^4	9.9561×10^{-5}
454	Every other DOF	9.6801×10^3	1.0889×10^{-4}
305	Every 3	6.4916×10^3	1.3217×10^{-4}
228	Every 4	4.8524×10^3	1.4489×10^{-4}
184	Every 5	3.9226×10^3	1.4002×10^{-4}
154	Every 6	3.2676×10^3	1.5042×10^{-4}
133	Every 7	2.8330×10^3	1.6167×10^{-4}
116	Every 8	2.4648×10^3	1.6482×10^{-4}
104	Every 9	2.2098×10^3	2.0764×10^{-4}
94	Every 10	1.9636×10^3	2.8395×10^{-4}
85	Every 11	1.7797×10^3	2.9328×10^{-4}
79	Every 12	1.6700×10^3	2.1074×10^{-4}
73	Every 13	1.5483×10^3	2.7683×10^{-4}

4. Sensitivity Study

The robustness of the methodology with changing parameters has been studied. The effect of changing number of sensors, the number of monitored natural frequencies, and the comparative study of the new proposed method to the existing OSP methods are presented in this section.

4.1 Effect of number of sensors on optimization variables

The number of sensors affects the quality of information which is collected by the sensors. Intuitively, the MRE, as well as the MCI are expected to be improved by increasing the number of sensors. Table 2 presents the results obtained for different number of evenly placed strain sensors (at the mid-point of beam elements) and accelerometers (at the element nodes) along the girder of the long span bridge presented in Section 3. The expected trend of improvement can be observed in the table with few exceptions (e.g., reduction from 228 sensors to 184 sensors and also reduction from 85 to 79sensors). The inconsistency in the increase of the mean error with the decrease of the number of strain sensor reflects that the location of the sensors is just as important as the number of sensors deployed. This inconsistency is not seen in the MCI index. The MCI index is used to differentiate between 10 modes of interest, using a large number of sensors, which introduces redundancy in the system, making the dependence of selection of appropriate sensor locations for the modal identification slightly less important. This indicates the importance of performing sensor placement optimization in achieving good quality of modal identification as well as improved extrapolation.

4.2 Effect of modes of interest on accuracy of interpolaton

The SEREP method makes use of the modal matrices for predicting the response of the structure at un-instrumented locations. This accuracy of interpolation is directly related to the number of vibration modes used for the reconstruction of the structure response. The purpose of this study was to ascertain the number of modes that are to be extracted for a realistic interpolation of the response of the structure at locations without any sensors.

Table 3 gives the performance of SEREP method for the case of 184 evenly placed sensors (both accelerometers and strain sensors) for different number of modes of interest. This specific case with 184 sensors was chosen as it gives the least mean relative error metric per sensor. A similar performance of the SEREP method is expected when different number of sensors is used.

As can be seen from the Table 3, the mean error is reduced as the number of modes used for SEREP increases. A higher number of modes allows more accurate depiction of the deformed shape and allows a better estimate. Thus there is a need to extract as many modes of vibration as possible to allow accurate expansion. But, the accurate extraction of the higher modes is a challenge using the ambient excitations due to wind or traffic loading. The energy content of the higher modes is masked by the measurement noise and may lead to inaccuracies. So the number of modes extracted is limited to the first few modes, more specifically the first 10 bending modes of vibration, for the purpose of this study.

Table 3 Performance of SEREP method for different number of modes

Number of modes	Sum of MCI per mode	Mean Error
2	1.1635×10^2	3.8520
3	1.7634×10^2	2.1364
4	2.3321×10^2	0.9638
5	2.6524×10^2	2.3789×10^{-2}
6	3.1226×10^2	6.4002×10^{-2}
7	3.2676×10^2	7.3942×10^{-3}
8	3.4330×10^2	1.1167×10^{-3}
9	3.7648×10^2	8.8382×10^{-4}
10	3.9226×10^2	1.4002×10^{-4}

4.3 Comparitive study of sensor placement strategies

The proposed methodology of sensor placement was compared with some established methods of sensor placement optimization. This study allows us to ascertain the effectiveness of the proposed methodology. The study was carried out keeping the upper limit of the number of sensors at 200. Similar results are expected for other number of sensors as well. The number 200 was chosen, as the optimized sensor placement gives a mean relative error below 1×10^{-4}. Table 4 gives a comparison of the performance of the sensor placements for integrated sensor placement (combined strain sensors and accelerometer), strain sensors alone and accelerometers alone.

It can be seen that for the same number of sensors the combined sensor placement results in improved Modal Clarity Index and reduced Mean Relative Error both, thus highlighting the need for the integrated treatment of the OSP problem. The combined sensor GA method yields better ratio than the individual methods. It can be seen that compared to the individually optimized strain sensor placement, the mean relative error in the joint optimized case is higher. However, this loss in accuracy has led to a better MCI value. Similarly in the case of the individually optimized accelerometers, the MCI performance is better than the combined placement method, but the mean error is much higher. Thus, the combined sensor placement optimization is recommended for optimal use of sensors.

It should also be noted that the optimization methods where both types of sensors are used yield better results than when a single type of sensors is used. This trend is apparent even in the evenly placed sensor configuration, where no optimization is undertaken. This gives more support to the idea of joint optimization.

For the convenience of presentation the Bridge is divided in to five sections as shown in Fig. 4. Fig. 5 indicates the candidate sensor locations for the strain sensors and the accelerometers. The strain was measured at the bottom of the girder at the middle of the elements, while the accelerations were measured at the nodes. The Sensor Deployment for each of the optimization strategies is shown in Fig. 6. The number of sensors of different types in each of the sections shows a definite trend, which is expected.

Table 4 Comparative Performance of Sensor Placement Strategies

Strategy	MCI	Mean Relative Error	Ratio (MCI/MRE)
Combined Sensor GA	0.73	6.56	1
Combined Sensors EFI	0.78	13.83	0.68
Only Accelerometers	1	22.65	0.27
Only Strain Sensors	0.07	1	0.19
Evenly Placed	0.26	11.27	0.53

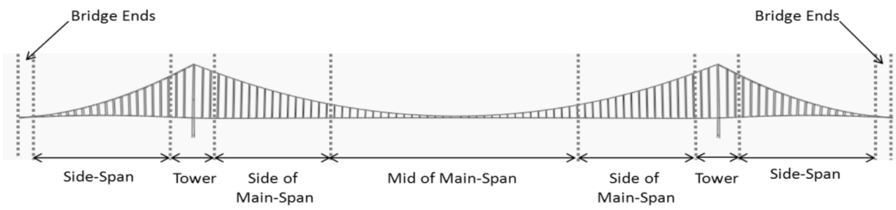

Fig. 4 Bridge Section Nomenclature

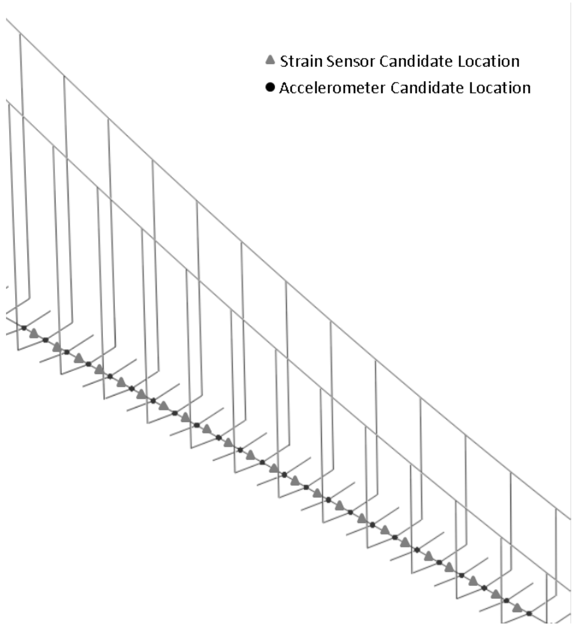

Fig. 5 Candidate locations for strain and accelerometer sensors

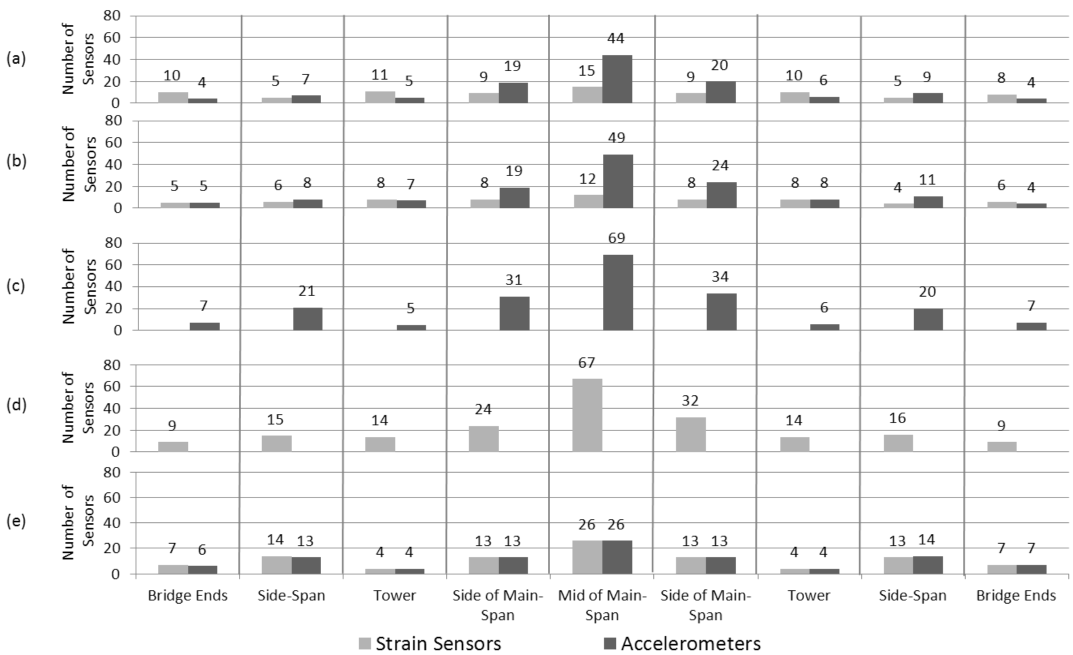

Fig. 6 Sensor Deployment using different strategies, (a) Combined Sensor Placement, (b) Combined Sensors EFI, (c) Only Accelerometers, (d) Only Strain Sensors and (e) Evenly Placed

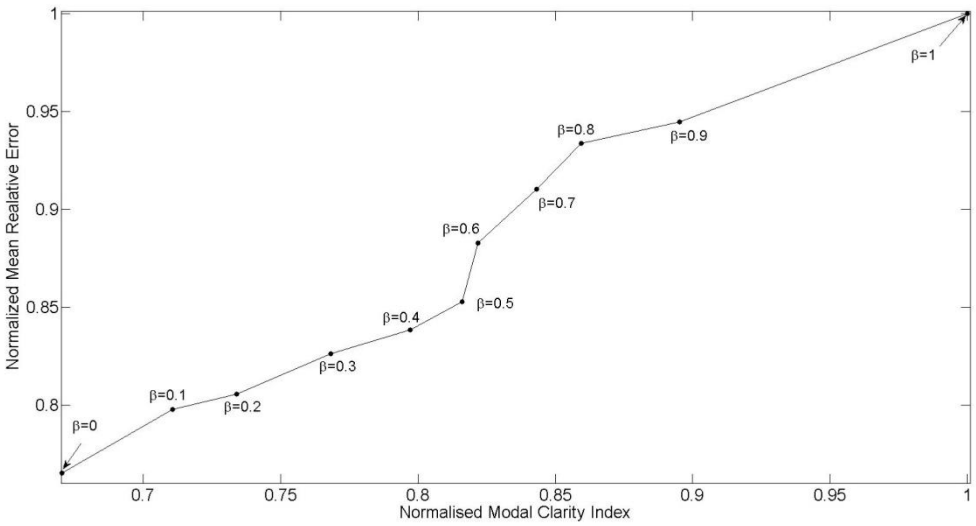

Fig. 7 Pareto Front for Normalized Modal Clarity Index v/s Normalized Mean Relative Error

4.4 Effect of scalarizing factor on the optimization metrics

The multi-objective optimization is simplified through the use of a scalarizing parameter. The two optimization principles are linearized and then combined to form one metric for optimization. This optimization through scalarizing allows faster convergence and makes easy decision making on the trade-offs between the contradicting optimization principles. The results of this optimization need to be generated for all relative weighing values for the scalarizing parameter ranging from 0 to 1. The optimised solution for the different values of the scalarizing parameter (β) can be best shown through the use of Pareto front. Fig. 7 shows one such plot for the different values of β ranging from 0 to 1 at intervals of 0.1. Based on this Pareto-front and knowing the application demands, the bridge owners may make the decision on the sensor placement which needs to be deployed for the fulfilment of the specific application demands. The Pareto-front generated is not optimal but in fact near-optimal as the GA was used for the optimization.

5. Conclusions

The paper proposes a methodology to optimize the location of multi-type sensors (strain sensors and accelerometers), for maximum modal identification (modal clarity) and minimum mean error (error in mode expansion). The methodology incorporates four significant steps, namely selection of application demands, combination of heterogeneous quantities (strain data and accelerometer data in the form of deflection mode shapes), comparison of different heterogeneous sensor placements and optimization for a selected fitness function.

The application demands chosen for this study are Modal Identification which is commonly used for sensor deployment to ensure that spatial aliasing does not take place. SEREP based extrapolation is used to improve the resolution of damage detection. In order to ensure accurate expansion, mean error is chosen as the principle for optimizing. These principles not only do they ensure proper modal identification but they also aid in the accurate damage detection.

The combination of the measured quantities is carried out through the use of a scaling factor to overcome the ill-conditioned nature of the modal matrix. The scaling factor is chosen keeping in mind the theoretical formulation of the least squares approach and the practical limitations of measuring the noise variance of the sensors before the actual deployment. Once the combined modal matrix is formed the sensor placements are compared based on two metrics derived from the application demands. The proposed metrics are Modal Clarity Index for Modal Identification and Mean Error for the accurate expansion of mode shapes. In order to facilitate the use of GA the two objectives are combined through scalarizing parameter to give one fitness function which is used in the optimization. The scalarizing parameter works like a weighing parameter, and hence, the relative importance of the two conflicting optimization principles can be varied. By changing the weighing factor, many sensor placements are obtained, which can be best presented in the form of a Pareto- front. The Pareto-front allows an informed decision making. The application demands often dictate the thresholds for the modal identification and the mode shape expansion and as such, through the Pareto-front one can make the decision for the best sensor deployment given the requirements and the thresholds for the application at hand.

The paper first presents some sensitivity studies, for the effect of number of sensors on the optimization principles and the effect of extracted mode shapes on the accuracy of interpolation. From these studies it is evident that the optimization of the location of the sensors is an essential step in order to ensure optimal use of the resources. In addition, it also corroborates the intuition that the accuracy of the extrapolation will increase with the increase in the mode shapes extracted. Based on these studies and practical limitations in exciting the higher modes, the number of modes extracted was decided to be 10. The parameters obtained from the sensitivity studies are then applied for a specific case to ascertain the improved performance of the proposed methodology.

The study shows that the use of integrated heterogeneous network allows optimal use of the resources. Furthermore, the use of heterogeneous networks gives a better fitness value than the optimized placements achieved through some established methods for sensor placement for homogenous sensors.

The paper shows the merits of treating the placement of different types of sensors as one optimization problem as the information acquired through the use of these sensors is complementary. In addition it outlines a methodology which allows to overcome the issues combining different measurements through the proper use of statistical tools. The results obtained through the combination are still comparable to real quantities like error in estimation and the similarity or difference in modal vectors.

Furthermore, a promising methodology using GA for multi-objective optimization of sensor placement for SHM using strain sensor and accelerometers is presented. This research points at a wide area of research in the multi-type sensor placement optimization where data from different sensors can be seamlessly fused to give more information on the condition of the structure.

The proposed methodology needs to be validated on an experimental setup. Once the validation is carried out it has promise in studying the effect of ambient condition changes, like the temperature, wind, humidity on the performance of the structure. In addition the study can be further extended to include other principles for optimization and applied to fatigue estimation and corrosion assessment through appropriate selection of the cost function.

Acknowledgements

The authors would like to acknowledge the European Commission for funding SmartEN (Grant No. 238726) under the Marie Curie ITN FP7 program, as the research work presented here is supported by this program. The authors would also like to thank COWI, for providing valuable information for the modeling of the Great Belt Bridge for the purpose of this study. The opinions expressed in this paper do not necessarily reflect those of the sponsors.

References

ABAQUS (2011), Analysis User's Manual, version 6.11 ed.

Chakraborty, S. and DeWolf, J.T. (2006), "Development and implementation of a continuous strain monitoring system on a multi-girder composite steel bridge", *J. Bridge Eng*., **11**(6), 753-762.

Chan, T.H.T., Yu, Y., Wong, K.Y. and Li, Z.X. (2008), "Condition-assessment-based finite element modeling of long-span bridge by mixed dimensional coupling method", *J. Mech. Mater*., 260-270.

COWI Internal Report (2000), *East Bridge, Technical Services, Superstructure. Vurdering af Accelerationer for Storebœ lt Hœ ngebro*, Teknisk rapport .

Doebling, S.W., Farrar, C.R. and Prime, M.B. (1998), "A summary review of vibration-based damage identification methods", *Shock Vib. Dig*., **30**(2), 91-105.

Ewins, D.J. (2000), *Modal testing; theory, practice and application*, Research Studies Press

Fedorov, V. and Hackl, P. (1994), "Optimal experimental design: spatial sampling", *Calcutta Statistical Association Bulletin*, **44**, 173-174.

Guyan, R.J. (1965), "Reduction of stiffness and mass matrices", *AIAA J*., **3**(2), 380.

Haupt, L.R. and Haupt, S.E. (2004), *Practical genetic algorithms*, 2nd Ed., John Wiley & Sons Inc.

Heo, G., Wang, M.L. and Satpathi, D. (1997), "Optimal transducer placement for health monitoring of long span bridge", *Soil Dynam. Earthq. Eng*., **16**(7-8), 495-502.

Kammer, D.C. (1991), "Sensor placement for on-orbit modal identification and correlation of large space structures", *J. Guid. Control Dynam*., **14**(2), 251-259.

Kidder, R.L. (1973), "Reduction of structural frequency equations", *AIAA J*., **11**(6), 892.

Larsen, A. (1993), "Aerodynamic aspects of the final design of the 1624 m suspension bridge across the Great Belt", *J. Wind Eng. Ind. Aerod*., **48**, 261-285.

Law, S.S., Li, X.Y., Zhu, X.Q. and Chan, S.L. (2005), "Structural damage detection from wavelet packet sensitivity", *Eng. Struct*., **27**(9), 1339-1348.

Levine-West, M., Mihnan, M. and Kissil, A. (1994), "Evaluation of mode shape expansion techniques for prediction- experimental evaluation", *AIAA J*., **34**(4), 821-829.

Natarajan, S., Howells, H., Deka, D. and Walters, D. (2006*), Optimization of sensor placement to capture riser VIV response*, OMAE, Hamburg, Germany.

O'callahan, J.C., Avitabile, P. and Riemer, R. (1989), "System equivalent reduction expansion process", *Proceeding of the 7th International Modal Analysis Conference*, Las Vegas, USA.

Ö rtel, T., Wagner, J.F. and Saupe, F. (2012), "Integrated motion measurement illustrated by a cantilever beam", *J. Mech. Syst. Signal Pr*., **34**(1-2), 131-145.

Papadimitriou, C. (2004), "Optimal sensor placement methodology for parametric identification of structural systems", *J. Sound Vib*., **278**(4-5), 923-947.

Rytter, A. (1993), *Vibrational-based inspection of civil engineering structures*, PhD Dissertaion, University of Aalborg.

Sim, S.H. (2011), *Decentralized identification and multimetric monitoring of civil infrastructure using smart sensors*, (Doctoral dissertation, University of Illinois).

Talebinejad. I, Fischer, C. and Ansari, F. (2011), "Numerical evaluation of vibration-based methods for damage assessment of cable-stayed bridges", *Comput. -Aided Civil Infrastruct. E.*, **26**(3), 239-251.

Uhl, T. (2009), "Structural Health Monitoring (SHM)-a new interdisciplinary approach to damage detection in mechanical structures", *Pomiary, Automatyka, Kontrola*, **55**(9), 703-706.

Unger, J., Teughels, A. and De Roeck, G. (2005), "Damage detection of a prestressed concrete beam using modal strains", *J. Struct. Eng. - ASCE*, **131**(9), 1456-1463.

Weight, A.J. (2009), "Critical analysis of the Great Belt East Bridge, Denmark", *Proceedings of Bridge Engineering 2 Conference*, April 2009, University of Bath, Bath, UK

Worden, K. and Burrows, A.P. (2001), "Optimal Sensor Placement for fault detection", *Eng. Struct.*, **23**(8), 885-901.

FC