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Crack location in beams by data fusion of fractal dimension features of laser-measured operating deflection shapes

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Abstract. The objective of this study is to develop a reliable method for locating cracks in a beam using data fusion of fractal dimension features of operating deflection shapes. The Katz's fractal dimension curve of an operating deflection shape is used as a basic feature of damage. Like most available damage features, the Katz's fractal dimension curve has a notable limitation in characterizing damage: it is unresponsive to damage near the nodes of structural deformation responses, e.g., operating deflection shapes. To address this limitation, data fusion of Katz's fractal dimension curves of various operating deflection shapes is used to create a sophisticated fractal damage feature, the 'overall Katz's fractal dimension curve'. This overall Katz's fractal dimension curve has the distinctive capability of overcoming the nodal effect of operating deflection shapes so that it maximizes responsiveness to damage and reliability of damage localization. The method is applied to the detection of damage in numerical and experimental cases of cantilever beams with single/multiple cracks, with high-resolution operating deflection shapes acquired by a scanning laser vibrometer. Results show that the overall Katz's fractal dimension curve can locate single/multiple cracks in beams with significantly improved accuracy and reliability in comparison to the existing method. Data fusion of fractal dimension features of operating deflection shapes provides a viable strategy for identifying damage in beam-type structures, with robustness against node effects.

Keywords: damage detection; crack location; multiple cracks; operating deflection shape; data fusion; fractal dimension; laser measurement

1. Introduction

Structural damage detection has been a research focus in civil, aerospace, mechanical, and military fields for the last few decades (Ren and Roeck 2002, Su *et al*. 2002, Xu *et al*. 2007, Sohn *et al*. 2008, Cao and Qiao 2009, Li *et al*. 2010, Wang and Wu 2011, Farhidzadeh *et al*. 2013, Li *et*

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al. 2013, Fayyadh and Razak 2013). In particular, vibration-based damage identification has been widely investigated, with the major task of developing methods for damage location and quantification (Fugate *et al*. 2001, Wang *et al*. 2006, Cao *et al*. 2012). Damage location, preceding damage quantification, is a critical component of vibration-based damage identification (Xu *et al*. 2007, Wu and Wang 2011, Cao *et al*. 2013). For damage location, the particular modal parameter, mode shape, is somewhat superior to the modal parameters of natural frequency and modal damping, because a mode shape conveys spatial characteristics of the structure being inspected (Ren and Roeck 2002, Parloo *et al*. 2003, Zhong and Oyadiji 2011, Roy and Ray-Chaudhuri 2013), whereas the other two lack structural spatial information. Several damage features suitable for damage location have been derived from mode shapes, such as the curvature mode shape (Pandey *et al*. 1991, Cao and Qiao 2009, Xiang *et al*. 2013), strain energy mode shape (Shi *et al*. 2000, Sazonov and Klinkhachorn 2005), and wavelet coefficients of mode shape (Cao *et al*. 2012, 2013).

In relatively recent years, the fractal dimension curve of a mode shape has been recognized as a promising feature for characterizing damage from the new perspective of fractal dynamics (Hadjileontiadis *et al*. 2005, Cao *et al*. 2006, Wang and Qiao 2007, Cao and Qiao 2009, Li *et al*. 2011, An and Ou 2012, Bai *et al*. 2012, Yang *et al*. 2012, Zhou *et al*. 2013, Zhang *et al*. 2013, Bai *et al*. 2014). This feature allows instant determination of damage location by its singular peak. Hadjileontiadis *et al*. (2005) proposed the basic method of using the fractal dimension curve of a lower-order mode shape to locate and quantify damage in beams. Bai *et al*. (2012) improved this method by using an affine transformation to preprocess mode shapes, expanding the scope of application from lower-order mode shapes to higher-order mode shapes for detecting slight damage in beams.

Despite the proven ability of a fractal dimension curve to locate damage (Hadjileontiadis and Doukab 2007, Qiao and Cao 2008), its efficacy is affected by the mode shape adopted for damage detection (Salawu and Williams 1994). Notably, dependence on the selected mode shape is a common characteristic not limited to the fractal dimension curve but observed for most damage features. In essence, a mode shape has positional sensitivity to damage, and that positional sensitivity is determined by the magnitude of curvature of the mode shape at the damage location (Cao *et al*. 2011). If the damage is located at the node of a particular mode shape with zero curvature therein, it will have no influence on the mode shape. This phenomenon is referred to as the node effect of a mode shape (Inman 2000). With node effects for a mode shape, the associated fractal dimension curve is not reactive to any damage, and is accordingly unsuited for damage localization. In general, as damage approaches a node, the efficacy of the fractal dimension curve in portraying damage is progressively impaired. In structural damage diagnosis applications, the location of damage is commonly unknown in advance, due to the inverse problem of damage detection, and therefore it is difficult to choose an appropriate mode shape with a high magnitude of curvature at the damage location for use in damage localization. That being the case, locating damage relying on a single fractal dimension curve from a sole mode shape is unreliable in practical damage diagnosis.

To deal with this limitation, this study explores data fusion (Li *et al*. 2008, Su *et al*. 2009, Federico 2013) of various fractal dimension curves to improve the reliability of damage localization for beam-type structures, with robustness against node effects. With the proposed data fusion, the most favorable damage feature of a certain fractal dimension curve will dictate the fused result of all fractal dimension curves, largely overcoming the node effect. Moreover, operating deflection shapes (ODSs; Schwarz and Richardson 1999, Pai and Young 2001, Waldron *et al*. 2002, Xu *et al*. 2013) rather than mode shapes are used to create various fractal dimension curves. Compared with mode shapes, a greater number of ODSs can be obtained from a cracked beam, providing more possibilities of damage being remote from nodes in one or several ODSs which produce the most prominent damage feature, dictating the fused result of damage detection. With the aid of a scanning laser vibrometer (SLV), a series of ODSs can easily be acquired from a beam subjected to various excitation frequencies. The proposed method is here numerically verified and further experimentally validated using cracked cantilever beams.

2. Sophisticated fractal damage feature

A sophisticated fractal damage feature is formulated in a sequential manner: affine transformation; the Katz's fractal dimension (KFD) curve; and the overall KFD curve generated by the technique of data fusion.

2.1 Affine transformation

For a measured ODS $S:(x_i, y_i)$, variables x_i and y_i are individually rescaled to the interval [0, 1] to achieve the normalized form of $S:(x_i, y_i)$: $S^*:(x_i^*, y_i^*)$, with the asterisk denoting normalization. $S^*:(x_i^*, y_i^*)$ is treated by an affine transformation

$$\begin{Bmatrix} x_i' \\ y_i' \end{Bmatrix} = A \begin{Bmatrix} x_i^* \\ y_i^* \end{Bmatrix}, \quad A = \begin{bmatrix} 1 & 0 \\ \sin\theta & (\cos\theta)/k \end{bmatrix} \tag{1}$$

where $S':(x_i', y_i')$ is the affine transformation of $S^*:(x_i^*, y_i^*)$, and A is the affine transformation matrix, with k and θ being adjustable parameters. A wide spectrum of values for k and θ is acceptable to derive $S':(x_i', y_i')$ that can eliminate the local extrema of the $S^*:(x_i^*, y_i^*)$ while preserving its topological characteristics containing damage information. In what follows, $k = 100$ and $\theta = 60^\circ$ are adopted for all numerical and experimental cases. As clarified by Bai *et al*. (2012), removal of extrema using the affine transformation is necessary preprocessing for using the KFD analysis of a structural deformation to portray damage, due to the fact that a local extremum of the structural deformation easily evokes a false warning of the occurrence of damage.

2.2 KFD curve

The KFD is a measure of the complexity of a 1D signal from the perspective of fractal dynamics. An ODS of a beam is a particular 1D signal. Let $W_J':(x_j', y_j')$ be an arbitrary segment of $S':(x_i', y_i')$, covering n sampling points. The KFD is defined on $W_J':(x_j', y_j')$ as by Katz (1988)

$$KFD(W_J') = \frac{\log_{10}(n)}{\log_{10}(n) + \log_{10}(d/L)} \tag{2}$$

where L is the total length of the segment, the sum of the distances between successive sampling points, and d is the maximum distance between the first point and any other point. The KFD displays its great sensitivity to any subtle change in the complexity of the signal being inspected. A subtle change in complexity of a structural response manifests an unexpected minor perturbation of the structure. Damage is a perturbation occurring to a beam, causing a change in the complexity of ODSs of the beam; therefore damage can be detected by using the KFD to inspect change in the complexity of the ODS.

Given that $W_J^{'}:(x_j^{'},y_j^{'})$ is a sliding window, by driving this sliding window to cover $S^{'}:(x_i^{'},y_i^{'})$ step by step and evaluating the KFD using the sampling points within the window at each step, a KFD curve consisting of a sequence of KFD values can be produced as the sliding window traverses $S^{'}:(x_i^{'},y_i^{'})$. This operation is expressed by

$$\Gamma_{S'} = \Im(S^{'}:(x_i^{'},y_i^{'})) = \Im\left\{W_J^{'}(x_j^{'},y_j^{'})\right\}_{J=1}^{M} \tag{3}$$

where $\Im$ denotes the operator that can sequentially evaluate KFD for sliding windows $W_J^{'}:(x_j^{'},y_j^{'})$, $J=1,2,\cdots,M$, ($M = N\text{-}n$, N is the total number of sampling points) and $\Gamma_{S'}$ is the KFD curve of $S^{'}:(x_i^{'},y_i^{'})$. $\Gamma_{S'}$ quantifies the local complexity of the signal $S^{'}:(x_i^{'},y_i^{'})$. Any singularity of $\Gamma_{S'}$ signifies a change in the local complexity of $S^{'}:(x_i^{'},y_i^{'})$, suggesting a perturbation occurring to the structure under investigation. This relation establishes the basic rationale for using the KFD curve of an ODS to diagnose damage in a beam.

To facilitate data fusion for damage identification, a KFD curve is rescaled by the typical standardization method

$$\Gamma'_{S'} = \frac{\Gamma_{S'} - \mu(\Gamma_{S'})}{\sigma(\Gamma_{S'})} \tag{4}$$

where $\mu(\Gamma_{S'})$ and $\sigma(\Gamma_{S'})$ are the mean and the standard deviation of the $\Gamma_{S'}$ and $\Gamma'_{S'}$ is the standardized $\Gamma_{S'}$. This rescaling does not alter the singularity characteristics of the original KFD curve, and it renders various KFD curves comparable at the common scale to lay the foundation for data fusion.

2.3 Overall KFD Curve

Ideally, the KFD curve of an ODS can characterize damage by its singular peak. However, when the damage is located near the node of an ODS, causing an insignificant change to the complexity of the ODS, the KFD curve usually fails to reflect the damage. To accommodate this deficiency, the concept of data fusion is introduced to tailor KFD curves for damage characterization. As an alternative to a single KFD curve, a group of KFD curves arising from multiple ODSs at different excitation frequencies are fused to produce an overall KFD curve. Although various techniques of data fusion are available in the literature (Federico 2013), the simplest method of averaging is adopted as it can achieve sound effects:

$$\bar{\Gamma} = \frac{1}{K}\sum_{k=1}^{K}\Gamma'_{S'_k} \tag{5}$$

where K is the number of KFD curves for data fusion, $\Gamma'_{S'_k}$ is the standardized KFD curve of S'_k in light of Eq. (4), and $\bar{\Gamma}$ is the overall KFD curve as a result of data fusion.

With data fusion, the ODSs corresponding to different excitation frequencies commonly have distinct node locations; moreover, separate nodes of ODSs can be easily obtained by a set of different excitation frequencies. Hence, damage overlooked by a KFD curve due to the node effect should be detected by another one or several KFD curves, finally forming a damage signature in the overall KFD curve.

3. Numerical demonstration

3.1 Finite element model

Detection of damage based on the proposed method is demonstrated using a cantilever beam with a double-edged fine crack (Fig. 1). The beam has the dimensions of length (L) 1.2 m, width (B) 0.02 m, and depth (H) 0.02 m, and its Young's modulus and material density are taken as 206 GPa and 7850 kg/m^3, respectively. The crack is specified primarily by two parameters: crack location ratio $\beta = L_c/L$, with L_c the crack location distance from the fixed end, crack depth ratio $a = 2e/H$, with e the reduction in depth from each surface. The crack extends along the beam span for 0.1% length of the beam. At the free end of the beam, a transverse mono-frequency excitation, $F_0\sin(\Omega t+\psi)$, is exerted to excite the beam. The modes of the beam are superposed by the following regime to form the ODSs (Inman 2000)

$$S(\Omega) = \sum_i \frac{W_i(\omega_i)[W_i(\omega_i)F]}{\omega_i^2 - \Omega^2} \tag{6}$$

where $W_i(\omega_i)$ is the ith-order mode shape of the beam at the natural frequency ω_i, $S(\Omega)$ is the ODS at excitation frequency Ω, and F is the magnitude of excitation.

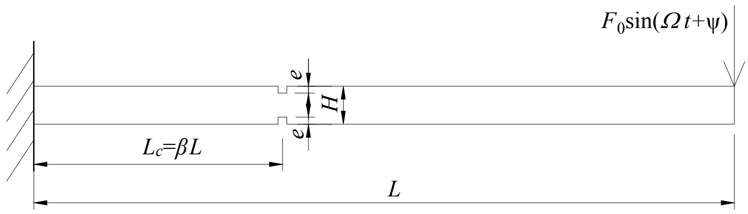

Fig. 1 A cantilever beam with a crack

The numerical beam is built using 1000 Euler–Bernoulli beam finite elements, with the crack modeled using a specific element of reduced size *e* for each side in depth. Various transverse excitations, with $F_0 = 1$ and $\psi = 60^\circ$ in common and distinguished by different values of Ω, are individually exerted to generate a group of ODSs. Table 1 lists the crack cases used for demonstration of the method.

Table 1 Numerical crack scenarios of cantilever beam

Crack scenario	α=0.1		
	β_1	β_2	β_3
I	0.3	/	/
II	0.2	0.5	0.8

3.2 Single crack detection

Use of the data fusion of KFD curves to locate a single crack is examined using Crack Scenario I. For this crack scenario, four frequency points between the 5th and 8th natural frequencies, labeled Ω(5000Hz), Ω(6500Hz), Ω(8000Hz), and Ω(10000Hz) in the displacement frequency response function (FRF) diagram (Fig. 2), are used as excitation frequencies to individually excite the beam to generate ODSs (Fig. 3). Without loss of generality, we prefer excitation frequencies distant from the main peaks of the FRF to obtain particular ODSs with much lower dynamic energy, conveying a smaller quantity of damage information than ODSs at frequency points near the main peak. Inspection of the slight damage information provides an opportunity to assess the performance of the proposed method comprehensively.

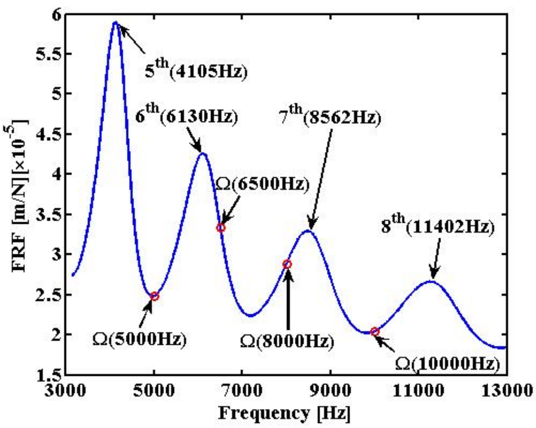

Fig. 2 Four excitation frequencies 5000 Hz, 6500 Hz, 8000 Hz, and 10000 Hz in-between the 5th and 8th natural frequencies for Crack Scenario I

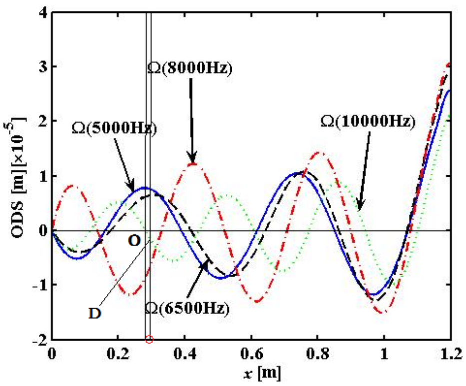

Fig. 3 Four ODSs for Crack Scenario I subjected to excitation frequencies 5000 Hz, 6500 Hz, 8000 Hz, and 10000 Hz, respectively

The procedure described in Section 2 is implemented to create a sophisticated fractal damage feature. For the four ODSs (Fig. 3), Fig. 4 shows the associated affine transformed ODSs with the extrema clearly eliminated but with damage information preserved. For these affine transformed ODSs, Fig. 5 presents the standardized KFD curves using Eq. (4), among which the curves individually responding to Ω(5000Hz), Ω(6500Hz), Ω(8000Hz) bear abruptly rising singular peaks, whereas that for Ω(10000HZ) lacks any singular peak. A singular peak signifies the change in the complexity of that ODS, clearly pinpointing the location of the crack. That being the case, the three KFD curves with a singular peak can locate the crack, but the KFD curve without a singular peak fails to detect it. As illustrated in Fig. 3, this failure of crack detection can be approximately attributed to the node effect of the crack marked "D" being near the node of the ODS, labeled "O". Without prior knowledge of damage in practical damage diagnosis, it is unavoidable to employ an ODS involving the node effect to identify damage, leading to a false result of damage identification. Hence, a single KFD curve of a single ODS cannot produce a trustworthy result of damage detection.

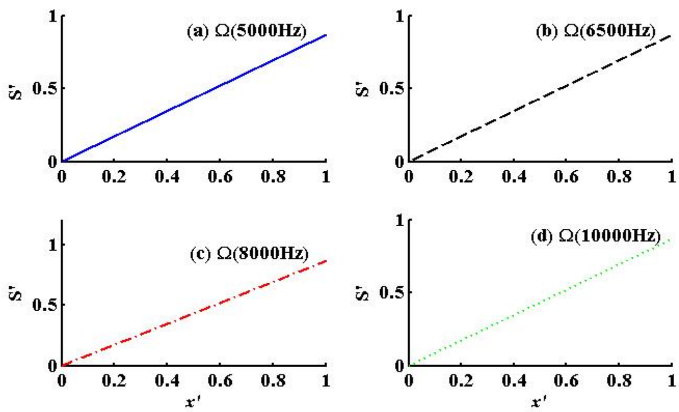

Fig. 4 Affine transformed ODSs for Crack Scenario I subjected to excitation frequencies 5000 Hz (a), 6500 Hz (b), 8000 Hz (c), and 10000 Hz (d)

As an alternative to a single KFD curve, all the standardized KFD curves (Fig. 5) are fused according to Eq. (5) to generate an overall KFD curve (Fig. 6). Dramatically, a prominent singular peak dominates the overall KFD curve, unambiguously indicating the location of the crack. Thus, the overall KFD curve locates the crack reliably and exclusively.

3.3 Multiple crack detection

Use of the data fusion of KFD curves to locate multiple cracks is investigated using Crack Scenario II. For this crack scenario, four frequencies: 3500, 5500, 7500, and 9500 Hz, distant from the relevant 5^{th} – 8^{th} natural frequencies, are arbitrarily selected as excitation frequencies to individually excite a beam with three cracks, with the resulting ODSs shown in Fig. 7.

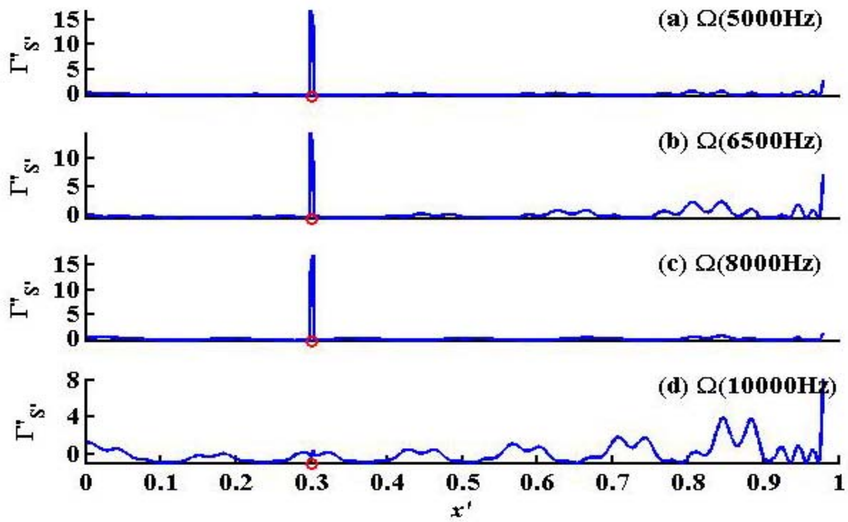

Fig. 5 Standardized KFD curves of ODSs for Crack Scenario I subjected to excitation frequencies 5000 Hz (a) 6500 Hz (b), 8000 Hz (c), and 10000 Hz (d). The red circle indicates the actual crack location

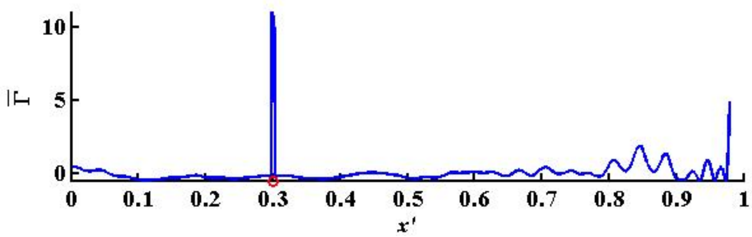

Fig. 6 Overall KFD curve generated by data fusion

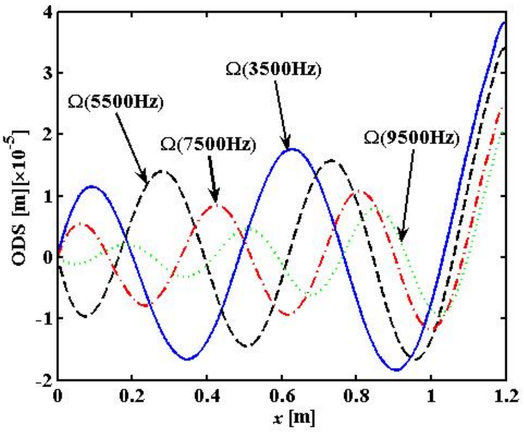

Fig. 7 Four ODSs for Crack Scenario II subjected to excitation frequencies 3500 Hz, 5500 Hz, 7500 Hz, and 9500 Hz

For the ODSs in Fig. 7, after normalization, the affine transformation is performed to eliminate the extrema of each normalized ODS. Following this processing, the KFD curves (Fig. 8) are obtained from the affine transformed ODSs. In general, a sole KFD curve cannot account for three cracks at the same time. This case entails the use of data fusion to produce a sophisticated overall damage feature. Use of Eq. (5), fusion of all the standardized KFD curves, produces an overall KFD curve (Fig. 9) in which three prominent peaks unambiguously indicate the three cracks, with their locations accurately identified.

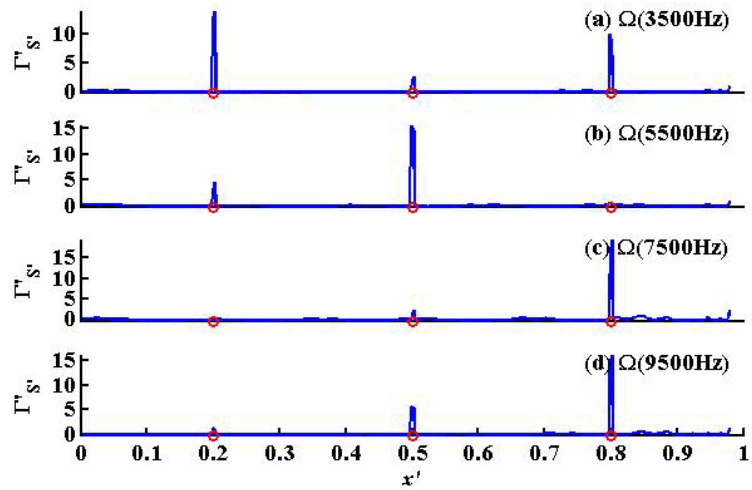

Fig. 8 Standardized KFD curves of ODSs for Crack Scenario II subjected to excitation frequencies 3500 Hz (a) 5500 Hz (b), 7500 Hz (c), and 9500 Hz (d)

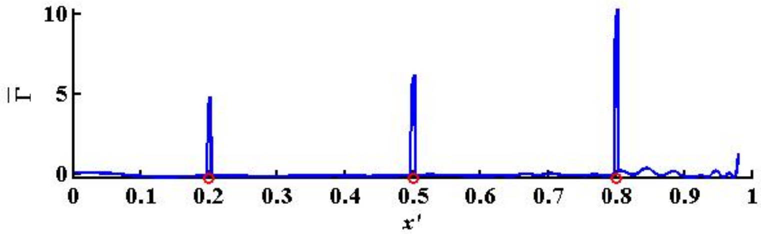

Fig. 9 Overall KFD curve resulting from data fusion

4. Experimental validation

4.1 Experimental setup

Cantilever steel beams of length 1000 mm, width 20 mm and depth 10 mm are used to create two test specimens (Table 2): Specimen I, containing a single-sided through-width crack located 300 mm (β=0.3) from the fixed end; Specimen II (Fig. 10), bearing three cracks located 130 mm (β_1=0.13), 300 mm (β_2=0.3) and 570 mm (β_3=0.57) from the fixed end, respectively. Each crack is 1 mm wide (along the beam span) and 2 mm deep (α=0.2).

An electromechanical shaker is used to excite the test specimen near its fixed end, harmonically and perpendicularly. While the specimen is vibrating, a SLV (Polytec PSV-400) is employed to register the velocities at sampling points evenly distributed along the mid-line of the specimen. The synchronous velocities at 121 measurement points constitute the ODSs of the specimen. Figure 11 shows the experimental setup for measuring ODSs using the SLV.

Table 2 Test specimens of cracked beam

Test specimen	α=0.2		
	β_1	β_2	β_3
I	0.3	/	/
II	0.13	0.3	0.57

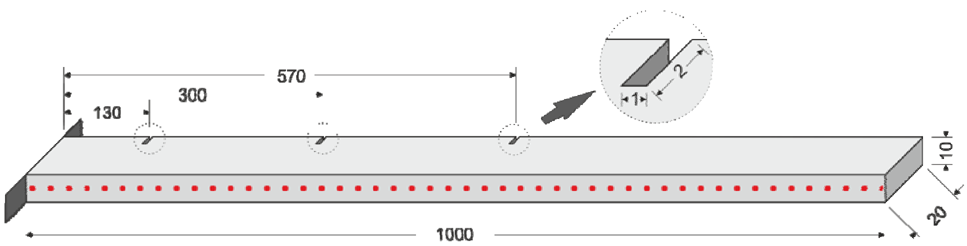

Fig. 10 Dimensions in millimeters of test specimen with three cracks

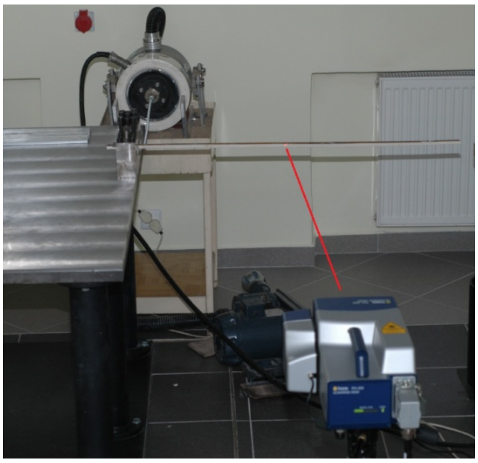

Fig. 11 Experimental setup using SLV to measure ODSs of cracked beam excited by electromechanical shaker

4.2 Results

For Specimen I with a single crack, eight ODSs at excitation frequencies 290, 1903, 3139, 3962, 4770, 5641, 8689, and 9742, as shown in Fig. 12, are acquired. For these ODSs, the associated standardized KFD curves (Fig. 13) can be roughly categorized into three groups: In Figs. 13(f) and 13(h), each KFD curve bears a singular peak, explicitly pinpointing the location of the damage; in Figs. 13(c), 13(d), and 13(g), each KFD curve contains a slight singular peak that is distinguishable but badly compromised by other peaks, from which the crack cannot be reliably determined; in Figs. 13(a), 13(b), and 13(e), no peak of any KFD curve appears prominent at the crack location, resulting in failure to detect damage. In practical damage diagnosis, without prior knowledge of the existence of a crack, it is of little validity to identify a crack relying on a sole KFD curve from a single ODS.

In contrast, the overall KFD curve (Fig. 14) derived from the data fusion of all the standardized KFD curves (Fig. 13) bears a striking singular peak, distinguishable from the other fluctuations of the curve, unambiguously indicating the occurrence and location of the crack.

For Specimen II with three cracks, ten ODSs are acquired at excitation frequencies 291, 1393, 1927, 3124, 4053, 5073, 6640, 7725, 8825, and 12250 Hz, as shown in Fig. 15. For these ODSs, Fig. 16 depicts the associated standardized KFD curves. In general, a given KFD curve can signify null, one, or two cracks; few KFD curves show three singular peaks that can account for the three cracks. Hence, without prior information as to the existence of a crack, it is invalid to attempt to identify three cracks simultaneously relying on a sole KFD curve from a single ODS.

In contrast to any single KFD curve, the overall KFD curve (Fig. 17) arising from the data fusion of all the standardized KFD curves (Fig. 16) is dominated by three prominent singular peaks standing out from the fluctuations of the curve. These peaks demonstrably locate the locations of the three cracks without ambiguity. Clearly, data fusion endows the overall KFD curve arising from various ODSs with substantially improved validity in locating damage compared to the sole KFD curve from any single ODS.

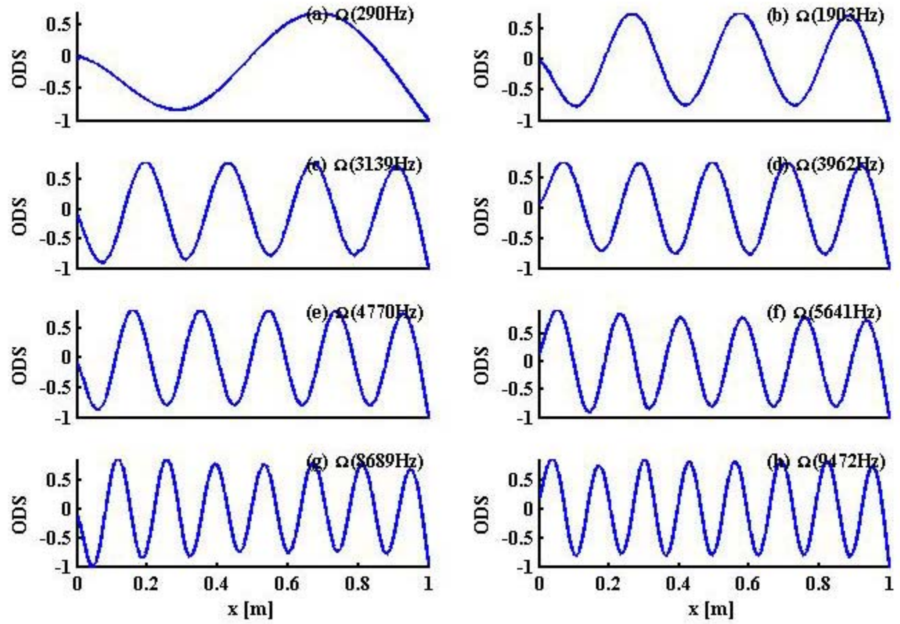

Fig. 12 ODSs for Specimen I subjected to excitation frequencies 290 Hz (a), 1903 Hz (b), 3139 Hz (c), 3962 Hz (d), 4770 Hz (e), 5641 Hz (f), 8689 Hz (g), and 9742 Hz (h)

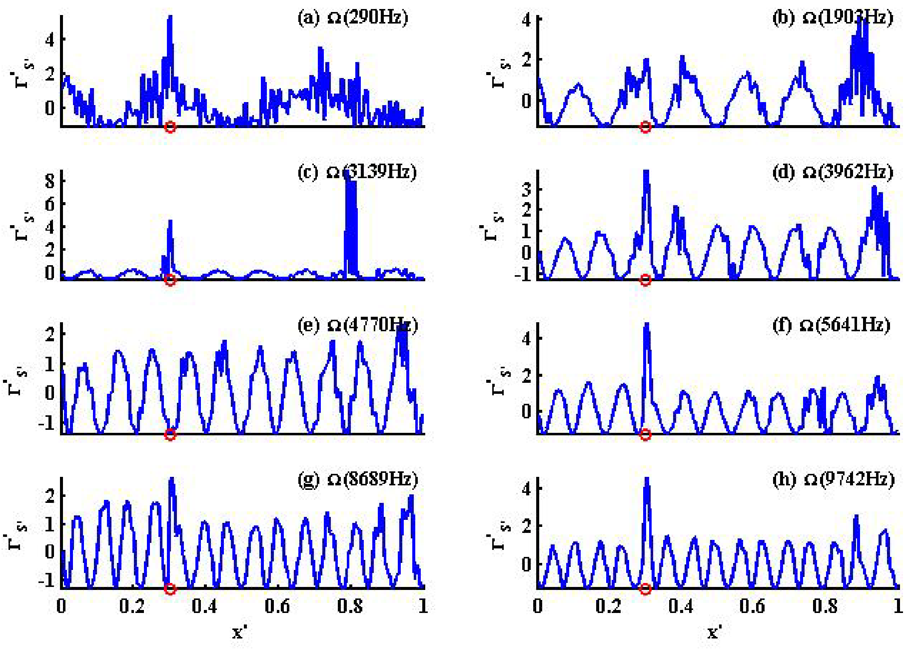

Fig. 13 Standardized KFD curves of ODSs for Specimen I subjected to excitation frequencies 290 Hz (a), 1903 Hz (b), 3139 Hz (c), 3962 Hz (d), 4770 Hz (e), 5641 Hz (f), 8689 Hz (g), and 9742 Hz (h)

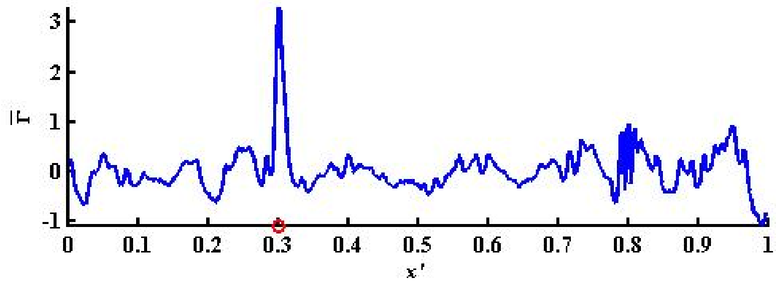

Fig. 14 Overall KFD curve for Specimen I with a single crack

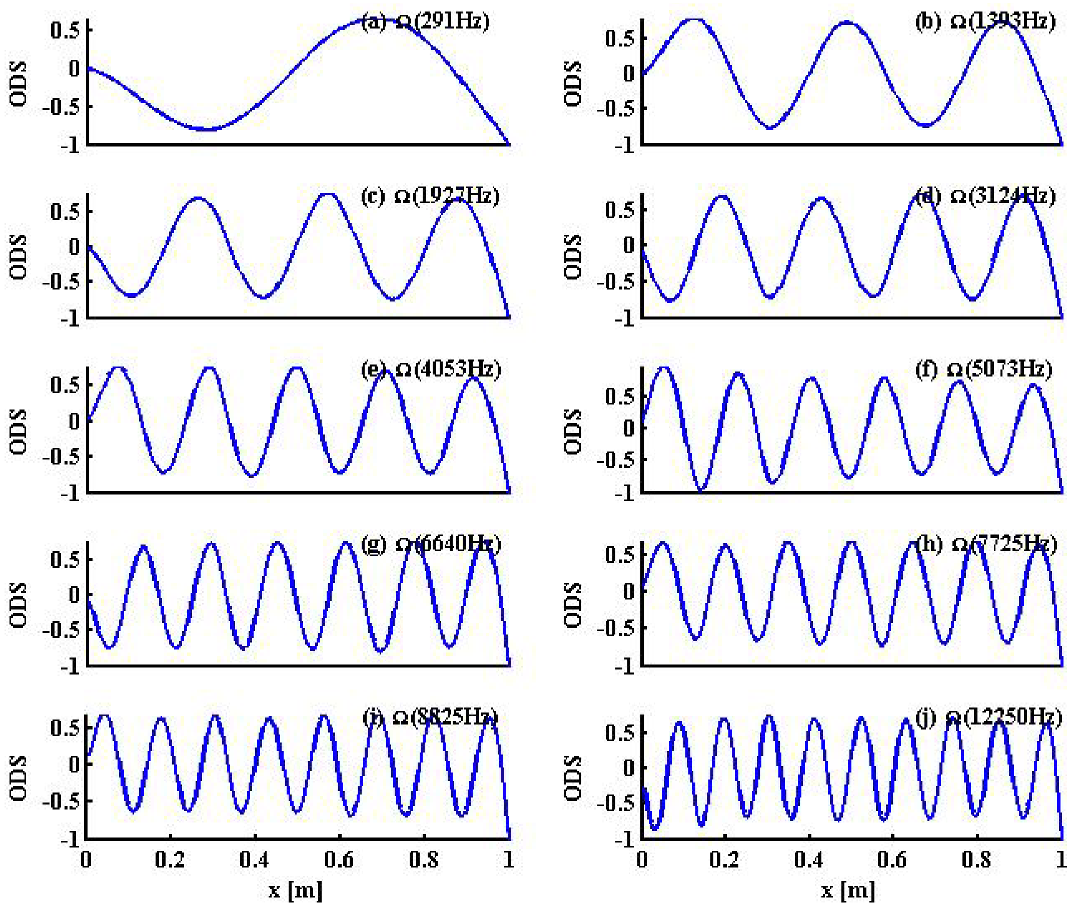

Fig. 15 ODSs for Specimen II subjected to excitation frequencies 291 Hz (a), 1393 Hz (b), 1927 Hz (c), 3124 Hz (d), 4053 Hz (e), 5073 Hz (f), 6640 Hz (g), 7725 Hz (h), 8825 Hz (i), and 12250 Hz (j)

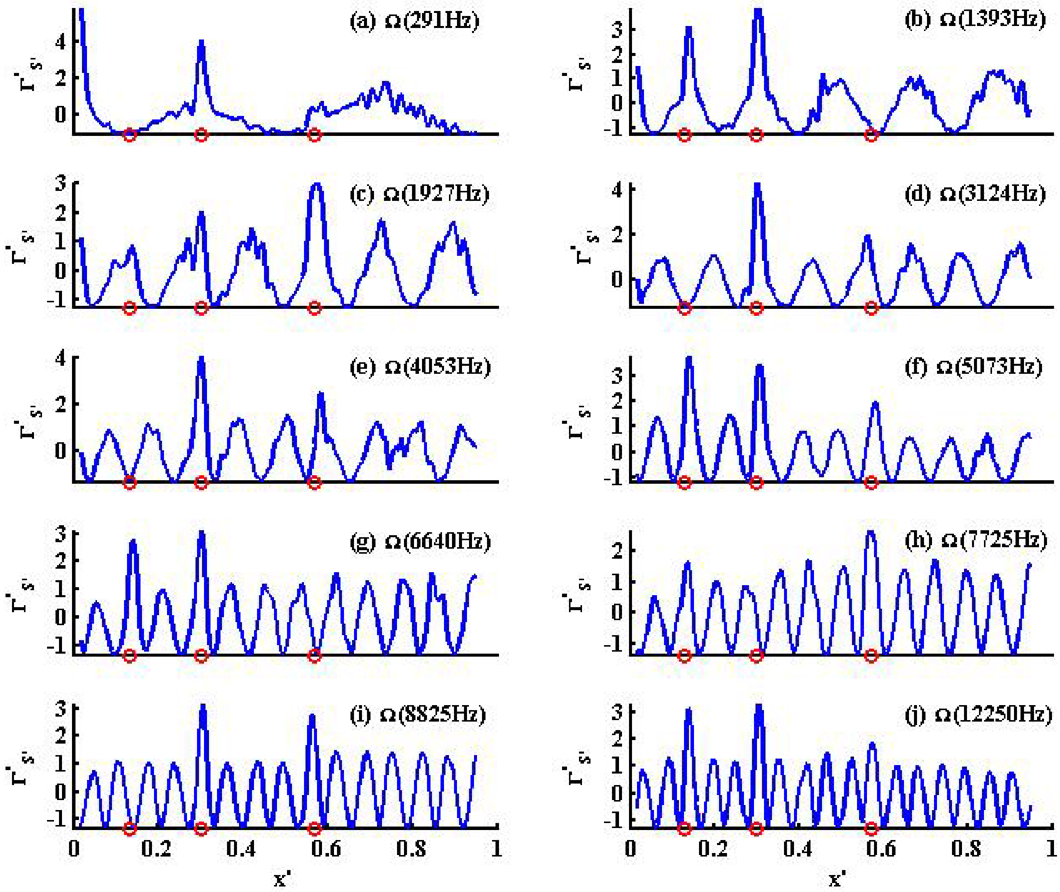

Fig. 16 Standardized KFD curves of ODSs for Specimen II subjected to excitation frequencies 291 Hz (a), 1393 Hz (b), 1927 Hz (c), 3124 Hz (d), 4053 Hz (e), 5073 Hz (f), 6640 Hz (g), 7725 Hz (h), 8825 Hz (i), and 12250 Hz (j)

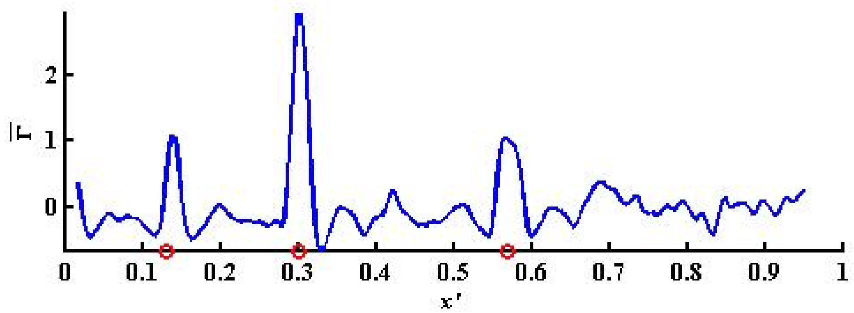

Fig. 17 Overall KFD curve for Specimen II with three cracks

5. Conclusions

A method of data fusion of fractal dimension curves is proposed for damage location in beam-type structures. Fusion of the fractal dimension curves is used to improve the reliability of damage detection. Rather than the mode shape, the ODS is employed as a generalized dynamic response for use in damage characterization. The performance of the proposed method is demonstrated by crack detection in numerical cases of cantilever beams with single/multiple cracks. The applicability of the method is experimentally validated on two cracked steel beams, using a SLV to acquire ODSs of the beams. The results show that the proposed method can pinpoint the location of the damage by tolerating the node effect of ODSs, without requiring the use of a healthy ODS serving as the baseline, and without prior knowledge of either the material properties or the boundary conditions of the structure.

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