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A novel transmissibility concept based on wavelet transform for structural damage detection

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Abstract. A novel concept of transmissibility based on a wavelet transform for structural damage detection is presented in this paper. The main objective of the research was the development of a method for detecting slight damage at the incipient stage. As a vibration-based approach, the concept of transmissibility has attracted considerable interest because of its advantages and effectiveness in damage detection. However, like other vibration-based methods, transmissibility-based approaches suffer from insensitivity to slight local damage because of the regularity of the traditional Fourier transform. Therefore, the powerful signal processing techniques must be found to solve this problem. Wavelet transform that is able to capture subtle information in measured signals has received extensive attention in the field of damage detection in recent decades. In this paper, we first propose a novel transmissibility concept based on the wavelet transform. Outlier analysis was adopted to construct a damage detection algorithm with wavelet-based transmissibility. The feasibility of the proposed method was numerically investigated with a typical six-degrees-of-freedom spring-mass system, and comparative investigations were performed with a conventional transmissibility approach. The results demonstrate that the proposed transmissibility is more sensitive than conventional transmissibility, and the former is a promising tool for structural damage detection at the incipient stage.

Keywords: structural health monitoring; damage detection; transmissibility; wavelet transform; outlier analysis

1. Introduction

Over recent decades, vibration-based damage detection methods have been widely applied in civil, mechanical, and aerospace engineering (Yi *et al.* 2011a, Yi *et al.* 2013). These methods are based on the fact that the modal parameters (i.e., natural frequencies, mode shapes, and modal damping) are functions of the physical properties of a given structure (i.e., mass, damping, and stiffness). Therefore, a change in the modal parameters may reflect a change in the physical properties, indicating damage occurrence, damage location, and damage severity. Some researchers have used natural frequencies to detect damage in structures (Cawley and Adams 1979, Ostachowicz and Krawczuk 1990, Biswas *et al.* 1990, Lee and Chung 2000) because of their convenient acquisition and high accuracy. However, it is difficult to determine the location of damage with natural frequencies for at least three reasons. First, damage occurrence at different

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locations may produce the same frequency change. Second, the change in natural frequency may be very small at the early stage of damage occurrence and thus may be undetectable. Third, the natural frequencies tend to be affected by the temperature of the environment. Therefore, to overcome these difficulties, other modal parameters such as mode shapes are adopted to detect the structural damage. However, in general, only the first several modes can be accurately obtained in experimental modal analysis, and the lower modes are not sensitive to slight local damage; thus, damage indices that are more sensitive need to be constructed to detect such damage. The curvature of mode shapes, modal flexibility, and modal strain energy, etc., may improve the accuracy of damage detection. These damage indices contain local information on the structure and have been found to be more damage-sensitive than the natural frequencies and mode shapes. Kim *et al*. (2003) used natural frequencies and mode shapes of the first two bending modes of a beam-type structure to evaluate frequency-based and mode-shape-based methods. Pandey *et al*. (1991) proposed using changes in the curvature of mode shapes to detect and localize damage. Shi *et al*. (2000) demonstrated the effectiveness of using modal strain energy to localize single and multiple damage in a frame structure. Catbas *et al.* (2008) utilized deflections and curvatures derived from modal flexibility to detect damage on a steel grid and a three-span bridge. Park and Park (2005) proposed a method to detect damage within a substructure by means of frequency response functions. Yang (2011) presented a method for structural damage identification based on flexibility disassembly. And some researchers (Yi *et al.* 2011b, Yi *et al.* 2012) addressed the sensor placement for structural health monitoring, which is a critical issue for vibration-based damage detection. The literature review of such damage detection methods can be found in Doebling *et al.* (1998), Teughels and De Roeck (2005), Alvandi and Cremona (2006), and Yan *et al.* (2007).

As one vibration-based approach, the concept of transmissibility has also attracted considerable interest in engineering fields. Transmissibility-based approaches have been successfully applied in system identification, structural modification, damage detection, etc. The transmissibility concept for a system with multiple degrees of freedom (DOF) is described in Ribeiro *et al.* (2000) and Maia *et al.* (2001). Transmissibility is defined as a function that represents the relationship between the responses at two coordinates of a structure and is originally used to estimate the responses from the unknown coordinates. The main advantages of transmissibility are that no modal identification needs to be performed and no analytic or numerical model of the structure is necessary (Maia *et al.* 2011). Recently, the transmissibility concept has undergone rapid development and has been used for various purposes. Some experiments have shown the feasibility of estimating the frequency response function of structures by means of this concept (Almeida *et al.* 2010, Urgueira *et al.* 2011). A statistical method for damage detection based on transmissibility has been proposed in Canales *et al.* (2009); Steenackers *et al*. (2007) illustrated the effectiveness of using the transmissibility concept for finite-element model updating; Devriendt (2008) proposed the transmissibility function to identify modal parameters; and Law *et al.* (2011) demonstrated the effectiveness of reconstructing the structural response based on the transmissibility concept. More applications of damage identification with transmissibility can be found in Sampaio *et al.* (2001), Johnson and Adams (2002), Maia *et al.* (2011), and Zhu *et al.* (2011). In fact, transmissibility reflects the dynamic stiffness or flexibility of the structures by means of the Fourier transform in the frequency domain. In damage detection, transmissibility-based approaches suffer from the same drawbacks as those of all Fourier-based approaches, i.e., the measured low-frequency-band characteristics are not sensitive to slight local damage in structures, because the Fourier transform is global and describes the overall regularity of signals while lacking the ability to find the local perturbations in structural stiffness caused by slight damage.

One effective way to solve this problem is to use wavelet transform, which is a newly emerging mathematical and signal processing tool. The advantage of wavelet transform over the traditional Fourier transform lies in the former's ability to have an adjustable window focus. Instead of using sine and cosine functions as bases, wavelet transform adopts local functions, which are dilated and shifted from their mother wavelet, and decomposes the original signal into multiple levels of details and approximations. Because of this time-frequency multi-resolution property, it is known as a "mathematical microscope" and has been widely used in damage identification and structural health monitoring. Wavelet-based damage detection can be classified into three categories: (1) variation of wavelet coefficients, (2) local perturbation of wavelet coefficients in a space domain, and (3) reflected waves caused by local damage (Kim and Melhem 2004). Wavelet transform not only detects abrupt loss of stiffness in a structure, but also estimates the cumulative damage. In addition, it can be used to perform the detection, localization, and quantification of damage. Gurley and Kareem (1999) summarized the potential application of wavelet transforms in earthquake, wind, and ocean engineering. Hou *et al.* (2000) demonstrated that wavelet transform had the ability to detect the damage occurrence in time domain when a simple structural model was subjected to an earthquake event. Sun (2002) presented a structural damage assessment based on a wavelet packet transform. Hong *et al.* (2002) used a continuous wavelet transform to estimate the Lipschitz exponent for identifying the location and extent of damage in a beam. Bayissa *et al.* (2008) proposed a damage detection approach for beam-type structures based on a combination of a continuous and a discrete wavelet transform. Jiang *et al.* (2012) investigated the slopes of mode shapes and proposed a method for detecting cracks in beams based on a complex continuous wavelet transform.

The objective of our study was to develop a novel transmissibility concept based on wavelet transform for detecting slight damage at the incipient stage. First, a novel damage index, designated as wavelet-based transmissibility, was theoretically formulated by means of a wavelet transform in the motion equation of the discretized structural system. The proposed transmissibility provides refined information on the dynamic stiffness or flexibility with an adaptive window in the wavelet domain, which is sensitive to the local change induced by structural damage. Second, outlier analysis was introduced to develop an algorithm for damage detection. The squared Mahalanobis distances of the wavelet-based transmissibility between the initial and unknown states were calculated for statistically recognizing the structural damage. Finally, to verify the feasibility of the proposed method, numerical simulations were performed with a six-degrees-of-freedom (6-DOF) mass–spring system. The detection ability of the conventional transmissibility was also investigated with the same system.

2. Theoretical developments

2.1 Wavelet transform and multi-resolution analysis

For the random function $f(t) \in L^2(R)$, the continuous wavelet transform is defined as

$$W_f(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} f(t) \bar{\Psi}(\frac{t-b}{a}) dt \tag{1}$$

$$\Psi_{a,b}(t) = \frac{1}{\sqrt{a}} \Psi(\frac{t-b}{a}) \quad a,b \in R, a \neq 0 \tag{2}$$

where $\Psi(t)$ is a mother wavelet function, and a and b denote the dilation and translation parameters, respectively. The bar over $\Psi(t)$ represents its complex conjugate.

In practice, the continuous wavelet needs to be replaced with a discrete wavelet transform.

$$a = 2^j; \; b = 2^j k \quad j,k \in Z \tag{3}$$

where a and b are in a dyadic form, and Z is the set of positive integers. Substituting Eq. (3) into Eq. (2), the discrete wavelet function can be rewritten as

$$\Psi_{j,k}(t) = 2^{-j/2} \Psi(2^{-j} t - k) \tag{4}$$

Here, $\Psi_{j,k}(t)$ constitutes an orthonormal basis for $L^2(R)$. By using the orthonormal basis, the wavelet expansion of $f(t)$ and the coefficients of the wavelet expansion are defined as

$$f(t) = \sum_j \sum_k \alpha_{j,k} \Psi_{j,k}(t) \tag{5}$$

and

$$\alpha_{j,k} = \int_{-\infty}^{+\infty} f(t) \bar{\Psi}_{j,k}(t) dt \tag{6}$$

In the discrete wavelet transform, the signals can be represented by its approximations and details based on multi-resolution analysis, which is carried out with the Mallat algorithm (Mallat 1989).

The detail coefficient (higher-frequency part) at level j is defined as

$$D_j = \sum_{k \in Z} \alpha_{j,k} \Psi_{j,k}(t) \tag{7}$$

and the approximation coefficient (lower-frequency part) at level j is defined as

$$A_J = \sum_{j > J} D_j \tag{8}$$

It can be seen that the lower-frequency part is further decomposed, whereas the higher-frequency part is reserved. Eq. (8) can also be rewritten as

$$A_{J-1} = A_J + D_J \tag{9}$$

The original signal is represented by

$$f(t) = A_J + \sum_{j \leq J} D_j \tag{10}$$

Eq. (10) shows a tree structure of the original signal, which can be reconstructed from its details and approximations, revealing important information. It also implies that the analysis can end at the *J*-th resolution level and that the original signal can be reconstructed using the approximation at the *J*-th level and all the details from the first level to the *J*-th level. More information can be found in Chui (1992).

2.2 Wavelet-based transmissibility

The motion differential equation of an *n*-DOF structural system can be given by

$$M\ddot{x}(t)+C\dot{x}(t)+Kx(t)=F(t) \tag{11}$$

where *M*, *C*, and *K* denote the mass matrix, damping matrix, and stiffness matrix, respectively; $x(t)$ denotes the response; and $F(t)$ denotes the applied force. By using Eq. (5), the response and applied force of the *n*-DOF system can be respectively expressed as

$$x(t)=\begin{pmatrix}\sum_j\sum_k \alpha_{j,k}^{1}\Psi_{j,k}(t)\\ \sum_j\sum_k \alpha_{j,k}^{2}\Psi_{j,k}(t)\\ \vdots \\ \sum_j\sum_k \alpha_{j,k}^{N}\Psi_{j,k}(t)\end{pmatrix} \tag{12}$$

and

$$F(t)=\begin{pmatrix}\sum_j\sum_k \beta_{j,k}^{1(F)}\Psi_{j,k}(t)\\ \sum_j\sum_k \beta_{j,k}^{2(F)}\Psi_{j,k}(t)\\ \vdots \\ \sum_j\sum_k \beta_{j,k}^{N(F)}\Psi_{j,k}(t)\end{pmatrix} \tag{13}$$

where $\alpha_{j,k}^{s}$ is the wavelet coefficient of the dynamic response at the *s*-th DOF and $\beta_{j,k}^{r(F)}$ is the wavelet coefficient of the applied force at the *r*-th DOF.

Substituting Eqs. (12) and (13) into Eq. (11), the equation of motion can be rewritten as

$$M\begin{pmatrix}\sum_j\sum_k \alpha_{j,k}^{1}\ddot{\Psi}_{j,k}(t)\\ \sum_j\sum_k \alpha_{j,k}^{2}\ddot{\Psi}_{j,k}(t)\\ \vdots \\ \sum_j\sum_k \alpha_{j,k}^{N}\ddot{\Psi}_{j,k}(t)\end{pmatrix}+C\begin{pmatrix}\sum_j\sum_k \alpha_{j,k}^{1}\dot{\Psi}_{j,k}(t)\\ \sum_j\sum_k \alpha_{j,k}^{2}\dot{\Psi}_{j,k}(t)\\ \vdots \\ \sum_j\sum_k \alpha_{j,k}^{N}\dot{\Psi}_{j,k}(t)\end{pmatrix}+K\begin{pmatrix}\sum_j\sum_k \alpha_{j,k}^{1}\Psi_{j,k}(t)\\ \sum_j\sum_k \alpha_{j,k}^{2}\Psi_{j,k}(t)\\ \vdots \\ \sum_j\sum_k \alpha_{j,k}^{N}\Psi_{j,k}(t)\end{pmatrix}=\begin{pmatrix}\sum_j\sum_k \beta_{j,k}^{1(F)}\Psi_{j,k}(t)\\ \sum_j\sum_k \beta_{j,k}^{2(F)}\Psi_{j,k}(t)\\ \vdots \\ \sum_j\sum_k \beta_{j,k}^{N(F)}\Psi_{j,k}(t)\end{pmatrix} \tag{14}$$

Multiplying $\Psi_{j,k}(t)$ on both sides of Eq. (14), and using the orthogonality of the wavelet, leads to

$$\left\{M\int\ddot{\Psi}_{j,k}(t)\Psi_{j,k}(t)dt+C\int\dot{\Psi}_{j,k}(t)\Psi_{j,k}(t)dt+K\right\}\begin{pmatrix}\alpha_{j,k}^{1}\\ \alpha_{j,k}^{2}\\ \vdots\\ \alpha_{j,k}^{N}\end{pmatrix}=\begin{pmatrix}\beta_{j,k}^{1(F)}\\ \beta_{j,k}^{2(F)}\\ \vdots\\ \beta_{j,k}^{N(F)}\end{pmatrix} \tag{15}$$

Let

$$r_{j,k}=\int\ddot{\Psi}_{j,k}(t)\Psi_{j,k}(t)dt \tag{16}$$

$$s_{j,k}=\int\dot{\Psi}_{j,k}(t)\Psi_{j,k}(t)dt \tag{17}$$

$$XW=\begin{pmatrix}\alpha_{j,k}^{1}\\ \alpha_{j,k}^{2}\\ \vdots\\ \alpha_{j,k}^{N}\end{pmatrix} \tag{18}$$

$$FW=\begin{pmatrix}\beta_{j,k}^{1(F)}\\ \beta_{j,k}^{2(F)}\\ \vdots\\ \beta_{j,k}^{N(F)}\end{pmatrix} \tag{19}$$

Substituting Eqs. (16)-(19) into Eq. (15) gives

$$\left\{Mr_{j,k}+Cs_{j.k}+K\right\}\bullet XW=FW \tag{20}$$

Let

$$KW=\left\{Mr_{j,k}+Cs_{j,k}+K\right\} \tag{21}$$

then

$$XW=KW^{-1}\bullet FW=HW\bullet FW \tag{22}$$

$$XW_{U}=HW_{UA}\bullet FW_{A} \tag{23}$$

and

$$XW_{K}=HW_{KA}\bullet FW_{A} \tag{24}$$

where U, K, and A denote the unmeasured coordinates, measured coordinates, and coordinates where forces are applied, respectively. Upon eliminating FW_A between Eq. (23) and (24), it follows that

$$XW_U = HW_{UA} \bullet HW_{KA}^{+} \bullet XW_K \tag{25}$$

where HW_{KA}^{+} is the pseudo-inverse of HW_{KA}. For the pseudo-inverse to exist, the number of K coordinates must be greater or equal than the number of A coordinates.

Finally

$$TW_{UK}^{(A)} = HW_{UA} \bullet HW_{KA}^{+} = \frac{\left\{\alpha_{j.k}^{U}\right\}}{\left\{\alpha_{j.k}^{K}\right\}} \tag{26}$$

is defined as the wavelet-based transmissibility. According to Eq. (26), the proposed transmissibility relates to the dynamic stiffness of the structural system in the wavelet domain. Obviously, a change in the dynamic stiffness due to damage inevitably alters the wavelet-based transmissibility. Using the local zoom capability of the wavelet transform, we can capture slight local changes in the stiffness by means of the wavelet-based transmissibility.

3. Damage detection algorithm

As indicated in section 2, wavelet-based transmissibility can be extracted as a damage index to reflect changes in the structural characteristics in a wavelet domain. The purpose of damage detection is to identify deviation of the damage indices from the initial or healthy state to damage states. We established an outlier analysis based on a statistical pattern recognition approach for detecting the structural damage. This is in fact a type of discordance derived from the statistical discipline of outlier analysis that is used to signal deviance from the norm. Outlier analysis is well established in the field of statistics but has not been extensively applied to damage detection. However, existing work has validated its feasibility for damage detection, particularly for cases of noisy measured data or a changing environment (Worden *et al.* 2000, Sohn *et al.* 2001).

The idea of outlier analysis is simple. During the normal operation of a structure, measurements are recorded, and features are extracted from data that characterize normal or healthy conditions. After training of the diagnostic procedure in question, subsequent data can be examined to determine whether the features deviate significantly from the norm. That is, outlier detection is a technique for deciding whether measurements from a system or structure indicate departure from previously established normal conditions. One way to identify possible outliers is to calculate the distance from each point to a "center" of the data. An outlier would then be a point with a distance larger than some predetermined value. A conventional measurement of quadratic distance from a point to a location in the multivariate setting is given by the Mahalanobis distance as

$$D_{\zeta} = (\{x_{\zeta}\} - \{\bar{x}\})^{\mathrm{T}}[S]^{-1}(\{x_{\zeta}\} - \{\bar{x}\}) \tag{27}$$

where $\{x_{\zeta}\}$ is the potential outlier datum, $\{\bar{x}\}$ is the mean vector of the sample observation, and $[S]$ is the sample covariance matrix. Here, T indicates transpose. Generally, the one-outlier displaying component β is calculated to determine which dimensions contribute most to the discordance of the outlier, and makes the outlier protrude as far as possible from the data mass. It is given by

$$\{\beta\} = [S]^{-1}(\{x_{\varsigma}\} - \{\bar{x}\}) \tag{28}$$

Based on the outlier analysis, the damage detection procedure may comprise the following steps.

(1) In the initial or healthy state, the dynamic responses at the sensor locations are obtained and analyzed by the wavelet transform using Eq. (26). The wavelet-based transmissibility between two locations is extracted as a damage index for damage detection.

(2) The damage indices are sampled at *p* regularly spaced points on the time range and copied for *n* observation times with the same corrupted noise level to construct a $p \times n$ matrix, of which the mean vector and covariance matrix are calculated with the exclusive methods. Then, the Mahalanobis distances of the $p \times n$ matrix for this reference state are calculated by means of Eq. (27) with the largest of the *n* Mahalanobis distances stored.

(3) To determine whether an observation is an outlier or an inlier, a threshold value needs to be set against which the discordance value can be compared. The threshold value is obtained by testing the $p \times n$ matrix for a large number of trials, with all the largest Mahalanobis distances stored and sorted in terms of magnitude. Then, the critical values for 1% tests of discordance are given by the Mahalanobis distances in the array above which 1% of the trials occur.

(4) For the unknown state of the measured structure, the dynamic responses at the same location with the healthy state are measured. For each pattern to be tested, step (2) is repeated until all the exclusive Mahalanobis distances of damage patterns are calculated and stored. The outliers, denoted as those Mahalanobis distances larger than the threshold value, indicate the damage occurring in the structure.

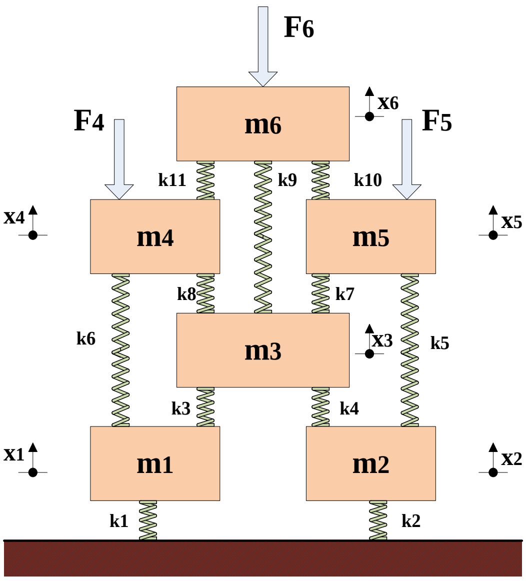


Fig. 1 6-DOF mass–spring system

4. Feasibility investigations

The feasibility of the proposed method was verified with a typical 6-DOF spring–mass vibrating system shown in Fig. 1. The vibrating system consisted of 6 masses and 11 springs, with their characteristics listed in Table 1. The damage cases simulated by the stiffness reduction of various springs are listed in Table 2. In cases 1 and 2, damage occurred at springs that were connected to the boundary. Cases 3 and 4 simulated damage occurring at springs between the inner DOFs, and the damage scenarios in cases 5 and 6 represent the stiffness reductions in the springs interconnecting the DOFs where external forces were applied.

Table 1 Characteristics of vibrating system for healthy pattern

Mass (kg)	Stiffness (kN/m)	
m1 = 7	k1 = 1	k7 = 8
m2 = 7	k2 = 1	k8 = 3
m3 = 4	k3 = 4	k9 = 6
m4 = 3	k4 = 2	k10 = 3
m5 = 6	k5 = 7	k11 = 5
m6 = 8	k6 = 2	

Table 2 Damage cases

Case number	Location of the damaged spring	Stiffness reduction			
		Healthy pattern	Damage pattern 1	Damage pattern 2	Damage pattern 3
1	k1	0	0.1%	1%	10%
2	k2	0	0.1%	1%	10%
3	k4	0	0.1%	1%	10%
4	k8	0	0.1%	1%	10%
5	k10	0	0.1%	1%	10%
6	k11	0	0.1%	1%	10%

4.1 Damage detection using wavelet transmissibility function

In this case study, a step force was employed to excite the vibrating system and applied at DOF 4, 5, and 6 simultaneously. The sampling time was 6.4 s, and the sampling frequency was 40 Hz. Figs. 2 and 3 show the acceleration response at DOF 5 and DOF 6, respectively.

The acceleration response of each DOF was analyzed by the wavelet transform. Selection of an appropriate type of wavelet function and the choice of its number of vanishing moments is essential for effective utilization of the wavelet analysis. A variety of wavelet functions have been proposed and tested for this purpose including orthogonal, Daubechies, symlet, Coiflet, biorthogonal, and reverse biorthogonal wavelets. Based on trial and error analysis, the “db1” wavelet function was used as the mother wavelet because of its orthogonal property and simplicity.

The level of the multi-resolution analysis was chosen as 3 in this study. Generally, local damage causes the subtle changes in every frequency band. The approximation coefficients represent the intrinsic smooth versions of the measured responses, which are not sensitive to the slight damage. Hence, the detail coefficients of the signal decompositions were extracted to damage-sensitive features for damage detection. The detail coefficients at each level were calculated and used as independent data sets in the outlier analysis. TW56 (wavelet transmissibility function between DOF 5 and 6) at different levels was calculated with Eq. (26), and the healthy pattern of TW56 at level 1 was plotted as shown in Fig. 4. To reduce the number of dimensions for improving computational efficiency, the averaged outlier-displaying component for level 1 was calculated and plotted as shown in Fig. 5. The selection of dimensions depends on the value of the outlier displaying components as referred to in Eq. (28). Both healthy and damage patterns were copied 1000 times with a noise level of 5%. The four data sets were concatenated to give a 4000-observation testing data set. Then, the exclusive Mahalanobis distances for each of these 4000 observations were calculated with Eq. (27). In Figs. 6-11, the outlier analysis for the different damage scenarios were performed to detect the damage occurrence. In these figures, the horizontal axis represents the number of the outlier analysis. As indicated previously, the first 1000 data points are the 1000 times noise perturbation of the measured wavelet-based transmissibility for the healthy state. 1001-2000, 2001-3000 and 3001-4000 data points are the perturbations of the measured data for each different damage scenario, respectively.

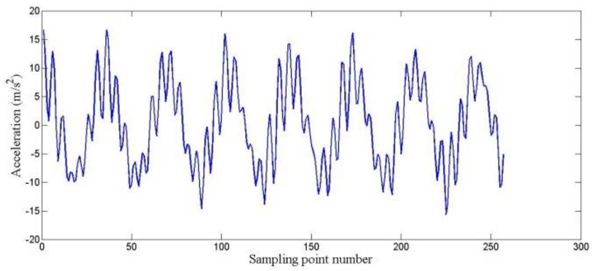


Fig. 2 Acceleration response at DOF 5

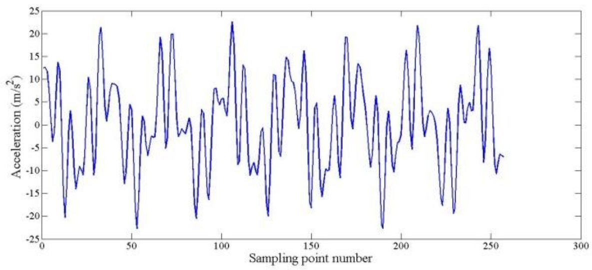


Fig. 3 Acceleration response at DOF 6

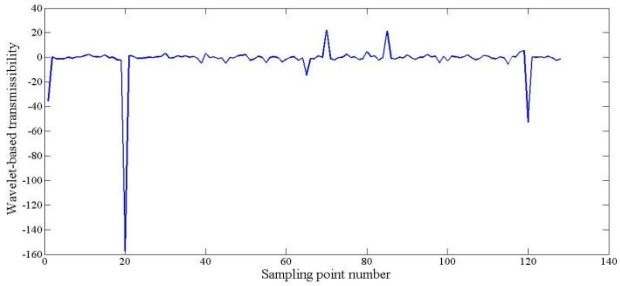


Fig. 4 Wavelet-based transmissibility between DOF 5 and 6 for healthy data (at level 1)

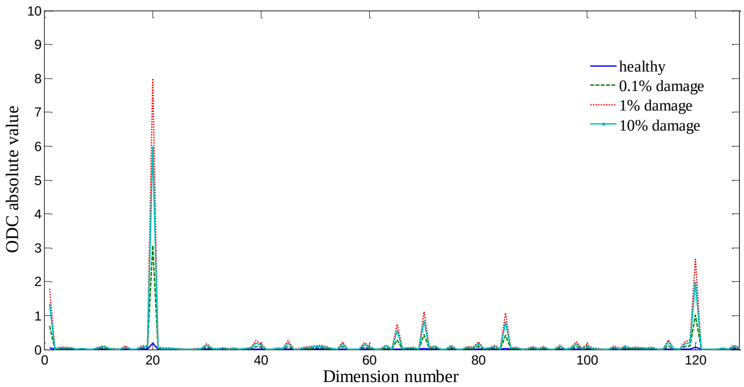
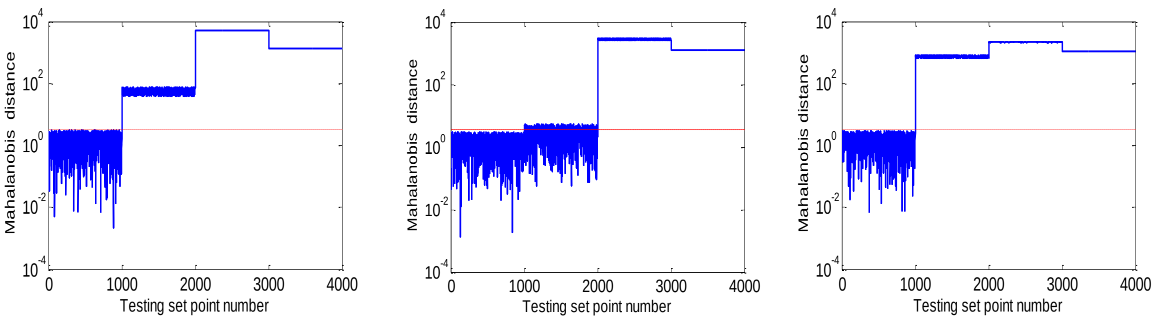


Fig. 5 Averaged outlier-displaying component for wavelet transmissibility



(a) (b) (c)

Fig. 6 Mahalanobis distances for case 1: (a) level 1 details, (b) level 2 details and (c) level 3 details (--- Threshold value)

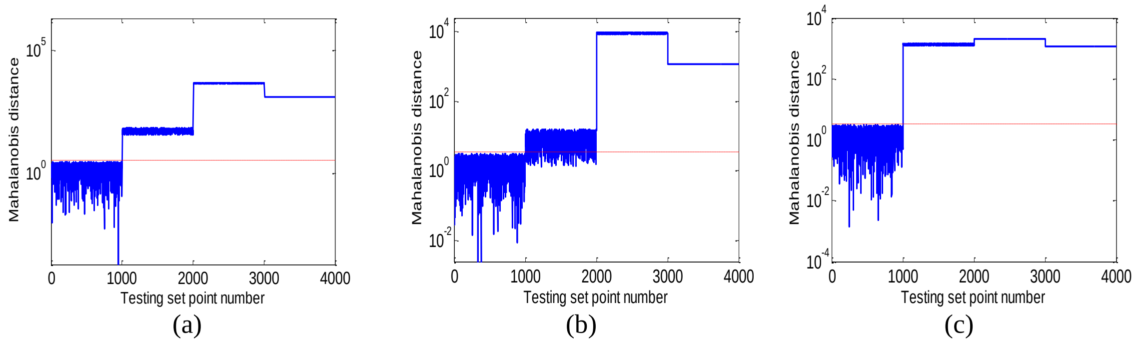


Fig. 7 Mahalanobis distances for case 2: (a) level 1 details, (b) level 2 details and (c) level 3 details (--- Threshold value)

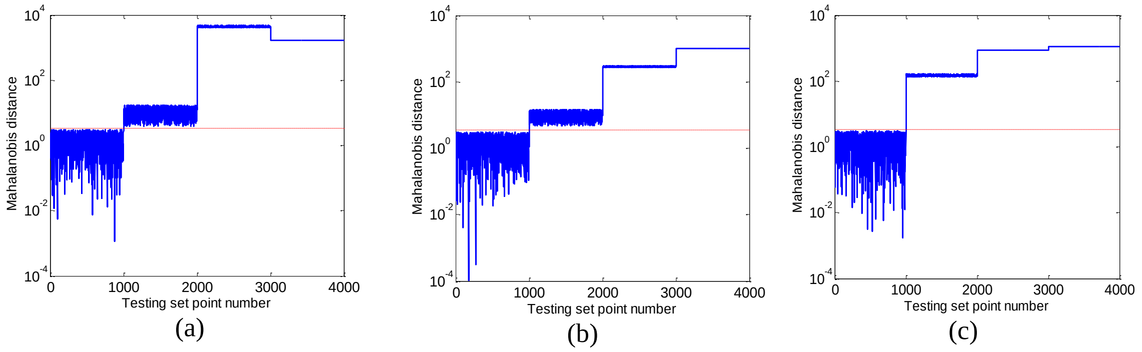


Fig. 8 Mahalanobis distances for case 3: (a) level 1 details, (b) level 2 details and (c) level 3 details (--- Threshold value)

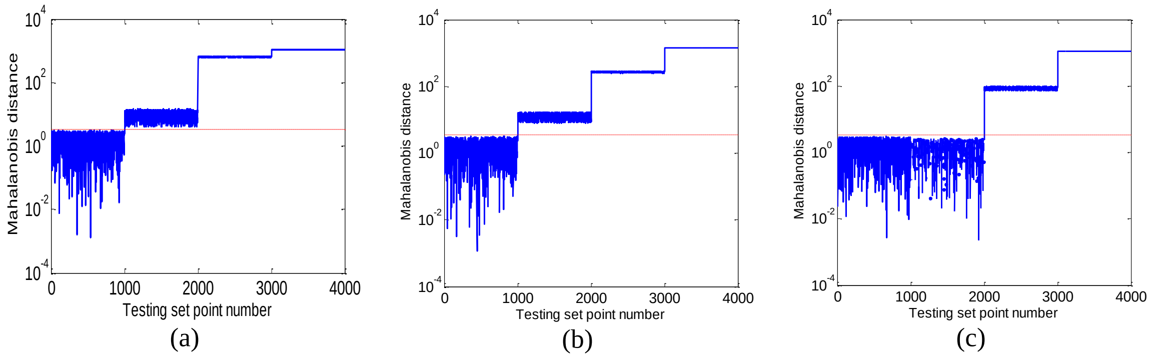


Fig. 9 Mahalanobis distances for case 4: (a) level 1 details, (b) level 2 details and (c) level 3 details (--- Threshold value)

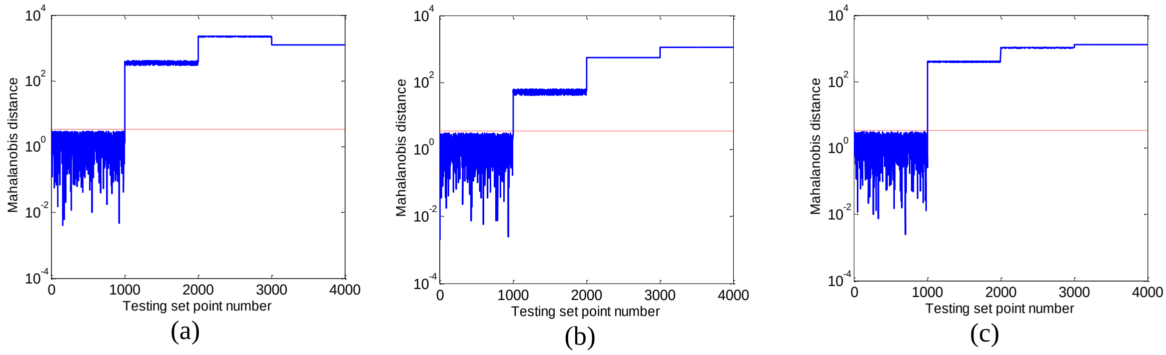


Fig. 10 Mahalanobis distances for case 5: (a) level 1 details, (b) level 2 details and (c) level 3 details (--- Threshold value)

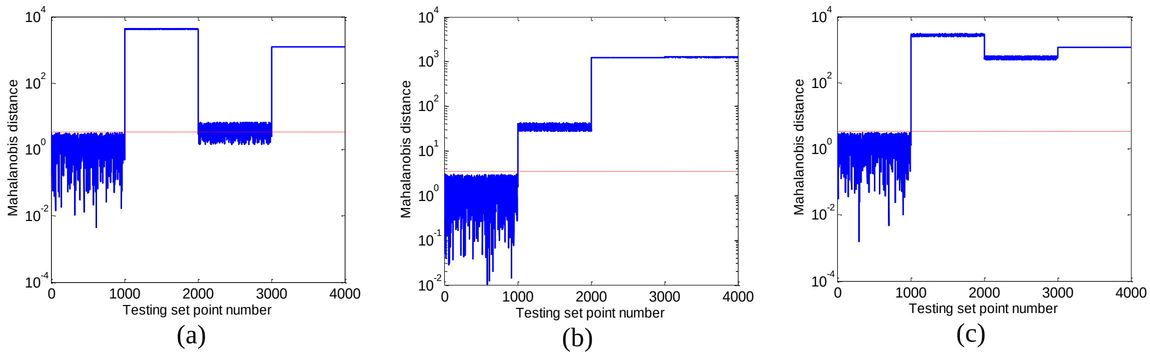


Fig. 11 Mahalanobis distances for case 6: (a) level 1 details, (b) level 2 details and (c) level 3 details (--- Threshold value)

Figs. 6-11 show the exclusive Mahalanobis distances for 4000 observations at each level in various cases. As can be seen from these figures, the healthy data sets (first 1000 observations) were all correctly labeled as inliers, and the damage could be diagnosed as outliers through observing the different levels in each damage case. This shows that the wavelet-based transmissibility is very sensitive to damage, and so even a slight stiffness reduction (e.g., 0.1%) occurring in the system can be detected.

It was also observed that different damage severities result in different Mahalanobis distances, and different level details reveal different distributions of Mahalanobis distances when the same damage occurs. Since different damage severities affect the measured signals in different frequency bands, changes in the Mahalanobis distances are not proportional to the damage severities.

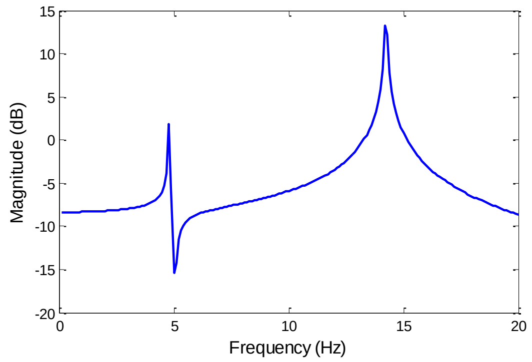


Fig. 12 Transmissibility function for healthy data

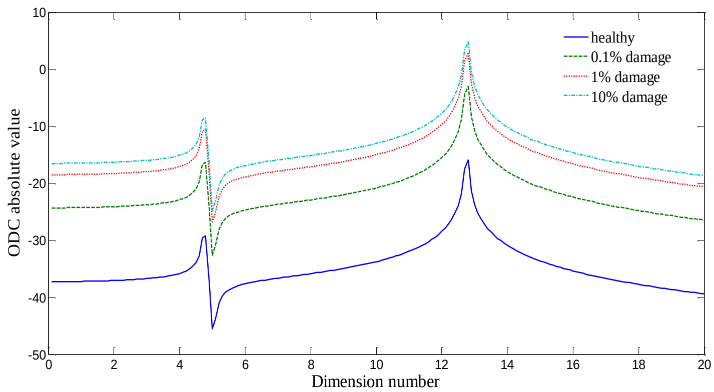


Fig. 13 Averaged outlier-displaying component for transmissibility

4.2 Damage detection with a conventional transmissibility function

Comparative investigations were also performed with a conventional transmissibility approach. The identical 6-DOF spring–mass system with the same healthy and damage pattern was used for this case. The forces were applied at DOF 4, 5, and 6. T56 (transmissibility function between DOF 5 and 6) was calculated and plotted as shown in Fig. 12. Then, it was sampled at 200 spaced points on the frequency range to obtain the healthy pattern to be used in the analysis. After the averaged outlier-displaying component for T56 was calculated and plotted as shown in Fig. 13, the data set was reduced so that it contained only the information from dimensions 46–53 and 140–145. The same damage detection procedure introduced in the previous section was performed again to give the computational results for different damage cases.

Fig. 14 shows that the damage was not detected using the conventional transmissibility approach. For damage cases 1 and 2, the transmissibility matrix T56 showed little change because the contribution of the boundary stiffness was less than that of other spring stiffnesses to the whole stiffness matrix of the system. For damage cases 3 and 4, the damage was too slight to be detected. For damage cases 5 and 6, the transmissibility matrix T56 did not change, because the spring was interconnecting the DOF where the external loads were applied. It was found that the outlier analysis might not be very effective in detecting damage in these damage cases with the conventional transmissibility matrix, whereas the use of wavelet transmissibility may yield a good detection result. This is because the conventional transmissibility approach is based on Fourier transform, which leads to regularity of the local changes in the measured signal. Thus, it masks the slight variation in the signal due to local damage.

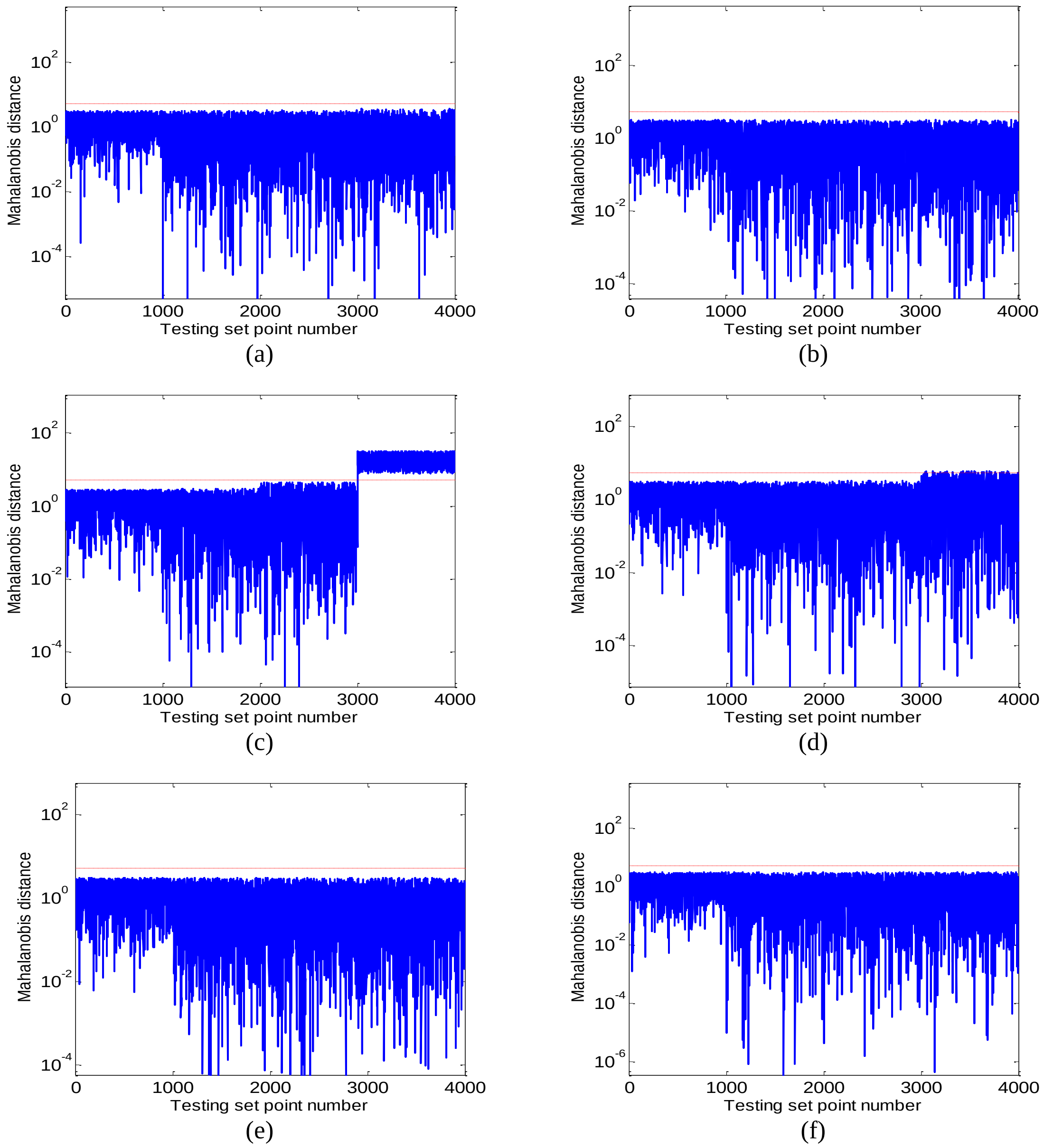


Fig. 14 Mahalanobis distances for (a) case 1, (b) case 2, (c) case 3, (d) case 4, (e) case 5 and (f) case 6 (--- Threshold value)

5. Conclusions

In this paper, a novel transmissibility concept based on a wavelet transform for damage detection was presented. To detect the slight damage at the incipient stage, wavelet transform was employed in a damage detection algorithm as a form of transmissibility. The wavelet-based transmissibility was theoretically formulated and designated as a damage index, and so, not only was it able to capture subtle information in the measured signals, but also it did not require the measurement of input force. Outlier analysis was adopted to develop the damage detection algorithm with the wavelet-based transmissibility, and the feasibility of the proposed method was demonstrated with a numerical simulation. Moreover, comparative work was performed with conventional transmissibility. The results indicate that the proposed method is capable of detecting very slight damage and is more sensitive to damage than conventional transmissibility. Although the proposed methodology has shown great potential in the numerical simulation, further study should be conducted with experimental investigations on more complex structures and damage patterns.

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References

Almeida, R.A.B., Urgueira, A.P.V. and Maia, N.M.M. (2010), "The use of transmissibility properties to estimate FRFs on modified structures", *Shock Vib*., **17**(4), 563-577.

Alvandi, A. and Cremona, C. (2006), "Assessment of vibration-based damage identification techniques", *J. Sound Vib.*, **292**(1), 179-202.

Bayissa, W.L., Haritos, N. and Thelandersson, S. (2008), "Vibration-based structural damage identification using wavelet transform", *Mech. Syst. Signal Pr.*, **22**(5), 1194-1215.

Biswas, M., Pandey, A.K. and Samman, M.M. (1990), "Diagnostic experimental spectral/modal analysis of a highway bridge", *Int. J. Anal. Exper. Modal* ., **5**(1), 33-42.

Canales, G., Mevel, L. and Basseville, M. (2009), "Transmissibility based damage detection", *Proceedings of the 27th International Modal Analysis Conference (IMAC–XXVII)*, Orlando, Florida, USA.

Catbas, F.N., Gul, M. and Burkett, J.L. (2008), "Conceptual damage-sensitive features for structural health monitoring: laboratory and field demonstrations", *Mech. Syst. Signal Pr.*, **22**(7), 1650-1669.

Cawley, P. and Adams, R.D. (1979), "The location of defects in structures from measurements of natural frequencies", *J. Strain Anal. Eng.*, **14**(2), 49-57.

Chui, C.K. (1992), *An introduction to wavelets*, Academic, San Diego.

Devriendt, C. and Guillaume, P. (2008), "Identification of modal parameters from transmissibility measurements", *J. Sound Vib.*, **314**(1), 343-356.

Doebling, S.W., Farrar, C.R. and Prime, M.B. (1998), "A summary review of vibration-based damage identification methods“, *Shock Vib*., **30**(2), 91-105.

Gurley, K. and Kareem, A. (1999), "Applications of wavelet transforms in earthquake, wind and ocean engineering", *Eng. Struct.*, **21**(2), 149-167.

Hong, J.C., Kim, Y.Y., Lee, H.C. and Lee, Y.W. (2002), "Damage detection using the Lipschitz exponent estimated by the wavelet transform: applications to vibration modes of a beam", *Int. J. Solids Struct.*, **39**(7), 1803-1816.

Hou, Z., Noori, M.N. and Amand, R. St. (2000), "Wavelet-based approach for structural damage detection", *J. Eng. Mech - ASCE*, **126**(7), 677-683.

Jiang, X., Ma, Z.J. and Ren, W.X. (2012), "Crack detection from the slope of the mode shape using complex continuous wavelet transform", *Comput. Aided Civil Infrastruct. Eng*., **27**(3), 187-201.

Johnson, T.J. and Adams, D.E. (2002), "Transmissibility as a differential indicator of structural damage", *J. Vib. Acoust.*, **124**(4), 634-641.

Kim, H. and Melhem, H. (2004), "Damage detection of structures by wavelet analysis", *Eng. Struct.*, **26**(3), 347-362.

Kim, J.T., Ryu, Y.S., Cho, H.M. and Stubbs, N. (2003), "Damage identification in beam-type structures: frequency-based method vs mode-shape-based method", *Eng. Struct.*, **25**(1), 57-67.

Law, S.S., Li, J. and Ding, Y. (2011), "Structural response reconstruction with transmissibility concept in frequency domain", *Mech. Syst. Signal Pr.*, **25**(3), 952-968.

Lee, Y. and Chung, M. (2000), "A study on crack detection using eigenfrequency test data", *Comput. Struct.*, **77**(3), 327-342.

Maia, N.M.M., Almeida, R.A.B., Urgueira, A.P.V. and Sampaio, R.P.C. (2011), "Damage detection and quantification using transmissibility", *Mech. Syst. Signal Pr.*, **25**(7), 2475-2483.

Maia, N.M.M., Silva, J.M.M. and Ribeiro, A.M.R. (2001), "The transmissibility concept in multi-degree-of-freedom systems", *Mech. Syst. Signal Pr.*, **15**(1), 129-137.

Mallat, S.G. (1989), "A theory for multiresolution signal decomposition: The wavelet representation", *IEEE T. Pattern Anal.*, **11**(7), 674-693.

Ostachowicz, W.M. and Krawczuk, M. (1990), "Vibration analysis of a cracked beam", *Comput. Struct.*, **36**(2), 245-250.

Pandey, A.K., Biswas, M. and Samman, M.M. (1991), "Damage detection from changes in curvature mode shapes", *J. Sound Vib.*, **145**(2), 321-332.

Park, N.G. and Park, Y. (2005), "Identification of damage on a substructure with measured frequency response functions", *J. Mech. Sci. Technol.*, **19**(10), 1891-1901.

Ribeiro, A.M.R., Silva, J.M.M. and Maia, N.M.M. (2000), "On the generalisation of the transmissibility concept", *Mech. Syst. Signal Pr.*, **14**(1), 29-35.

Sampaio, R., Maia, N.M.M., Ribeiro, A. and Silva, J. (2001), "Transmissibility techniques for damage detection". *Proceedings of 19th International Modal Analysis Conference (IMAC XIX)*, Kissimmee, Florida, USA, 1524-1527.

Shi, Z., Law, S. and Zhang, L. (2000), "Structural damage detection from modal strain energy change", *J. Eng. Mech.- ASCE*, **126**(12), 1216-1223.

Steenackers, G., Devriendt, C. and Guillaume, P. (2007), "On the use of transmissibility measurements for finite element model updating", *J. Sound Vib.*, **303**(3), 707-722.

Sun, Z. (2002), "Structural damage assessment based on wavelet packet transform", *J. Struct. Eng.- ASCE*, **128**(10), 1354-1361.

Teughels, A. and De Roeck, G. (2005), "Damage detection and parameter identification by finite element model updating", *Arch. Comput. Method E.*, **12**(2), 123-164.

Urgueira, A.P.V., Almeida, R.A.B. and Maia, N.M.M. (2011), "On the use of the transmissibility concept for the evaluation of frequency response functions", *Mech. Syst. Signal Pr.*, **25**(3), 940-951.

Worden, K., Manson, G. and Fieller, N. (2000), "Damage detection using outlier analysis", *J. Sound Vib.*, **229**(3), 647-667.

Yan, Y.J., Cheng, L., Wu, Z.Y. and Yam, L.H. (2007), "Development in vibration-based structural damage detection technique", *Mech. Syst. Signal Pr.*, **21**(5), 2198-2211.

Yang, Q.W. (2011), "A new damage identification method based on structural flexibility disassembly", *J. Vib. Control*, **17**(7), 1000-1008.

Yi, T.H., Li, H.N. and Gu, M. (2011a), "Characterization and extraction of global positioning system multipath signals using improved particle filtering algorithm", *Meas. Sci. Technol.*, **22**(7), DOI 075101:

1-11.
Yi, T.H., Li, H.N. and Gu, M. (2011b), "A new method for optimal selection of sensor location on a high-rise building using simplified finite element model", *Struct. Eng. Mech.*, **37**(6), 671-684.
Yi, T.H., Li, H.N. and Gu, M. (2013), "Experimental assessment of high-rate GPS receivers for deformation monitoring of bridge", *J. Int. Meas. Confed.*, **46**(1), 420-432.
Yi, T.H., Li, H.N. and Zhang, X. (2012), "Sensor placement on Canton Tower for health monitoring using asynchronous-climbing monkey algorithm", *Smart Mater. Struct.*, **21**(12), DOI 125023: 1-12.
Zhu, D., Yi, X. and Wang, Y. (2011), "Sensitivity analysis of transmissibility functions for structural damage detection", *Proceedings of the SPIE*, 79832M.