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Damage identification using chaotic excitation

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Abstract. Vibration-based damage detection methods are popular for structural health monitoring. However, they can only detect fairly large damages. Usually impact pulse, ambient vibrations and sine-wave forces are applied as the excitations. In this paper, we propose the method to use the chaotic excitation to vibrate structures. The attractors built from the output responses are used for the minor damage detection. After the damage is detected, it is further quantified using the Kalman Filter. Simulations are conducted. A 5-story building is subjected to chaotic excitation. The structural responses and related attractors are analyzed. The results show that the attractor distances increase monotonously with the increase of the damage degree. Therefore, damages, including minor damages, can be effectively detected using the proposed approach. With the Kalman Filter, damage which has the stiffness decrease of about 5% or lower can be quantified. The proposed approach will be helpful for detecting and evaluating minor damages at the early stage.

Keywords: minor damage; damage identification; chaotic excitation; attractor; Kalman Filter

1. Introduction

Structural health monitoring of civil engineering structures is a fundamental issue for structural safety and integrity, due to the fact that they will deteriorate just after they were built and put into services. The failure of structures will not only result in severe economic lost but may threaten the lives of people. Hence maintaining safety and reliable civil engineering structures for daily use is an extremely important issue which has received considerable attention in literature in recent years. In this process, damage detection becomes the key issue. In practice, damage was defined as the changes introduced into a system which adversely affected its current or future performance (Farrar 2001). Therefore changes in structural parameters have been extensively applied as effective tools for damage detection. Many methods are developed for this purpose. Among which, the vibration-based damage identification (VBDI) methods draws extensive attention and are deeply developed. The basic concept of VBDI is that any degradation of structural properties will result in changes of its vibration parameters, such as natural frequencies, mode shapes, mode damping ratios, etc. Pandey *et al*. (1991) proposed mode shape curvature (MSC) method on the premise that, for a given moment applied to damaged and undamaged structures, a reduction of stiffness associated with damage will, in turn, lead to an increase in curvature. Stubbs *et al*. (1995) developed a methodology free from normalized criteria based on the comparison of modal strain

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energy before and after damage to locate and estimate the size of damage in flexural structures for which few mode shapes are available. Pandey *et al*. (1994) presented flexibility matrix method to detect the presence and location of structural damage based on changes in the measured flexibility of the structure. Zhang and Aktan (1995) combined certain aspects of the MSC method and change in flexibility method to develop the change in flexibility curvature method by considering the flexibility matrix as the translational displacement under a unit load at j^{th} DOF. The basic idea is that a localized loss of stiffness will produce a curvature increase at the same location. Extensive literature reviews and advances on VBDI have been reported by Doebling *et al*. (1996), Sohn *et al*. (2003), Carden and Fanning (2004), and Adewuyi and Wu (2009).

At present, modal parameter-based damage identification methods keep the dominant position and are widely applied in most engineering practice for structural health monitoring. However, some challenges still confront the practicality of these techniques. Since modal parameters are usually the “global” parameters which reflect the “global” behaviors, and therefore insensitive to the local damage. In many cases, even when the local damage becomes very severe, those global parameters still show little changes, due to the stress re-distribution within the structures, which makes the detection of the local and small damages extremely difficult. This fact can be trace back to 1993, when Pape (1993) tried to identify damaged parts using statistical methods and measured natural frequencies. Damages were detected by assessing resonant frequencies that fall outside the mean standard deviations, but the shortcoming of the approach was found for its inability to detect smaller defects. However, as we know, the deterioration of the civil engineering structures usually begins from the local and small damages. Small damages gradually develop and become large damages and at last cause failure of the structure. For the consideration of the structural safety and reliability, detecting small damages is essential and useful. Also, Adewuyi and Wu (2009) found that the popular modal parameter based methods could be easily degraded by noises. Their research proved that even when 2% of noise was added into the signals, the damage identification results became very poor. Therefore, in order to detect miner damages to ensure the safety and reliability of the structures, development of other approaches is necessary, in which chaos attractor-based analysis seems to be a promising way. However, in many times, detection alone may not be sufficient for the purpose of damage evaluation and structural maintenance. In this case, damage requires to be further quantified. In this paper, the damage after being detected will be further quantified using the Kalman Filter. Thus the damage can be detected and evaluated at the early stage and proper relevant measurement can be applied to ensure the safety of the structures.

2. Attractor based damage detection using chaotic excitation

Recently, some new damage detection techniques have been proposed by using chaotic excitation and attractor analysis. In the field of nonlinear dynamics, systems are often described via their state space. Given infinite time, an ensemble of trajectories evolving in the state space can trace out a dynamical attractor which may be thought of as a geometrical object in the space to which all trajectories belong. The attractor of the space actually contains useful information due to the fact that it reflects the invariant properties of the system, and therefore draws much attention to the application of system classification.

Basically, attractor-based approach requires the acknowledgement of each state variable, which will make it very inconvenient in practice. In steady, attractor reconstruction is often applied, with its advantages to allow only a small number of variables be observed in real applications. Attractor reconstruction is a technique to recreate a topologically equivalent picture to the original multi-dimensional system behavior. Considering a low-dimensional deterministic original system composed of *d* variables, attractor of the original system is obtained by plotting time series of *d* variables in *d* dimensional space. However, as mentioned, it is always the case that limited number of variables can only be observed. This limitation can be solved using the Taken's theorem (Takens *et al.* 1981). For a time series $x_1, x_2, \ldots, x_N$ of a single variable *x*, the embedding vector can be defined by

$$\mathbf{X}_i = (x_i, x_{i+\tau}, \cdots, x_{i+(m-1)\tau})^T \quad (i = 1,2,\cdots N_m) \tag{1}$$

where m is embedding dimension, τ is delay time and $N_m = N-(m-1)\tau$. By plotting (embedding) $\mathbf{X}_i$ in m dimensional space from index i of 1 to N_m, an attractor which is not the same with the original attractor itself but is topologically equivalent to it can be reconstructed by only a single variable.

Even though white noise is often used for VBDI analysis, it cannot be used for attractor-based analysis, because it is not deterministic and cannot yield deterministic responses. For applying the attractor-based analysis, the chaotic signal as the input is usually applied to be the excitation. Chaotic signals possess broad band frequency domain like noise, so that they can excite desirable number of modes. However, unlike the noise which is a random signal, chaotic signal is a low dimensional deterministic signal, so that it can provide deterministic and low dimensional responses. Also, with the deterministic chaotic excitations, noises can be significantly reduced, simply by stacking and averaging.

In the structural health monitoring field, attractors, as a special feature, were also used for the damage detection. Nichols *et al.* (2003) detected the damage by comparing the defined "features" based on attractors reconstructed from healthy and damaged structural responses using chaotic excitation. Sato *et al.* (2010) also proposed an attractor based damage detection method using chaotic excitation and recurrence analysis. Since damages, even small ones, can change the state of a system which can be amplified in the attractor space, damages can be detected by studying the amplified change of the attractor trajectories. In this paper, distances of attractors between a health system and a damaged one are used to describe the system change, and hence to detect damages. Distance of two attractors between a health system and a damaged one can be expressed as

$$Dis = \sum_i \left\| \mathbf{H}_A^i - \mathbf{D}_A^i \right\| \quad (i = 1,2,\cdots n) \tag{2}$$

Where $\mathbf{H}_A^i$ and $\mathbf{D}_A^i$ are two discrete points in attractor space for a healthy structure system and a damaged structure system respectively. Superscript *i* is the serial number, means the *i*th point, *n* is the point number, while subscript A stands for the attractor space. $\mathbf{H}_A^i$ and $\mathbf{D}_A^i$ can be further be expressed as $\mathbf{H}_A^i = (h_1^i, h_2^i, \cdots, h_m^i)$ and $\mathbf{D}_A^i = (d_1^i, d_2^i, \cdots, d_m^i)$, where m is the dimension of the attractor space.

Comparing the attractors of both healthy system and damaged system and calculating their distance, damage detection can be realized. The attractor-based damage detection process is as follows:

- Test the intact structure subjected to a chaotic excitation and collect its dynamic responses

as the baseline;
- Observe and collect the dynamic response of the same structure but may have damage (test structure with unknown health status), also subjected to the same chaotic excitation;
- Build the attractors in a higher dimensional attractor space using the attractor reconstruction method for the signals of both intact structure response and test structure response respectively;
- Calculate the distances of the two attractors;
- Evaluate the attractor gap between the intact and a test structures as well as evaluate its health condition.

This process can be described in the flow chart as shown in Fig. 1.

3. Popular attractor and corresponding chaotic signals

3.1 Lorenz attractor

As for the attractors, Lorenz attractor may be the most popular one. In 1963, Lorenz found the first chaotic attractor in a three dimensional autonomous system (Lü 2002)

$$\begin{cases} \dot{x} = a(y - x) \\ \dot{y} = cx - xz - y \\ \dot{z} = xy - bz \end{cases} \tag{3}$$

which is chaotic when a = 10, b = 8/3, c = 28.

The three dimensional Lorenz chaotic signals can be seen in Fig. 2, and corresponding constructed attractor can be found in Fig. 3.

As mentioned in chapter 2, attractors can be reconstructed by the signal in one direction, Fig. 4 shows a reconstructed Lorenz attractor by only using x direction signal, with the delay $\tau = 10$.

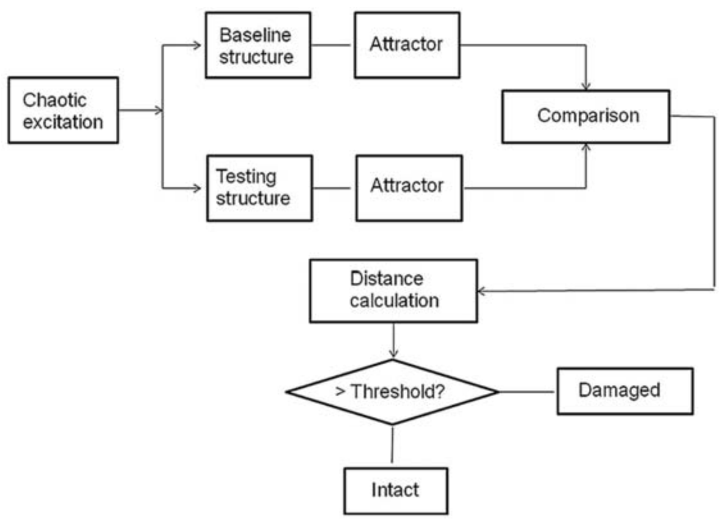

Fig. 1 Flow chart of the attractor-based structural health monitoring

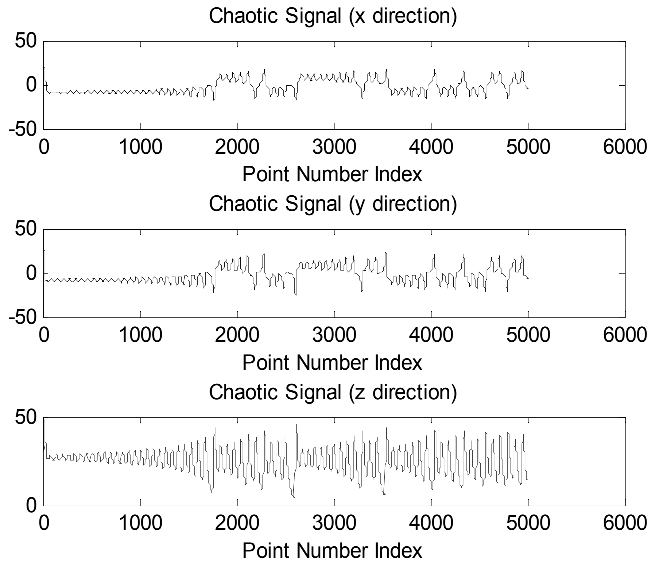

Fig. 2 Three dimensional Lorenz signals

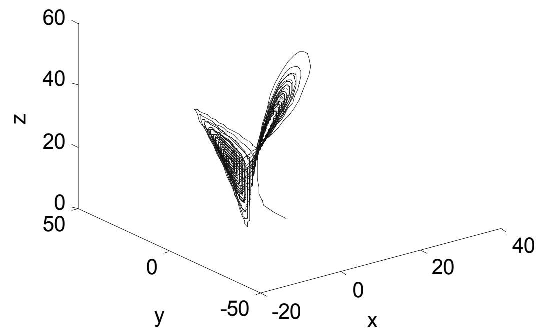

Fig. 3 Constructed Lorenz attractor

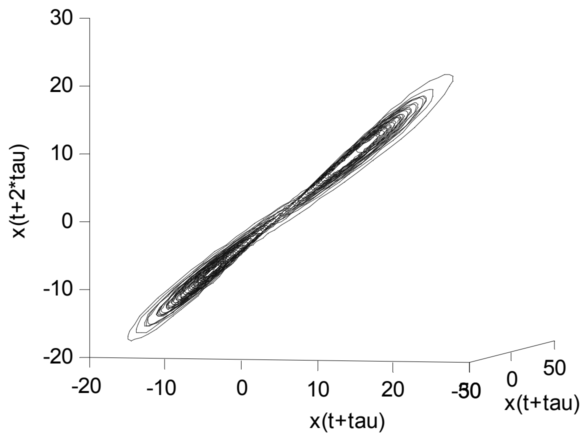

Fig. 4 Reconstructed Lorenz attractor by only using *x* direction signal, $\tau = 10$

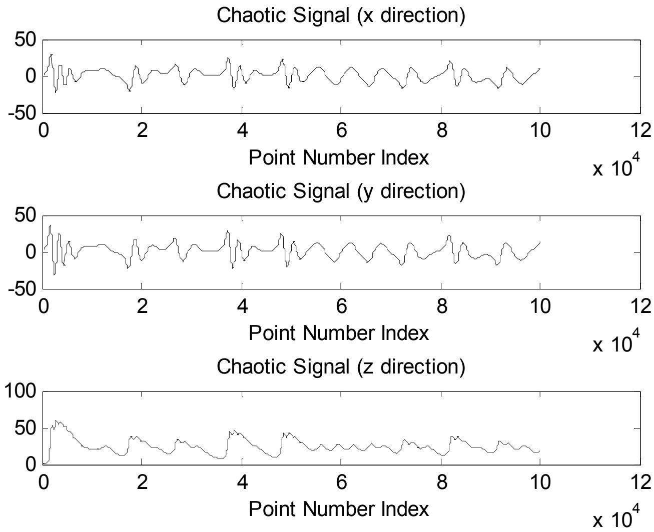

Fig. 5 Three dimensional Chen chaotic signal

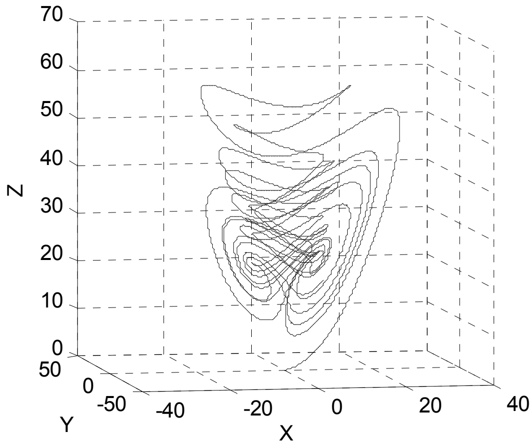

Fig. 6 Constructed Chen attractor

3.2 Chen attractor

Chen and Ueta (1999), however, found another chaotic attractor, which is also in a simple three dimensional autonomous system. Its chaotic system can be mathematically described as

$$\begin{cases} \dot{x} = a(y - x) \\ \dot{y} = (c - a)x - xz + cy \\ \dot{z} = xy - bz \end{cases} \tag{3}$$

where a = 35, b = 3, c = 28.

The three dimensional Chen chaotic signals can be seen in Fig. 5, and the corresponding Chen attractor can be found in Fig. 6.

3.3 Rössler attractor

In 1976, Otto Rössler designed the Rössler attractor (Rössler 1976), which has some similarities to the Lorenz attractor, but is simpler and has only one manifold. The defining equations of Rössler system are

$$\begin{cases} \dot{x} = -y - z \\ \dot{y} = x + ay \\ \dot{z} = b + z(x - c) \end{cases} \tag{4}$$

where a = 0.2, b = 0.2, c = 5.7

The three dimensional Rössler chaotic signals can be seen in Fig. 7, and the corresponding Rössler attractor can be found in Fig. 8.

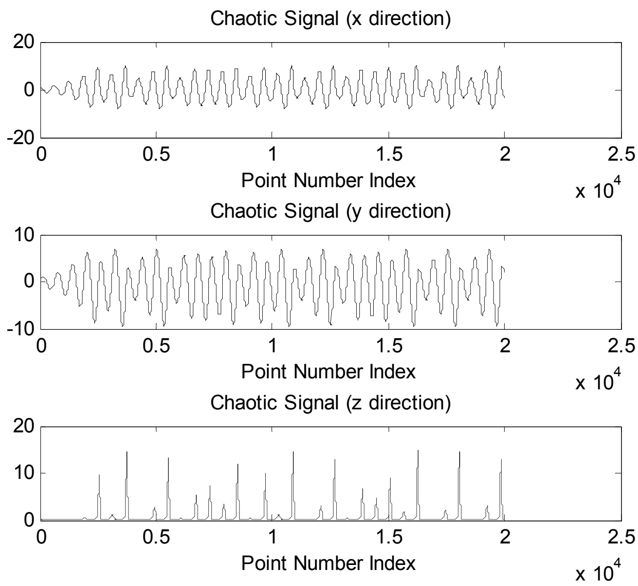

Fig. 7 Three dimensional Rössler chaotic signal

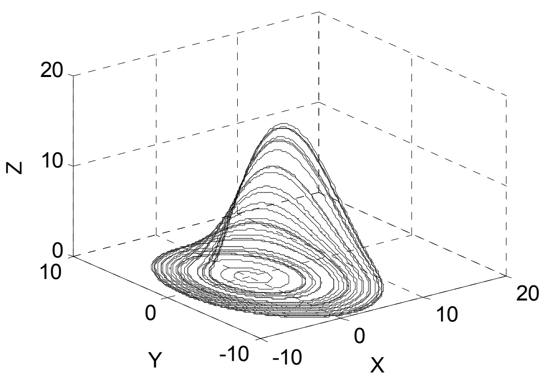

Fig. 8 Constructed Rössler attractor

3.4 Lü attractor

Vanecek and Celikovsky (1996) classified a generalized Lorenz system family and a Chen system by a condition on its linear part $A=[a_{ij}]$: $a_{12}a_{21}>0$ for generalized Lorenz system family and $a_{12}a_{21}<0$ for Chen's system. Lü (2002), however, proposed anther chaotic system which will satisfy the condition $a_{12}a_{21}=0$. The Lü system Equations are

$$\begin{cases} \dot{x}=a(y-x) \\ \dot{y}=-xz+cy \\ \dot{z}=xy-bz \end{cases} \tag{5}$$

where a = 36, b = 3 while c can vary. When $12.7<c<17.0$, the attractor generated by this system is similar to the Lorenz attractor, while similar to Chen attractor when $23.0<c<28.5$, but has a transitory shape when $18.0<c<22.0$.

4. Minor damage quantification using Kalman Filter

Kalman filtering is a powerful tool in the state estimating problem. It provides an efficient computational method to estimate the state of a process. After it was first proposed by R.E. Kalman in 1960, it was extensively studied and applied in many areas. In civil engineering filed, it is often used for the damage identification for structural health monitoring.

For a system, it has a state transfer function as

$$x_t=\Phi_{t-1}x_{t-1}+\Gamma_t w_t \tag{6}$$

where x_t is the state vector, Φ_t is the state transfer matrix, Γ_t is the noise effect matrix and w_t is the system noise vector.

For Structural monitoring, we will have the observation equation as

$$\mathrm{z}_t=\mathbf{H}_t x_t+v_t \tag{7}$$

where z_t is the observed vector, $\mathbf{H}_t$ is the observation matrix, and v_t the observation noise vector.

In the application to a system identification problem, state transfer matrix can be the unit matrix, i.e., $\Phi_t=\mathrm{I}$, while x_t is still the unknown parameter vector. If we further assume that the system noise is also zero, which means we only consider the noise in the observation vector, then the simplified Kalman filter can be obtained as

$$\overline{x}_t=\hat{x}_{t-1} \tag{8}$$

$$\mathbf{G}_t=\mathbf{P}_{t-1} \tag{9}$$

$$\hat{x}_t=\overline{x}_t+\mathbf{B}_t(z_t-\mathbf{H}_t\overline{x}_t) \tag{10}$$

$$\mathbf{P}_t=\mathbf{G}_t-\mathbf{K}_t\mathbf{H}_t\mathbf{G}_t \tag{11}$$

$$\mathbf{B}_t = \mathbf{G}_t \mathbf{H}_t^T (\mathbf{H}_t \mathbf{G}_t \mathbf{H}_t^T + \mathbf{R}_t)^{-1} \tag{12}$$

where $\mathbf{P}_t$ is the state vector covariance matrix, while $\mathbf{R}_t$ is the covariance matrix of the observation noise vector.

$$\mathbf{P}_t = \mathrm{E}[(x_t - \hat{x}_t)(x_t - \hat{x}_t)^T] \tag{13}$$

$$\mathbf{G}_t = \mathrm{E}[(x_t - \overline{x}_t)(x_t - \overline{x}_t)^T] \tag{14}$$

$$\mathbf{R}_t = \mathrm{E}[v_t v_t^T] \tag{15}$$

For a structure, its motion equation is

$$\mathbf{M}\ddot{y} + \mathbf{C}\dot{y} + \mathbf{K}y = F \tag{16}$$

Where **M** is the mass matrix, **C** is the damping matrix, **K** is the stiffness matrix, and F is the out excitation force vector.

Then we can have corresponding equation for the baseline structure

$$\sum_{k=1}^{n} m_{ik}^s \ddot{y}_k^s + \sum_{k=1}^{n} c_{ik}^s \dot{y}_k^s + \sum_{k=1}^{n} k_{ik}^s y_k^s = f_i \tag{17}$$

and the one for the damaged structure

$$\sum_{k=1}^{n} m_{ik}^d \ddot{y}_k^d + \sum_{k=1}^{n} c_{ik}^d \dot{y}_k^d + \sum_{k=1}^{n} k_{ik}^d y_k^d = f_i \tag{18}$$

Subtracting Eq. (18) from Eq. (17) yields

$$\sum_{k=1}^{n} (m_{ik}^s \ddot{y}_k^s - m_{ik}^d \ddot{y}_k^d) + \sum_{k=1}^{n} (c_{ik}^s \dot{y}_k^s - c_{ik}^d \dot{y}_k^d) + \sum_{k=1}^{n} (k_{ik}^s y_k^s - k_{ik}^d y_k^d) = 0 \tag{19}$$

Let

$$\Delta m_{ik} = m_{ik}^s - m_{ik}^d \tag{20}$$

$$\Delta c_{ik} = c_{ik}^s - c_{ik}^d \tag{21}$$

$$\Delta k_{ik} = k_{ik}^s - k_{ik}^d \tag{22}$$

Substituting Eqs. (20)-(22) into Eq. (19) gives

$$\sum_{k=1}^{n} \Delta m_{ik}^d \ddot{y}_k^d + \sum_{k=1}^{n} \Delta c_{ik}^d \dot{y}_k^d + \sum_{k=1}^{n} \Delta k_{ik}^d \ddot{y}_k^d = -\sum_{k=1}^{n} m_{ik}^s (\ddot{y}_k^s - \ddot{y}_k^d) - \sum_{k=1}^{n} c_{ik}^s (\dot{y}_k^s - \dot{y}_k^d) - \sum_{k=1}^{n} k_{ik}^s (y_k^s - y_k^d) \tag{23}$$

Because m_{ik}^s, c_{ik}^s, k_{ik}^s are known and $\ddot{y}_k^s$, $\dot{y}_k^s$, y_k^s as well as $\ddot{y}_k^d$, $\dot{y}_k^d$, y_k^d are all observed values, therefore Eq. (23) can be used as the observation equation in time marching integration scheme.

Eq. (23) can be rewritten to

$$b_i = \sum_{k=1}^{n} \Delta m_{ik}^{d} \ddot{y}_{k}^{d} + \sum_{k=1}^{n} \Delta c_{ik}^{d} \dot{y}_{k}^{d} + \sum_{k=1}^{n} \Delta k_{ik}^{d} \ddot{y}_{k}^{d} \tag{24}$$

where

$$b_i = -\sum_{k=1}^{n} m_{ik}^{s} (\ddot{y}_{k}^{s} - \ddot{y}_{k}^{d}) - \sum_{k=1}^{n} c_{ik}^{s} (\dot{y}_{k}^{s} - \dot{y}_{k}^{d}) - \sum_{k=1}^{n} k_{ik}^{s} (y_{k}^{s} - y_{k}^{d}) \tag{25}$$

Eq. (24) can then be expressed as

$$b_i = [\Delta m_{ik}]\{\ddot{y}^d\} + [\Delta c_{ik}]\{\dot{y}^d\} + [\Delta k_{ik}]\{y^d\} \tag{26}$$

For simplicity, we assume the damage only has the stiffness decrease, but no change for the mass and damping, then Eq. (26) can be simplified into

$$b_i = [\Delta k_{ik}]\{y^d\} \tag{27}$$

Since $[\Delta k_{ik}]\{y^d\}$ can be transformed into $[\mathrm{O}(y^d)]\{\Delta k\}$, where $[\mathrm{O}(y^d)]$ is a matrix composed from the components of displacement vector of the damaged structure while $\{\Delta k\}$ is the vector indicating the stiffness change of the structure, Eq. (27) can be rewritten as

$$b_i = [\mathrm{O}(y^d)]\{\Delta k\} \tag{28}$$

Eq. (28) is the observation equation, using the Kalman Filter Eqs. (8)-(12), the structural stiffness change $\{\Delta k\}$ can be obtained, thus the damage can be evaluated.

5. Simulations

In order to verify the feasibility of the attractor-based damage detection method using chaotic excitations, simulations are conducted. A model of a five-story structure is used for the numerical simuolations. The mass of each floor is assumed to be concentrated to a mass point as shown in Fig. 9, and defined to be 100 kg respectively, while the stiffness of each floor is defined to be 1×10^6 KN/m. An exciter is placed on the top of the structure to produce chaotic signals, as depicted in Fig. 9.

5.1 Damage detection

In the simulation, the sampling frequency is selected as 200 Hz, a chaotic signal in the *x* direction is generated by the Lorenz system. Fig. 10 shows the generated Lorenz signal in *x* direction. In order to avoid the initial transitional part of the signal, a part of the signal (10 s long) is segmented as the input external force, which can be seen in Fig. 11.

In the dynamic analysis of this structure, mode superposition method is applied. The modal damping ratio of each mode is adopted as 0.03. The dynamic responses of both acceleration and displacement at the 5th floor can be found in Figs. 12 and 13 respectively.

The output signals can be used for reconstruction of the attractors. In this case, the delay is adopted as $\tau = 10$. The reconstructed attractor using the 5th floor acceleration of the intact structure can be seen in Fig. 14, while the reconstructed attractor using the 5th floor acceleration of the damaged structure (5% stiffness reduction at 1st floor) can be seen in Fig. 15. It can be found that due to the damage of the structure, the reconstructed attractors changed a little from the intact one.

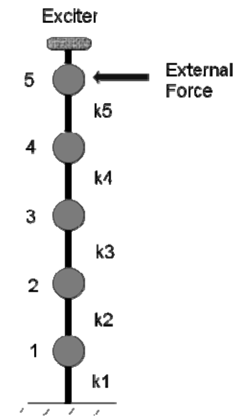

Fig. 9 A five-story model under a external force at the top

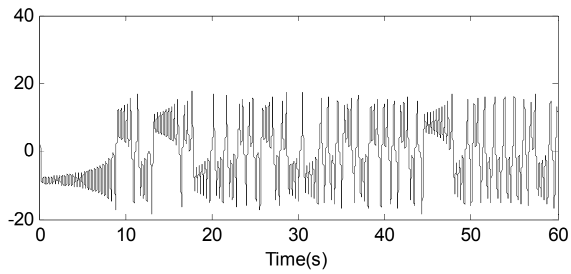

Fig. 10 Lorenz signal in x direction

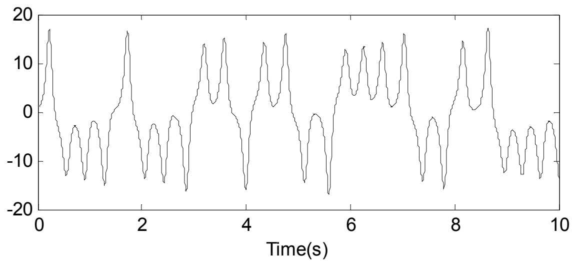

Fig. 11 Input external force

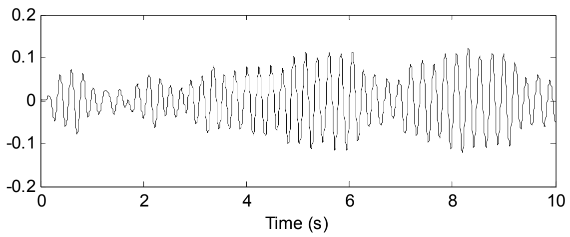

Fig. 12 Acceleration at the 5th floor of intact structure

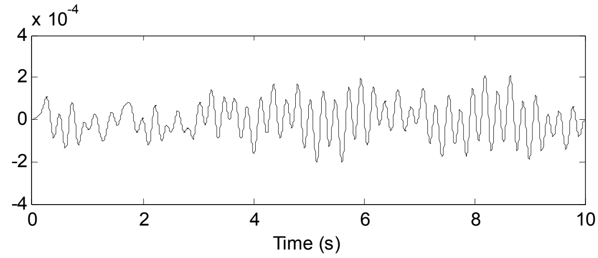

Fig. 13 Displacement at the 5th floor of intact structure

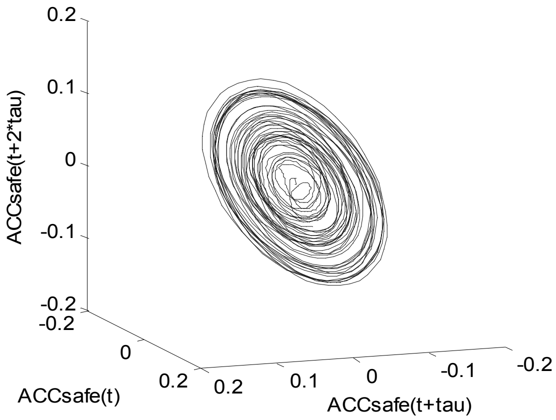

Fig. 14 Reconstructed attractor of 5th floor acceleration for intact structure

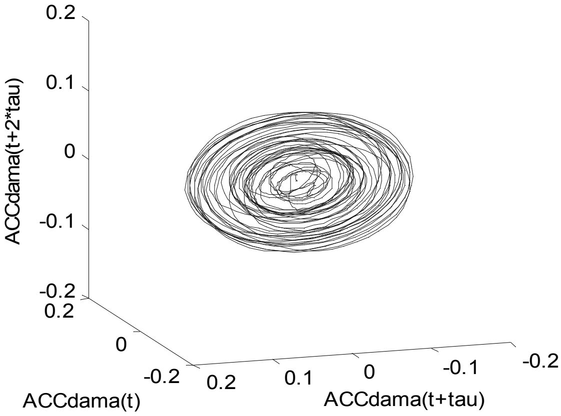

Fig. 15 Reconstructed attractor of 5th floor acceleration for damaged structure

Since the damage of the structure will cause the change of the attractor, the deviation of the attractors can be evaluated by calculating its distance. Fig. 16 shows the attractor distance between a damaged and the intact structure. In Fig. 16, the 5th floor is assumed to be damaged. It can be found that with the increase of the damage, the attractor distance increases monotonously at the same time for each floor.

It should be noted that above analysis is based on the assumption that the input signal is not polluted by the noise. However, in real application, noise is always inevitable. Hence the case with noise should be considered. In this paper, in order to study the noise effect, Gaussian distributed noise with the mean 0 and standard deviation 3 is added into the chaotic input signal. The input force signals with and without noise can be seen in Fig. 17. The noise to signal ratio is about 15%.

For damaged structure, mode superstition method is also applied. Even though the noise will affect the structural response greatly, its effect can be alleviated significantly by stacking structural responses for many times, and therefore the noise effect can be eliminated. This is because that the input chaotic signal is a deterministic signal and the noise is a random time process with zero mean. Fig.18 shows the structural acceleration responses at the 5th floor with and without noise. In the figure, the noisy responses are stacked by 1000 times and then averaged. It can be found that the response signals with and without noise are totally overlapped. As the result, the calculated attractors should also be the same. In this case, even with the noise considered, the attractor based damage detection algorithm still works.

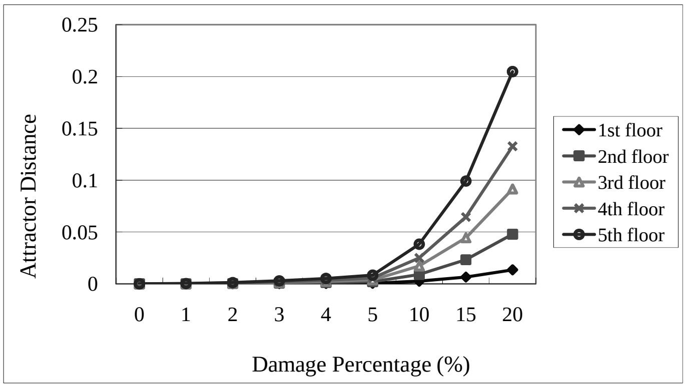

Fig. 16 Attractor distance with respect to the damage percentage when the 5th floor is damaged

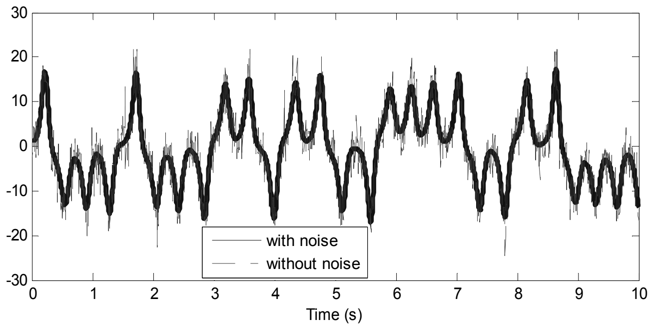

Fig. 17 The input force signals with and without noise

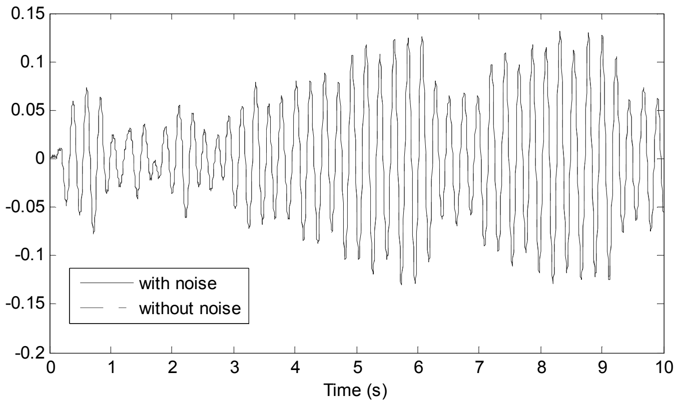

Fig. 18 Structural acceleration responses at the 5th floor with and without noise

5.2 Damage quantification

With the proposed attractor-based damage detection method using chaotic excitation, damages, especially the minor damages, can be detected. After the damage is detected, it can be further identified using the Kalman Filters, so that the damage can be evaluated correctly at the early stage, and give the guidance for the maintenance of the structure.

In this simulation, for the structure shown in Fig. 9, $[\mathrm{O}(y^d)]\{\Delta k\}$ in the observation equation is

$$[\mathrm{O}(y^d)]\{\Delta k\}=\begin{bmatrix} y_1^d - y_2^d & 0 & 0 & 0 & 0 \\ y_2^d - y_1^d & y_2^d - y_3^d & 0 & 0 & 0 \\ 0 & y_3^d - y_2^d & y_3^d - y_4^d & 0 & 0 \\ 0 & 0 & y_4^d - y_3^d & y_4^d - y_5^d & 0 \\ 0 & 0 & 0 & y_5^d - y_4^d & y_5^d \end{bmatrix}\begin{pmatrix} \Delta k_1 \\ \Delta k_2 \\ \Delta k_3 \\ \Delta k_4 \\ \Delta k_5 \end{pmatrix} \quad (29)$$

For using Kalman Filter, time step integration was applied. The duration of the time step is 0.005 s, the damping ratio is still 0.03, and the damping matrix is calculated as $\mathbf{C} = 2\xi\sqrt{\mathbf{MK}}$. Structural damages are identified using Kalman Filter subjected to the chaotic excitation at the top. The results are as follows

Table 1 shows the structural stiffness identification using the Kalman Filter for three different conditions. From which, it can be found that the Stiffness and its reduction, i.e., the damage can be identified correctly and precisely, even the damage rate is only 5%. As an example, Table 2 shows the stiffness identification for the 3rd column with respect to different damage rate. In these two tables, “Error” indicates the difference between the real damage rate and the identified damage rate (error = identified damage rate(%)-real damage rate(%)). It can be seen that minor damages can be identified effetely.

Table 1 Stiffness identification using Kalman Filter

Condition	story	Original stiffness (KN/mm)	Present stiffness (KN/mm)	Damage rate (%)	Identified stiffness (KN/mm)	Identified damage rate (%)	Error
Intact structure	5	1000	1000	0	999.5494	0.04506	0.04506
	4	1000	1000	0	999.3727	0.06273	0.06273
	3	1000	1000	0	999.4318	0.05682	0.05682
	2	1000	1000	0	999.3081	0.06919	0.06919
	1	1000	1000	0	999.4541	0.05459	0.05459
Damage at the top column	5	1000	950	5	949.6068	5.03932	0.03932
	4	1000	1000	0	999.3704	0.06296	0.06296
	3	1000	1000	0	999.4264	0.05736	0.05736
	2	1000	1000	0	999.2875	0.07125	0.07125
	1	1000	1000	0	999.4439	0.05561	0.05561
Damage at the 3rd column	5	1000	1000	0	999.5652	0.04348	0.04348
	4	1000	1000	0	999.3921	0.06079	0.06079
	3	1000	950	5	949.4987	5.05013	0.05013
	2	1000	1000	0	999.3124	0.06876	0.06876
	1	1000	1000	0	999.4464	0.05536	0.05536

Table 2 Stiffness identification for the 3rd column at different damage rate

Condition	story	Original stiffness (KN/mm)	Present stiffness (KN/mm)	Damage rate (%)	Identified stiffness (KN/mm)	Identified damage rate (%)	Error
Damage at the 3rd column	3	1000	990	1	989.4459	1.05541	0.05541
	3	1000	980	2	979.4596	2.05404	0.05404
	3	1000	970	3	969.473	3.0527	0.0527
	3	1000	960	4	959.486	4.0514	0.0514
	3	1000	950	5	949.4987	5.05013	0.05013
	3	1000	920	8	919.5348	8.04652	0.04652
	3	1000	900	10	899.5573	10.04427	0.04427
	3	1000	850	15	849.6079	15.03921	0.03921
	3	1000	800	20	799.6512	20.03488	0.03488

6. Conclusions

In this paper, a damage identification approach is proposed. Chaotic signals are applied as the input, i.e., the external excitation forces. The reconstructed attractors are built to describe the dynamic characteristics. The distance between the attractor of a testing structure and the one of a baseline intact structure is used to detect the damage. After its detection, it can be further quantified with the Kalman filter for evaluation. Simulations to a five story building model subjected to a chaotic signal are conducted. Results show that the proposed approach can effectively detect not only the certain level of damages but even a very minor damage. After that, damages of about 5% or lower can be quantified successfully. It shows that the proposed damage identification method can do the work very well. It can help to identify minor damages at the early stage. Thus the structure can be repaired and maintained in time and its safety and reliability can be ensured.

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