Smart Structures and Systems, Vol. 10, No. 4-5 (2012) 459-470 DOI: http://dx.doi.org/10.12989/sss.2012.10.4_5.459

Finite element model updating of Canton Tower using regularization technique

Truong Thanh Chung¹, Soojin Cho², Chung Bang Yun^{*3} and Hoon Sohn¹

¹Department of Civil and Environmental Engineering, KAIST, Korea

²Department of Civil and Environmental Engineering, University of Illinois at Urbana-Champaign, USA ³School of Urban and Environmental Engineering, UNIST, Korea

(Received December 4, 2011, Revised May 22, 2012, Accepted June 18, 2012)

Abstract. This paper summarizes a study for the modal analysis and model updating conducted using the monitoring data obtained from the Canton Tower of 610 m tall, which was established as an international benchmark problem by the Hong Kong Polytechnic University. Modal properties of the tower were successfully identified using frequency domain decomposition and stochastic subspace identification methods. Finite element model updating using the measurement data was further performed to reduce the modal property differences between the measurements and those of the finite element model. Over-fitting during the model updating was avoided by using an optimization scheme with a regularization term.

Keywords: Canton Tower; modal identification; finite element model updating; regularization

1. Introduction

Finite element (FE) modeling is a subject that has a long history development and has received widely acceptance in various disciplines of engineering: aerospace, civil, mechanical and electrical engineering. The development of modern computers led to the ability to construct large and intricate FE models, which has a profound impact in the process of engineering design and development. However, these FE models do not always reflect the measured data with sufficient accuracy. Zhang *et al.* (2000) pointed out several factors contribute to this discrepancy: (1) inaccuracy in the FE model discretization, (2) uncertainties in the geometry and boundary conditions, (3) variations in the material properties, (4) environmental variability (such as temperature and wind) and variability in the operational conditions (such as traffic) during measurement, and (5) errors associated with the measured signals and the post processing techniques.

FE model updating is a process of tuning the FE model so that the updated model gives a better reflection of the measured data. There are generally two approaches of FE model updating: global methods and local methods (Friswell *et al.* 1995). The global approach updates the system matrices so that the updated model can replicate exactly the measured response. The main drawback of this method is that the updated FE model does not keep the structural connectivity and the suggested corrections are not always physically meaningful. The local approach updates a pre-selected set of

^{*}Corresponding author, Chaired Professor, E-mail: cbyun@unist.ac.kr, Formerly at KAIST, Korea

Truong Thanh Chung, Soojin Cho, Chung Bang Yun and Hoon Sohn

physical parameters of the original model and, thereby, generates a updated model that is physically realistic.

This paper presents the FE model updating process of a super-tall structure: the 610 m-tall Canton Tower in China. First, the modal properties of the tower are identified from the measurement data, which were provided by the research group of Prof Ni. Y.Q from the Hong Kong Polytechnic University (Ni *et al.* 2012). More detail about the structure and the experiment to obtain the measurement data will be shown in the following section. Next, the FE model updating is performed using a local approach with updating parameters for equivalent beam segments in the reduced FE beam model of the tower. The FE model updating is posed as an optimization problem with the objective function defined in term of the discrepancy between the experimental modal data and those calculated from the FE model. Since this optimization process is generally ill-posed, i.e. a small error in measurements may lead to unreasonably large errors in the updating parameters (Ahmadian *et al.* 1998, Zhang *et al.* 2000, Friswell *et al.* 2001), an additional term so-called a regularization function (or penalty function) is added to the objective function. The main idea of introducing this regularization function is to prevent excessive and unrealistic changes of the updating parameters. The resultant of the FE model after updating will then be discussed to show the performance of the updating method.

2. The Canton Tower

The Canton Tower located in the city of Guangzhou, China, is a super-tall structure with a total height of 610 m. It consists of a main tower of 454 m tall and an antenna mast of 156 m tall. The



Fig. 1 Canton tower and positions of accelerometers: (a) side view, (b) positions and directions of accelerometers, (c) outer tube, (d) inner tube, (e) plan view of accelerometers and (f) examples of measured accelerations at Sensors 1 and 11

460

main tower is a tube-in-tube structure which comprises a reinforced concrete inner tube and a steel lattice outer tube (Figs. 1(a)-(d)). The outer tube consists of 24 concrete-filled-tube columns uniformly spaced in an oval configuration and inclined in the vertical direction. The dimensions of the outer tube decrease from 50×80 m at the ground to the minimum of 20.65×27.5 m at the waist (280 m) before increasing to 41×55 m at the top (454 m). The inner tube is also an oval with constant dimensions of 14×17 m. The antenna mast at the top of the main tower is a steel spatial structure with an octagonal cross-section of 14 m in the maximum diagonal.

A sophisticated structural health monitoring system consisting of over 700 sensors has been designed and implemented by the Hong Kong Polytechnic University(HKPU) for both in-construction and in-service monitoring (Ni *et al.* 2011). In order to obtain the tower's dynamic responses, 20 uniaxial accelerometers were deployed at 8 sections along the tower as shown in Fig. 1(b). Four uniaxial accelerometers were installed at Section 4 and 8, while two uniaxial accelerometers were installed at Section 4 and 8, while two uniaxial accelerometers were installed at Section 4 and 8, while two uniaxial accelerometers were installed at Section 4 and 8, while two uniaxial accelerometers were installed at Section 4 and 8, while two uniaxial accelerometers were installed in the direction in long and short axes of the inner tube oval as shown in Fig. 1(e). Examples of the ambient vibration acceleration data are shown in Fig. 1(f) measured by Sensors 1 and 11 for a period of 1 hour. Ambient vibration measurement was carried out for 24 hours starting at 18:00pm on January 19, 2010. The data is then subdivided into 24 data sets, each with



Fig. 2 Full and reduced FE models: (a) full FE model and (b) reduced model with equivalent beam elements

one hour length. Wind data, including direction and speed, and ambient temperature data were also recorded using an anemometer and a thermocouple installed at the top of the main tower.

The full FE model established by the HKPU is shown in Fig. 2(a). It contains 122,476 elements, 84,370 nodes, and 505,164 degrees of freedom in total. The size of the full model is too large for structural health monitoring and related studies, therefore, a reduced model was established based on this full model by the HKPU with the following assumptions: the floor systems are assumed as rigid body, each segment between two adjacent floors is modeled as an equivalent beam element, and the masses are lumped at the corresponding floors (Ni *et al.* 2012). Consequently, the whole structure is modeled as a cantilever beam with 37 beam elements and 38 nodes (Fig. 2(b)). The main tower consists of 27 elements and the mast is modeled as 10 elements.

3. Output-only modal identification

3.1 Frequency domain decomposition method

The relationship between the input excitation x(t) and the output response y(t) can be expressed in terms of spectral matrices

$$G_{vv}(\omega) = H(\omega)G_{vr}(\omega)H^{I}(\omega)$$
⁽¹⁾

where $G_{xx}(\omega)$ and $G_{yy}(\omega)$ are the power spectral density matrix of the input and the output, respectively, and $H(\omega)$ is the frequency response function matrix. Then using the frequency domain decomposition(FDD) method (Brincker et al. 2000), $G_{yy}(\omega)$ can be decomposed as

$$G_{vv}(\omega) = U(\omega)S(\omega)U'(\omega)$$
⁽²⁾

where $S(\omega)$ is a diagonal matrix holding the singular values, and $U(\omega)$ is a unitary matrix containing the singular vectors. Then the natural frequencies can be evaluated from the peaks of the 1st singular values plotted versus frequency, while the mode shapes are the corresponding 1st singular vectors at those frequencies.

3.2 Stochastic subspace identification method

The stochastic subspace identification (SSI) method (Overschee and De Moor 1996) starts by constructing and decomposing the block Hankel matrix H_{n_1,n_2} into an observability matrix O_{n_1} and an extended controllability matrix Γ_{n_2}

$$H_{n_1,n_2} = \begin{vmatrix} R_1 & \dots & R_{n_2} \\ \vdots & \ddots & \vdots \\ R_{n_1} & \dots & R_{n_1+n_2-1} \end{vmatrix} = O_{n_1} \Gamma_{n_2}$$
(3)

where R_k is the output covariance matrix. The observability matrix has a form as

$$O_{n_1} = \begin{bmatrix} C & CA & \dots & CA^{n_1-2} & CA^{n_1-1} \end{bmatrix}^T$$
(4)

where A is the discrete state matrix, C is the discrete observation matrix. By define $O_{n_1-1} = [C \ CA \ ... \ CA^{n_1-2}]^T$ and $O_{n_1}^{\uparrow} = [C \ ... \ CA^{n_1-2} \ CA^{n_1-1}]^T$, the following relationship is established and the state matrix A is obtained using pseudo-inverse technique.

$$O_{n_1}^{\uparrow} = O_{n_1 - 1}A \tag{5}$$

From the discrete system matrix A, the eigenvalue (λ_i) and eigenvector (ψ_i) are obtained and then the natural frequencies and mode shapes of the system can be found from the following relationships

$$\lambda_{C_i}, \lambda_{C_i}^* = -\xi_i \omega_i \pm j \omega_i \sqrt{1 - \xi_i^2}$$

$$\phi_i = C \psi_i$$
(6)

where $\lambda_{C_i} = \ln(\lambda_i)/\Delta t$ is the *i*th eigenvalue of the continuous system, Δt is the sampling time, and ξ_i is the modal damping ratio.

3.2 Results of modal analysis of Canton Tower

Modal analysis has been carried out using the measurement data from the Canton Tower. Each of 24 data sets of 1 hours has been analyzed separately, then the natural frequencies and modes shapes are averaged. Fig. 3 shows the stabilization chart of the SSI method plotted along with the first singular values of the FDD method.

Table 1 shows the natural frequencies identified using FDD and SSI methods in comparison with the analysis results using the reduced FE model with 37 beam elements. BX and BY denote the bending modes in X and Y directions; T denotes the torsional mode; the values in parenthesis are differences (%) with respect to those from the FE analysis. The lowest row indicates average difference between FE model and identified model in frequencies. Fig. 4 shows the identified mode shapes from the SSI and FDD methods compared with the FE analysis results. The mode shapes identified from two methods are nearly identical except for the torsional mode(mode 6). The



Fig. 3 Comparison of stabilization chart of SSI with the 1st singular value of FDD

463

Modes	Mode types -	Natural frequencies				
		FEM	FI	DD	SS	SI
1	BY	0.110	0.094	(-15.2)	0.094	(-15.1)
2	BX	0.159	0.138	(-12.8)	0.138	(-12.8)
3	BX	0.346	0.366	(5.7)	0.366	(5.8)
4	BY	0.369	0.424	(14.9)	0.425	(15.2)
5	BY	0.399	0.475	(18.9)	0.474	(18.6)
6	Т	0.461	0.506	(9.8)	0.506	(9.8)
7	BX	0.485	0.523	(7.8)	0.522	(7.7)
8	BX	0.738	0.795	(7.7)	0.796	(7.9)
9	BY	0.903	0.965	(6.9)	0.964	(6.8)
10	BX	0.997	1.150	(15.4)	1.149	(15.2)
11	BX	1.037	1.191	(14.8)	1.191	(14.9)
12	Т	1.122	1.251	(11.5)	1.249	(11.4)
Average difference		(11.8)		(11	.8)	

Table 1 Natural frequencies from FDD and SSI modal identification methods

*Note: The values in parentheses are differences (%) from those from the FE analysis



average different of the natural frequencies between the measurement and those from the FE model is approximately 11.8%.

4. Finite element model updating

4.1 Theory of finite element model updating

In FE model updating, an optimization problem is set-up to minimize the differences between the experimental and calculated modal data by adjusting the structural properties of the pre-selected members. In this paper, only the stiffness matrix is adjusted, and the mass matrix is assuming correct. The updated global stiffness matrix is represented by introducing updating parameters for

element-level stiffness matrices as

$$K = K_0 + \sum_{i=1}^{NP} \theta_i K_{0,i}$$
⁽⁷⁾

where K_0 is the global stiffness matrix of the initial FE model, $K_{0,i}$ is the stiffness matrix of the initial FE model of the *i*th beam element or element group, θ_i is the corresponding updating parameter, and *NP* is the number of the updating parameter. Let θ represent the vector of the updating parameters, $\theta = [\theta_1, ..., \theta_{NP}]^T$.

Define the residual vector as $r = [r_f r_{\phi}]^T$ where r_f represents the residual of frequencies and r_{ϕ} represents the residual of mode shapes as

$$r_i^f = \frac{\left| f_i - \tilde{f}_i \right|}{\tilde{f}_i}; \ r_i^{\varphi} = \frac{1 - \sqrt{MAC_i}}{\sqrt{MAC_i}}$$
(8)

where f_i and \tilde{f}_i denote the calculated and measured natural frequencies of the *i*th mode. The modal assurance criteria (MAC) values between the measured mode shapes and those of the FE analysis are defined as

$$MAC_{i} = \frac{\left(\varphi_{i}^{T}\tilde{\varphi}_{i}\right)^{2}}{\left(\varphi_{i}^{T}\varphi_{i}\right)\left(\tilde{\varphi}_{i}^{T}\tilde{\varphi}_{i}\right)}$$
(9)

where φ_i and $\tilde{\varphi}_i$ denote the calculated and measured mode shape vectors.

In this study the objective function is defined by combining the modal residual term and the regularization term as

$$J = J_{residual} + \lambda J_{regularization} \tag{10}$$

with

$$J_{residual} = r_f^T r_f + w r_{\varphi}^T r_{\varphi}$$
(11)

and

$$J_{regularization} = \theta^T \theta \tag{12}$$

The objective function in Eq. (10) consists of two parts: the first part is the residual function $J_{residual}$, and the second part is the regularization function (or penalty function) $J_{regularization}$. The trade-off between them is controlled by the regularization parameter λ . The residual function $J_{residual}$ consists of the residuals of frequencies and mode shapes where the relative important between them is represented by the vector of weighting factors w.

The regularization technique can be seen as the minimization of the residual function $J_{residual}$ while putting a restriction on the update parameters θ . In this study, the restriction is applied by adding a regularization term (or penalty term) which is a square norm of the updating parameter vector θ , so that excessive changes in the parameter updating may be prevented to obtain a feasible FE model.

The remaining question is how to identify the optimum value of the regularization parameter λ , since different results of updating parameter θ are obtained with different values of λ . In this paper, the *L*-curve method (Hansen and O'Leary 1993) is used to identify the optimum regularization parameter. *L*-curve is a plot between the residual function and the regularization function over a range of values of λ . Theoretically, when plotting the curve in log-log scale, it almost has the



characteristic of an L shape with a distinct corner separating the vertical and horizontal parts of the curve as in Fig. 5. The optimum value of the regularization parameter λ can be obtained at the corner of the L-curve, because this region gives the solution with a good compromise between achieving a small residual function and keeping the regularization function reasonably small.

4.2 Results of finite element model updating of Canton Tower

The reduced FE model of Canton Tower is updated using the modal analysis results. Since the mode shape of two rotational modes: mode 6 and mode 12 have poor match to the FE model, these two modes are excluded for FE model updating. In addition, it is well known that natural frequencies can be identified much more reliable than mode shapes, therefore the weighting



466

parameters w is chosen less than 1. In this paper, it is chosen w=0.02.

In order to reduce the number of the updating parameters and the complexity of the optimization problem, the FE model with 37 beam elements described in Section 2 is subdivided into 11 groups. The beam elements in each group are assumed to have the same updating parameters. The main tower is divided into 9 groups, each group has 3 elements; and the mast is divided into 2 groups, each group has 5 elements. In total, there are 11 group of elements. The bending stiffness in two directions *X* and *Y* are chosen for updating the FE model, which results in 22 updating parameters. The feasible range of each updating parameters is taken as $-0.5 < \theta_i < 0.5$.

To minimize the objective function defined in Eq. (10), the function *fmincon* with sequential quadratic programming algorithm (Boggs and Tolle 1995) within the Optimization Toolbox of MATLAB is used. This is a widely used method for non-linear constrained optimization. Model updating is carried out for a series of the values of regularization parameter λ which results an *L*-curve as in Fig. 6. The corner of the curve is not as clear as expected, but arguably, we can find the value λ =0.002 as the optimum regularization parameter. The updating parameters results are shown in Fig. 7 with two sets of results: with and without using the regularization technique (λ =0.002 and 0.000, respectively).

Modal properties of two updated models are shown in Tables 2 and 3. It can be observed that the



Fig. 7 Updating parameters of stiffness: (a) x-direction and (b) y-direction

	Natural frequencies (Hz)				
Modes	Experiment	Before update	After update	After update	
	(551)	_	(without regularization)	(with regularization)	
1	0.094	0.110 (15.1)	0.094 (0.0)	0.093 (-0.5)	
2	0.138	0.159 (12.8)	0.139 (0.7)	0.141 (1.8)	
3	0.366	0.346 (-5.8)	0.368 (0.5)	0.369 (0.7)	
4	0.425	0.369 (-15.2)	0.420 (-0.9)	0.406 (-4.1)	
5	0.474	0.399 (-18.6)	0.441 (-7.2)	0.434 (-8.5)	
6	0.506	0.461 (-9.8)	0.469 (-7.3)	0.469 (-7.3)	
7	0.522	0.485 (-7.7)	0.523 (0.1)	0.518 (-0.9)	
8	0.796	0.738 (-7.9)	0.802 (0.9)	0.791 (-0.5)	
9	0.964	0.903 (-6.8)	0.947 (-1.9)	0.958 (-0.8)	
10	1.149	0.997 (-15.2)	1.082 (-5.9)	1.063 (-7.6)	
11	1.191	1.037 (-14.9)	1.143 (-4.3)	1.124 (-5.6)	
12	1.249	1.122 (-11.4)	1.187 (-5.0)	1.137 (-9.0)	
Average differences		(11.8)	(2.9)	(3.9)	

Table 2 Natural frequencies of FE models before and after updating

*Note: The values in parentheses are differences (%) with respect to the experiments

Modes	MAC values					
	Before update	After update (without regularization)	After update (with regularization)			
1	0.88	0.95	0.89			
2	0.93	0.98	0.94			
3	0.86	0.95	0.93			
4	0.87	0.91	0.93			
5	0.83	0.95	0.92			
7	0.76	0.94	0.93			
8	0.75	0.83	0.82			
9	0.74	0.80	0.79			
10	0.68	0.86	0.82			
11	0.81	0.80	0.82			
Average	0.81	0.90	0.88			

Table 3 MAC values of FE models before and after updating

*Two torsional modes (Modes 6 and 12) are excluded from this Table

average difference of the updated frequencies from the measurements is significantly reduced from 11.8% (before update) to 2.9% (update without using the regularization technique) and 3.9% (update with using the regularization technique). MAC values also have considerable improvement after updating process, from 0.81 (before update) to 0.90 (update without using the regularization technique) and 0.88 (update with using the regularization technique).

By comparing two sets of the updating parameters without and with using regularization term (Fig. 7), we can see that a significantly changes of the updating parameters results only a small

improvement in the modal properties matching (Tables 2 and 3). This indicates that the objective function without regularization term suffers from the over-fitting issue. And therefore, it is reasonable to accept the set of the updating parameters with the regularization term. Of course subsequent field investigation is still required for the validation of the updated model presented here.

5. Conclusions

This paper analyses the data collected from the Canton Tower, a high-rise slender structure in China, which was established as an international benchmark problem for advanced structural health monitoring. Modal properties of structure were evaluated using two different output-only identification methods: FDD and SSI. The natural frequencies of the first 12 modes are found to differ from those of the FE model with 37 beam elements approximately 11.8% in average, which indicates the need for updating the FE model.

FE model updating using the measurement data was further performed to reduce the modal property differences between the measurements and those of the FE model. The special consideration of this paper is to avoid the over-fitting problem by applying a regularization technique to the objective function. To identify the optimum value of the regularization parameter, the *L*-curve method was used. The modal properties from the updated model are found excellent agreement with the measurement. However, further field investigation is needed to validate the updated model presented in this paper.

Acknowledgements

The authors gratefully acknowledge Professor Ni, Y.Q., Hong Kong Polytechnic University, Hong Kong, China for the generosity of sharing information related to the benchmark problem of the Canton Tower. This work is supported by Smart Infra-Structure Technology Center (R11-2002-101-03001-0) and the Leap Research Program (2011-0016470) of National Research Foundation (NRF) of Korea funded by Ministry of Education, Science & Technology (MEST).

References

- Ahmadian, H., Mottershead, J.E. and Friswell, M.I. (1998), "Regularisation methods for finite element model updating", *Mech Syst Signal Pr.*, **12**(1), 47-63.
- Boggs, P.T. and Tolle, J.W. (1995), "Sequential Quadratic Programming", Acta Numerica , 4, 1-51
- Brincker, R., Zhang, L. and Andersen, P. (2001), "Modal identification of output-only systems using frequency domain decomposition", *Smart Mater Struct*, **10**(3), 441-445.
- Friswell, M.I. and Mottershead, J.E. (1995), *Finite element model updating in structural dynamics*, Kluwer Academic Publishers.
- Friswell, M.I., Mottershead, J.E. and Ahmadian, H. (2001), "Finite-element model updating using experimental test data: parametrization and regularization", *Philos. T. R. Soc. A.*, **359**(1778), 169-186.
- Hansen, P.C. and O'Leary, D. (1993), "The use of the L-curve in the regularization of discrete ill-posed problems", SIAM J. Sci. Comput., 14(6), 1487-1503.
- Ni, Y.Q., Xia, Y., Liao, W.Y. and Ko, J.M. (2009), "Technology innovation in developing the structural health monitoring system for Guangzhou New TV Tower", *Struct Control Health Monit.*, **16**(1), 73-98.

- Ni, Y.Q., Xia, Y., Lin, W., Chen, W.H., and Ko, J.M. (2012), "SHM benchmark for high-rise structures: a reduced-order finite element model and field measurement data", *Smart Struct Syst*, in this issue.
- Overschee, V.P. and De Moor, B. (1996), Subspace identification for linear systems, Kluwer Academic Publisher. Zhang, Q.W., Chang, C.C., Chang, T.Y.P. (2000), "Finite element model updating for structures with parametric constraints", *Earthq. Eng. Struct. D.*, 29(7), 927-944.