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Numerical and experimental investigation for damage detection in FRP composite plates using support vector machine algorithm

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Abstract. Detection of damages in fibre reinforced plastic (FRP) composite structures is important from the safety and serviceability point of view. Usually, damage is realized as a local reduction of stiffness and if dynamic responses of the structure are sensitive enough to such changes in stiffness, then a well posed inverse problem can provide an efficient solution to the damage detection problem. Usually, such inverse problems are solved within the framework of pattern recognition. Support Vector Machine (SVM) Algorithm is one such methodology, which minimizes the weighted differences between the experimentally observed dynamic responses and those computed using the finite element model- by optimizing appropriately chosen parameters, such as stiffness. A damage detection strategy is hereby proposed using SVM which perform stepwise by first locating and then determining the severity of the damage. The SVM algorithm uses simulations of only a limited number of damage scenarios and trains the algorithm in such a way so as to detect damages at unknown locations by recognizing the pattern of changes in dynamic responses. A rectangular fiber reinforced plastic composite plate has been investigated both numerically and experimentally to observe the efficiency of the SVM algorithm for damage detection. Experimentally determined modal responses, such as natural frequencies and mode shapes are used as observable parameters. The results are encouraging since a high percentage of damage cases have been successfully determined using the proposed algorithm.

Keywords: support vector machine; natural frequencies; mode shapes; fibre reinforced plastic (FRP) composites; finite element analysis; damage detection

1. Introduction

Fibre Reinforced Plastics (FRP) is extensively used in weight sensitive applications, such as in the field of aerospace, marine, automotive, sports etc. due to its high specific strength and stiffness for almost last three decades. The application of FRP in infrastructure is somewhat recent and is attributed to its superior durability, fatigue strength and corrosion resistance, as compared to conventional materials. Usually pultruded sections in regular forms are used as structural components and are likely to replace most of their similar counterparts made of conventional

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isotropic materials in the near future. The possibility of using different anisotropic materials in layers and in desired orientations brings infinite design possibilities to cater to the need of particular applications in terms of geometric configurations and material property combinations.

Unlike the weight sensitive aerospace applications, these infrastructure facilities made of FRP are supposed to operate over long period of time and may deteriorate considerably, yet remaining serviceable. In case of layered FRP structures, such deterioration may be within the layers and may not be visible readily and needs non-destructive evaluation from time to time for their condition assessment and effective health monitoring. The material and structural fabrication are simultaneous operation for the manufacture of FRP structures, thus it inflicts uncertainties into the stiffness and strength as well as into its energy dissipation capabilities even during its inception. Thus, it appears that if long term condition assessment of FRP infrastructural facilities is in mind, the existing material properties need to be correctly assessed using a reliable non-destructive inverse technique. Although such effort may be somewhat easier for a newly built FRP structure, it will become more difficult for existing structures in the long run as it deteriorates. All the significant parameters of interest, such as the material elastic properties etc. need to be updated altogether to check if substantial changes into them have really taken place due to damage over long period of time.

Early detection of damages in structures, especially for layered fibre reinforced plastic type of composites is extremely important from the condition assessment and health monitoring point of view. Vibration based damage detection techniques are one of the most popular methodologies because of the fact that measurements can be made at convenient locations globally, whereas the localized damaged site need not be directly accessible. This is particularly advantageous for layered FRP structures in which damaged locations are mostly within layers and difficult to access for measurement. Experimental modal testing provides scope for the measurement of damage sensitive vibration characteristics, viz. natural frequencies and mode shapes etc. In such inverse methodologies the differences between the experimentally measured dynamic response quantities (i.e., natural frequencies, mode shapes etc.) and those computed from finite element modeling are minimized to estimate the parameters related to the stiffness loss. Such algorithms operate in stages- first to judge the existence and subsequently assess the location of such damages before finally estimating the quantitative extent of damage. The vibration based damage detection problems can be posed as statistical pattern recognition approach with the help of Support Vector Machine (SVM) algorithm in which simulations of a limited number of damage scenarios are used to train the algorithm so as to detect damages at unknown locations by recognizing the pattern of changes in measured dynamic responses of structures.

Damage detection methodologies using changes in natural frequencies and mode shapes has been surveyed by Salawu and Williams (1994). Doebling *et al.* (1998) into their review paper described different aspects of vibration based damage detection techniques. A more recent review paper by Carden and Fanning (2004) discusses vibration based condition monitoring with particular emphasis on structural engineering applications. Bai *et al*. (2014) used fractal dimension analysis for the mode shapes to investigate damages in structures. In this paper, mode shapes were decomposed using the stationary wavelet transformation, with the damage information and the noise separated into distinctly scaled values of mode shapes. A scanning laser vibrometer was used for the experimental investigation on a cracked aluminum beam.

Prior to the damage detection of any structure, it is to be ensured that the other important parameters such as the elastic material parameters and boundary conditions have much less uncertainty than the parameters related to the stiffness loss. The damage sensitive vibration response quantities, such as the natural frequency and the mode shape information have also been used to identify the different material parameters of layered composite structures using inverse approaches. Goni *et al*. (2015) investigated the determination of material elastic parameters along with the elastic boundary stiffness parameters from modal properties of FRP composites plates. Mondal *et al*. (2016) identified the in-plane as well as out-of-plane elastic properties of sandwich composite plates using a gradient based inverse eigensensitivity method (IEM) under free boundary conditions. Mondal and Chakraborty (2016) identified the in-plane as well as out-of-plane elastic properties of pultruded Glass Fibre Reinforced Plastic (GFRP) laminated composites using IEM. The in-plane parameters were estimated from composite plate samples, whereas the out-of-plane shear moduli were determined from beam samples.

Use of measured vibration responses to detect damages are plenty in current literature. Sampaio and Chan (2015) conducted experimental study on railway bridges to estimate the modal parameters. Garcia-Palencia *et al.* (2015) used frequency response functions (FRFs) to identify damages. Abderahmane et al. (2016) used finite element analysis combined with acoustic emission techniques to determine the interfacial failure of a carbon fiber reinforced polymer (CFRP) repair patch on a notched aluminum substrate. Li *et al*. (2014) and Nagarajaiah and Erazo (2016) reviewed various recent innovations and applications of the structural health monitoring methodologies.

There are plenty of literature available which discusses damage detection of FRP type of composites exclusively. Three different approaches- a graphical method, an artificial neural network approach and a surrogate-based optimization approach are compared for determining existence, location and extent of delamination in composite beams by Zhang *et al.* (2013). The problem of stiffness degradation of multidirectional symmetric laminates in the presence of off-axis cracks in symmetric laminates has been studied by Carraro and Quaresimin (2015). Very good agreement with experimental data has been observed. Harizi *et al.* (2015) proposed a methodology to map the heterogeneity of Glass Fiber-Reinforced Polymer laminated plates using ultrasonic waves in C-Scan mode. This can be easily used for the evaluation of damage caused during manufacturing or during service of such structures. Garcia D. and Trendafilova (2015) used the Singular Spectrum Analysis technique to assess damage in a two degrees of freedom mass-spring-damper system and subsequently applied to an experimental case study of a composite laminated beam. Yang and Oyadiji (2017) proposed a delamination detection approach using modal frequency surface and its wavelet coefficients by attaching point mass at different locations of a plate. The introduced local inertia greatly enhances the dynamic response at a delaminated section which produces significant frequency deviation. Meihong *et al*. (2017) determined damage location and severity for single and multiple damage sites in composite beams based on the curvature mode difference. Very simple impact hammer test was used for experimental verification. The amount of discrepancy in dynamic responses between a real composite structure and its numerical simulation can be modelled as noise added to structural dynamic parameters. A sensitivity analysis is carried out by Zhang *et al*. (2017) to determine the influence of such discrepancies on the prediction accuracy of delamination in composite beams. Soleimanpour and Ng (2017) detected delaminations in laminated composite beams using nonlinear guided waves. Chiachío *et al.* (2017) proposed a multilevel Bayesian inverse problem framework to deal with the sources of uncertainties associated with the measurement error and also due to the modelling errors due to inadequate assumptions in the context of ultrasound-based damage identification. The method is tested against a post-impact fatigue damage experiment in a cross-ply carbon-epoxy laminate.

Amongst the various damage detection methodologies using pattern recognition algorithms, SVM employs heuristic statistical learning mechanisms (Vapnik 2000). Such technique finds applications in diverse fields, such as in bio-informatics, computational linguistics, computer vision, etc. (Nello and Taylor 2000). Sohn *et al.* (2001) used multiple pattern recognition techniques for structural health monitoring, combining Auto-Regressive model with eXogenous (ARX) input. However, the use of SVM in structural damage detection is somewhat recent. He and Yan (2007) used SVM in combination with non-linear wavelets to identify damages in a single layer spherical lattice dome type of structure from measured ambient responses. Das *et al.* (2007) used SVM algorithm, employing time varying frequency data to identify damages in a carbon fiber reinforced plastic (CFRP) composite plate. Surface-mounted piezoelectric transducers were used to characterize the damages from measured waves. Park *et al.* (2008) used SVM to identify damage in a rail road track. Two types of damages were considered- a hole of 5 mm diameter at the web and a transverse cut of 5 mm in length with the same depth. Bornn (2009) presents an auto regressive SVM algorithm for damage detection using sensor output data from vibrating structure, implementing a nonlinear time-series model. Barthorpe and Worden (2011) employed a set of binary support vector machines for locating damages. The major contribution of the above work seems to be that a classifier trained for single-site damage is also made workable for multiple damages. Farrar and Worden (2012) described statistical pattern recognition techniques for the purpose of structural health monitoring. This is a valuable reference for comparison of various pattern recognition approaches. Kim *et al.* (2012) investigated damage detection of smart structures using discrete wavelet transform, autoregressive model and a SVM algorithm. Satpal *et al.* (2013) and Gupta *et al.* (2016) investigated the damage detection in isotropic aluminum beams and plates, honeycomb core sandwich panels using SVM, employing mode shapes. The data has been polluted with Gaussian random noise to observe the sensitivity of the algorithm against noise. Rother *et al.* (2014) used Morlet wavelet transform to assess the condition of a machine using SVM. The combination of continuous wavelet transform and SVM is found to be more efficient than the combination with the discrete wavelet transform and SVM for detecting machine faults. Satpal *et al*. (2015) determined damage locations in aluminium beams using SVM as classifier and regression employing both simulation data and experimental data. Cevik *et al*. (2015) provided considerable insight on various aspects of structural health monitoring using SVM. Case studies of damage detections were presented along with the determination of ultimate load capacity of beams and corbels. Shyamala *et al.* (2016) used SVM to identify damage in an isotropic beam using numerically simulated data. Damage detection in FRP composite structures using SVM and comparison of results with artificial neural networks (ANN) were presented by Farooq *et al.* (2012). Static strain data were numerically simulated adding Gaussian noise at 6 predefined locations. One undamaged structure and two damaged structures having small cracks were investigated, with varying material properties and loading conditions. It was found that the SVM performed better than the artificial neural network in almost all cases.

The literature available on SVM shows that the methodology is likely to be robust and can be used for damage detection of existing structures. However, limited literatures are available on damage detection of existing fibre reinforced plastic (FRP) composite structures using SVM. This needs to be explored along with the estimation of elastic parameters prior to the detection of damages in case the structure has already passed a considerable period of serviceable life. In the present investigation, the location and severity of damage in a fibre reinforced plastic composite plate is investigated employing SVM algorithm from experimentally determined natural frequencies and mode shapes. During simulations, the damage is modelled by reducing the elastic moduli, such as the Young's moduli in two orthogonal directions and the shear modulus of the material within an elemental volume which indirectly reflects the localized stiffness loss. Numerically simulated data are generated from a converged finite element model of the composite plate, simulating various damage scenarios. Further, experimental investigation has also been carried out to detect damages in a fabricated FRP composite plate successfully.

2. Mathematical formulation of the support vector machine algorithm

The present paper implements the regression form of SVM to detect damages in a fibre reinforced plastic composite plate. Several damage scenarios are simulated and the regression analysis finds the best fit function by minimizing the error between the model and the observed modal properties. The two datasets are assumed to be linearly independent in a two dimensional space for proper operation. This is practically true when a large number of observations could be made of the output measureable parameters in an over-determined sense. The benefit of using SVM is that, the linearly non-separable data can easily be distinguished by projecting the data space to a higher dimensional space by adding suitable kernel function. The kernel function thus converts the non-linearly separable data into a linearly separable data set in higher dimensions.

The methodology of SVM is basically a supervised statistical learning procedure which analyzes data for both classification and regression. The algorithm works depending upon the principle of structural risk minimization (SRM), which is found to be superior to the traditional empirical risk minimization (ERM) principle that tries to minimize the error between the actual (experimentally observed) value of a parameter and its predicted value. Such prediction may come from finite element modeling of structural behavior (Vapnik 2000). The SVM algorithm employing SRM technique is found to be superior for parameter estimation (such as stiffness loss in damage detection) as compared to the ERM techniques, even within the larger variations of the upper and lower bounds of the parameters. The basic aim of the present investigation is to implement the SVM technique to retain the above mentioned advantages, employing SRM for more robust estimation of damage sensitive parameters, such as the local stiffness loss due to damage in FRP composite plates.

The SVM algorithm classifies a structure as either damaged or undamaged, depending upon the defined loss of local stiffness. This is realized into the algorithm as the normalized distance from a hyper-plane defined in multi-dimensional space to distinguish a damaged state from an undamaged one. The subsequent portion of the algorithm is devoted to the regression form of the SVM algorithm to quantitatively estimate the severity of damage in FRP composite plate (Gunn 1998).

The algorithm works on a set of training data which can be represented as (y,x) where $x=(x_1,x_2,x_3 \ldots x_{n-1}, x_n)$ are the natural frequencies of the FRP plate in 'n' dimensional plane. Here 'y' is a scalar value indicating the class of the '***x***', representing the damage state realized through the local stiffness loss at designated locations. For the present formulation, y is normalized between -1 to +1, hence $y \in \{-1, 1\}$.

Fig. 1 gives a geometric interpretation of the data plotted in two dimensions as an example. It may be mentioned that in actual physical problem where a large number of sets of numerical and experimental modes are compared, the visualization will become difficult in two dimensional plots due to their eventual multidimensional characteristics. Let us take for example, H_1 and H_2 being the two particular classes of data. The idea is that, data points belonging to a particular class is expected to cluster amongst themselves and thus, making it easy to define a hyper-plane

distinguishing these two different states. In our damage detection problem, the two distinguished states for the structure are clearly either the damaged state or the undamaged state.
Thus the damage detection problem within the framework of SVM can be defined as determining an appropriate division or boundary between the clusters indicating either damaged or undamaged states. Considering Fig. 1, $\boldsymbol{x} \in R^2$, where, R is a set of real values of natural frequencies. The data points can be separated by a linear fit of the boundary. This is treated as a classifier for indicating damage state and can be represented as

$$\langle \boldsymbol{w}, \boldsymbol{x} \rangle + \boldsymbol{b} = 0 \tag{1}$$

Where, $\boldsymbol{w}$ is the weighting vector perpendicular to the straight boundary line and b is the perpendicular distance of the boundary line from the origin (called a bias). The above equations can further be represented as

$$\begin{aligned} &\langle \boldsymbol{w}, \boldsymbol{x} \rangle + b \geq 1, \ \ when \ \ y = 1 \\ &\langle \boldsymbol{w}, \boldsymbol{x} \rangle + b \leq -1, \ \ when \ \ y = -1 \end{aligned} \tag{2}$$

The distance d of a point from the hyperplane (called the margin) as shown in Figure 1, can be defined as

$$d = \frac{|\langle \boldsymbol{w}, \boldsymbol{x} \rangle + \boldsymbol{b}|}{\|\boldsymbol{w}\|} \tag{3}$$

The boundary which has the maximum margin from either side of the points is the optimal hyper-plane. For the clearly non-separable data, a functional has to be minimized to decide on which side of the hyper-plane the points under consideration belong.

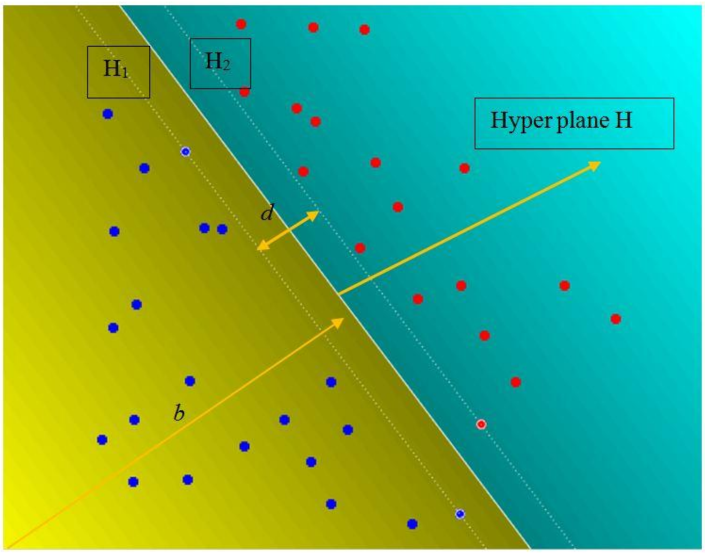

Fig. 1 Hyper planes dividing the data points belonging to two classes

$$\frac{1}{2}\|\boldsymbol{w}\|^2 + C\sum_{i=1}^{n}\xi_i \tag{4}$$

Here, C is the penalty function. The above equation should also satisfy the constraints $\left(\langle \boldsymbol{w}, \theta(\boldsymbol{x}) + b\rangle\right) \geq 1 - \xi, \xi \geq 0$. Here, $\theta(\boldsymbol{x})$ transforms the problem to higher dimensional space. The term ξ represents those data points which are noisy and can be termed as 'mis-classifiers', similar to an outlier. These points should be avoided during computation by adjustment of the penalty function C (Gunn 1998).

When the dataset is very large, extension of the problem to higher dimensional space eventually becomes complicated. Most of the practical damage detection problems are actually expected to fall into this category as damage can be at multiple locations simultaneously having varying extents. If a multiple damage scenario is to be tackled appropriately, the spatial distribution of stiffness loss at several points needs to be stored for further processing and the database becomes comparatively larger. The problem can be solved by choosing suitable Kernel functions efficiently, example of one such Kernel function can be expressed as

$$K(x, x') = \theta(x).\theta(x') \tag{5}$$

Then the resultant Lagrangian which needs to be minimized becomes

$$\min_{\alpha} \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_i y_i \alpha_j y_j K(x_i, x_j) + \sum_{k=1}^{n}\alpha_k \tag{6}$$

Here, i, j, k, are the indices, α is a coefficient and 'n' is ranging upto the number of damages. Here the kernel function does the transformation implicitly and gives a scalar product in the feature dimension space. The kernel used in the present investigation is a Gaussian type of a Radial Basis Function and can be expressed as

$$K(x, x') = \left(e^{-\frac{\|x - x'\|^2}{2\sigma^2}}\right) \tag{7}$$

The classification problem in SVM can be extended to the regression analysis by introduction of a loss function to enable the same framework of pattern recognition scheme described above for classification (whether damaged or undamaged) towards quantification of damage (estimation of severity of damage through stiffness loss). Effort to use SVM for both the purpose-first to locate the damage using classifier approach and then, extent of damage using regression are explained in subsequent sections.

3. Numerically smulated eample

A set of numerically simulated problems are presented in this section in order to explore the possible applications of the SVM algorithm to detect damages in structures. It may be mentioned that the methodology depends upon the availability of a correct baseline model of the structure at its undamaged state. For this, accurate assessment of the elastic parameters, such as stiffness and inertia properties are mandatory. The vibration response of a structure can be conveniently expressed in modal domain in terms of natural frequencies, mode shapes and modal damping factors. Since the influence of damping on the natural frequencies and mode shapes is comparatively less, inclusion of damping in the model is kept out of scope of the present investigation. It is assumed that the stiffness and mass properties of the undamaged structure is either accurately known a-priori or estimated through any suitable methodology before the damage detection using SVM is taken up.

The approach followed in this section using SVM algorithm is to identify the location of the damage first and then, to estimate the extent of the damage. The parameters selected for the present investigation are the material elastic constants of the composite structure. The damage is visualized as a local reduction of stiffness at selected locations. In practical circumstances where the exact values of the material elastic constants are not known, the same can be guessed initially from established standards or from manufacturer's manual or from engineering judgment, considering their statistical variances and nominal values. Else, the values can be obtained from experimental characterization tests of samples; however, this may be difficult for existing structures. The methodology relies upon comparison of experimental modal database with the numerically modeled modal properties to estimate the stiffness loss exclusively due to damage. Hence, the basic steps in the algorithm is to prepare an accurate and converged finite element model of the structure, to obtain experimental modal data as precise as possible, form the objective function from the differences in modal properties obtained by both approaches and finally, the application of SVM to determine the stiffness loss using parameter estimation as damage indicator. In case of numerically simulated examples, the 'experimental' modal data is also simulated using the same finite element model of the damaged structure.

3.1 Finite element modeling of the FRP composite plate

Finite element technique is almost universally used to model complicated structural forms with different material property combinations. This is particularly advantageous for layered FRP type of composites. An equivalent single layer theory (ESL) is most popular for modelling the FRP type of structures for the determination of global dynamic responses. The theory is based on the determination of the average in-plane stiffness parameters of a laminate from defined geometric properties and elastic material properties. The laminate consists of several individual lamina which are assumed to be quasi-homogeneous, orthotropic and connected at lamina interfaces by perfect bond. The principal assumption in modelling, implementing such equivalent single layer theory is that the displacements are assumed to vary through the thickness of the entire laminate according to a single expression and not allowing for possible discontinuities in the deflected shape at the interfaces of two individual laminas. Strains are assumed to be constant within a particular layer. The individual laminas having different orientations are stacked in sequences and the overall stiffness is found by integration after appropriate transformation, taking the geometrical mid-plane as the reference plane. The formulation for the global stiffness and mass matrices for composite structure follows standard procedures explained by Ishai and Daniel (2005) and the equations of equilibrium can be expressed as

$$[\boldsymbol{M}]\{\ddot{x}\}+[\boldsymbol{C}]\{\dot{x}\}+[\boldsymbol{K}]\{x\}=\{F(t)\} \tag{8}$$

An 8 noded quadratic shell element (S8R) incorporating shear deformation has been implemented using ABAQUS. The element has six degrees of freedom per node, thereby allowing both translation and rotation in all directions. The real mode eigenvalues and eigenvectors are obtained by solving the undamped equation of motion.

$$\left\{ \left[\boldsymbol{K}\right] - \omega_i^2 \left[\boldsymbol{M}\right] \right\} \left\{ \boldsymbol{U}_i \right\} = \left\{ 0 \right\} \tag{9}$$

There are several algorithms available in current practice to determine undamped eigenvalues and eigenvectors. Amongst them, two popular solution algorithms are the subspace iteration and Lanczos method. In Lanczos solver, the dimension of the subspace grows for making better approximation of the eigenvectors. Due to this reason, the Lanczos solver is selected and implemented using ABAQUS for the present application. Only free boundary condition is explored for this proof of concept study to eliminate errors accumulating due to imperfect boundary conditions.

The finite element program requires initial values of all the elastic parameters, density etc. for the FRP plate. The mass density is assumed to be 1960 kg/m^3. The nominal values of the in-plane material elastic constants are chosen as E_x=35 GPa, E_y=30 GPa, G_{xy}=7 GPa and the in-plane Poisson's ratio is assumed to be υ_{xy}=0.25 (Mondal and Chakraborty, 2014). Here, x and y are the longitudinal and transverse directions of the plate. The transverse shear moduli, G_{xz} and G_{yz} both are considered to be 5 GPa.

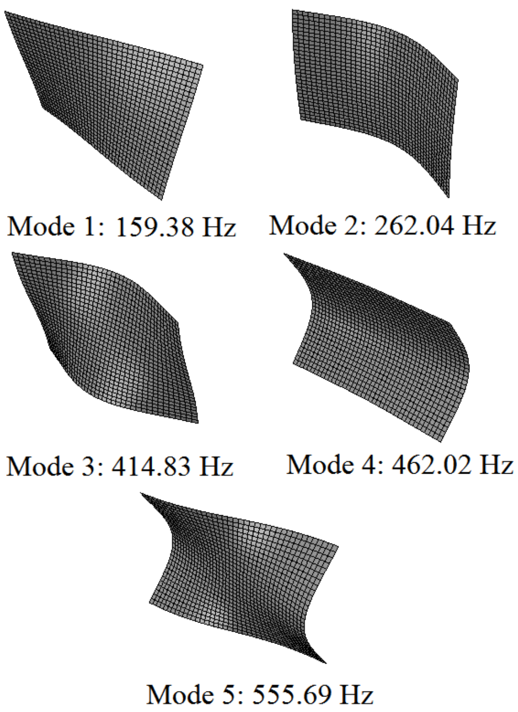

Fig. 2 First five mode shapes and corresponding frequencies of the GFRP composite plate

For this numerically simulated problem, ‘experimental’ modal data are generated using the same finite element model with the above nominal material properties and incorporating damage by reducing the stiffness at selected region locally. The aim of the SVM algorithm is to realize such damages through detection of stiffness loss at a specific location.

The plate investigated here is assumed to have dimensions 400 mm x 300 mm with thickness of 10 mm. Fig. 2 shows a typical set of mode shapes and corresponding frequencies of the undamaged FRP composite plate, modeled using finite element with chosen nominal values of material parameters as input.

3.2 Identifying damage location using SVM

The damage detection algorithm employing SVM is first tried as a classifier. A set of material property values are assumed within selected upper and lower bounds to generate the required data sets of dynamic responses to prepare adequate training data. For the plate considered here, the values of E_x and E_y are made to vary between 15 GPa to 45 GPa. The value of G_{xy} is selected between 3 GPa and 9 GPa. It may be noted that the selected upper and lower bounds are very much conservative as compared to most real applications. In any realistic situation the chances of the actual values of these parameters falling outside these limits are practically negligible. The material parameter sets are generated randomly within these bounds. The density of the plate is assumed to be 1960 kg/m3 and is assumed to have less uncertainty. The plate was divided into small size elements of size 20 mm x 20 mm to create at least 300 sets of elements as shown in Fig. 3. Damage is inflicted into any of these elements by reducing the stiffness within its elemental area to the desired severity. The values of E_X, E_y, G_{xy} are reduced simultaneously in such cases. In a similar manner huge data sets of various damage cases are formed and the data are stored.

For possible damage location 1 to 50 and 251 to 300, 125 combinations of E_x, E_y and G_{xy} for these 100 locations (i.e., 12500 possibilities in total) and for the remaining 200 locations (i.e., from 51 to 250) with 72 combinations of E_x, E_y, G_{xy} (i.e., 14400 possibilities in total) involving first 4 modes of vibration are used to generate the required model for the SVM algorithm. The numbers 125 and 72 are chosen arbitrarily at the moment and later found to be appropriate for predicting the damages.

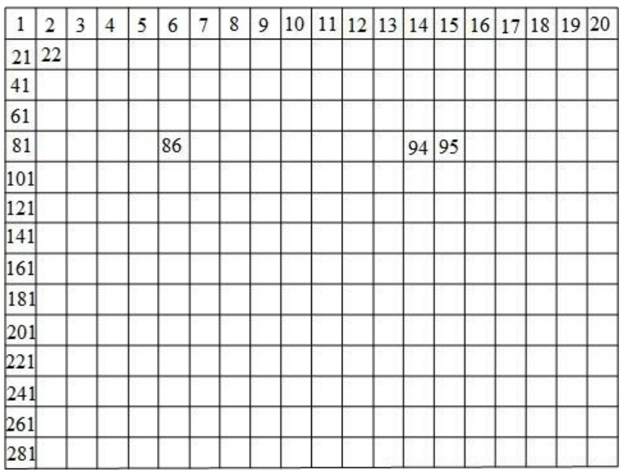

Fig. 3 Discretization of FRP Plate into 300 damage locations

Thus, it needs some trial and error for individual problems to be tackled in various cases to fix the number of combinations of material parameters for generating data sets. A total 26900 cases (14400+12500) are considered as training data sets for the present problem. The intention behind having such a large data set is due to the fact that the damage state of the plate is affected by several combinations of independent material parameters. The optimum values of cost parameter *C* and radial basis function Kernel parameter σ were then obtained by cross-validation technique. The trained data has been split into 10 numbers of sets. Out of these 10 sets, 9 sets of data were used to generate SVM model using cost parameter C (varying between limits 1 to 10000) as well as, the radial basis function Kernel parameter σ (varying between 0.00001 to10). The remaining 10^{th} set (comprising of 2690 data) were used for the validation of the model by selecting different combination of C and σ. It was observed that the optimum value of C and σ are 4000 and 0.008 respectively for the particular example chosen for the present investigation.

A sample size of 6000 was selected randomly for each location as training data from the total overall available 26900 sample data. This is done arbitrarily to reduce the computational effort. The studies involved generating model using natural frequencies alone, mode shapes alone and also using their weighted combinations. For test data (actually numerically simulated damage data which is not considered earlier as training data), 10 locations were chosen in a random manner as shown in Fig. 3. The damages were induced by reducing the values of the in-plane elastic parameters (E_x, E_y, G_{xy}) locally for an element. Further, each location was run for 3 different sets of reduced in-plane parameters, thereby resulting in 30 sample test data.

The actual damages are caused at the location 94 and 95 as indicated in Fig. 3. The results of the damage location prediction using SVM have been given in Table 1, using natural frequencies alone, mode shapes alone as well as, using their weighted combinations. For example, $2\varpi + 1\,\phi$ means, the weight to frequency is 2, whereas the weight to the mode shape is 1.

It may be readily observed from Table 1 that there are large variations within the success rates achieved using various combinations of frequencies and mode shapes for locating the damage. The highest success rate of prediction of correct location of damage was achieved with a combination of twice the weight to the natural frequency with a weight of unity for the mode shapes. This has been done with trial and error for different combinations of weights. However, still with a high percentage of success rates of the SVM, the detection of the location of damage can be done with confidence.

Table 1 Prediction of location of damage using test data (numerically simulated)

Training data	Prediction rate (%) for 30 samples of test data
Only first 10 natural frequencies(ϖ)	24
Only first 10 mode shapes(ϕ)	70
$1\times$ natural frequencies (ϖ) $+ 1\times$ mode shapes (ϕ)	73
$2\times$ natural frequencies (ϖ) $+ 1\times$ mode shapes (ϕ)	80
$5\times$ natural frequencies (ϖ) $+ 1\times$ mode shapes (ϕ)	60

Table 2 Estimation of severity of damage in terms of stiffness losses (numerically simulated)

Training data sets	Error in Prediction rate(%) for 30 samples of test data		
	E_x	E_y	G_{xy}
Only first 10 natural frequencies	10.25	11.32	12.5
Only first 10mode shapes	5.5	5.64	4.6
$1\times$ natural frequencies $(\varpi)+1\times$ mode shapes (ϕ)	5.62	4.24	6.21
$2\times$ natural frequencies $(\varpi)+1\times$ mode shapes (ϕ)	12.21	11.581	8.21
$5\times$ natural frequencies $(\varpi)+1\times$ mode shapes (ϕ)	20.12	22.21	12.01

3.3 Estimation of severity of damage using SVM

After the determination of location, the determination of the severity of damages has been taken up. The errors in predicting the severity of damages are presented in Table 2 and it can be observed that the results are quite reliable with various combinations of frequencies and mode shapes. It may also be noted that the error is least while using the mode shapes information; this is perhaps due to the localized information available in the elements of the mode shapes even if they are normalized. Such information is difficult to be obtained from much globalized natural frequency information alone. It may appear that incorporation of natural frequency information is actually detrimental for predicting the severity. However, incorporation of natural frequency still is necessary to improve the prediction rate of the location of damage. Also, the error for severity prediction is less (around 12% maximum) in the particular case study.

4. Experimental investigation on a composite plate

A pultruded FRP composite plate having the same dimensions as described in the numerically simulated example has been modal tested to determine the natural frequencies and modes shapes using an impact hammer (8207 of B&K) and accelerometers (4507 of B&K) to measure forces and accelerations directly. The plate was suspended from flexible rubber threads to replicate the free boundary conditions. This eliminates the possibility of error due to improper fixity. The frequency of oscillation of the suspended FRP plate is much lower than the frequencies of vibration of the plate within the frequency range of interest, thereby justifying the imposed suspension system as somewhat acceptable replication of the free boundary condition. In case such plates are supported elastically on the boundaries, as the case may be in practice, the boundary stiffness parameters need to be updated along with the material parameters using model updating technique using SVM. The plate was divided into 10 x 10 elements and accelerations at a particular point were measured, whereas the point of application of the force changed sequentially to all the available 121 finite element node points. For the acquisition of the responses, the accelerometer was placed at point 80 as shown in Fig. 4 and the roving hammer procedure was adopted traversing at all the 121 points. The time domain force and acceleration response data were Fourier transformed in a spectrum analyzer and their division gives the FRFs using PULSE LABSHOP modal testing software (B&K). The software MEScope was used to determine the natural frequencies, mode shapes and modal damping values from the measured FRFs using curve fitting. Fig. 4 shows the excitation and response measurement of the FRP plate in free boundary conditions. First the undamaged plate is tested. The plate is then progressively damaged by removing materials by making cuts of square shapes of sizes 10 mm, 20 mm and 40 mm subsequently, extending through half of the thickness of the plate. Fig. 5 compares the first 5 measured mode shapes of the undamaged and the damaged FRP plate with cut of square shape of 40 mm size through half of the depth. Fig. 6 shows the percentage variations of frequencies for various sizes of the cut within the plate, taking the undamaged plate as reference. It may be observed that the dynamic responses of the plate become more sensitive, as the size of the damage increases. Moreover, such variations are mode dependent, thereby providing opportunity for the SVM type of pattern recognition based damage detection algorithm to inversely determine the location as well as, the severity of damages.

The nominal material parameters assumed to model the plate using finite element have been updated with respect to the observed modal parameters of the undamaged plate using SVM. The modal data thus obtained and the updated material properties using SVM are considered as the base line material and structural property data for the plate. The error in the estimation of E_x was less than 1% for 60% of the total cases considered and the error in the estimation of E_y and G_{xy} was less than 1% for 80% of the cases considered. The maximum error for the determination of E_x was about 10% of the nominal values, whereas for E_y and G_{xy} the maximum error is 5% of the nominal values of the baseline model respectively. Thus it can be concluded that the user of the algorithm can easily detect the outliers and rely upon the successfully predicted values. Next, the finite element model was used to generate training data for various combinations of values of E_x ,E_y ,G_{xy}.

Table 3 shows that there are large variations within the success rates achieved using various combinations of frequencies and mode shapes for locating the damage. The highest success rate of prediction of correct location of damage achieved was, like the numerical simulation, with a combination of twice the weight to the natural frequency with a weight of unity for the mode shapes.

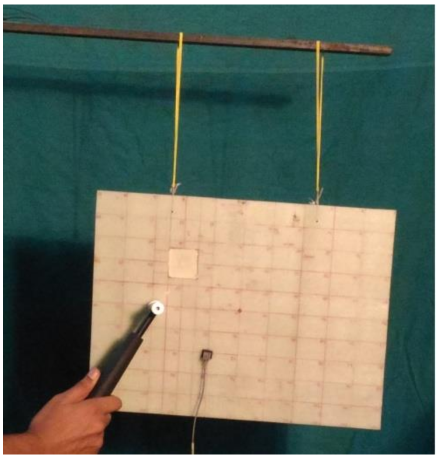

Fig. 4 Modal testing of the damaged pultruded FRP plate

Table 3 Prediction of location of damage using test data (Experimental)

Training data	Predicted damage location from the test data
Only first 10 natural frequencies(ϖ)	19(edge of plate but nearby damage region)
Only first 10 mode shapes (ϕ)	81 (close to actual damage area)
$1\times$ natural frequencies (ϖ) $+ 1\times$ mode shapes (ϕ)	87 (Correctly identified the damage location)
$2\times$ natural frequencies (ϖ) $+ 1\times$ mode shapes (ϕ)	88 (Correctly identified the damage location)
$5\times$ natural frequencies (ϖ) $+ 1\times$ mode shapes (ϕ)	92 (center of plate and nearby actual damage area)

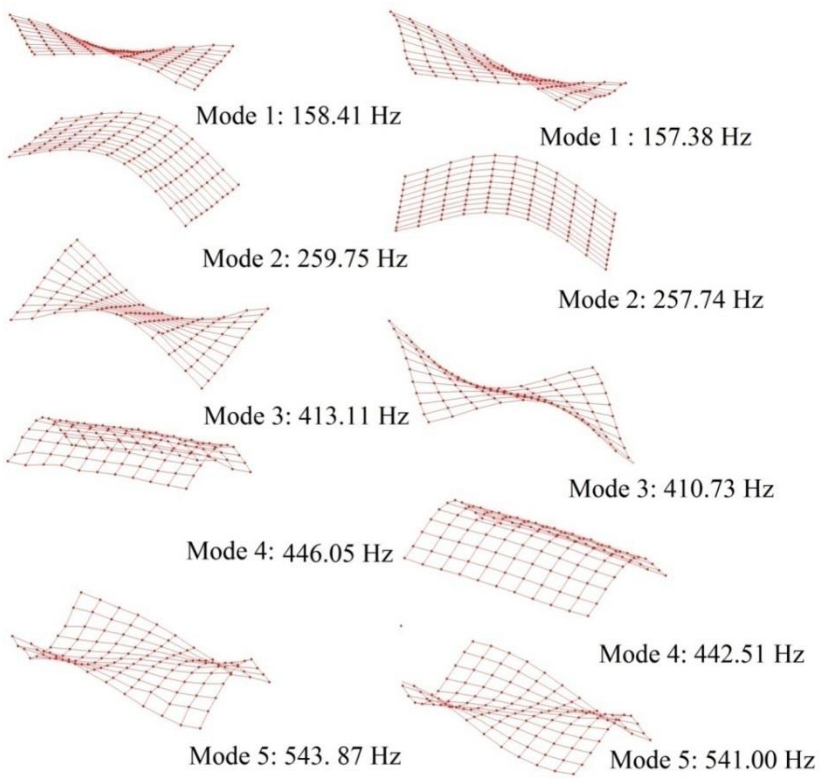

Fig. 5 Experimentally obtained mode shapes (undamaged and damaged mode shapes)

This has been done with trial and error for different combinations of weights. It may be mentioned that the weights to the natural frequencies and mode shapes have been kept same in Table 1(numerical) and Table 3 (experimental). It is clearly seen that the global natural frequencies alone are not very efficient damage indicator, so also the weighted combination of frequencies and mode shapes where natural frequencies are weighed substantially more than the mode shapes.

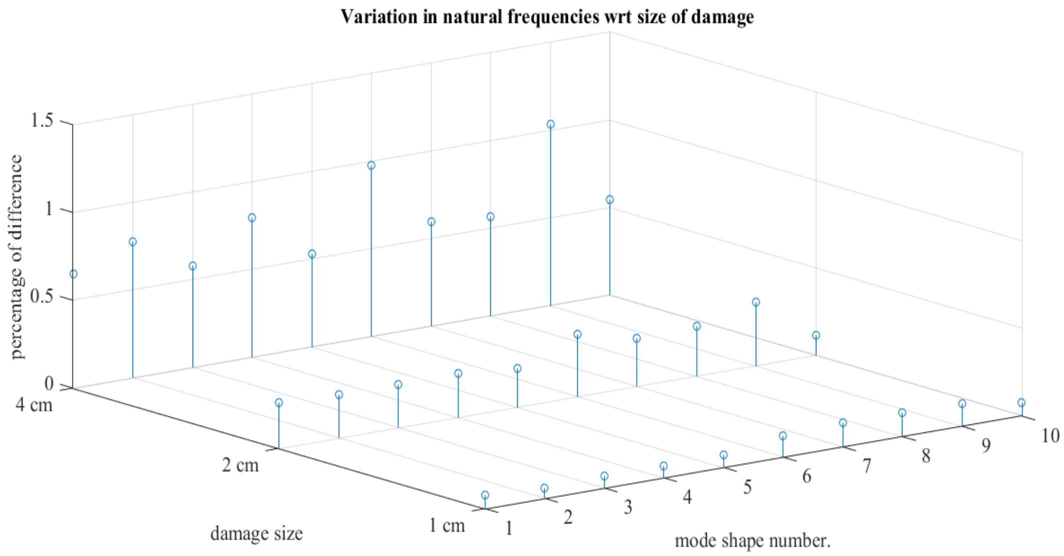

Fig. 6 Variations of natural frequencies with respect to damage sizes (experimental)

In most real applications, it is practicable only to measure the first few flexural modes. Also, deployment of sensors has its own practical limitations- usually displacement or accelerations are measured at selected locations only due to high cost and also sensors take up physical space and affects the dynamics of the body itself. In most practical cases a limited number of modes are measured and displacements sampled at limited coordinates in particular direction (say transverse direction only, rotational degrees of freedom are almost never measured). The above is known as modal and coordinate measurement sparsity and has been accepted as more realistic. Obviously, the present methodology of damage detection of FRP plate employing SVM will perform much better if this restriction of coordinate and modal sparsity is relaxed, i.e. more measurement points and increased number of measured modes are made available. Methodologies in current literature which demand a very large number of modes to be measured with displacements sampled at a large number of coordinates have less scope of being practically applied.

The authors have put the proposed methodology into test to extreme level of modal and coordinate sparsity, only 4 modes are taken into account. The methodology still works as desired thus indicating the robustness.

It may also be noted that, since a symmetric rectangular plate with respect to the centroidal axes is chosen, the reporting for the location of damage for such plate using frequency and symmetric mode shape information may be interchanged between the elements symmetrically located within the plate boundaries. Incorporation of higher asymmetric modes can bring additional uniqueness to the solution of the problem. The investigations related to the severity of damages are taken up next and the results are presented in Table 4.

It may once again be observed that the accuracy of the determination of the severity is well within the acceptable limits. Even with a high percentage of frequency information, the percentage error level never exceeds 30% for this particular problem.

Table 4 Estimation of severity of damage in terms of stiffness losses (Experimental)

Material parameter of undamaged plate (GPa)		Material parameter of damaged size (4×4cm) GPa
E_x	32.00	19.72
E_y	31.0	22.03
G_{xy}	5.73	4.188

The SVM algorithm as a classifier detects if any element has a stiffness loss or not. Into the numerically simulated problem in absence of any measurement noise discussed in this paper, the regressor form of the SVM algorithm detects the amount of stiffness loss into those elements perfectly. In case of actual experimentation, since the damage created to the structure is substantial, the regressor part of the SVM is still able to estimate the extent of damage into these selected elements. The discretization of the structure should be adequate so that the damaged extent can be covered by elements completely. Since the user is expectedly unaware of both the damage location and extent initially, this may not be completely true if some elements partially cover the damaged portion of the structure. In the present investigation, the damaged element has been assumed to be either damage free or completely damaged with varying extents in numerically simulated example, whereas in the actual experimentation also, damaged portion has been formed extending full extent of any element. In actual practice, the error can only be minimized by refining the mesh sizes in finite element modelling.

4. Conclusions

The location and severity of damages in a fibre reinforced plastic composite plate has been determined using Support Vector Machine (SVM) algorithm through numerical simulations and actual experimentations. The method could detect the location of damages with high degree of confidence as the outliers could be easily eliminated. The experimental investigations also demonstrate that the location of the inflicted damages could be detected with high confidence level. For practical application, it will be mandatory to have an updated finite element model in terms of material properties to determine the severity of damages. It has been consistently observed that the localized information of the mode shape is actually quite useful in determining the severity of damages whereas, properly weighted frequency information in addition to the weighted mode shape information improves the accuracy of determination of both location and severity.

References

ABAQUS v6.10 (2010), Dassault Systèmes Simulia Corp.

Abderahmane, S., Mokhtar, B.M., Smail, B., Wayne, S.F., Zhang, L., Belabbes, B.B. and Boualem, S. (2017), "Experimental and numerical disbond localization analyses of a notched plate repaired with a CFRP patch", *Struct. Eng. Mech*., **63**(3), 361-370.

Bai, R.B., Ostachowicz, W., Cao, M.S. and Su, Z. (2014), "Crack detection in beams in noisy conditions using scale fractal dimension analysis of mode shapes", *Smart Mater. Struct.*, **23**, 065014. (10pp). doi:10.1088/0964-1726/23/6/065014.

Barthorpe, R.J. and Worden, K. (2011), “Classification of multi-site damage using support vector machines”, *J. Phys.: Conference Series*, **305**(1), 012059.

Bornn, L., Farrar, C.R., Park, G. and Farinholt, K. (2009), “Structural health monitoring with autoregressive support vector machines”, *J. Vib. Acoust.*, **131**(2), 021004.

Carden, E.P. and Fanning, P. (2004), “Vibration based condition monitoring: A review”, *Struct. Health Monit*., **3**(4), 355-377, DOI: 10.1177/1475921704047500

Carraro, P.A. and Quaresimin, M. (2015), “A stiffness degradation model for cracked multidirectional laminates with cracks in multiple layers”, *Int. J. Solids Struct.*, **58**, 34-51.

Cevik, A., Kortoglu, A.E., Bilgehan, M., Gulsan, M.E. and Albegmprli, H.M. (2015), “Support vector machines in structural engineering: a review”, *J. Civil Eng. Management*, **21**(3), 261-281.

Chiachío, J., Bochud, N., Chiachío, M., Cantero, S. and Rus, G. (2017), “A multilevel Bayesian method for ultrasound-based damage identification in composite laminates”, *Mech. Syst. Signal Pr.*, **88**, 462-477

Daniel, I.M. and Ishai, O. (2005), Engineering Mechanics of Composite Materials 2nd Ed., *Oxford University Press*, USA.

Das, S., Srivastava, A.N. and Chattopadhyay, A. (2007), “Classification of damage signatures in composite Plates using one-class SVMs”, *IEEE Aerospace Conference*, 1-19. doi:10.1109/AERO.2007.352912.

Doebling, S.W., Farrar, C.R. and Prime, M.B. (1998), “A summary review of vibration based damage identification methods”, *Shock Vib. Digest*, **30**(2), 91-105.

Farooq, M., Zheng, H., Nagabhushana, A., Roy, S., Burkett, S., Barkey, M., Kotru, S. and Sazonov, E. (2012), “Damage detection and identification in smart structures using SVM and ANN”, Proceedings of SPIE - The International Society for Optical, 8346:40, doi: 10.1117/12.915189.

Farrar, C.R. and Worden, K. (2012), *Structural health monitoring: A machine learning perspective*, Wiley Publications, ISBN: 978-1-119-99433-6.

Garcia-Palencia, A., Santini-Bell, E., Mustafa Gul, M. and Catbas, N. (2015), “A FRF–based algorithm for damage detection using experimentally collected data”, *Struct. Monit. Maint*., **2**(4), 399-418.

Garcia, D. and Trendafilova, I. (2015), “A multivariate data analysis approach towards Vibration analysis and vibration-based damage assessment: Application for delamination detection in a composite beam”, *J. Sound Vib.*, **338**, 76-90

Goni, S.A., Mondal, S. and Chakraborty, S. (2015), “A new gradient based step size controlled inverse eigen sensitivity algorithm for identification of material and boundary parameters of plates”, *J. Vib. Control*, 1-15.

Gunn, S.R. (1998), Support vector machines for classification and regression Technical report, *Image Speech & Intelligent Systems Group, University of Southampton UK.*

Gupta, S., Satpal, S.B., Banerjee, S. and Guha A. (2016), “Vibration based health monitoring of honey comb core sandwich panels using support vector machine”, *Int. J. Smart Sens. Intell. Syst.*, **9**(1), 215-232.

Harizi, W., Chaki, S., Bourse, G. and Ourak, M. (2015), “Mechanical damage characterization of glass fiber-reinforced polymer laminates by ultrasonic maps”, *Compos.: Part B*, **70** ,131-137

He, H.X. and Yan, W.M. (2007), “Structural damage detection with wavelet support vector machine: Introduction and applications”, *Struct. Control Health Monit.*, **14**, 162-176. DOI: 10.1002/stc.150.

He, M., Yang, T. and Du, Y. (2017), “Non-destructive identification of composite beams damage based on the curvature mode difference”, *Compos.Struct.*, **176**, 178-186.

Kim, Y., Chong, J.W., Chon, K.H. and Kim, J. (2012), “Wavelet-based AR–SVM for health monitoring of smart structures”, *Smart Mater. Struct.*, **22**(1), 015003.

Li, H.N., Yi, T.H., Ren, L., Li, D.S. and Huo, L.S. (2014),“Reviews on innovations and applications in structural health monitoring for infrastructures”, *Struct. Monit. Maint.*, **1**(1),1-45.

Mondal, S. and Chakraborty, S. (2014), “Identification of material parameters of pultruded FRP Composite plates using finite element model updating”, *Proceedings of the 58th Congress (an international conference) of Indian Society of Theoretical and Applied Mechanics* (ISTAM) , IIEST Shibpur, India, December.

Mondal, S. and Chakraborty, S. (2016), “Identification of in-plane and out-of-plane elastic parameters of orthotropic composite structures”, *Proceedings of the 23rd International Congress on Sound and*

Vibration, ICSV23, Athens (Greece), 10-14 July.
Mondal, S. and Chakraborty, S., Mitra, N. (2016), "Estimation of elastic parameters of sandwich composite plates using a gradient based finite element model updating approach", *Proceedings of the Conferences on Smart Materials, Adaptive Structures and Intelligent Systems (ASME 2016 SMASIS)*, Stowe, Vermont, USA, September 28-30.
Nagarajaiah, S. and Erazo, K., (2016), "Structural monitoring and identification of civil infrastructure in the United States", *Struct. Monit. Maint.*, **3**(1), 51-69. DOI: 10.12989/smm.2016.3.1.051.
Nello, C. and Taylor, J.S. (2000), *An introduction to support vector machines and other kernel-based learning methods* , Cambridge university press, ISBN: 9780521780193.
Park, S., Inman, D.J., Lee, J.J. and Yun, C.B. (2008), "Piezoelectric sensor-based health monitoring of railroad tracks using a two-step support vector machine classifier", *J. Infrastruct. Syst.*, **14**(1), 80-88.
Rother, A., Jelali, M. and Soffker, D. (2014), "Development of a fault detection approach based on SVM", Applied to Industrial Data *Le Cam, Vincent and Mevel, Laurent and Schoefs, Franck. EWSHM - 7th European Workshop on Structural Health Monitoring, Nantes, France, July*.
Salawu, O.S. and Williams, C. (1994), "Damage location using vibration mode shapes", *Proceedings of the 12th International Modal analysis Conference*, **2251**, 933-941.
Sampaio, R. and Chan, T.H.T, (2015), "Modal parameters identification of heavy-haul railway RC bridges – experience acquired", *Struct. Monit. Maint.*, **2**(1), 1-18. DOI: 10.12989/smm.2015.2.1.001
Satpal, S.B., Guha, A. and Banerjee, S. (2015), "Damage identification in aluminium beams using support vector machine: Numerical and experimental studies", *Struct. Control Health Monit*, DOI: 10.1002/stc.1773
Satpal, S.B., Khandare, Y., Banerjee, S. and Guha, A. (2013), "Application of support vector machine in health monitoring of plate structures", *Proceedings of the 2013 World Congress, Advance in Structural Engineering and Mechanics(ASEM13)*, Jeju, *Korea*, 8-12.
Shyamala, P., Mondal, S. and Chakraborty, S. (2016), "Detection of damage in beam from measured natural frequencies using support vector machine algorithm", Advance in Dynamics, *Vibration and Control*, 06-310, *Narosa Publishing House*, New Delhi.
Sohn, H., Farrar, C.R., Hunter, N.F. and Worden, K. (2001), "Structural health monitoring using statistical pattern recognition techniques", *J. Dynam. Syst. Measure.Control*, **123**(4), 706-711.
Soleimanpour, R. and Ng, C.T. (2017), "Locating delaminations in laminated composite beams using nonlinear guided waves", *Eng. Struct.*, **131**, 207-219.
Vapnik, V. (2000), "The nature of statistical learning theory", *Information science and statistics*, Springer, 2nd Ed.
Yang, C. and Oyadiji, S.O. (2017), "Delamination detection in composite laminate plates using 2D wavelet analysis of modal frequency surface", *Comput. Struct.*, **179**, 109-126.
Zhang, Z., Shankar, K., Ray, T., Morozov, E.V. and Tahtali, M. (2013), "Vibration-based inverse algorithms for detection of delamination in composites", *Compos. Struct*., **102**, 226-236.
Zhang, Z., Zhan, C., Shankar, K., Morozov, E.V., Singh, H.K. and Ray, T. (2017), "Sensitivity analysis of inverse algorithms for damage detection in composites", *Compos. Struct.*, **176**, 844-859

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