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Detection of structural damage via free vibration responses by extended Kalman filter with Tikhonov regularization scheme

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Abstract. It is a challenging problem of assessing the location and extent of structural damages with vibration measurements. In this paper, an improved Extended Kalman filter (EKF) with Tikhonov regularization is proposed to identify structural damages. The state vector of EKF consists of the initial values of modal coordinates and damage parameters of structural elements, therefore the recursive formulas of EKF are simplified and modal truncation technique can be used to reduce the dimension of the state vector. Then Tikhonov regularization is introduced into EKF to restrain the effect of the measurement noise for improving the solution of ill-posed inverse problems. Numerical simulations of a seven-story shear-beam structure and a simply-supported beam show that the proposed method has good robustness and can identify the single or multiple damages accurately with the unknown initial structural state.

Keywords: extended Kalman filter; structural damage identification; Tikhonov regularization; ill-posed inverse problem

1. Introduction

A large volume of model-based methods have been developed for damage identification of engineering structures in the last two decades. Damage location and severity can be indicated by the degradation of element stiffness in model-based methods. These damage evaluation methods can be classified as frequency domain methods and time domain methods loosely (Friswell and Mottershead 1995). The structural modal parameters are taken as the input of the identification algorithm in the frequency domain. However, measurement vibration signals are used to identify the model physical parameters and damages in the time-domain-based identification algorithm. The time series of structural dynamical response can provide more information for damage detection and prevent the errors from modal parameters extraction. The least-squares estimation (LSE) (Loh *et al.* 2000), restoring force method (REM) (Masri and Caughey 1979), extended Kalman filter (EKF) (Jazwinski 1970) are common damage identification algorithms in the time domain. The damage identification methods based on EKF have gained more attention during recent years since satisfactory performance in restrain the interference of input and output signal noise.

The Kalman filter and its variants have been widely used to estimate the structural state and

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identify system parameters (Hoshiya and Saito 1984, Corigliano and Mariani 2004, Gao and Lu 2006, Zheng *et al.* 2004). When the external excitations are not measured or not available, Yang *et al*. (2007) proposed a parameter identification method by deriving the analytical recursive solution for EKF with unknown excitation. Adaptive tracking techniques based on EKF are proposed to identify the time-varying system parameters and track the abrupt changes of structural damages (Yang *et al.* 2006, Zhou *et al.* 2008).

For complex structures, identification algorithms based on EKF confront some challenges (Straser and Kiremidjian 1996) about convergence and accuracy because of the tremendous increase of computational costs and intrinsic ill-posedness of inverse problem.

Reduction of the structural degrees of freedom (dofs) is a major technique to solve the problem. Substructure method is one of efficient techniques to reduce dofs by dividing the initial problem into smaller problems of manageable size (Lei *et al.* 2012, Xing *et al.* 2014). Modal transformation and truncation is another effective one (Liu *et al*. 2009). The state vector of EKF is constructed by truncating instantaneous modal coordinates and damage parameters to replace actual displacement, velocity and acceleration.

Recently, ELSheikh *et al.* (2013) introduces regularization method into the Kalman filter to improve the ill-posedness of inverse problems. The Kalman gain matrix is regularized by using truncated singular value decomposition (SVD) to filter out noisy correlations, and then the convergence rate of the algorithm is enhanced effectively.

The motivation for the present study is to develop an improved EKF for reducing the computational costs and solving the ill-posed inverse problem. We use the initial values of truncated modal coordinates and the structural damage parameters to construct the state vector of EKF. Therefore linearization of state equations is omitted and the error of linearization is prevented. Moreover the Kalman gain matrix is regularized by Tikhonov regularization to restrain the interference of noise in the ill-posed problem. A seven-story shear-beam structure and a simply-supported beam are taken as numerical examples, and noisy synthetic data are utilized to detect structural parameters. The parameter identification results illustrate the efficiency and robustness of the proposed algorithm.

2 Mathematical model of the system and EKF

2.1 Basic equation of motion

Based on the finite element method, the equation of motion of a structure can be written as

$$\boldsymbol{M}\ddot{\boldsymbol{q}}(t) + \boldsymbol{C}\dot{\boldsymbol{q}}(t) + \boldsymbol{K}\boldsymbol{q}(t) = \boldsymbol{F}(t) \tag{1}$$

where $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are structural mass, damping and stiffness matrices, respectively. $\boldsymbol{q}(t)$, $\dot{\boldsymbol{q}}(t)$ and $\ddot{\boldsymbol{q}}(t)$denote the vectors of displacement, velocity and acceleration response, respectively. $\boldsymbol{F}(t)$ is the external force vector. The vector $\boldsymbol{q}(t)$ can be represented by general modal coordinates $\boldsymbol{p}(t)$

$$\boldsymbol{q}(t) = \boldsymbol{\Phi}\boldsymbol{p}(t) \tag{2}$$

where $\boldsymbol{\Phi}$ is the mass normalized modal matrix and satisfy the following equations

$$\boldsymbol{\Phi}^T\boldsymbol{M}\boldsymbol{\Phi} = \boldsymbol{I} \tag{3}$$

$$\boldsymbol{\Phi}^T\boldsymbol{K}\boldsymbol{\Phi} = \boldsymbol{\Lambda} \tag{4}$$

with $\boldsymbol{\Lambda} = \mathrm{diag}(\omega_1^2 \quad ... \quad \omega_\mathrm{n}^2)$ and ω_n is the n$^{\text{th}}$ eigen-frequency of the undamped system. Therefore, the motion Eq. (1) can be transformed into

$$\ddot{\boldsymbol{p}} + \boldsymbol{\Gamma}\dot{\boldsymbol{p}} + \boldsymbol{\Lambda}\boldsymbol{p} = \boldsymbol{\Phi}^T\boldsymbol{F} \tag{5}$$

where $\boldsymbol{\Gamma} = \boldsymbol{\Phi}^T\boldsymbol{C}\boldsymbol{\Phi}$.

For a real structure, ambient vibration is the most accessible data that can be acquired since the measurement requires no expensive exogenous excitation. Many techniques such as the random decrement (RD) signature technique can be used to obtain decayed free vibration signals from random structure responses. Therefore, the damage identification method based on damped free vibration signal is studied in this paper.

The ratio of damping is defined as $\xi_n = C_n/(2\omega_n M_n)$. Free vibration solution of Eq. (5) can be obtained as

$$p_n(t) = e^{-\xi_n\omega_n t}\left(p_{n0}\cos(\omega_{dn}t) + \frac{\dot{p}_{n0}+\xi_n\omega_n p_{n0}}{\omega_{dn}}\sin(\omega_{dn}t)\right) \tag{6}$$

where $\omega_{dn} = \omega_n\sqrt{1-\xi_n^2}$ is the n$^{\text{th}}$ eigen-frequency of the damped system, p_{n0} and $\dot{p}_{n0}$ are the initial values of modal coordinates and velocity.

2.2 Extended Kalman filter approach

The Kalman filter is a linear optimal recursive estimator designed for linear time-varying dynamic systems, and then the extended Kalman filter is proposed for nonlinear system by using the linearized models around any working point. In the proposed EKF algorithm for structural damage identification, we choose the unknown initial values of general modal coordinate and velocity, the change of the element stiffness and damping ratio to construct the state vector $\boldsymbol{\theta}$ in EKF.

$$\boldsymbol{\theta} = \{p_{10}, \quad \dot{p}_{10}, \quad \cdots, \quad p_{N0}, \quad \dot{p}_{N0}, \quad \alpha_1, \quad \cdots, \quad \alpha_m, \quad \beta_1, \quad \cdots, \quad \beta_N\}^\mathrm{T} \tag{7}$$

where N is the number of selected structural modal coordinates, m is the number of damage parameters about stiffness. Since the structural vibration responses contain only few low order modal components usually, high order modal coordinates can be abandoned so that N is much less than the number of structural dofs, In this paper the structural damages are represented as the reduction of element elastic modulus α_i and the change of modal damping ratio β_i.

$$\alpha_i = (E_i^0 - E_i)/E_i^0 \tag{8a}$$

$$\beta_j = (\xi_j - \xi_j^0)/\xi_j^0 \tag{8b}$$

where E_i^0 and E_i are the initial and current elastic modulus of i$^{\text{th}}$ element, ξ_i^0 and ξ_i represent the initial and current j$^{\text{th}}$ modal damping ratio.

According to the definition (7) of the state vector, the general solution of motion equations is used to form the observation equation in EKF, and the identification problem of structural damage parameters can be expressed as

$$\dot{\boldsymbol{\theta}} = \boldsymbol{0} \tag{9}$$

$$\boldsymbol{z}(x,t) = f(\boldsymbol{\theta},x,t) + \boldsymbol{e} \tag{10}$$

where $\boldsymbol{z}$ is the observation vector, $\boldsymbol{e}$ is zero-mean white noises with covariance matrix $\boldsymbol{R}$, the function $f(\boldsymbol{\theta},x,t)$ is the theory solution of structural response at the measure point x. When the free vibration response is taken as the input data to detect damages, the theory expression of the response function $f(\boldsymbol{\theta},x,t)$ can be simplified as

$$f(\boldsymbol{\theta},x,t) = \sum_{n=1}^{N}\{\phi_n(x)\}p_n(t) =$$

$$\sum_{n=1}^{N}\{\phi_n(x)\}\left(e^{-\xi_n\omega_n t}\left(p_{n0}\cos(\omega_{dn}t) + \frac{\dot{p}_{n0}+\xi_n\omega_n p_{n0}}{\omega_{dn}}\sin(\omega_{dn}t)\right)\right) \tag{11}$$

where $\phi_n(x)$ is n^{th} modal vector.

For parameter identification system consisted of Eqs. (9) and (10), the standard EFK algorithm can be simplified as

$$\boldsymbol{\theta_{j+1}} = \boldsymbol{\theta_j} + \boldsymbol{K_{j+1}}(\boldsymbol{z_{j+1}} - f(\boldsymbol{\theta_j})) \tag{12}$$

$$\boldsymbol{P_{j+1}} = (\boldsymbol{I} - \boldsymbol{K_{j+1}}\boldsymbol{H_{j+1}})\boldsymbol{P_{j+1}} \tag{13}$$

$$\boldsymbol{K_{j+1}} = \boldsymbol{P_j}\boldsymbol{H}_{\boldsymbol{j+1}}^{\mathrm{T}}(\boldsymbol{H_{j+1}}\boldsymbol{P_j}\boldsymbol{H}_{\boldsymbol{j+1}}^{\mathrm{T}} + \boldsymbol{R_{j+1}})^{-1} \tag{14}$$

where $\boldsymbol{P_j}$ is the covariance matrix of the estimation error at time j, $\boldsymbol{K_{j+1}}$ is called the Kalman gain matrix, $\boldsymbol{I}$ is the unit matrix, and $\boldsymbol{H_{j+1}}$ is the Jacobian matrix of the nonlinear function $f(\boldsymbol{\theta},x,t)$

$$\boldsymbol{H}_{j+1} = \partial f(\boldsymbol{\theta}_{j+1},x,t_{j+1})/\partial\boldsymbol{\theta}_{j+1} \tag{15}$$

and then the sensitivity of nature frequencies and modes to damage parameters, $\partial\omega_n/\partial\alpha_i$ and $\partial\phi_n/\partial\alpha_i$, in Eq. (15) can be calculated according to the undamaged or initial finite element model (Weber *et al*. 2009). Of course, the updated finite element model is suggested to be used to compute more accurate sensitivity which contributes to obtain more ideal identification results.

Because the state vector $\boldsymbol{\theta}$ is constant vector, and only the nonlinear observation Eq. (10) need to be linearized, the proposed EKF may reduce the error from the linearized state and observation equations simultaneously in classical EKF.

2.3 Extended Kalman filter with Tikhonov regularization

In recursive formulas (12)-(14) of the EKF algorithm, Kalman gain matrix $\boldsymbol{K}_{j+1}$ decides the influence of measurement innovation on state estimation values. The value of $\boldsymbol{K}_{j+1}$ depends on three factors: the precision of previous estimate values ($\boldsymbol{P_j}$), the accuracy of the measurement innovation ($\boldsymbol{R_{j+1}}$), and the relationship between observation and state vector ($\boldsymbol{H_{j+1}}$). Since the inverse matrix of $(\boldsymbol{H_{j+1}}\boldsymbol{P_j}\boldsymbol{H}_{\boldsymbol{j+1}}^{\mathrm{T}} + \boldsymbol{R_{j+1}})$ is needed to compute the gain matrix $\boldsymbol{K}_{j+1}$, we consider that the ill-posedness of inverse problem may lead to the singularity of the matrix $(\boldsymbol{H_{j+1}}\boldsymbol{P_j}\boldsymbol{H}_{\boldsymbol{j+1}}^{\mathrm{T}} + \boldsymbol{R_{j+1}})$. Therefore, the correction item $\boldsymbol{K_{j+1}}(\boldsymbol{z_{j+1}} - f(\boldsymbol{\theta_j}))$ is affected easily by measurement noise, and the accuracy of damage identification is degraded remarkably. The Tikhonov regularization will be introduced to estimate the Kalman gain matrix.

For the solution of ill-posed problems with the form $\boldsymbol{Az} = \boldsymbol{b}$, Tikhonov regularization is a best-known regularization scheme. By adding a penalty term on the norm of the update parameters, Tikhonov regularization translates the calculation of $\boldsymbol{Az} = \boldsymbol{b}$ into the solution of following functional minimum problem

$$J = \|\boldsymbol{Az} - \boldsymbol{b}\|_2^2 + \lambda\|\boldsymbol{z}\|_2^2 \tag{16}$$

where $\| \quad \|_2^2$ is the square of vector 2-norm, and λ is the regularization parameter used to balance the residual function and the penalty function. The least-square solution of functional (16) is

$$\boldsymbol{z} = \widetilde{\boldsymbol{A}}^{-1}\boldsymbol{b} = (\boldsymbol{A}^T\boldsymbol{A} + \lambda\boldsymbol{I})^{-1}\boldsymbol{A}^T\boldsymbol{b} \tag{17}$$

where

$$\widetilde{\boldsymbol{A}}^{-1} = (\boldsymbol{A}^T\boldsymbol{A} + \lambda\boldsymbol{I})^{-1}\boldsymbol{A}^T \tag{18}$$

is the pseudo-inverse of the matrix $\boldsymbol{A}$ with Tikhonov regularization. Using singular value decomposition, the matrix $\boldsymbol{A}$ can be decomposed

$$\boldsymbol{A} = \boldsymbol{USV}^T \tag{19}$$

where $\boldsymbol{U}$ and $\boldsymbol{V}$ are orthogonal matrices, the diagonal matrix $\boldsymbol{S}$ contains the singular values, and the diagonal elements is a non-negative entries $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_k \geq 0$. Substituting the Eq. (19) into Eq. (17), the solution can be shown as

$$\boldsymbol{z} = (\boldsymbol{VSU}^T\boldsymbol{USV}^T + \lambda\boldsymbol{VV}^T)^{-1}\boldsymbol{VSU}^T\boldsymbol{b} = \boldsymbol{V}(\boldsymbol{S}^2 + \lambda\boldsymbol{I})^{-1}\boldsymbol{SU}^T\boldsymbol{b} = \sum_{\boldsymbol{i}=\boldsymbol{1}}^{\eta}\frac{\sigma_i\boldsymbol{u}_i^T\boldsymbol{b}}{\sigma_i^2+\lambda}\boldsymbol{v}_{\boldsymbol{i}} \tag{20}$$

where $\boldsymbol{u}_{\boldsymbol{i}}$ and $\boldsymbol{v}_{\boldsymbol{i}}$ are the column vector of orthogonal matrices $\boldsymbol{U}$ and $\boldsymbol{V}$, respectively, and η is the order of matrix $\boldsymbol{A}$. In ill-posed problems, small singular values σ_i may amplify significantly the measurement errors in vector $\boldsymbol{b}$ if the regularization parameter λ is zero. $\frac{\sigma_i}{\sigma_i^2+\lambda}$ cannot be infinity by selecting appropriate regularization parameter, and the effects of the small singular values on the solution vector $\boldsymbol{z}$ can be reduced. When the EKF is used to detect structural damages, the regularizing the inverse of the matrix $(\boldsymbol{H}_{\boldsymbol{j+1}}\boldsymbol{P}_{\boldsymbol{j}}\boldsymbol{H}_{\boldsymbol{j+1}}^{\mathrm{T}} + \boldsymbol{R}_{\boldsymbol{j+1}})$ in the Kalman gain matrix is essential to avoid the small singular values dominate the update process. According to the Eq. (18), the Kalman gain matrix with Tikhonov regularization can be expressed as

$$\boldsymbol{K}_{\boldsymbol{j+1}} = \boldsymbol{P}_{\boldsymbol{j}}\boldsymbol{H}_{\boldsymbol{j+1}}^{\mathrm{T}}\widetilde{\boldsymbol{Q}}^{-1} \tag{21}$$

where $\widetilde{\boldsymbol{Q}}^{-1} = (\boldsymbol{Q}^T\boldsymbol{Q} + \lambda\boldsymbol{I})^{-1}\boldsymbol{Q}^T$, $\boldsymbol{Q} = \boldsymbol{H}_{\boldsymbol{j+1}}\boldsymbol{P}_{\boldsymbol{j}}\boldsymbol{H}_{\boldsymbol{j+1}}^{\mathrm{T}} + \boldsymbol{R}_{\boldsymbol{j+1}}$, and the value of regularization parameter λ is determined by L-curve method (Hansen and O'Leary 1993) in this paper. Since the Kalman gain matrix K is time dependent, the optimal value of regularization parameter timed dependent too, and it is time consuming if the L-curve method is used in each time step. In fact, the regularization parameter can be determined only by using the Kalman gain matrix K in last time step. The value of regularization parameter obtained in this way may not be optimal, but it is enough to achieve satisfactory identification results. Thus the damage identification algorithm based on EKF with Tikhonov regularization can be achieved by the Eqs. (12)- (13) and (21).

3. Application of EKF with Tikhonov regularization

3.1 Seven-story shear-beam structure

An idealized seven-story linear shear-beam type building is taken as a numerical example to illustrate the proposed algorithm. The structure model is represented as a linear spring-mass-damper system with floor masses m_i, inter-story stiffnesses k_i, and modal damping ratios $\xi_i (i = 1, \cdots, 7)$, where the first spring is connected to the ground.

For numerical simulations, structural damage is introduced by the change of partial spring stiffness and modal damping ratio shown in Table 1. To simulate measurements, free vibration responses of seven masses during 5s with sampling frequency 1000 Hz are calculated and random noise with a normal distribution is added. Three noise levels, 5%, 10%, and 15% are considered, where the noise levels denote the standard deviation of the noise.

The identification results presented in Table 2 show that the changes of inter-story stiffness and modal damping ratios can be detected correctly even the noise level is high. With the increase of measurement noise, errors of detecting the stiffness and damping grow more slowly. Relative errors of identified stiffness and damping ratio are less than 1% and 3%, respectively. In Fig. 1, the convergence curves for each variable in the state vector of EKF are given when the noise level is 15%. Both structural parameters and initial values of modal coordinates converge very fast. Moreover, the identified structural response compares well with the simulated noisy response (the time segment from 1s to 3s is intercepted only for clear displaying), as shown in Fig. 2. Consequently, tracking the change of the identified stiffness and damping can detect structural damage with good accuracy and robustness by the proposed algorithm.

3.2 Simply supported beam

The efficiency of the proposed algorithm is proved in the previous example, and then a more complex simply-supported beam with the I-steel section is illustrated to show the superiority of the algorithm.

In the finite element model shown in Fig. 3, the beam is divided into 10 elements with equal length. Each element has two nodes, and each node has three dofs in the horizontal, vertical and rotational directions, respectively. The properties of the beam are as follows: mass density $\rho = 7850\ \mathrm{kg/m^3}$, elastic modulus $E = 210\mathrm{GPa}$, cross section area $A = 67.05\ \mathrm{cm^2}$ and length of the beam $l = 10$ m.

Table 1 Stiffness reduction and damping parameters of seven-story shear-beam structure

	k_1(kN/m)	k_2(kN/m)	k_3(kN/m)	ξ_1	ξ_2	ξ_3
undamaged structure	350	350	350	0.020	0.020	0.020
damaged structure	280	245	315	0.022	0.024	0.021
	$\alpha_1 = 20\%$	$\alpha_2 = 30\%$	$\alpha_3 = 10\%$	$\beta_1 = 10\%$	$\beta_2 = 20\%$	$\beta_3 = 5\%$

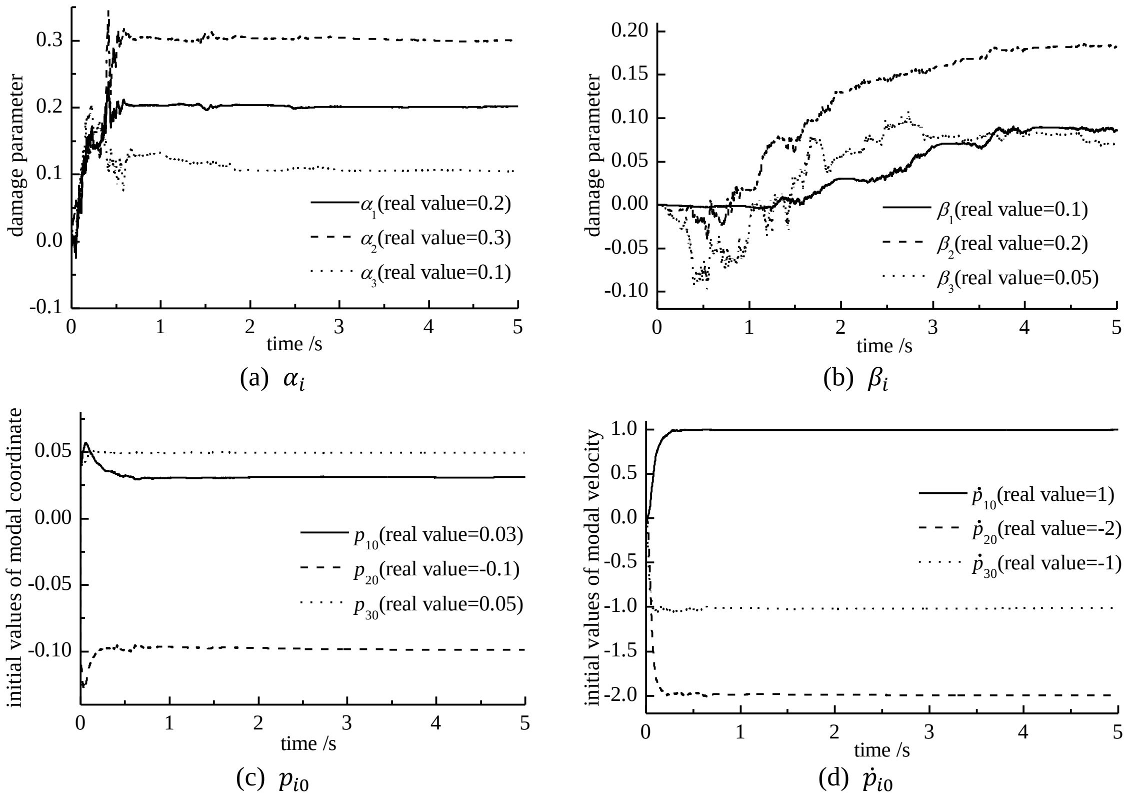

Fig. 1 Convergence curve of state variables (noise level is 15%)

Table 2 Estimation results of seven-story shear-beam structure

real value		identified value					
		noise 5%	relative error	noise 10%	relative error	noise 15%	relative error
k_1(kN/m)	280.0	280.0	-0.01%	278.4	-0.57%	279.4	-0.20%
k_2(kN/m)	245.0	245.3	0.13%	245.2	0.06%	244.8	-0.06%
k_3(kN/m)	315.0	313.9	-0.35%	314.1	-0.29%	313.4	-0.52%
k_4(kN/m)	350.0	351.5	0.44%	352.6	0.73%	352.2	0.63%
k_5(kN/m)	350.0	349.1	-0.27%	349.8	-0.05%	349.6	-0.13%
k_6(kN/m)	350.0	349.5	-0.13%	351.1	0.33%	349.2	-0.24%
k_7(kN/m)	350.0	351.3	0.38%	349.8	-0.05%	353.2	0.90%
ξ_1	0.022	0.0217	-1.49%	0.0217	-1.16%	0.0217	-1.21%
ξ_2	0.024	0.0237	-1.13%	0.0238	-0.99%	0.0237	-1.45%
ξ_3	0.021	0.0211	0.51%	0.0212	0.90%	0.0214	2.00%

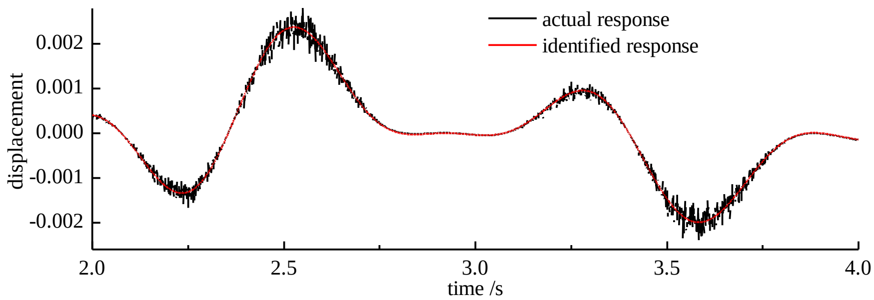

Fig. 2 Comparison of actual and identified displacement response at mass1 (noise level is 15%)

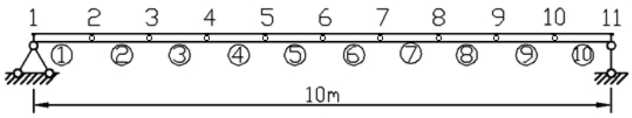

Fig. 3 Simulated simply-supported beam

In the numerical example, only free vibration responses in the vertical direction of each node are measured and the damping is neglected. To consider measurement noise pollution, all the input time series are simulated by superimposing the structural responses computed by ANSYS software with the corresponding stationary Gauss white noise.

Based on the presented algorithm, damage identification of the beam with single and multiple damages are analyzed respectively. Three noise levels, 0%, 5% and 10%, are considered in the numerical cases. In order to reduce the dimension of state variables, only first three modal coordinates are used to identify the structural damages.

3.2.1 Identification of single damage

The single damage is simulated by a reduction of 30% in the elastic modulus of the 5^{th} element. Identified local damage under different noise level is shown in Fig. 4. Similar identification results are obtained although the different noises are added into the simulated input signal. Even the noise level reaches 10%, the relative errors of the identified elastic modulus of damage element are less than 1.8%.

The convergence curves of the initial values of modal coordinates in the state vector of EKF are shown in Fig. 5. Fast convergences of identified initial values mean the proposed EKF algorithm has obtained the initial state of the whole structure and relieves the difficulty in estimating the unknown initial state.

Next, the effect of regularization is taken into account in damage identification. Fig. 6 compares the identified results between the classical EKF and the proposed regularized EKF. Because classical EKF do not combine the regularization process to suppress noise interference, the corresponding identified results obtained by EKF alone show obvious oscillation and cannot indicate the location and severity of damage.

When more and smaller elements are divided in the finite element model, the sensitivity of structural responses to elemental damage is decreased, and then the ill-posedness of identification problems is strengthened. Therefore, the identified results degrade obviously due to the interference of noise. Combined with Tikhonov regularization, the improved EKF suppresses noise interference successfully and provides an effective method to solve ill-posed inverse problem.

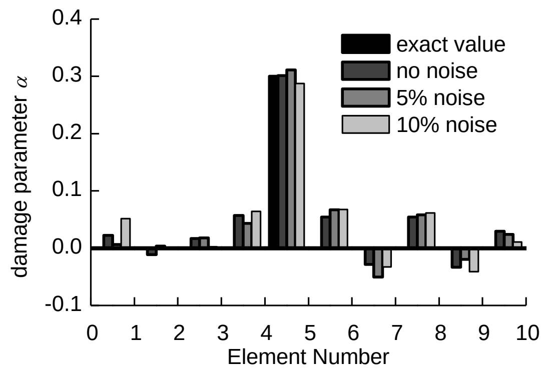

Fig. 4 Single damage identification results under different noise levels

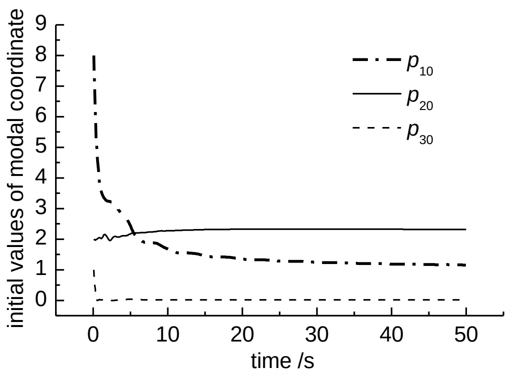

Fig. 5 Convergence curve of the initial values of modal coordinates in the state vector of EKF

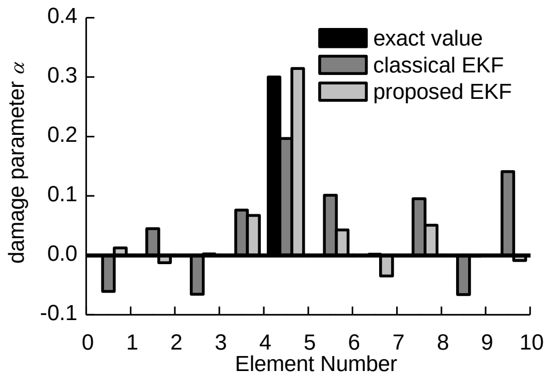

Fig. 6 The influence of Tikhonov regularization to damage identification results

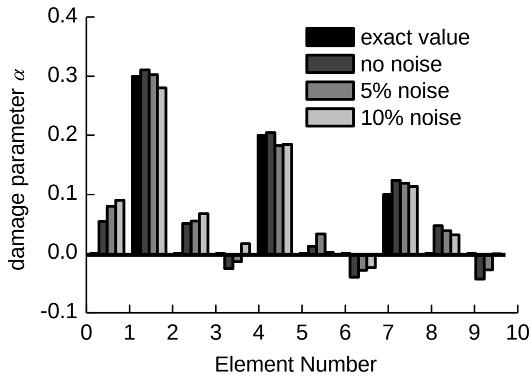

Fig. 7 Multi-damage identification results under different noise levels

3.2.2 Identification of multiple damages

Three local damages are simulated by the elastic modulus reduction of 30%, 20%, 10% in the 2nd, 5th and 8th element, respectively. As shown in Fig. 7, multiple damages of the structure can be identified by the proposed method accurately. Errors of identified results may increase slightly with the increment of noise level, and the ideal precision of identified parameters is maintained even in the case with 10% noise. The maximal relative error of the elastic modulus of damage elements is only 2.87%. Fig. 8 shows that the convergence curves of damage parameters in damage elements in the noise level 5%. It is observed that the proposed algorithm can detect and locate the structural damages from the degradation of element stiffness parameters accurately.

3.2.3 Influence of the number of measurement points

More measurements offer more structure information for damage identification, but limited acceleration responses are measured in real structures because of limited acceleration sensors, so the influence of the number of measurement points to identification results is analyzed in this section.

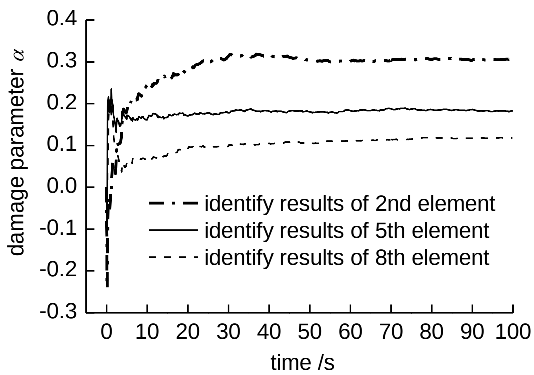

Fig. 8 Convergence curve of the damage parameters under 5% noise level

Table 3 Identification success rates with different numbers of measurement points

Number of measurement points	Measurement points	Identification success rates
9	all	99%
8	2,3,4,5,7,8,9,10	93%
7	2,3,4,6,8,9,10	87%
6	2,3,4,8,9,10	78%
5	2,3,6,9,10	30%
4	2,3,9,10	9%
3	2,6,10	0%

Considering the example in section 3.2.1, we select different numbers of measurement points and use 100 groups random noise-contamined structural responses to detect damages with 5% noise level. The result with identified damage parameter $\alpha_5 \in [0.25,0.35]$ in the damage element and $\alpha_i \in [-1.0,1.0]$ in undamaged elements is taken as successful identification. The statistical results of identification success rates with different numbers of measurement sensors are listed in Table 3. The success rates of damage identification in this case decrease slowly with the reduction of the numbers of measurement points when the sensors are enough, and there is a significant reduction in success rates when the number of measurement points is less than 6. It means that the number of required sensors has a minimum value to obtain the correct results and more measurements provide more accurate results.

3.2.4 Effect of modal truncation

Modal truncation technique is used in proposed method to improve the efficiency of identification algorithm. The proper number of modal reduction needs to be determined by actual structure and dynamical responses. In Fig. 9, the success rates of damage identification are displayed as a function of the number of modal truncation, when the example in section 3.2.1 in 10% noise level is considered. Because the dynamical responses mainly include the first three modes in this case, it can be observed that the structural damages can be identified accurately only when the number of modal truncation is greater than or equal to 3. However, retaining more modes will enlarge the dimension of state variables and increase the computational complexity. Fig. 10 shows the computational time of the identification algorithm versus the number of modal truncation. Hence, we can know the main vibration modes by frequency spectrum analysis of the structural responses, and then determine the proper number of modal reduction.

4. Conclusions

The problem of damage detection and localization has been treated by the proposed EKF with Tikhonov regularization in time domain. Using the damage parameters and the initial values of modal coordinates to construct the state vector of EKF, the recursive formulas of EKF are simplified and the linearization error of the state equation is avoided. Moreover the dimension of the state vector in EKF can be reduced by modal truncation technique effectively. For ill-posed damage identification problems, the Kalman gain matrix is filtered using Tikhonov regularization to restrain the interference of the measurement noise. Several numerical examples validate that the proposed algorithm can accurately estimate structural initial state, dynamical response and the structural parameters with limited response output measured. Then the structural damage can be identified by tracking the degradation of elemental elastic modulus with satisfactory accuracy and robustness.

The proposed method can be combined with substructure technique to investigate structural damage identification of large size structural systems, and can be combined with load identification method to consider the general forced vibration responses. Relevant work will be presented in future.

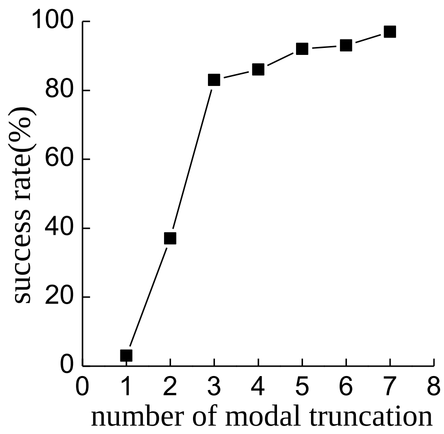

Fig. 9 Success rate of identification versus the number of modal truncation

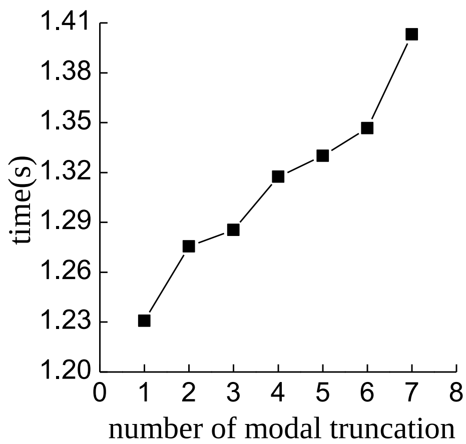

Fig. 10 Computation time versus the number of modal truncation

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