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The mixed substructure synthesis method with physics-impedance-modal parameter

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Abstract. Based on the concept of the parameter-mixed synthesis, this paper presents a mixed formulation of the substructure synthesis method in terms of the physics-impedance-modal parameter. An example is given to show the validity of this method.

Key words: mixed synthesis; dynamic substructure.

1. Introduction

Complicated structural systems can often be modeled by breaking the system into a number of simple components or substructures. The substructures are first modeled separately, and then the separated models are coupled together to form the composite structure system. This technique is generally known as substructure synthesis (Craig *et al.* 1976, Merovitch *et al.* 1981). As a rule the substructure synthesis method often refers to the modal synthesis method which addresses the dynamic characteristic of the structure via the modal parameters (such as the modal mass, the modal stiffness and the modal damping etc.). The method has been extensively used to research all kinds of complex structures. Recently it also has been extended to the sensitivity analysis of complex structure (Liu *et al.* 1993) and the dynamic analysis of flexible and rigid-flexible multibody system (Lim *et al.* 1994, Liu *et al.* 1994, Liew *et al.* 1996).

However besides the modal parameters, the impedance parameter (transfer function) and the physics parameter (i.e., the mass, stiffness and damping of the structure etc.) can also be used to characterize the structure or the substructures. Therefore according to the different kinds of parameters used in the substructures of the system, the dynamic substructure synthesis method can be classified into three categories.

1.1. The single parameter method

For this method, all the substructures of a system are characterized by the same kind of the parameter. Thus it is evident that the single parameter method can include (i) the physics parameter method (such as the finite element method and the transfer matrix method (Yee and

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Tsuei 1989, etc.), (ii) the impedance parameter method (i.e., the Building Block approach (Liu *et al.* 1993) or the dynamic stiffness method (Liao and Tse 1993)) and (iii) the modal parameter method (Craig 1987) (the modal synthesis method).

1.2. The two-parameter method

When some of the substructures are characterized by one of the above three kinds of parameters and the other are defined by one of the other, the corresponding substructure method is called the two-parameter method which can really contain (i) the physics-impedance method (Wang 1995), (ii) the physics-modal method (Yu and Peng 1990) and (iii) the impedance-modal method (Nagamatsu 1985).

1.3. The three parameter method

If a system includes simultaneously the substructures defined respectively by the physics parameter, the impedance parameter and the modal parameter, the equation of the system will then contain three kinds of parameters (the physics, impedance and modal parameter). Therefore the dynamic substructure synthesis method is the mixed method with three kinds of parameters.

Based on the above concept, in section 2 we present a mixed formulation of the substructure synthesis method in terms of the physics-impedance-modal parameter. In section 3 an example is given in order to show the validity of this method.

2. Formulation

Consider a (see Fig. 1). They are connected through the interface j and k. Suppose that the equations of substructure A, C and B are respectively defined by the modal parameters, impedance parameters (i.e., transfer function) and physical parameters. Therefore the corresponding dynamic substructure formulation is discussed as follows.

First, the modal parameter equation of the substructure A is written as

$$[\bar{m}]_A[\ddot{\bar{q}}]_A + [\bar{k}]_A[\bar{q}]_A = [\bar{p}]_A \tag{1}$$

where $[\bar{m}]_A$, $[\bar{k}]_A$ and $[\bar{p}]_A$ are the modal mass matrix, the modal stiffness matrix and the modal load vector respectively.

Defining $[\bar{q}]_A$ and $[\bar{p}]_A$ through

$$\left.\begin{aligned}[\bar{q}]_A &= [Q]_A e^{i\omega t}\\ [\bar{p}]_A &= [F]_A e^{i\omega t}\end{aligned}\right\} \tag{2}$$

and substituting (2) into (1), we obtain

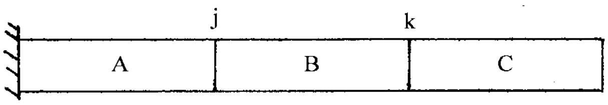

Fig. 1 Structure consisting of three substructure

$$[\bar{k}]_A - \omega^2[\bar{m}]_A)\{Q\}_A = [F]_A \tag{3}$$

If the matrixes in Eq. (3) are partitioned into submatrixes in terms of the interface coordinate and the interior coordinate, Eq. (3) can be stated as

$$\begin{bmatrix} [\bar{k}_{ii}]_A - \omega^2[\bar{m}_{ii}]_A & [\bar{k}_{ij}]_A - \omega^2[\bar{m}_{ij}]_A \\ [\bar{k}_{ji}]_A - \omega^2[\bar{m}_{ji}]_A & [\bar{k}_{jj}]_A - \omega^2[\bar{m}_{jj}]_A \end{bmatrix} \begin{bmatrix} \{Q_i\}_A \\ \{Q_i\}_A \end{bmatrix} = \begin{bmatrix} \{F_i\}_A \\ \{F_j\}_A \end{bmatrix} \tag{4}$$

where i, j show the interior coordinates and the interface coordinates respectively.

Next, substructure B is described by the physical parameters. Its dynamical equation is

$$[m]_B\{\ddot{x}\} + [k]_B\{x\}_B = \{p\}_B \tag{5}$$

Assuming

$$\left.\begin{aligned} \{x\}_B &= \{X\}_B e^{i\omega t} \\ \{p\}_B &= \{F\}_B e^{i\omega t} \end{aligned}\right\} \tag{6}$$

and substituting (6) into (5), we have

$$([k]_B - \omega^2[m]_B)\{X\}_B = \{F\}_B \tag{7}$$

Eq. (7) can be written in a partitioned form as

$$\begin{bmatrix} [k_{ii}]_B - \omega^2[m_{ii}]_B & [k_{ij}]_B - \omega^2[m_{ij}]_B & [k_{ik}]_B - \omega^2[m_{ik}]_B \\ [k_{ji}]_B - \omega^2[m_{ji}]_B & [k_{jj}]_B - \omega^2[m_{jj}]_B & [k_{jk}]_B - \omega^2[m_{jk}]_B \\ [k_{ki}]_B - \omega^2[m_{ki}]_B & [k_{kj}]_B - \omega^2[m_{kj}]_B & [k_{kk}]_B - \omega^2[m_{kk}]_B \end{bmatrix} \begin{Bmatrix} [X_i]_B \\ [X_j]_B \\ [X_k]_B \end{Bmatrix} = \begin{Bmatrix} [F_i]_B \\ [F_j]_B \\ [F_k]_B \end{Bmatrix} \tag{8}$$

where i, j and k are the interior coordinates and the interface coordinates respectively.

Then the impedance parameter equation of the substructure C is defined as

$$[Z]_C[X]_C = [F]_C \tag{9}$$

After being partitioned it is

$$\begin{bmatrix} [Z_{ii}]_C & [Z_{ik}]_C \\ [Z_{ki}]_C & [Z_{kk}]_C \end{bmatrix} \begin{Bmatrix} [X_i]_C \\ [X_k]_C \end{Bmatrix} = \begin{Bmatrix} [F_i]_C \\ [F_k]_C \end{Bmatrix} \tag{10}$$

Now, assuming the displacement compatiablity conditions and the force equilibrium conditions at the interfaces of the substructures are

$$\left.\begin{aligned} [x_j]_A &= [x_j]_B \\ [x_k]_B = [x_k]_C \quad &\text{or} \quad [X_k]_B = [X_k]_C \end{aligned}\right\} \tag{11}$$

and

$$\left.\begin{aligned} [p_j]_A - [p_j]_B &= 0 \\ [p_k]_B - [p_k]_C = 0 \quad &\text{or} \quad [F_k]_B = [F_k]_C \end{aligned}\right\} \tag{12}$$

where $[X]$ and $[F]$ are respectively the Fourier transform of $[x]$ and $[p]$.

For the substructure A, the transformation relationship between the modal coordinate $[\bar{q}]_A$, $[\bar{Q}]_A$ and the physical coordinate $[x]_A$ is

$$[x]_A = [\phi]_A \{\bar{q}\}_A = [\phi]_A \{Q\}_A e^{i\omega t} \tag{13}$$

Considering the partitioned form

$$\begin{Bmatrix} \{x_i\}_A \\ \{x_j\}_A \end{Bmatrix} = \begin{bmatrix} [\phi_i]_A \\ [\phi_j]_A \end{bmatrix} \{Q\}_A e^{i\omega t} \tag{14}$$

we can obtain

$$\{x_j\}_A = \{\phi_j\}_A \{Q\}_A e^{i\omega t} \tag{15}$$

Eq. (6) of substructure B is partitioned as

$$\left.\begin{aligned} \begin{Bmatrix} \{x_i\}_B \\ \{x_i\}_B \\ \{x_i\}_B \end{Bmatrix} &= \begin{Bmatrix} \{X_i\}_B \\ \{X_j\}_B \\ \{X_k\}_B \end{Bmatrix} \\ \begin{Bmatrix} \{p_i\}_B \\ \{p_j\}_B \\ \{p_k\}_B \end{Bmatrix} &= \begin{Bmatrix} \{F_i\}_B \\ \{F_j\}_B \\ \{F_k\}_B \end{Bmatrix} \end{aligned}\right\} \tag{16}$$

from which the interface displacement and forces can be separated as

$$\left.\begin{aligned} \{x_j\}_B &= \{X_j\}_B e^{i\omega t} \\ \{x_k\}_B &= \{X_k\}_B e^{i\omega t} \\ \{p_j\}_B &= \{F_j\}_B e^{i\omega t} \\ \{p_k\}_B &= \{F_k\}_B e^{i\omega t} \end{aligned}\right\} \tag{17}$$

From Eqs. (11) and (15), we have

$$\{\phi_j\}_A \{Q\}_A = \{X_j\}_B \tag{18}$$

Suppose the generalized displacement vector of the whole structure is expressed as

$$\{Q\} = \{Q\}_A^T \{X_i\}_B^T \{X_i\}_B^T \{X_i\}_C^T$$

and the transformation relationship stating the substructure coordinates to the structural generalized coordinates as

$$\begin{Bmatrix} \{Q\}_A \\ \{X_i\}_B \\ \{X_j\}_B \\ \{X_k\}_B \\ \{X_i\}_C \\ \{X_k\}_C \end{Bmatrix} = \begin{bmatrix} [I] & 0 & 0 & 0 \\ 0 & [I] & 0 & 0 \\ [\phi_j]_A & 0 & 0 & 0 \\ 0 & 0 & [I] & 0 \\ 0 & 0 & 0 & [I] \\ 0 & 0 & [I] & 0 \end{bmatrix} \begin{Bmatrix} \{Q\}_A \\ \{X_i\}_B \\ \{X_k\}_B \\ \{X_i\}_C \end{Bmatrix} \tag{19}$$

Connecting the Eqs. (4), (8), (10) and considering the Eq. (19) we obtain the mixed equation of the physics-impedance-modal parameter for the whole structure as

$$[Z]\{Q\}=\{F\} \tag{20}$$

where the coefficient matrix is $[Z]$ which can be written as

$$[Z]=\begin{bmatrix} [\bar{k}]_A-\omega^2[\bar{m}]_A+[\phi]_A^T([k_{jj}]_B-\omega^2[m_{jj}]_B)[\phi_j]_A & [\phi_j]_A^T[k_{ji}]_B-\omega^2[m_{ji}]_B & [\phi_j]_A^T([k_{ik}]_B-\omega^2[m_{ik}]_B) & 0 \\ ([k_{ij}]_B-\omega^2[m_{jj}]_B)[\phi_j]_A & ([k_{ii}]_B-\omega^2[m_{ii}]_B) & [k_{ik}]_B-\omega^2[m_{ik}]_B & 0 \\ ([k_{ij}]_B-\omega^2[m_{jj}]_B)[\phi_j]_A & ([k_{ki}]_B-\omega^2[m_{ki}]_B) & [k_{ki}]_B-\omega^2[m_{ki}]_B & [Z_{ki}]_C \\ 0 & 0 & [Z_{ik}]_C & [Z_{ii}]_C \end{bmatrix} \tag{21}$$

and $\{F\}$ is the generalized load vector, showed as

$$\{F\}=\begin{Bmatrix} \{F\}_A+[\phi_j]_A^T\{F_j\}_B \\ \{F_i\}_B \\ \{F_k\}_B+\{F_k\}_C \\ \{F_i\}_B \end{Bmatrix} \tag{22}$$

3. Numerical example

In order to demonstrate the proposed method, a spring-mass system showed in Fig. 2 is employed as an example.

The system consists of three substructures. The modal equation of the substructure A

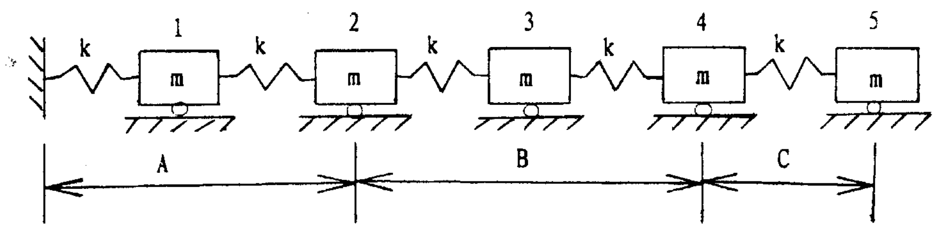

Fig. 2 Spring-mass system

$$\begin{bmatrix} 3.618m & 0 \\ 0 & 1.3819m \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} 1.382k & 0 \\ 0 & 3.618k \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = 0$$

For the substructure B, the corresponding dynamical equation of the physical parameter is

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} \begin{bmatrix} \ddot{x}_2 \\ \ddot{x}_3 \\ \ddot{x}_4 \end{bmatrix} + \begin{bmatrix} k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & k \end{bmatrix} \begin{bmatrix} x_2 \\ x_3 \\ x_4 \end{bmatrix} = 0$$

The impedance matrix of the substructure C is

$$[H] = \begin{bmatrix} H_{44} & H_{45} \\ H_{54} & H_{55} \end{bmatrix} = \begin{bmatrix} k & -k \\ -k & k-\omega_n^2 m \end{bmatrix}$$

The coordinate transformation matrix is determined by the interface conditions as

$$[T] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1.618 & -0.618 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Therefore the coefficient matrix of the system is

$$[Z] = \begin{bmatrix} 4k-3.618\omega_n^2 m & -k & -1.618k & 0 & 0 \\ -k & 4k-1.3819\omega_n^2 m & 0.618k & 0 & 0 \\ -1.618k & 0.618k & 2k-\omega_n^2 m & -k & 0 \\ 0 & 0 & -k & 2k-\omega_n^2 m & -k \\ 0 & 0 & 0 & -k & k-\omega_n^2 m \end{bmatrix}$$

Table 1 The first five ordered eigenfrequencies

	The result of this paper	The result of (Wang 1995)
ω_{n1}	2.0259Hz	2.02589Hz
ω_{n2}	5.9126Hz	5.91354Hz
ω_{n3}	9.32275Hz	9.32211Hz
ω_{n4}	11.97970Hz	11.97550Hz
ω_{n5}	13.66140Hz	13.65860Hz

Letting $[Z]=0$, we can obtain the natural frequencies of the system. Assuming $k=1000$ kg/mm, $m=5$ kg, the first five ordered eigenfrequencies are computed and listed in Table 1. The results agree well with those provided by Wang (1995).

In above example, the substructure A, B and C are respectively described by the modal, impedance and physical parameters. Using the coordinate transformation matrix $[T]$, we form the coefficient matrix $[Z]$ of the whole structure with three kinds of parameter. At last, by solving $[Z]$, the first five orders of natural frequency of the structure are obtained.

4. Conclusions

In this investigation, the mixed synthesis formulation in terms of the parameter of three kinds has been developed for the dynamic substructure method. This is an universal method. All the other methods (such as the modal synthesis method, the Block Building method, the finite element method etc.) are the special case of this new method. Because the different parameter equations can be used to model the different substructures in a system considering their distinguished features, this new method is specially suited for a large complicated structure system. For example, when establishing the analysis model of a complex structure (such as a large mechanical equipment) by the substructure synthesis, some substructures of it can be modeled by finite elements and modal parameters. But for some substructures, because of the complication of their construction, we can only obtain their transfer functions by the vibration measurement. In this case, the analysis model of this complex structure system can be established by combining directly transfer functions of these substructures and finite elements, modal parameters of the other substructures using the method of this paper.

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