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Effects of elastic foundation on the dynamic stability of cylindrical shells

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Abstract. A formulation for the dynamic stability analysis of cylindrical shells resting on elastic foundations is presented. In this previously not studied problem, a normal-mode expansion of the partial differential equations of motion, which includes the effects of the foundation as well as a harmonic axial loading, yields a system of Mathieu-Hill equations the stability of which is analyzed using Bolotin's method. The present study examines the effects of the elastic foundation on the instability regions of the cylindrical shell for the transverse, longitudinal and circumferential modes.

Key words: dynamic stability; elastic foundation; cylindrical shell.

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1. Introduction

Especially in recent years, static and dynamic analysis of beams and columns on elastic foundations has been extensively studied. The influence of a partially tangential force and elastic foundation on the elastic instability of a uniform beam was investigated for clamped boundary conditions (Lee *et al.* 1996). A general solution to vibrations of beams on variable Winkler elastic foundation was presented where the exact solution of the dynamic response of the beam was obtained by considering the reaction force of the foundation on the beam as the external force acting on the beam (Zhou 1993). The dynamic stability of a tapered cantilever beam on an elastic foundation subjected to a follower force was analyzed using the Lagrangian approach and the assumed mode method (Lee 1996). Circular beams with variable cross-sections on Winkler-like elastic foundations under arbitrary loading were analyzed using the finite element method (Aköz and Kadıoglu 1996). To simulate bridges, runways, rails, roadways, pipelines, etc., a method was presented to perform the deterministic and random vibration analysis of a Rayleigh-Timoshenko beam on an elastic foundation (Chang 1994). The transfer matrix method was used to investigate the influence of a Winkler elastic foundation on the non-conservative instability of uniform Timoshenko beams (Lee and Yang 1993). Also investigated was the influence of a Winkler elastic foundation and the slenderness ratio on the non-conservative instability of cantilever non-uniform Timoshenko beams of rectangular cross-section (Lee and Yang 1994).

The free vibration analysis of beams on a two-parameter elastic foundation has also been

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studied. The vibrational behavior of uniform beams on a two-parameter elastic foundation with initial stress was examined using the finite element formulation (Naidu and Rao 1995). The free vibrations of Timoshenko beams on two-parameter elastic foundations in which two variants of the equation of motion are deduced, where the second foundation parameter is a function of either the total rotation of the beam or a function of the rotation due to bending has also been examined (De Rosa 1995). The effects of non-linear elastic foundations on the dynamic responses of beams has also been studied. One study looks into the effects of a non-linear elastic foundation on the mode shapes of buckling and free vibration of uniform beams (Raju and Rao 1993a). By the method of perturbation, the static deflection analysis was carried out for a general elastically end restrained non-uniform beam resting on a non-linear elastic foundation subjected to axial and transverse forces, governed by a non-linear fourth order non-homogeneous ordinary differential equation with variable coefficients (Kuo and Lee 1994). The buckling and free vibration of uniform beams on non-linear elastic foundations was also evaluated using the finite element method (Naidu and Rao 1996).

To a far lesser extent than beams and columns, the dynamic analyses of plates and shells on elastic foundations have also been studied. The Rayleigh-Ritz approach has been used to study the free vibration characteristics of uniform simply supported beams and rectangular plates of constant thickness resting on a uniform elastic foundation with externally applied loads (Raju and Rao 1993b). The non-linear static and dynamic response analyses of a clamped, rectangular composite plate resting on a two-parameter elastic foundation was also studied by applying Galerkin's method (Chandrasekharappa 1992). The fundamental solutions and the boundary element method for obtaining numerical solutions of non-linear Reissner plates on an elastic foundation have been presented (Qin 1993). Also presented was a modified variational principle based on a hybrid Trefftz finite element model for analysis of Reissner plates on an elastic foundation (Qin 1995). The static and dynamic responses of a thin circular plate on an elastic foundation of Winkler-type that reacts in compression only has also been analyzed using Galerkin's method (Guler and Celep 1995). A systematic way for the derivation of Kirchhoff plate-elastic foundation interaction by mixed-type formulation using the Gateaux differential has been presented (Omurtag *et al.* 1997).

The postbuckling and vibration behavior of flat and shallow curved panels resting on a Winkler foundation has been examined using a higher-order shear deformable theory which encompassed a number of effects such as transverse shear, geometric non-linearities and initial geometric imperfections (Librescu and Lin 1997). The static and dynamic analyses of shells of revolution on an elastic foundation was analyzed by employing a finite element method based on variational formulation (Eslami and Ayatollahi 1993). The bending analysis of a thin isotropic ellipsoidal shell of small ellipticity, resting on a Winkler-type elastic foundation and subjected to uniform internal pressure has been carried out (Paliwal *et al.* 1993). A simple approximate large deflection analysis of shallow spherical shells on Pasternak-type elastic foundations subjected to a concentrated load at its centre was developed (Paliwal 1994). The free vibration analysis of circular cylindrical shells on Winkler and Pasternak foundations was examined for shells of simply-supported boundary conditions (Paliwal *et al.* 1996).

A fair amount of work into the dynamic stability of cylindrical shells have been carried out recently including some by the present authors (Lam and Ng 1998, Ng and Lam 1998) and (Ng *et al.* 1998a, 1998b). However, a literature search showed that a study on the dynamic stability of circular cylindrical shells resting on elastic foundations is not available. A study that includes an elastic foundation would be interesting as it would shed light on the effects of the elastic foundations on the instability regions.

2. Theory and formulation

The cylindrical shell on an elastic foundation is in the coordinate system as shown in Fig. 1. The periodic extensional axial load per unit length is given by

$$N_a(x,t) = N_o + N_s \cos Pt \tag{1}$$

where P is the frequency of excitation in radians per unit time.

The equations of motion in terms of force and moment resultants for a cylindrical shell on an elastic foundation of zero damping can be written as

$$L_x(u,v,w) - k_1 u = \rho h \frac{\partial^2 u}{\partial t^2}$$

$$L_y(u,v,w) - k_2 v = \rho h \frac{\partial^2 v}{\partial t^2}$$

$$L_z(u,v,w) - k_3 w + \frac{\partial}{\partial x}\left(N_a \frac{\partial w}{\partial x}\right) = \rho h \frac{\partial^2 w}{\partial t^2}$$

where k_1, k_2 and k_3 (N/m^3) are the spring rates of the elastic foundation that act in the u, v and w directions respectively and according to Donnell's theory for thin shells. The mass density is ρ and E is the elastic modulus.

It is important to note here that the forces from the elastic foundation act on the neutral plane of the shell. This is a fair assumption as far as the transverse motion w is concerned. Strictly speaking, the u in the $k_1 u$ term of Eq. (2a) should be replaced by $(h/2)\ \partial u/\partial x$ and the v in the $k_2 v$ term of Eq. (2b) should be replaced by $(h/2R)\ \partial v/\partial \theta$. This is not usually done as it would create an illusion of accuracy not warranted by the elastic foundation concept as a whole.

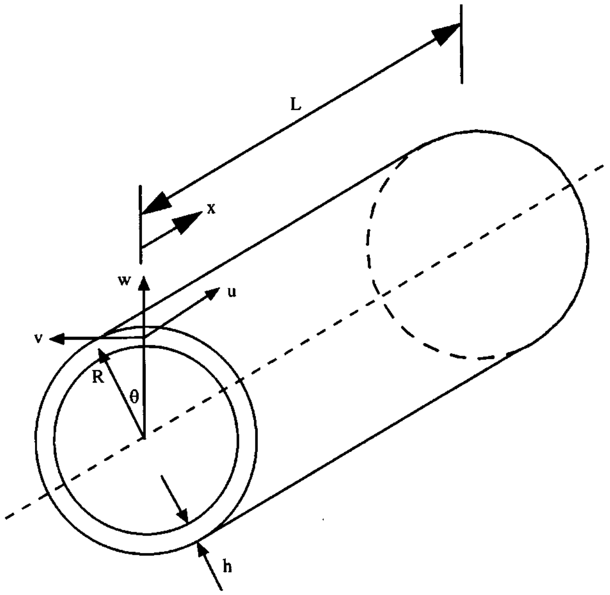

Fig. 1 Coordinate system of the cylindrical shell

The differential operators L_x, L_y and L_z are defined here according to Donnell's theory as described in the Appendix.

Assuming the shell to be simply-supported, there exists a solution for the equations of motion in the form

$$u_{mn} = A_{mn} e^{i\omega t} \cos\lambda_m x \cos n\theta \tag{3}$$
$$v_{mn} = B_{mn} e^{i\omega t} \sin\lambda_m x \sin n\theta \tag{4}$$
$$w_{mn} = C_{mn} e^{i\omega t} \sin\lambda_m x \cos n\theta \tag{5}$$

where n represents the number of circumferential waves, m the number of axial half-waves in the corresponding standing wave pattern and $\lambda_m = m\pi/L$. ω represents the natural frequency of the cylindrical shell under constant axial loading N_o.

The equations of motion can be solved using an eigenfunction expansion of the normal modes of free vibration of a cylindrical shell under a constant axial load N_o with the oscillating component N_s=0. Substitution of the above three equations into the equations of motion which are also a set of three coupled homogeneous equations yields a cubic frequancy equation when the determinant is equated to zero.

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{Bmatrix} A_{mn} \\ B_{mn} \\ C_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{6}$$

where the C_{ij}'s are defined in the Appendix. Thus, for each m and n, there exists three roots corresponding to the transverse, axial and circumferential modes.

To solve the equations of motion that include the oscillating component N_s, a solution is sought in the form shown below where all the modes are superimposed.

$$u_{mnj} = \sum_{j=1}^{3} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mnj} q_{mnj}(t) \cos\lambda_m x \cos n\theta \tag{7}$$
$$v_{mnj} = \sum_{j=1}^{3} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} B_{mnj} q_{mnj}(t) \sin\lambda_m x \sin n\theta \tag{8}$$
$$w_{mnj} = \sum_{j=1}^{3} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} C_{mnj} q_{mnj}(t) \sin\lambda_m x \cos n\theta \tag{9}$$

where $q_{mnj}(t)$ is a generalized coordinate.

Substituting the above three equations into the equations of motion and simplifying yields

$$\sum_{j=1}^{3} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (\ddot{q}_{mnj} + \omega_{mnj}^2 q_{mnj})\, \alpha_{mnj} \cos\lambda_m x \cos n\theta = 0 \tag{10}$$
$$\sum_{j=1}^{3} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (\ddot{q}_{mnj} + \omega_{mnj}^2 q_{mnj})\, \beta_{mnj} \sin\lambda_m x \sin n\theta = 0 \tag{11}$$
$$\sum_{j=1}^{3} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (\ddot{q}_{mnj} + \omega_{mnj}^2 q_{mnj}) \sin\lambda_m x \cos n\theta - \frac{1}{\rho_t} \lambda_m \cos Pt \sum_{j=1}^{3} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} q_{mnj} \frac{\partial}{\partial x} (N_s \cos\lambda_m x) \cos n\theta = 0 \tag{12}$$

where

$$\alpha_{mnj} = \frac{A_{mnj}}{C_{mnj}}, \qquad \beta_{mnj} = \frac{B_{mnj}}{C_{mnj}} \tag{13}$$

Making use of the orthogonality condition, we multiply Eq. (10) by $\alpha_{rsi}\cos\lambda_r x \cos s\theta$, Eq. (11) by $\beta_{rsi}\sin\lambda_r x \sin s\theta$, and Eq. (12) by $\sin\lambda_r x \cos s\theta$. This yields a set of equations

$$\boldsymbol{M}_{IJ}\ddot{\boldsymbol{q}}_J + (\boldsymbol{K}_{IJ} - \cos Pt\boldsymbol{Q}_{IJ})\boldsymbol{q}_J = 0 \tag{14}$$

where $\boldsymbol{M}_{IJ}$, $\boldsymbol{K}_{IJ}$ and $\boldsymbol{Q}_{IJ}$ are diagonal matrices and $\ddot{\boldsymbol{q}}_J$ and $\boldsymbol{q}_J$ are column vectors consisting of the $\ddot{q}_{mnj}$'s and q_{mnj}'s respectively.

The subscripts r, s, i, m, n, j, I and J have the following ranges

$$\begin{gathered} r, s, m, n = 1, 2, 3, 4, \cdots, N \\ i, j = 1, 2, 3 \\ I, J = 1, 2, 3, 4, \cdots, (N \times N \times 3) \end{gathered} \tag{15}$$

where I and J contain all possible combinations of r, s, i and m, n, j respectively.

The diagonal matrices $\boldsymbol{M}_{IJ}$, $\boldsymbol{K}_{IJ}$ and $\boldsymbol{Q}_{IJ}$ are given as

$$\begin{aligned} \boldsymbol{M}_{IJ} = \int_0^L \int_0^{2\pi} (&\alpha_I\alpha_J\cos\lambda_r x \cos s\theta \cos \lambda_m x \cos n\theta + \beta_I\beta_J \sin \lambda_r x \sin s\theta \sin \lambda_m x \sin n\theta \\ &+ \sin\lambda_r x \cos s\theta \sin\lambda_m x \cos n\theta)\, d\theta\, dx = \begin{cases} \frac{1}{2}\pi L(1 + \beta_I\beta_J + \alpha_I\alpha_J) & \text{if } I = J \\ 0 & \text{if } I \neq J \end{cases} \end{aligned} \tag{16}$$

$$\boldsymbol{K}_{IJ} = \omega_J^2 \boldsymbol{M}_{IJ} \tag{17}$$

$$\begin{aligned} \boldsymbol{Q}_{IJ} &= \frac{1}{\rho_t}\lambda_m \int_0^L \int_0^{2\pi} \frac{\partial}{\partial x}(N_s\cos\lambda_m x \cos n\theta)\sin\lambda_r x \cos s\theta\, d\theta\, dx \\ &= \begin{cases} -\frac{1}{2}\pi L \frac{1}{\rho_t}\lambda_r\lambda_m N_s & \text{if } I = J \\ 0 & \text{if } I \neq J \end{cases} \end{aligned} \tag{18}$$

3. Stability analysis

Eq. (14) is in the form of a second order differential equation with periodic coefficients of the Mathieu-Hill type. The regions of unstable solutions are separated by periodic solutions having period T and $2T$ with $T=2\pi/P$. The solutions with period $2T$ are of greater practical importance as the widths of these unstable regions are usually larger than those associated with solutions having period T. Applying Bolotin's method (see Bolotin 1964), a first approximation of the periodic solutions with period $2T$ can be sought in the form

$$\boldsymbol{f} = \boldsymbol{a}\sin\frac{Pt}{2} + \boldsymbol{b}\cos\frac{Pt}{2} \tag{19}$$

where $\boldsymbol{a}$ and $\boldsymbol{b}$ are arbitrary vectors.

Substituting Eq. (19) into Eq. (14) and equating the coefficients of the $\sin Pt/2$ and $\cos Pt/2$

terms, a set of linear homogeneous algebraic equations in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$ can be obtained. The conditions for non-trivial solutions are given by

$$\det\left[\begin{pmatrix} K_{IJ} - \frac{1}{2}Q_{IJ} - \frac{1}{4}P^2 M_{IJ} & 0 \\ 0 & K_{IJ} + \frac{1}{2}Q_{IJ} - \frac{1}{4}P^2 M_{IJ} \end{pmatrix}\right] = 0 \tag{20}$$

Rearranging the above equation, the standard form of a generalized eigenvalue problem is obtained

$$\det\left[\begin{pmatrix} K_{IJ} - \frac{1}{2}Q_{IJ} & 0 \\ 0 & K_{IJ} + \frac{1}{2}Q_{IJ} \end{pmatrix} - P^2 \begin{pmatrix} \frac{1}{4}M_{IJ} & 0 \\ 0 & \frac{1}{4}M_{IJ} \end{pmatrix}\right] = 0 \tag{21}$$

where $\mathbf{0}$ is a $N \times N$ null matrix. As the matrices $\boldsymbol{K}$, $\boldsymbol{Q}$ and $\boldsymbol{M}$ are diagonal, the eigenvalues of Eq. (21) can be immediately deduced.

4. Numerical results and discussion

A shell resting on an elastic foundation can be viewed as resting on elastic springs having spring constants k_1, k_2 and k_3 acting in the u, v and w directions respectively. For an elastic foundation of homogeneous material and uniform thickness that is defined by a modulus of elasticity E_f and a Poisson's ratio ν_f, an approximate first-order estimate is such that k_1 and k_2 are proportional to the foundation shear modulus G_f and k_3 is proportional to the foundation elastic modulus E_f. In the present analysis, the special situation is adopted where $k_1 = k_2 = k_3 = k$, which is not an unreasonable approximation if one wishes to explore the overall influence of an elastic foundation on a shell. The non-dimensional excitation frequency parameter p is defined as

$$p = P\left(\frac{\rho R^2(1-\nu^2)}{E}\right)^{1/2} \tag{22}$$

The values of η_o are chosen to be in terms of η_{cr} which is the critical buckling load of a simply-supported circular cylindrical shell subjected to static compressive axial load. For cylindrical shells of short to intermediate length, as are the cases used here, the buckling load as given by Timoshenko and Gere (1961) is

$$P_{cr} = \frac{Eh^2}{[3(1-\nu^2)]^{1/2}R} \tag{23}$$

and can be nondimensionalized as

$$\eta_{cr} = P_{cr}\left(\frac{1-\nu^2}{Eh}\right) \tag{24}$$

If v is taken to be 0.3,

$$\eta_{cr} = 0.5507 \frac{h}{R} \tag{25}$$

Each unstable region is bounded by two curves originating from a common point from the p axis with η_s=0. The two curves appear at first glance to be straight lines but are in fact two very slightly "outward" curving plots. For the sake of tabular presentation, the angle subtended, Θ, is introduced. It is calculated based on the arctangent of the right-angled triangle, *abc*, obtained by halving the whole unstable region as shown in Fig. 2. This angle gives a good measure of the size of the unstable region as calculations done with the smaller similar triangle, *ab'c'* (see Fig. 2), are within 0.2%.

As there are presently no results in open literature for the dynamic stability of cylindrical shells resting on elastic foundations, comparison of results in this study is made with those of Paliwal *et al.* (1996) for free vibration analysis of cylindrical shells resting on Winkler foundations. This comparison is to ensure that elastic foundation effects have been correctly integrated into the present formulation. The comparison is shown in Table 1. Present results in this free vibration study are obtained by finding the roots of the C_{ij} matrix of Eq. (6). The results of Paliwal *et al.* (1996) used for comparison are obtained by solving the frequency determinant defined in that paper. It is observed from Table 1 that good agreement is achieved. Present results are generally slightly higher than those of Paliwal *et al.* (1996) and is due to the different manner in which k is considered in the formulation. This will be discussed subsequently.

The results for the dynamic stability in the transverse, longitudinal and circumferential modes are presented in Tables 2 to 4 for an isotropic cylindrical shell of Poisson's ratio, v=0.3, and linear parameters L/R=2 and h/R=0.01 with the axial loading being η_o=0.5 η_{cr}. Table 2 presents the results for modes (1,1), (1,2) and (1,3). Table 3 presents the results for modes (2,1), (2,2) and (2,3) and Table 4 presents the results for modes (3,1), (3,2) and (3,3). Fig. 3 gives a graphical illustration for the case of mode (1,1). In Fig. 3, the three diagrams are of the same scale so as to

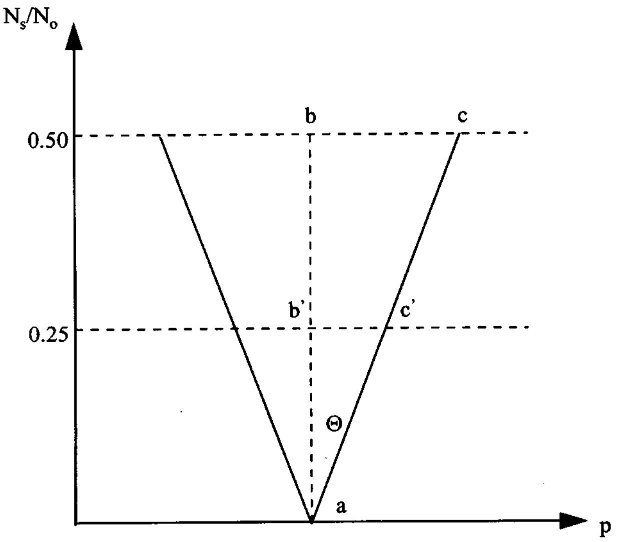

Fig. 2 An unstable region in the N_s/N_o-p plane

Table 1 Comparison of frequency parameter $\omega\left(\rho R^2(1-\nu^2)/E\right)^{1/2}$ for a cylindrical shell resting on an elastic foundation and having linear parameters of h/R=0.01, L/R=2 under no external loading

			Present	Paliwal *et al.* (1996)
$k=5\times10^{-5}$	$(m,n)=(1,1)$	transverse	0.57285303	0.57283785
		longitudinal	1.24794099	1.24792574
		circumferential	1.94826188	1.94825045
	$(m,n)=(1,2)$	transverse	0.32764303	0.32762793
		longitudinal	1.60244035	1.60242642
		circumferential	2.65637441	2.65636585
	$(m,n)=(1,3)$	transverse	0.19635443	0.19634044
		longitudinal	2.05685248	2.05684089
		circumferential	3.49471603	3.49470933
$k=1\times10^{-4}$	$(m,n)=(1,1)$	transverse	0.57289667	0.57286631
		longitudinal	1.24796103	1.24793052
		circumferential	1.94827471	1.94825185
	$(m,n)=(1,2)$	transverse	0.32771932	0.32768913
		longitudinal	1.60245595	1.60242810
		circumferential	2.65638382	2.65636670
	$(m,n)=(1,3)$	transverse	0.19648171	0.19645375
		longitudinal	2.05686463	2.05684146
		circumferential	3.49472319	3.49470978

Table 2 Unstable regions for a cylindrical shell resting on an elastic foundation and having linear parameters of h/R=0.01, L/R=2 and loading $\eta_o=1/2\eta_{cr}$. Axial half-wave number m=1

$(m,n)=(1,1)$		Transverse	Longitudinal	Circumferential
$k=0$	Pt. of origin p	1.1533206	2.4971456	3.8968799
	Angle subtended $\Theta\times10^{-4}$	38.289150	6.5378380	1.9126257
$k=5\times10^{-5}$	Pt. of origin p	1.1611840	2.5007872	3.8992144
	Angle subtended $\Theta\times10^{-4}$	38.030287	6.5283192	1.9114806
$k=1\times10^{-4}$	Pt. of origin p	1.1689946	2.5044234	3.9015475
	Angle subtended $\Theta\times10^{-4}$	37.776604	6.5188419	1.9103376
$(m,n)=(1,2)$		Transverse	Longitudinal	Circumferential
$k=0$	Pt. of origin p	0.6715663	3.2053060	5.3129618
	Angle subtended $\Theta\times10^{-4}$	80.882591	2.2872763	1.1599082
$k=5\times10^{-5}$	Pt. of origin p	0.6849827	3.2081438	5.3146743
	Angle subtended $\Theta\times10^{-4}$	79.307661	2.2852532	1.1595344
$k=1\times10^{-4}$	Pt. of origin p	0.6981414	3.2109791	5.3163863
	Angle subtended $\Theta\times10^{-4}$	77.821289	2.2832354	1.1591610
$(m,n)=(1,3)$		Transverse	Longitudinal	Circumferential
$k=0$	Pt. of origin p	0.4221632	4.1138360	6.9895402
	Angle subtended $\Theta\times10^{-5}$	1420.9714	7.7761903	6.1245746
$k=5\times10^{-5}$	Pt. of origin p	0.4431950	4.1160474	6.9908420
	Angle subtended $\Theta\times10^{-5}$	1354.5873	7.7720124	6.1234341
$k=1\times10^{-4}$	Pt. of origin p	0.4632729	4.1182577	6.9921436
	Angle subtended $\Theta\times10^{-5}$	12967.152	7.7678412	6.1222942

Table 3 Unstable regions for a cylindrical shell resting on an elastic foundation and having linear parameters of h/R=0.01, L/R=2 and loading $\eta_o=1/2\eta_{cr}$. Axial half-wave number m=2

(m,n)=(2,1)		Transverse	Longitudinal	Circumferential
k=0	Pt. of origin p	1.7210080	3.9732339	6.6377789
	Angle subtended $\Theta\times 10^{-5}$	1485.1342	29.744932	5.9145634
$k=5\times 10^{-5}$	Pt. of origin p	1.7262875	3.9755235	6.6391497
	Angle subtended $\Theta\times 10^{-5}$	1480.6123	29.727801	5.9133422
$k=1\times 10^{-4}$	Pt. of origin p	1.7315509	3.9778119	6.6405203
	Angle subtended $\Theta\times 10^{-5}$	1476.1315	29.710700	5.9121218
(m,n)=(2,2)		Transverse	Longitudinal	Circumferential
k=0	Pt. of origin p	1.3440866	4.5439867	7.5204159
	Angle subtended $\Theta\times 10^{-5}$	1847.0903	37.901876	7.3644303
$k=5\times 10^{-5}$	Pt. of origin p	1.3508400	4.5459889	7.5216258
	Angle subtended $\Theta\times 10^{-5}$	1837.9206	37.885183	7.3632456
$k=1\times 10^{-4}$	Pt. of origin p	1.3575599	4.5479903	7.5228355
	Angle subtended $\Theta\times 10^{-5}$	1828.8860	37.868513	7.3620615
(m,n)=(2,3)		Transverse	Longitudinal	Circumferential
k=0	Pt. of origin p	1.0183185	5.2747960	8.7838517
	Angle subtended $\Theta\times 10^{-5}$	2458.8999	26.137372	7.0168727
$k=5\times 10^{-5}$	Pt. of origin p	1.0272160	5.2765209	8.7848876
	Angle subtended $\Theta\times 10^{-5}$	2437.8609	26.128828	7.0160453
$k=1\times 10^{-4}$	Pt. of origin p	1.0360370	5.2782452	8.7859234
	Angle subtended $\Theta\times 10^{-5}$	2417.3529	26.120292	7.0152181

Table 4 Unstable regions for a cylindrical shell resting on an elastic foundation and having linear parameters of h/R=0.01, L/R=2 and loading $\eta_o=1/2\eta_{cr}$. Axial half-wave number m=3

(m,n)=(3,1)		Transverse	Longitudinal	Circumferential
k=0	Pt. of origin p	1.8841598	5.7224602	9.6582479
	Angle subtended $\Theta\times 10^{-5}$	3187.5130	9.3507182	3.2278347
$k=5\times 10^{-5}$	Pt. of origin p	1.8889834	5.7240502	9.6591901
	Angle subtended $\Theta\times 10^{-5}$	3179.4471	9.3481208	3.2275199
$k=1\times 10^{-4}$	Pt. of origin p	1.8937946	5.7256398	9.6601321
	Angle subtended $\Theta\times 10^{-5}$	3171.4421	9.3455256	3.2272051
(m,n)=(3,2)		Transverse	Longitudinal	Circumferential
k=0	Pt. of origin p	1.6742565	6.1209159	10.272028
	Angle subtended $\Theta\times 10^{-5}$	3528.2675	22.160097	4.0356894
$k=5\times 10^{-5}$	Pt. of origin p	1.6796830	6.1224024	10.272914
	Angle subtended $\Theta\times 10^{-5}$	3516.9966	22.154716	4.0353414
$k=1\times 10^{-4}$	Pt. of origin p	1.6850920	6.1238886	10.273800
	Angle subtended $\Theta\times 10^{-5}$	3505.8330	22.149340	4.0349935
(m,n)=(3,3)		Transverse	Longitudinal	Circumferential
k=0	Pt. of origin p	1.4307637	6.6992358	11.219003
	Angle subtended $\Theta\times 10^{-5}$	4090.6765	25.056484	4.6458033
$k=5\times 10^{-5}$	Pt. of origin p	1.4371098	6.7005940	11.219814
	Angle subtended $\Theta\times 10^{-5}$	4072.8852	25.051405	4.6454674
$k=1\times 10^{-4}$	Pt. of origin p	1.4434281	6.7019520	11.220625
	Angle subtended $\Theta\times 10^{-5}$	4055.3240	25.046329	4.6451317

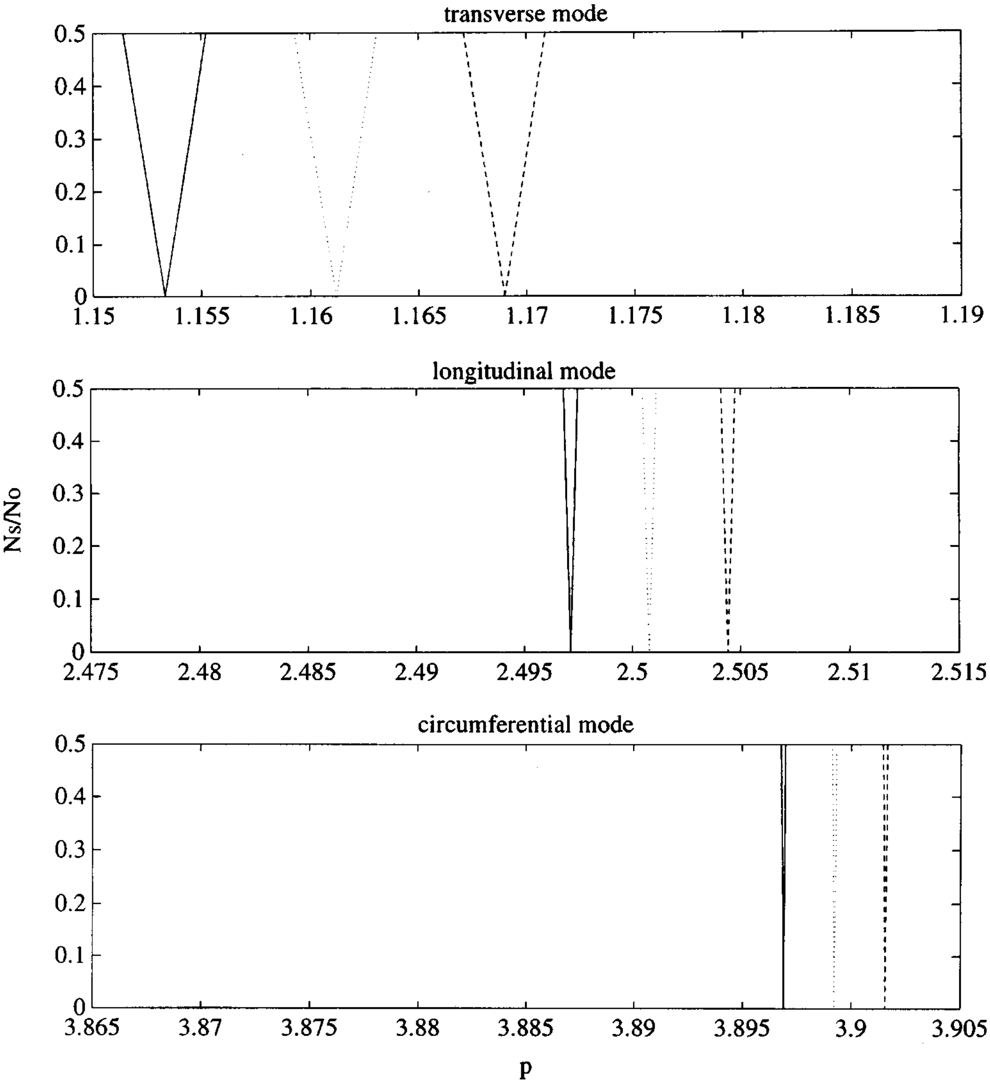

Fig. 3 Unstable regions for a cylindrical shell resting on an elastic foundation and with linear parameters of h/R=0.01, L/R=2 and loading $\eta_o=1/2\eta_{cr}$. (m, n)=(1, 1). '——', k=0, '······', $k=5\times10^{-5}$, '-----', $k=1\times10^{-4}$

give a better gage to the relative shifts in the unstable regions between the three modes. The higher modes of n>3 are not considered due to limitations of Donnell's theory for high circumferential wave numbers in short to moderate length cylindrical shells.

The results show that the sizes of the unstable regions associated with the transverse mode are much larger than those associated with the longitudinal and circumferential modes. It is observed from the results that the unstable regions associated with the three modes generally shift to the right with reduction in sizes as the value of k is increased. This is to be expected as the elastic foundation causes the overall shell stiffness to be increased. It is also observed that for any particular mode, the percentage shift in the point of origin corresponds to the percentage decrease in the size of the unstable region. Also, the right shift in the points of origin and the decrease in the sizes of the unstable regions are found to be proportional to the increase in the k values.

The results show that the value of k influences the transverse mode considerably. The longitudinal and circumferential modes are also influenced but to a much lesser extent. This observation is in line with the conclusions of Paliwal *et al.* (1996) for the free vibration analysis of stationary circular cylindrical shells on a Winkler foundation. To relate the present work to vibration analysis, one only has to look at the points of origin of the unstable regions which correspond to twice the natural frequency of that mode. The present work shows that the dynamic stability results associated with longitudinal and circumferential modes are also influenced to a larger extent by k when compared with the free vibration results presented by Paliwal *et al.* (1996) in which the longitudinal and circumferential modes remained almost unaffected. This can easily be traced to the formulation of Paliwal *et al.* (1996) which neglected the effects of k in the u and v directions thus isolating the longitudinal and circumferential modes to the effects of the elastic foundation to a certain degree.

An interesting observation from the results of Tables 2 to 4 is the influence of increasing k values on the results of the transverse mode of vibration for different values of the circumferential wave number n. It is observed that for a particular axial wave number m, increase in points of origin and decrease in the sizes of the unstable regions due to increased k values are more pronounced for modes with higher n values. This observation also applies for the longitudinal and circumferential modes but the phenomenon is not as pronounced. The converse of the previous observation is true for increasing values of axial wave number m. In this instance, the effects of increased k values (for a fixed n) is diminished for higher axial wave numbers m. As in the previous observation, this observation applies for all three modes of vibration but is most pronounced for the transverse mode.

5. Conclusions

The dynamic stability of simply-supported cylindrical shells resting on elastic foundation has been examined. For a particular combination of m and n, the effects of the foundation is always more pronounced on the transverse mode than on the longitudinal and circumferential modes. The effects of the foundation on the dynamic instability regions are also different for different combinations of m and n. For a fixed m, higher n values will be more affected by changes in the foundation stiffness. For a fixed n, lower m values will be more affected by changes in the foundation stiffness.

References

Aköz, A.Y. and Kadloglu, F. (1996), "The mixed finite element solution of circular beam on elastic foundation", *Computers and Structures*, **60**, 643-651.

Bolotin, V.V. (1964), *The Dynamic Stability of Elastic Systems*, Holden-Day, San Francisco.

Chandrasekharappa, G. (1992), "Nonlinear static and dynamic damped response of a composite rectangular plate resting on a two-parameter elastic foundation", *Computers and Structures*, **44**, 609-618.

Chang, T.P. (1994), "Deterministic and random vibration of an axially loaded Timoshenko beam resting on an elastic foundation", *Journal of Sound and Vibration*, **178**, 55-66.

De Rosa, M.A. (1995), "Free vibrations of Timoshenko beams on two-parameter elastic foundation", *Computers and Structures*, **57**, 151-156.

Eslami, M.R. and Ayatollahi, M.R. (1993), "Modal analysis of shell of revolution on elastic foundation", *International Journal of Pressure Vessels and Piping*, **56**, 351-568.

Guler, K. and Celep, Z. (1995), "Static and dynamic responses of a circular plate on a tensionless elastic foundation", *Journal of Sound and Vibration*, **183**, 185-195.

Kuo, Y.H. and Lee, S.Y. (1994), "Deflection of nonuniform beams resting on a nonlinear elastic foundation", *Computers and Structures*, **51**, 513-519.

Lam, K.Y. and Ng, T.Y. (1998), "Dynamic stability analysis of cylindrical shells subjected to conservative periodic axial loads using different thin shell theories", *Journal of Sound and Vibration*, (in press).

Lee, H.P. (1996), "Dynamic stability of a tapered cantilever beam on an elastic foundation subjected to a follower force", *International Journal of Solids and Structures*, **33**, 1409-1424.

Lee, S.Y., Lin, J.C. and Hsu, K.C. (1996), "Elastic instability of a beam resting on an elastic foundation subjected to a partially tangential force", *Computers and Structures*, **59**, 983-988.

Lee, S.Y. and Yang, C.C. (1993), "Non-conservative instability of a Timoshenko beam resting on Winkler elastic foundation", *Journal of Sound and Vibration*, **162**, 177-184.

Lee, S.Y. and Yang, C.C. (1994), "Non-conservative instability of non-uniform beams resting on an elastic foundation", *Journal of Sound and Vibration*, **169**, 433-444.

Librescu, L. and Lin, W.Q. (1997), "Postbuckling and vibration of shear deformable flat and curved panels on a non-linear elastic foundation", *International Journal of Non-Linear Mechanics*, **32**, 211-225.

Naidu, N.R. and Rao, G.V. (1995), "Vibrations of initially stressed uniform beams on a two-parameter elastic foundation", *Computers and Structures*, **57**, 941-943.

Naidu, N.R. and Rao, G.V. (1996), "Free vibration and stability behaviour of uniform beams and columns on nonlinear elastic foundation", *Computers and Structures*, **58**, 1213-1215.

Ng, T.Y. and Lam, K.Y. (1998), "Dynamic stability analysis of cross-ply laminated cylindrical shells using different thin shell theories", *Acta Mechanica*, (in press).

Ng, T.Y., Lam, K.Y. and Reddy, J.N. (1998a), "Dynamic stability analysis of cross-ply laminated cylindrical shells", *International Journal of Mechanical Sciences*, (in press).

Ng, T.Y., Lam, K.Y. and Reddy, J.N. (1998b), "Parametric resonance of a rotating cylindrical shell subjected to periodic axial loading", *Journal of Sound and Vibration*, (in press).

Omurtag, M.H., Ozutok, A. and Aköz, A.Y. (1997), "Free vibration analysis of Kirchhoff plates resting on elastic foundation by mixed finite element formulation based on Gateaux differential", *International Journal for Numerical Methods in Engineering*, **40**, 295-317.

Paliwal, D.N., Gupta, R. and Jain, A. (1993), "Stress analysis of ellipsoidal shell on an elastic foundation", *International Journal of Pressure Vessels and Piping*, **56**, 229-242.

Paliwal, D.N. (1994), "Large deflection in a shallow spherical shell on an elastic foundation under a concentrated load at the centre", *International Journal of Pressure Vessels and Piping*, **57**, 131-133.

Paliwal, D.N., Pandey, R.K. and Nath, T. (1996), "Free vibrations of circular cylindrical shell on Winkler and Pasternak foundations", *International Journal of Pressure Vessels and Piping*, **69**, 79-89.

Qin, Q.H. (1993), "Nonlinear analysis of Reissner plates on an elastic foundation by the BEM", *International Journal of Solids and Structures*, **30**, 3101-3111.

Qin, Q.H. (1995), "Hybrid-Trefftz finite element method for Reissner plates on an elastic foundation", *Computer Methods in Applied Mechanics and Engineering*, **122**, 379-392.

Raju, K.K. and Rao, G.V. (1993a), "Effect of a non-linear elastic foundation on the mode shapes in stability and vibration problems of uniform columns/beams", *Journal of Sound and Vibration*, **160**, 369-371.

Raju, K.K. and Rao, G.V. (1993b), "Vibrations of initially stressed beams and plates around transition values of elastic foundation stiffness", *Journal of Sound and Vibration*, **161**, 378-384.

Zhou, D. (1993), "General solution to vibrations of beams on variable Winkler elastic foundation", *Computers and Structures*, **47**, 83-90.

Appendix

$$L_x = \frac{Eh}{R^2(1-\nu^2)}\left\{R^2\frac{\partial^2 u}{\partial x^2} + \frac{1}{2}(1-\nu^2)\frac{\partial^2 u}{\partial \theta^2} + \frac{R}{2}(1+\nu)\frac{\partial^2 v}{\partial x \partial \theta} + \nu R\frac{\partial w}{\partial x}\right\}$$

$$L_y = \frac{Eh}{R^2(1-\nu^2)}\left\{\frac{R}{2}(1+\nu)\frac{\partial^2 u}{\partial x \partial \theta} + \frac{R^2}{2}(1-\nu)\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial \theta^2} + \frac{\partial w}{\partial \theta}\right\}$$

$$L_z = \frac{Eh}{R^2(1-\nu^2)}\left\{-\nu R\frac{\partial u}{\partial x} - \frac{\partial v}{\partial \theta} - w - K\left(R^4\frac{\partial^4 w}{\partial x^4} + 2R^2\frac{\partial^4 w}{\partial x^2 \partial \theta^2} + \frac{\partial^4 w}{\partial \theta^4}\right)\right\} \tag{A1}$$

$$\begin{aligned}
C_{11} &= (\lambda_m R)^2 + n^2(1-\nu)/2 - \gamma\omega^2 + \bar{k}_1\\
C_{12} &= -n\,\lambda_m R(1+\nu)/2\\
C_{13} &= -\nu\lambda_m R\\
C_{21} &= -n\,\lambda_m R(1+\nu)/2\\
C_{22} &= n^2 + (\lambda_m R)^2(1-\nu)/2 - \gamma\omega^2 + \bar{k}_2\\
C_{23} &= n\\
C_{31} &= -\nu\lambda_m R\\
C_{32} &= n\\
C_{33} &= 1 + K((\lambda_m R)^2 + n^2)^2 - \gamma\omega^2 + \bar{k}_3 + (\lambda_m R)^2\eta_o
\end{aligned} \tag{A2}$$

$$\gamma = \rho\frac{R^2(1-\nu^2)}{E}$$

$$K = \frac{h^2}{12R^2}$$

$$\bar{k}_j = k_j\frac{R^2(1-\nu^2)}{Eh} \qquad j = 1, 2, 3 \tag{A3}$$