Structural Engineering and Mechanics, Vol. 8, No. 2 (1999) 119-136
DOI: http://dx.doi.org/10.12989/sem.1999.8.2.119

Factors governing redistribution of moment in continuous prestressed concrete beams

V.K.R. Kodur†

Institute for Research in Construction, National Research Council Canada, Montreal Road, Ottawa, Ontario, Canada

T.I. Campbell‡

Department of Civil Engineering, Queen's University, Kingston, Ontario, Canada

Abstract. The failure load of a continuous prestressed concrete beam depends partially on the amount of redistribution of moment that occurs prior to failure. Results from a parametric study, carried out using a nonlinear finite element computer program, are presented to demonstrate the influences of various factors on redistribution of moment in two-span, continuous bonded prestressed concrete beams. Trends in the data from the numerical studies are compared with those from a theoretical expression for percentage of redistribution, and it is shown that the redistribution of moment occurring in a continuous prestressed concrete beam is a function of number of parameters.

Key words: redistribution of moment; continuous beam; prestressed concrete; nonlinear analysis; secondary moment; parametric study; bonded post-tensioned beams.

1. Introduction

The bending moments in a continuous prestressed concrete beam can be predicted using a linear elastic analysis, provided the load level is such that the elastic limit is not exceeded in any of the constituent materials. When the elastic limit is exceeded, at any particular load level, the bending moments in the beam will differ from those predicted by a linear analysis. The difference, for a particular load level, between the actual moment at a section and that determined by a linear analysis is referred to as the amount of redistribution of moment. In order to determine the actual amount of redistribution of moment a nonlinear analysis has to be carried out. The extent of redistribution of moment can be full, partial or nil, and depends on a number of factors (Kodur 1992).

In practice, design codes for concrete structures usually recommend the use of an elastic analysis, and either ignore the nonlinear effect totally or recognise it by applying a somewhat arbitrary adjustment to the design elastic moments (CSA 1994, ACI 1995, SAA 1994). This arbitrary adjustment for redistribution of moment is based on cross-sectional ductility as measured

† Research officer
‡ Professor

by factors such as ratio of neutral axis depth to effective depth (c/d), or reinforcement index (ω) at the support section. There still exists debate on the extent of redistribution permitted by different codes of practice (Kodur 1992, Cohn and Lounis 1991). The provisions for redistribution of moment and the treatment of secondary moments in the CSA, ACI and SAA codes were the subject of controversy in some fairly recent studies (Cohn 1986, Sveinson and Dilger 1991, Warner and Yeo 1986, Wyche *et al.* 1992). Some of these studies have recommended the use of structural ductility, as opposed to cross-sectional ductility, in defining the permissible redistribution of moment.

The influence of ductility on redistribution of moment has been extensively studied by previous researchers through laboratory tests and analytical studies on non-prestressed and prestressed concrete beams (Kodur 1992). However, the effects of some parameters, such as span to depth ratio, cross-sectional shape, position and type of loading, secondary moment, partial prestressing index, confinement of concrete and tension-stiffening, have not been fully investigated. This is due to certain physical limitations, such as dimensions of test beams and practical difficulty in applying uniformly distributed load, making it impossible to study fully the effects of some of the these parameters on redistribution of moment by means of laboratory tests. However, these limitations can be overcome using computer simulation to conduct a parametric study of beams having a wide range of characteristics.

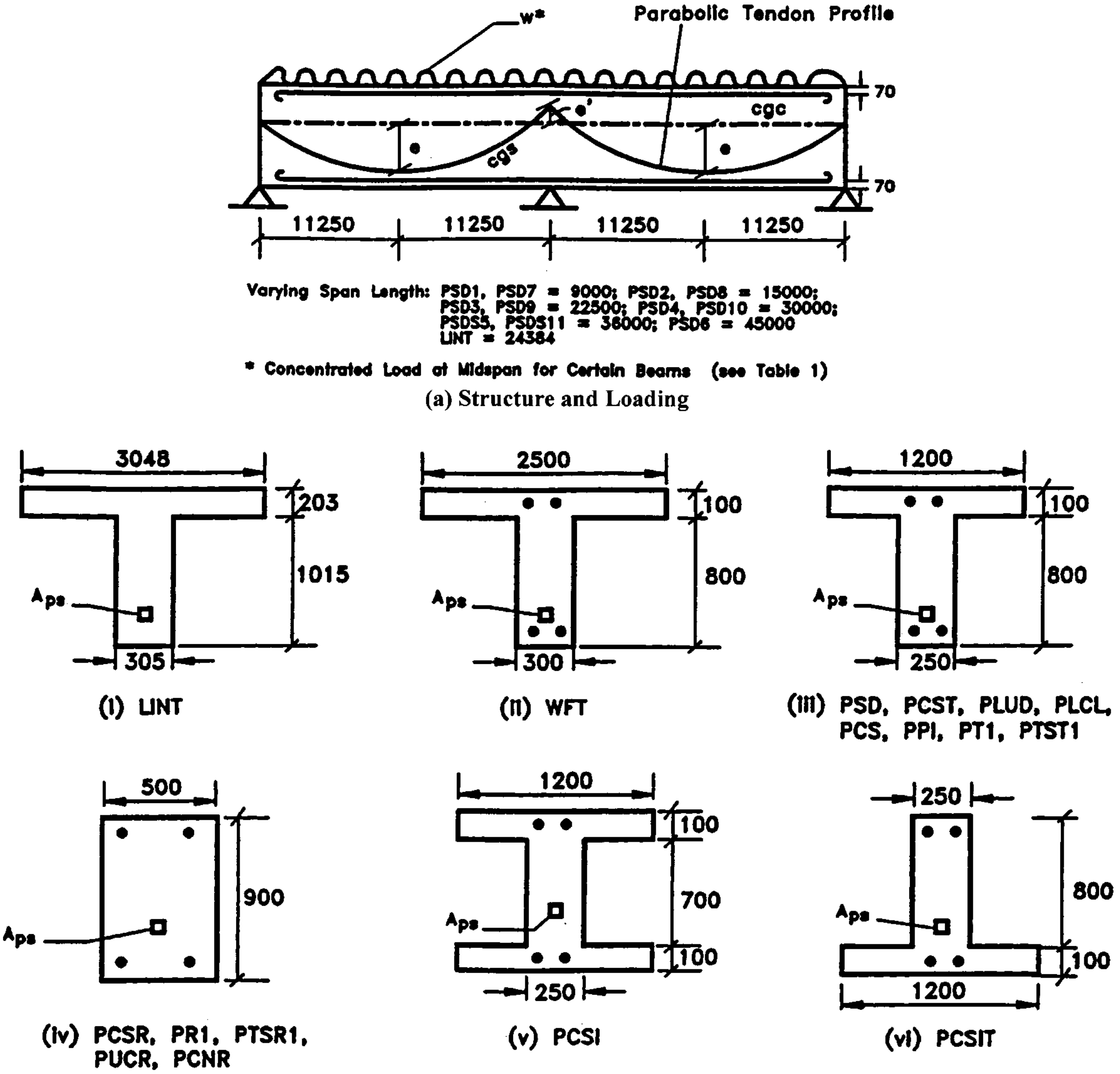
2. Parametric study

Kodur (1992) utilized the computer program NAPCCB to carry out a parametric study on a series of continuous prestressed concrete beams. This program is based on a macroscopic finite element approach and uses a curvature incrementing technique to trace the response of a bonded prestressed concrete beam over the entire loading range from prestressing to collapse. The validity of NAPCCB has been established by comparing its predictions with test data and other analytical predictions (Campbell and Kodur 1990, Kodur and Campbell 1995). In general, good agreement was obtained with regard to the failure load, but NAPCCB tended to underestimate the deformation at failure.

NAPCCB can accommodate reinforced, partially prestressed and fully prestressed concrete beams, and is capable of accounting for confinement of concrete, moment shedding (softening of concrete), tension-stiffening of concrete, strain-hardening of non-prestressed reinforcement and shear cracking. However, shear cracking was not taken into account in this study since the influence of shear deformation has been studied by Kodur (1992) and shown to be insignificant in prestressed concrete beams having a span/depth ratio greater than 20.

The influences of different parameters were examined by analysing sixty-eight, two-span, prestressed concrete beams having a wide range of characteristics. Fig. 1 shows details of the beams, while Table 1 lists various properties of the beams according to the designations used by Kodur (1992) for identification. The parameters varied included span-depth ratio (Beams PSD), cross-sectional shape (Beams PCST, PCSR, PCSI, PCSIT), type of loading (Beams PLCL, PLUD), concrete strength (Beams PCS), partial prestressing index (Beams PPI), confinement of concrete (Beams PUCR and PCNR), span stiffness (WFT, NFT), secondary moment (LINT, WFT, NFT), and tension-stiffening effect in concrete (Beams PT1, PTST1, PR1 and PTSR1), respectively.

All beams were symmetric over two spans with a span length of 22.5 m except for LINT beams which had a span length of 24.38 m, and beams PSD1 to PSD11 where the span length



(a) Structure and Loading

(b) Cross Section

Note: All Dimensions are in mm
Not to Scale

Fig. 1 Layout of beams used in parametric study

was varied from 9 m to 45 m to give a span to overall depth ratio, (L/h), ranging from 10 to 50. All beams, except Beams PSD, PLCL and PR1 and PTSR1, which were subjected to a concentrated load at the centre of each span, were loaded symmetrically with uniformly distributed load. Bonded post-tensioning was provided in all beams and varying amounts of non-prestressed reinforcement were provided. Details on the amounts of non-prestressed reinforcement are shown in Table 1, while the properties of the concrete, non-prestressed and prestressed reinforcement, as well as the amounts of prestressed reinforcement, are given in Table 2. The material properties were kept constant for all beams, except for beams PCS5 to PCS12 and LINT where the properties of concrete were varied. The maximum compressive strain in the concrete at

Table 1 Details and results of analyses for beams (continued on next page)

Beam	Span			Support			Span (c/d)	Support (c/d)	M_B (kN·m)
	e mm	A_s mm^2	A_s' mm^2	e' mm	A_s mm^2	A_s' mm^2			
PCSIT1	216	400	10000	407	1500	400	0.212	0.193	3054
PCSIT2	216	400	10000	407	3000	400	0.212	0.297	3054
PCSIT3	216	400	10000	407	5300	400	0.212	0.446	3054
PCSIT4	216	400	10000	407	8000	400	0.212	0.589	3054
PLCL1*	389	2000	400	0	400	11500	0.232	0.271	3257
PLCL2*	389	2000	400	0	400	5900	0.235	0.432	3257
PLCL3*	389	2000	400	0	400	2600	0.220	0.647	3257
PLCL4*	389	2000	400	0	2000	400	0.235	0.796	3257
PLUD1	389	2000	400	0	400	11500	0.232	0.271	3257
PLUD2	389	2000	400	0	400	5900	0.235	0.432	3257
PLUD3	389	2000	400	0	400	2600	0.235	0.647	3257
PLUD4	389	2000	400	0	2000	400	0.235	0.796	3257
PCS1	388	1500	400	155	400	10000	0.197	0.229	3291
PCS2	388	1500	400	155	400	6500	0.197	0.306	3291
PCS3	388	1500	400	155	400	4000	0.197	0.468	3291
PCS4	388	1500	400	155	400	1600	0.197	0.602	3291
PCS5	388	1500	400	155	400	10000	0.137	0.218	3374
PCS6	388	1500	400	155	400	6500	0.137	0.267	3374
PCS7	388	1500	400	155	400	4000	0.137	0.380	3374
PCS8	388	1500	400	155	400	1600	0.137	0.502	3374
PCS9	388	4500	400	155	400	10000	0.195	0.218	4219
PCS10	388	4500	400	155	400	6500	0.195	0.267	4219
PCS11	388	4500	400	155	400	4000	0.195	0.380	4219
PCS12	388	4500	400	155	400	1600	0.195	0.502	4219
PPI1	388	3500	400	155	400	4700	0.333	0.426	3779
PPI2	388	6000	400	155	4300	4700	0.333	0.426	3792
PPI3	388	9500	400	155	7300	4700	0.333	0.426	3527
PPI4	388	13000	400	155	11300	4700	0.333	0.426	3153
PUCR	388	400	4300	312	400	4300	0.213	0.220	3160
PCNR	388	400	4300	312	400	4300	0.233	0.240	3140
PT1	388	1500	400	155	400	10000	0.197	0.229	3291
PTST1	388	1500	400	155	400	10000	0.197	0.223	3290
PR1*	389	2000	400	0	400	2600	0.220	0.650	3257
PTSR1*	389	2000	400	0	400	2600	0.223	0.654	3256

failure was assumed to be 0.004 in all cases, except in beam PCNR where confinement of concrete was considered resulting in an ultimate strain of 0.0066 in the concrete. Additional information on the beams is given by Kodur (1992), and Kodur and Campbell (1996).

Results from the NAPCCB analysis, namely failure loads, secondary moment at central support, and (c/d) ratios and moment capacities of the critical sections, are given for all beams in Table 1. The three loads given are the failure loads based on plastic analysis (W_{pl} for concentrated load or w_{pl} for distributed load), nonlinear analysis (W_{col} or w_{col}), and elastic analysis (W_{le} or w_{le}). These failure load levels are indicated in Fig. 2 which shows a plot of load vs. moment (support and span) for a two-span beam in which less than full redistribution of moment occurs at failure. The moments M_B and M_C correspond to the moment capacities of the span and the support critical sections, respectively, while M_{sec} is the secondary moment at the central support section based on

Table 1 (Continued from previous page)

Beam	M_C kN·m	M_{sec} kN·m	MR	w_{le}** kN/m	w_{col}** kN/m	w_{pl}** kN/m	$PAR1$	x %
PCSIT1	3377	18	0.353	53.64	65.50	72.49	0.629	18.18
PCSIT2	3746	73	0.247	60.34	67.52	74.97	0.491	10.81
PCSIT3	4243	148	0.132	69.38	70.87	78.30	0.167	2.17
PCSIT4	4673	223	0.050	77.38	73.98	81.18	−0.890	−4.82
PLCL1*	2107	748	0.179	676.79	759.24	766.31	0.921	14.17
PLCL2*	2033	718	0.223	652.14	743.92	759.73	0.853	16.00
PLCL3*	1747	730	0.356	576.40	656.70	753.30	0.450	13.79
PLCL4*	1659	757	0.391	572.56	623.67	726.49	0.332	11.50
PLUD1	2107	748	0672	45.12	54.37	66.21	0.439	21.74
PLUD2	2033	718	0.732	43.48	52.43	66.63	0.387	21.80
PLUD3	1747	730	0.941	39.14	46.32	64.56	0.283	20.65
PLUD4	1659	757	1.006	38.17	44.24	63.92	0.236	18.80
PCS1	2786	845	0.344	57.38	65.01	72.39	0.508	14.77
PCS2	2739	841	0.361	56.57	64.19	72.08	0.491	14.96
PCS3	2567	837	0.426	53.79	60.73	70.92	0.405	14.61
PCS4	2270	832	0.560	49.02	55.96	68.83	0.350	16.21
PCS5	2801	851	0.368	57.71	66.25	73.85	0.529	16.17
PCS6	2771	846	0.380	57.16	65.68	73.65	0.517	16.28
PCS7	2669	842	0.418	55.49	63.56	72.96	0.462	16.07
PCS8	2463	838	0.503	52.16	60.01	71.58	0.404	16.78
PCS9	2802	743	0.720	56.02	71.04	87.64	0.475	25.32
PCS10	2771	744	0.734	55.54	70.98	84.73	0.529	26.07
PCS11	2669	744	0.785	53.93	67.97	86.75	0.428	24.97
PCS12	2463	744	0.899	50.68	63.47	85.37	0.369	24.73
PPI1	2635	760	0.622	53.65	64.74	79.34	0.432	21.03
PPI2	2996	572	0.544	57.03	63.25	81.97	0.249	12.59
PPI3	3057	322	0.513	53.40	59.35	78.06	0.201	9.25
PPI4	3186	33	0.437	50.02	52.34	72.82	0.102	2.83
PUCR	3048	474	0.341	55.65	65.06	72.02	0.575	16.34
PCNR	3020	474	0.343	56.64	66.05	71.48	0.636	17.90
PT1	2786	845	0.344	54.33	64.79	72.39	0.579	14.41
PTST1	2790	845	0.342	54.33	65.01	72.39	0.591	14.77
PR1*	1747	730	0.355	576.40	656.70	753.30	0.450	13.79
PTSR1*	1748	730	0.354	576.00	660.00	753.00	0.470	14.28

*Beams with concentrated load
**for concentrated load W_{le}, W_{col}, W_{pl} in kN
1 in=25.4 mm; 1 kip/ft=14.63 kN/m; 1 kip·ft=1.356 kN·m

an elastic analysis. The actual failure load of a beam is taken as that obtained from the nonlinear analysis, while the elastic failure load is computed based on results from NAPCCB in the linear elastic range. The plastic collapse load is computed using the ultimate moment capacities of the critical sections obtained from the moment-curvature relationships for these sections generated in NAPCCB. For beams subjected to concentrated load, the load point is the critical region in the span, while for beams subjected to uniformly distributed load the location of the critical section in the span was assumed to be at approximately 0.4 L from the end support.

Table 2 Material properties of beams

Material	Property	
Concrete[1,2,3]	f_c'=40	E_c=40 000
	f_t'=3.79	ε_{cu}=0.004
Nonprestressed	f_y=400	ε_y=0.002
Reinforcement	E_s=200 000	ε_{su}=0.1542
Prestressed	A_{ps}=2376	f_{se}=1209
Reinforcement[4]	f_{pu}=1860	ε_{pu}=0.05

f and E in MPa, A_{ps} in mm^2; 1 in=25.4 mm, 1 ksi=6.895 MPa

[1]f_c'=34.5, f_t'=2.93, E_c=29 923, f_{se}=1037 and A_{ps}=2580 for beams LINT1, LINT2, LINT3, LINT4

[2]f_c'=50, f_t'=4.24 and E_c=50 000 for beams PCS5, PCS6, PCS7, PCS8, PCS9, PCS10, PCS11, PCS12

[3]ε_{cu}=0.0066 for beam PCNR

[4]A_{ps}=1782, 990, 99 for beams PPI2, PPI3 and PPI4, respectively

3. Measurement of redistribution of moment

The extent of redistribution of moment that occurs prior to failure was measured using the approach outlined by Moucessian and Campbell (1988). This approach measures redistribution of moment in terms of the plastic adaptation ratio, $PAR1$, which is given by the following relationship:

$$PAR\,1 = \frac{W_{col} - W_{le}}{W_{pl} - W_{le}} \quad \text{or} \quad \frac{w_{col} - w_{le}}{w_{pl} - w_{le}} \tag{1}$$

where W_{col} and w_{col}, W_{le} and w_{le}, W_{pl}, and w_{pl} are the failure loads based on nonlinear, elastic and plastic analyses, respectively.

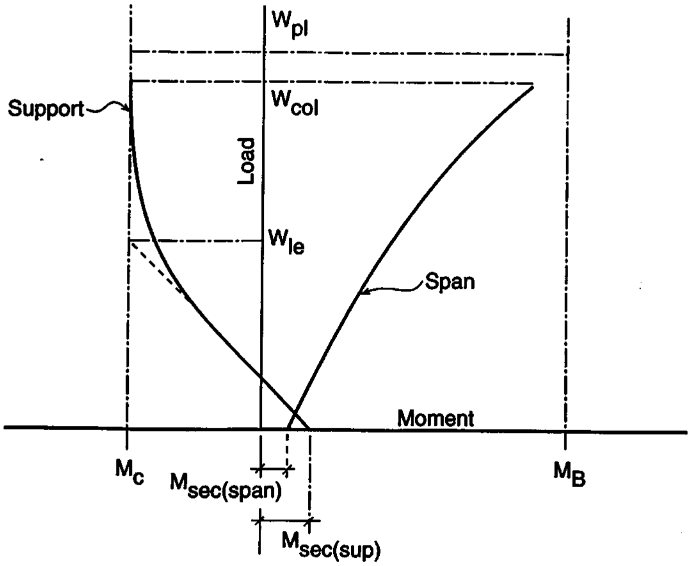


Fig. 2 Typical load-moment relationships for a two-span continuous prestressed concrete beam

*PAR*1 defines the level of W_{col} or w_{col} relative to W_{le} or w_{le}, and W_{pl} or w_{pl}, thereby giving a good representation of the degree of redistribution of moment that occurs at failure (Fig. 2). For full redistribution, where W_{col} (or w_{col})=W_{pl} (or w_{pl}), *PAR*1=1, while for no redistribution, where W_{col} (or w_{col})=W_{le} (or w_{le}), *PAR*1=0.

The parameter *PAR*1 was evaluated for each beam using Eq. (1) and the relevant values of *PAR*1 are given in Table 1. The *PAR*1 values are less than unity indicating that partial redistribution occurred in all the beams. It should be noted also in Table 1, that the value of *PAR*1 is less than zero for some beams. These beams had a high *c*/*d* value at the support section and a low *c*/*d* value at the critical span section. As a result the more critical section is the span section, as opposed to central support section, and consequently redistribution of moment is from the span to the support section, rather than from the support to the span section, in these beams.

The segment idealization recommended by Campbell and Kodur (1990) was used in modelling the beams for the nonlinear analysis. It may be argued that the recommended segment length of 0.25 times the depth of the section is too short to give a proper representation of a plastic hinge in a beam, and as a result the deformation in the failure region over the central support is underestimated leading to a lower predicted degree of redistribution (Bazant *et al.* 1987). The effect of variation in the length of the segment, from the depth of the beam (900 mm) to 250 mm, on the values of *PAR*1 for the four PCST beams is indicated in Fig. 3. It can be seen that, while *PAR*1 increases with segment length, the trend that *PAR*1 decreases with increasing *c*/*d* at the support is consistent for all segment lengths. Consequently, while the values of *PAR*1 reported in this paper may be regarded as conservative, nevertheless the trends are correct.

4. Factors influencing redistribution of moment

By equating the total available rotation at the central support region with the inelastic rotation required to achieve a percentage of redistribution in a symmetric two-span prestressed concrete beam (Kodur and Campbell 1996) derived the following expression for *PAR*1:

$$PAR1 = \frac{\frac{2}{3}EI\left(\frac{1}{EI_C} - \frac{1}{EI_{Cy}}\right)\left(\frac{d}{L} + 0.1m_1\right) + \frac{M_{sec}}{M_C}}{MR} \tag{2}$$

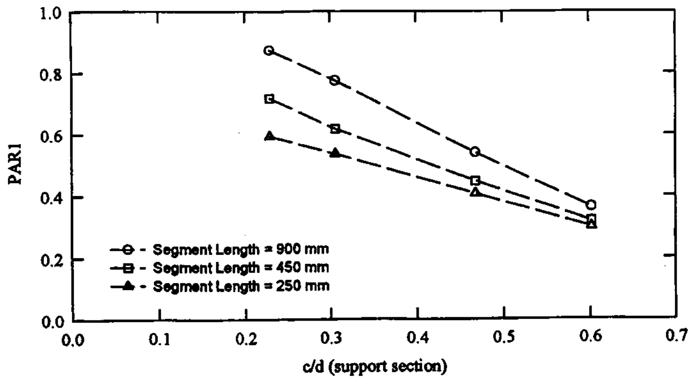


Fig. 3 Influence of segment length on *PAR*1

In Eq. (2) EI is a measure of the flexural stiffness of the span, EI_C is the flexural stiffness at failure of the support critical section, EI_{Cy} is the flexural stiffness of the support section at first yield of the reinforcement, d is the distance from the extreme compression fibre to the centre of the tension force at the support section, L is the span length, M_{sec} is the secondary moment at the central support section, M_C is the ultimate moment capacity of the support section, and m_1 is the fraction of the span length between the hinging region and the adjacent point of contraflexure. The moment ratio MR is defined as:

$$MR = \left(\frac{M_C + M_{sec}}{M_C}\right)\left(\frac{T_f(M_B + aM_C)}{a(1-a)s_1(M_C + M_{sec})} - 1\right) \tag{3}$$

where M_B and M_C are the ultimate moment capacities of the span and support critical sections, respectively, a is the ratio of the distance of the span critical section from the end support to the span length when the span and centre support section ultimate strengths are developed simultaneously, s_1 is a factor used in defining the bending moment at the central support (s_1=16/3 for concentrated load at mid-span and s_1=8 for uniformly distributed load), and T_f = 1.0 for a concentrated load and 2.0 for a uniformly distributed load in each span.

Kodur and Campbell (1996) have shown that the values of $PAR1$ from Eq. (2) compare well with those derived from NAPCCB for beams subjected to concentrated load, but tended to be higher for beams with uniformly distributed load. From Eq. (2) it can be inferred that:

1. An increase in the span-depth ratio, L/d, will result in decreased redistribution of moment.
2. The greater the value of EI_C, corresponding to a higher c/d ratio at the support section, the lower will be the amount of redistribution.
3. The extent of redistribution of moment is influenced by the type of loading, as reflected by the factors a and s_1 in the expression for MR (Eq. (3)).
4. Concrete strength and partial prestressing index, whose effects are accounted for indirectly in the computation of MR through the values of M_B and M_C, also influence the extent of redistribution.
5. The amount of redistribution of moment increases with stiffness of the span (EI), plastic hinge length (reflected by the term 0.1 m_1) and secondary moment, M_{sec}, having the same sign as M_c.
6. The effect of confinement of concrete will be reflected in the stiffness of the span and support sections.

The variation of $PAR1$ with different parameters, as found from the parametric study, is discussed in the following sections.

4.1. Span-depth ratio

The effect of span-overall depth ratio, L/h, on the redistribution of moment was investigated for two cases, one with concentrated load only (PSD1 to PSD6) and the other for concentrated load together with uniformly distributed load due to self weight of beam (PSD7 to PSD11). The secondary moment due to prestress remained the same for all the beams (Table 1).

Results from the analyses for both cases of loading are shown in Fig. 4, where it can be seen that $PAR1$ decreases with increase in L/h ($\approx L/d$) but at a decreasing rate. Further, for a particular value of L/h the value of $PAR1$ is smaller for the beam loaded with both concentrated and

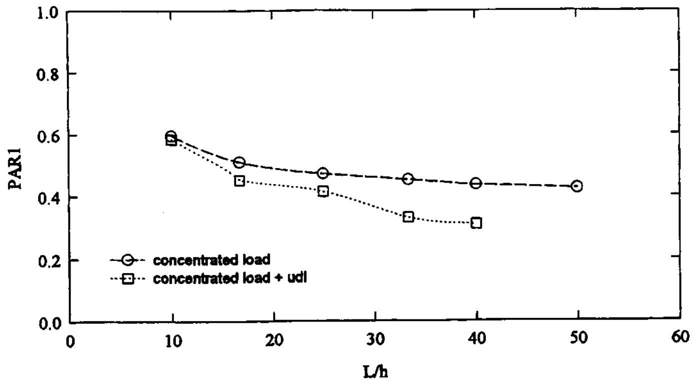


Fig. 4 Effect of span to depth ratio on *PAR*1

uniformly distributed load, indicating that the extent of redistribution of moment depends on the type of loading. Also, the decrease in *PAR*1 when self weight is considered is greater for higher values of L/h since the moment due to self weight becomes more significant in beams with larger spans. The influence of type of loading on *PAR*1 is discussed later.

It should be noticed that the majority of test beams used in laboratory investigations in the past had L/h values less than ten, and in most of these beams apparent full redistribution of moment was reported (Kodur 1992). In practice the span to depth ratio of prestressed concrete beams will normally be in the range of 20-30, and hence the effect of span-depth ratio should be considered in determining the amount of redistribution of moment.

4.2. Cross-sectional shape and c/d of support section

Fig. 5 shows the variation of *PAR*1 with c/d at the central support section for two-span beams with rectangular (PCSR), T (PCST), I (PCSI) and inverted T (PCSIT) cross sections. The beams were proportioned such that the strengths and the c/d values of the critical sections were of similar magnitude for the different cross sections. The value of c/d at the span critical section was maintained at about 0.2, while the value of c/d at the support critical section was varied from about 0.2 to 0.6, as indicated in Table 1.

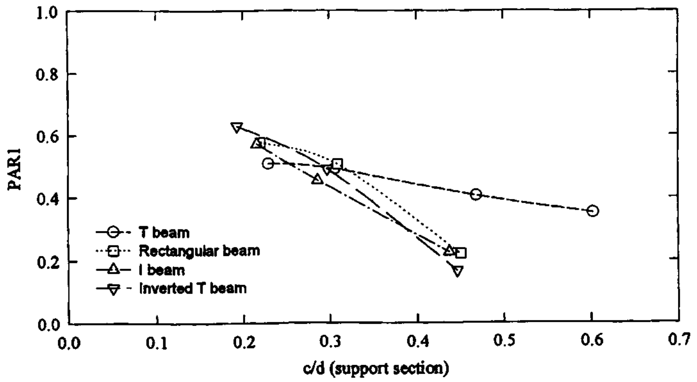


Fig. 5 Effect of cross-sectional shape on *PAR*1

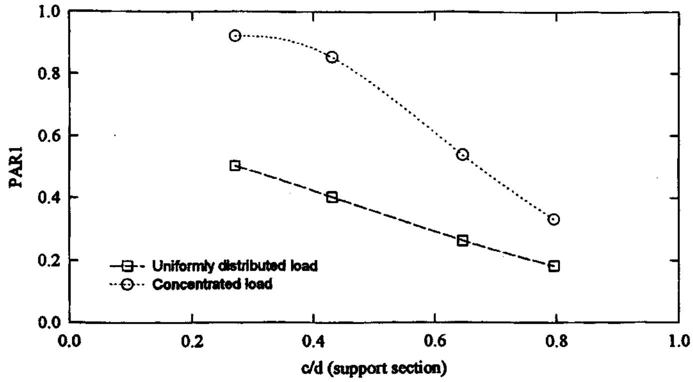


Fig. 6 Effect of loading type on *PAR*1

It can be seen from Fig. 5 that *PAR*1 decreases with the increase in c/d at the support section. This trend is reflected in Eq. 2 where a higher c/d value at the support section leads to a smaller value of $\left(\frac{1}{EI_C} - \frac{1}{EI_{Cy}}\right)$, thereby resulting in a lower value of *PAR*1. Fig. 5 also shows that the T cross section has the highest *PAR*1 values for c/d greater than 0.3, and that the other three sections have similar *PAR*1 values for all c/d values. For the rectangular, I and inverted T cross sections negative *PAR*1 values were obtained when c/d at the support critical section was equal to 0.6, indicating that reverse redistribution occurred in these beams. The higher *PAR*1 values in the beams with a T cross section can be attributed to increased ductility induced by the high percentage of compression reinforcement at the support section (Gattesco and Cohn 1989).

4.3. Loading type

The variation of PAR1 with c/d at the central support is shown in Fig. 6 for equivalent beams subjected to two types of loading, namely a concentrated load at the centre of each span (PLCL) and a uniformly distributed load (PLUD). It can be seen that the beams subjected to uniformly distributed load exhibited lower redistribution of moment than those subjected to concentrated load, and that the reduction of PAR1 with increasing c/d is greater for the concentrated loading.

The variation of *PAR*1 with loading type is due mainly to the different plastic rotation capacities. The rotation capacity of a hinging region increases with increase in distance from the hinge to the point of contraflexure (Park and Paulay 1975), denoted by $m1$ in Eq. (2). The point of contraflexure in a beam subjected to concentrated load at mid-span is further from the central support than in an equivalent beam subjected to uniformly distributed load, resulting in a larger rotation capacity which permits a greater amount of redistribution of moment, as indicated by Eq. (2).

4.4. Prestressing

4.4.1. Secondary moment

The effect of secondary moment on the extent of redistribution of moment was investigated by analysing three groups of beams (LINT, WFT and NFT) having varying values of secondary moment. The beams in each group had a similar c/d ratio at the support critical region but a

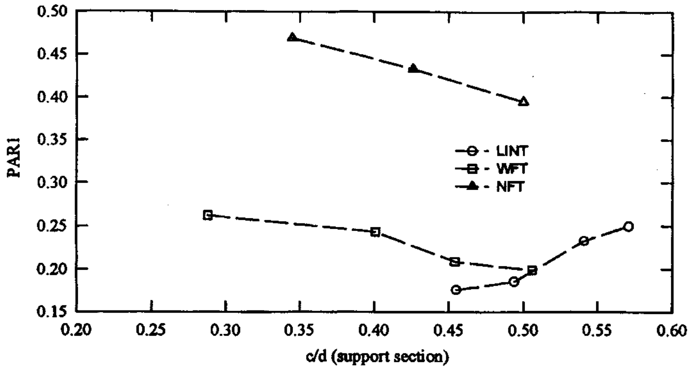


Fig. 7 Variation of *PAR*1 with *c*/*d* at support for varying M_{sec} and span stiffness

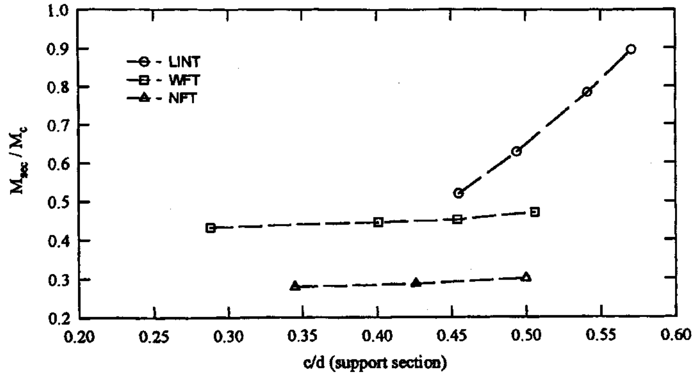


Fig. 8 Variation of secondary moment to ultimate moment ratio with *c*/*d*

varying *c*/*d* ratio at the span critical region. The *PAR*1 values are plotted as a function of *c*/*d* at the support section for three groups of beams in Fig. 7.

For the LINT beams *PAR*1 increased with *c*/*d* at the support section despite the decrease in sectional ductility associated with increasing *c*/*d* ratio. This trend can be attributed to the influence of secondary moment. The secondary moment was significant in all four beams, with the ratio M_{sec}/M_C increasing from 0.520 to 0.890 with *c*/*d* ratio at the support, as can be seen from Fig. 8. For all these beams, the moment capacity of the span critical section was found to be close to the yield moment, and thus $E_{IC} \approx EI_{Cy}$. Consequently, the amount of inelastic action is very small for these beams and majority of redistribution results from the M_{sec} component shown in Eq. (2). In the WFT and NFT beams, where the secondary moment did not change significantly from beam to beam, as indicated in Fig. 8, *PAR*1 decreased with *c*/*d*. The larger values of *PAR*1 in the NFT beams may be attributed to the higher span stiffness in these beams as discussed later.

4.4.2. Partial prestressing index

The effect of the level of prestress on the extent of redistribution of moment was studied by analysing four two-span beams having varying degrees of prestress. Beam PPI1 was fully prestressed, beams PPI2 and PPI3 were partially prestressed, and beam PPI4 contained non-prestressed reinforcement only. The amounts of prestressed and non-prestressed reinforcement were varied to give different values of the Partial Prestressing Index ($PPI=(\omega/(\omega+\omega_p))$. The beams

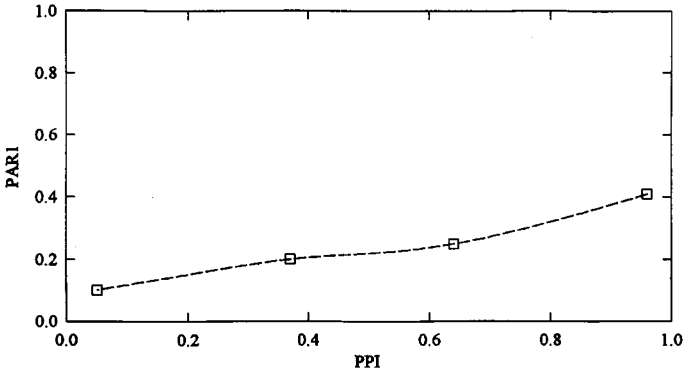


Fig. 9 Effect of *PPI* on *PAR*1

were proportioned such that the *c*/*d* values at the span and the support critical regions did not change significantly from beam to beam Table 1).

Fig. 9 shows that *PAR*1 increases with partial prestressing index at the support section. The increase in *PAR*1 values, despite the decrease in cross-sectional ductility resulting from the increase in the *PPI* ratio, (Cohn and Bartlett 1982, Naaman *et al.* 1986) can be attributed to the increase in secondary moment with *PPI*. It can be seen from Table 1 that the secondary moment, at the central support, is maximum for the fully prestressed beam and almost zero for the non-prestressed beam (the small value of secondary moment for reinforced beam is due to the presence of small amount of fictitious prestressed reinforcement necessary to facilitate the NAPCCB analysis).

4.5. Span stiffness

The influence of span stiffness on the extent of redistribution of moment was investigated by analysing two groups of beams (WFT and NFT). In each group, the value of *c*/*d* at the span critical section was kept constant, while the value of *c*/*d* at the support critical section was varied (Table 1). The higher *PAR*1 in the NFT beams, having higher *c*/*d* values at span section than the WFT beams, can be attributed to the variation of stiffness along the span length.

Fig. 10 shows the variation with load of EI/EI_C, the ratio of stiffness at the critical span section

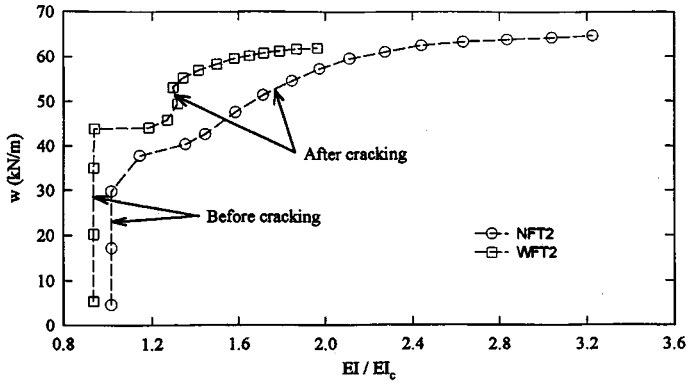


Fig. 10 Variation of stiffness ratio with load

to that at the central support section, for Beams WFT2 and NFT2. In both beams the stiffness ratio is constant until cracking occurs after which the stiffness ratio increases with increase in load. At failure the ratio is approximately 2 for Beam WFT2, while for Beam NFT2 it is around 3.2. Eq. (2) indicates that a larger value of the stiffness ratio (EI/EI_C) at failure will lead to a higher degree of redistribution of moment. This shows that the span stiffness plays a significant role in determining the extent of redistribution of moment.

4.6. Concrete strength

The effect of concrete strength on redistribution of moment, was investigated by analysing twelve prestressed concrete beams (PCS1 to PCS12). Beams PCS1 to PCS4 had a concrete strength of 40 MPa, while beams PCS5 to PCS12 had a concrete strength of 50 MPa. Table 1 shows that the c/d value of the span critical section decreased from 0.197 in beams PCS1 to PCS4, to 0.138 in beams PCS5 to PCS8, when f_c' was increased from 40 to 50 MPa, respectively.

The variation of *PAR*1 with c/d at the support critical section for two concrete strengths is shown in Fig. 11. It can be seen that the value of *PAR*1 decreases only by a small amount with increase in concrete strength. In order to examine the sole effect of concrete strength four beams (PCS9 to PCS12) having a concrete strength of 50 MPa were analysed, where the c/d of the span section in a beam was the same as that in the corresponding beam with f_c'=40 MPa (PCS1 to PCS 4). It can be seen from Fig. 11 that the decrease in *PAR*1 with increase in concrete strength is noticeable for beams having similar c/d values at the two critical sections. This result is due to the decrease in ductility of sections with increasing concrete strength. The CEB-FIP Model Code (CEB-FIP 1990) recognises the influence of concrete strength by specifying a decrease in redistribution of moment with increase in concrete strength.

4.7. Confinement of concrete

Confinement of concrete, provided by the lateral hoop reinforcement, increases the ductility of a section (Park and Paulay 1975). The effect of confinement of concrete was investigated by analysing two prestressed concrete continuous beams PUCR and PCNR. In the unconfined case,

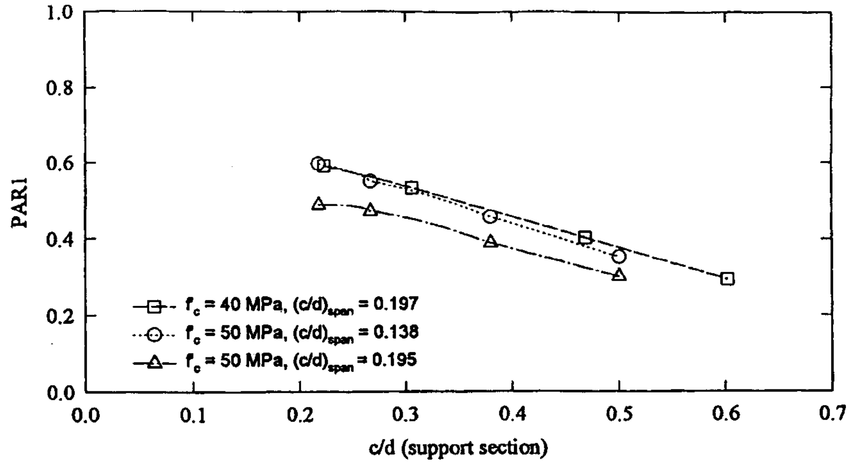


Fig. 11 Effect of concrete strength on *PAR*1

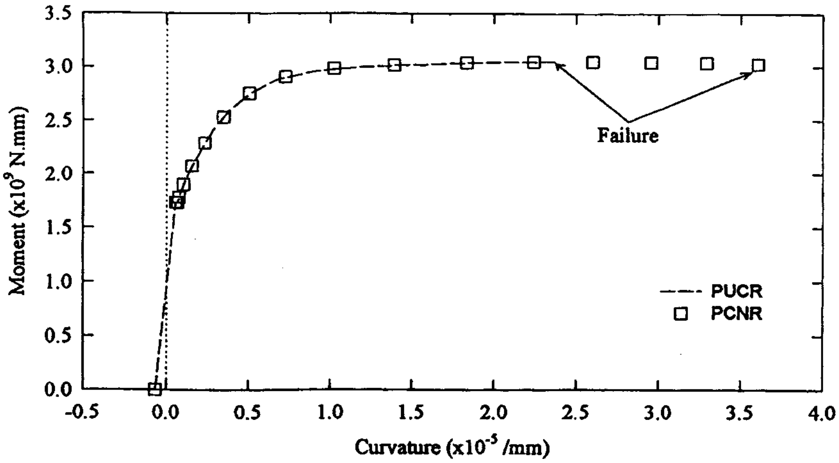


Fig. 12 Effect of confinement of concrete on moment-curvature relationship of support section

beam PUCR, the ultimate concrete strain was taken as 0.004, while for the confined case, beam PCNR, the stress-strain curve for concrete confined by transverse reinforcement (Park *et al.* 1982) was used and the resulting value of the ultimate strain in concrete was found to be 0.0066.

Fig. 12 shows a comparison of the moment-curvature relationships at the support section for the two beams. The *PAR*1 value is increased from 0.539 in beam PUCR to 0.636 in beam PCNR (Table 1). It should be noted that the increase in the *PAR*1 value occurs despite a small increase in (c/d) at the support critical section. The contribution of confinement of concrete to the redistribution of moment was noted in laboratory investigations reported by Bhatia (1984) and Moucessian (1986).

4.8. Tension-stiffening of concrete

Generally, the contribution of concrete in tension subsequent to flexural cracking is neglected in the design and analysis of concrete structures. When the contribution of concrete in tension is accounted for a member becomes stiffer, resulting in a reduced deformation of the member under load. However, the influence of tension-stiffening on the extent of redistribution of moment does not appear to have been investigated for continuous prestressed concrete beams.

To investigate the variation of *PAR*1 with tension-stiffening, four beams were analysed. Table 1 shows the *PAR*1 values for beams PTST1 and PTSR1 in which tension-stiffening was included in the analysis, and for beams PT1 and PR1 in which tension-stiffening was neglected. It can be seen that the *PAR*1 values do not change significantly when the tension stiffening effect is included. An examination of the computed deflections indicated that the deflection at ultimate reduced by about five percent when tension stiffening was included. However, there is no significant change in the failure load and hence in *PAR*1, indicating that the tension stiffening effect has negligible influence on redistribution of moment.

5. Design implications

The above discussion indicates that the extent of redistribution of moment in continuous prestressed concrete beams is influenced by a number of factors. However, the majority of current codes of practice use only one or two factors, based on cross-sectional ductility, in defining the permissible redistribution of moment. Hence there is a need for a rational approach for determining the permissible amount of redistribution of moment by accounting for the influence of various parameters and based on structural ductility. The fact that the majority of the continuous beams tested in previous laboratory investigations attained full redistribution of moment (Kodur 1992) may be attributed to the use of favourable factors in the test beams, such as rectangular cross section, low span-depth ratio (in the range of 7-15) and concentrated load. Full redistribution of moment cannot be expected in practice where the beams usually have a span-depth ratio of 20-30, are subjected to uniformly distributed load and are of T, I or inverted T cross section.

The theoretical expression for evaluating the redistribution of moments (Eq. (2)) is somewhat complex in nature, and since the derivation is based on a number of simplifying assumptions, its applicability may be limited in practice. However the relationship is useful in identifying the various parameters which influence the extent of redistribution of moment in a two-span prestressed concrete beam. Results from the numerical studies, presented in this paper, were used to develop the following equations for calculating the percentage of redistribution of moment, x, occurring at failure of a continuous prestressed concrete beam.

For beams with concentrated load:

$$x = 60\left(1 - e^{-\left(\frac{MR}{0.7}\right)}\right) \qquad 0 \le x \le 60 \tag{4}$$

For beams with udl:

$$x = 45\left(1 - e^{-\left(\frac{MR}{0.7}\right)}\right) - 10 \qquad 0 \le x \le 30 \tag{5}$$

where x is related to $PAR1$ by:

$$PAR1 = \frac{\left(\frac{x}{100 - x}\right)}{MR} \tag{6}$$

Eq. 4 is found to be an excellent fit to $PAR1$ values derived from NAPCCB, however the data to which Eq. (5) is fitted are more scattered (Kodur and Campbell 1996).

The above equations for redistribution of moment, are based on two parameters, the percentage of redistribution (x), and the moment ratio (MR) as given by Eq. (3), and take into account the characteristics of the whole beam, including secondary moment. This ensures that the extent of redistribution is related to overall structural characteristics of the beam.

6. Conclusions

The extent of redistribution moment that can be attained before the failure of a continuous prestressed concrete beam depends on span-depth ratio, cross-sectional shape, loading type, partial prestressing index, stiffness of span, magnitude and nature of secondary moments, concrete strength and confinement of concrete, in addition to ductility of critical sections. Thus the extent of redistribution of moment should be related to overall structural characteristics of the beam rather than to just one critical section.

From the results of the parametric study reported in this paper, the following points may be summarised.

1. The extent of redistribution of moment decreases with increase in span-depth ratio.
2. Beams with a T cross section attain a higher degree of redistribution than comparable beams with rectangular, I and inverted T cross sections.
3. A beam subjected to uniformly distributed load exhibits a higher degree of redistribution than does a comparable beam with concentrated loading.
4. An increase in partial prestressing index, accompanied by a beneficial increase in secondary moment, results in larger redistribution of moment.
5. The higher the ratio of span stiffness to support stiffness, the higher will be the redistribution of moment.
6. The greater the secondary moment, of the same sign as that of ultimate moment at the support section, the higher will be the redistribution of moment.
7. The lower the concrete grade, the higher is the redistribution of moment.
8. Confinement of concrete, provided by hoop reinforcement, helps in achieving a higher degree of redistribution.
9. Tension stiffening effect in concrete does not contribute to redistribution of moment.

Acknowledgements

The authors wish to acknowledge the financial assistance provided by the Natural Sciences and Engineering Research Council of Canada under Grant No. A8255.

References

ACI Committee 318. (1995), "Building code requirements structural concrete (ACI318-95) and commentary (ACI 318R-95)", American Concrete Institute, Detroit, MI.

Bazant, Z. P., Pan, J. and Pijaudier-Cabot, G. (1987), "Softening in reinforced concrete beams and frames", *Journal of Structural Engineering,* **113**(12), 2333-2347.

Bhatia, S. (1984), "Continuous prestressed concrete beams in the inelastic range", M.Sc. Thesis, Department of Civil Engineering, Queen's University, Kingston, Ontario, Canada.

Campbell, T. I. and Kodur, V. K. R. (1990), "Deformation controlled nonlinear analysis of prestressed concrete continuous beams", *PCI JOURNAL,* **35**(5), 42-90.

CSA (Canadian Standards Association) (1994), "Design of concrete structures", A23.3-94, Rexdale, Canada.

Cohn, M. Z. (1986), "Continuity in prestressed concrete", *Partial Prestressing, From Theory to Practice,* **1**, Martinus Nijhoff Publishers, Boston, USA, 189-256.

Cohn, M. Z. and Bartlett, M. (1982), "Computer-simulated flexural tests of partially prestressed concrete sections", *ASCE Journal of the Structures Division,* **108**(ST 12), 2747-2765.

Cohn, M. Z. and Lounis, Z. (1991), "Moment redistribution in standard concrete codes", *Canadian Journal of Civil Engineering,* **18**(2), 92-108.

Gattesco, N and Cohn, M. Z. (1989), "Computer-simulated tests on moment redistribution – Part 1: Ultimate limit state considerations", Study E Richere – No. 11, Politecnico de Milano, Italia, 269-299.

CEB-FIP (1990), "CEB-FIP model code 1990", Comite Euro-International du Beton, Lausanne, Switzerland.
Kodur, V. K. R. (1992), "Redistribution of moment and the lifluence of secondary moment on continuous prestressed concrete beams", Ph.D. Thesis, Department of Civil Engineering, Queen's University, Kingston, Ontario, Canada.
Kodur, V. K. R. and Campbell, T. I. (1996), "Evaluation of moment redistribution in a two-span continuous prestressed concrete beam", *American Concrete Institute (ACI) Structural Journal*, **93**(6), 721-728.
Kodur, V. K. R. and Campbell, T. I. (1995), "A computer program for the non-linear analysis of prestressed concrete continuous beams", 'External Prestressing in Structures', AFPC, Paris, France, 75-86.
Moucessian, A. (1986), "Nonlinearity and continuity in prestressed concrete beams", Ph.D. Thesis, Department of Civil Engineering, Queen's University, Kingston, Ontario, Canada.
Moucessian, A. and Campbell, T. I. (1988), "Prediction of the load capacity of two-span continuous prestressed concrete beams", *PCI JOURNAL*, **33**(2), 130-151.
Naaman, A. E., Harajili, M. H. and Wight, J. K. (1986), "Analysis of ductility in partially prestressed concrete flexural members", *PCI Journal*, **31**(3), 64-87.
Park, R. and Paulay, T. (1975), *Reinforced Concrete Structures*, John Wiley and Sons, New York, N.Y.
Park, R., Priestley, M. J. N. and Scott, B. D. (1982), "Stress-strain behaviour of concrete confined by overlapping hoops at low and high strain rates", *ACI Journal, Proceedings*, **79**(1), 13-27.
SAA (Standards Association of Australia), (1994), "Concrete structures", AS 3600, Homebush, Australia.
Sveinson, T., and Dilger, W. H. (1991), "Moment redistribution in reinforced concrete structures", *Progress in Structural Engineering*, Kluwer Academic Publishers, Boston, MA, 51-70.
Warner, R. F. and Yeo, M. F. (1986), "Ductility requirements for partially prestressed concrete", *Partial Prestressing, From Theory to Practice*, **II**, Martinus Nijhoff Publishers, Boston, MA, 316-326.
Wyche, P., Uren, G. and Reynolds, G. (1992), "Interaction between prestress secondary moments, moment redistribution, and ductility – A treatise on the Australian concrete code", *ACI Structural Journal*, **89**(1), 57-70.

Notations

A_{ps}	= area of prestressed reinforcement
A_s	= area of non-prestressed tensile reinforcement
A_s'	= area of non-prestressed compressive reinforcement
a	= ratio of the distance of the span critical section from the end support to the span length, when the span and center support section ultimate strengths are developed simultaneously
c	= distance from extreme compression fibre to the neutral axis at the ultimate limit state
d	= distance from extreme compression fibre to the centre of tension force
E_c	= modulus of elasticity of concrete
E_s	= modulus of elasticity of non-prestressed reinforcement
EI	= flexural stiffness
e	= eccentricity of prestressing tendon
f_c'	= specified compressive strength of concrete
f_{pu}	= ultimate stress in prestressed reinforcement
f_{se}	= effective stress in prestressed reinforcement
f_t'	= modulus of rupture of concrete
f_y	= yield stress in nonprestressed reinforcement
h	= overall depth of the section
L	= span length
M	= moment
M_B	= ultimate moment capacities of the span critical section

M_C	= ultimate moment capacities of the support critical section
M_{sec}	= secondary moment due to prestress
MR	= Moment Ratio
m_1	= fraction of the span length between the hinging region and the adjacent point of contraflexure
PPI	= partial prestressing index
$PAR1$	= Plastic Adaptation Ratio (based on failure loads)
s_1	= variable factor used in defining the bending moment of the central support (for concentrated load at the mid-span s_1=16/3; for uniformly distributed loading s_1=8)
T_f	= 1.0 for a concentrated load, and 2.0 for a uniformly distributed load in each span
W	= concentrated load
w	= uniformly distributed load
x	= percentage redistribution of moment
ε_{cu}	= strain at ultimate in concrete
ε_{pu}	= strain at ultimate in prestressed reinforcement
ε_{su}	= strain at ultimate in nonprestressed reinforcement
ε_y	= strain at yield in nonprestressed reinforcement
ω	= reinforcing index for non-prestressed reinforcement as defined in ACI 318-95
ω_p	= reinforcing index for prestressed reinforcement as defined in ACI 318-95

Conversion factors

1 in	= 25.4 mm
1 kip	= 4.448 kN
1 ksi	= 6.895 MPa
1 kip-ft	= 1.356 kN·m