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Stochastic analysis of seismic structural response with soil-structure interaction

S. Sarkani†

School of Engineering and Applied Science, The George Washington University, Washington, D.C. 20052, U.S.A.

L.D. Lutes‡

Department of Civil Engineering, Texas A&M University, College Station, TX 77843, U.S.A.

S. Jin‡†

School of Engineering and Applied Science, The George Washington University, U.S.A.

C. Chan‡‡

Haynes Whaley Associates, Inc., Houston, TX, U.S.A.

Abstract. The most important features of linear soil-foundation-structure interaction are reviewed, using stochastic modeling and considering kinematic interaction, inertial interaction, and structural distortion as three separate stages of the dynamic response to the free-field motion. The way in which each of the three dynamic stages modifies the spectral density of the motion is studied, with the emphasis being on interpretation of these results, rather than on the development of new analysis techniques. Structural distortion and inertial interaction analysis are shown to be precisely modeled as linear filtering operations. Kinematic interaction, though, is more complicated, even though it has a filter-like effect on the frequency content of the motion.

Key words: filtering; seismic; soil-structure interaction; stochastic models; transfer functions.

1. Introduction

The goal in most studies of seismic response of structures is the determination of numerical values for the structural distortions based on presumed characteristics of the seismic input motion. The rather complex dynamics problem is more easily understood and analyzed, though, when the

† Professor
‡ Wiley Professor
‡† Research Associate
‡‡ Structural Engineer

interaction of the soil, foundation and structure is split up into two parts, called kinematic and inertial interaction, and the determination of structural distortions is separated from the interaction problem. The purpose of this paper is to summarize the mathematical characterization of each of these three stages of dynamic response, using stochastic processes and stochastic fields (Lutes and Sarkani 1997) to represent the various stages of motion, and relatively simple, linear models to represent the behavior of the soil and structure. The primary emphasis will not be on the development of new analysis techniques, but on interpretation of the results.

Where possible the concept of a linear filter will be exploited in interpreting the various aspects of the problem. In particular, the structural dynamics and inertial interaction aspects of the problem can be exactly modeled as filters which modify the spectral density of the vector signal passing through. The nature of the filter representing inertial interaction strongly depends on the dynamic characteristics of both the structure and the soil-foundation system. Finding these characteristics for the structure is a fairly straightforward problem, which is usually expedited by the use of eigenanalysis. The dynamic charateristics of the soil-foundation system, though, are often acquired from interpolation of tabulated or graphical results from detailed numerical analysis. Simple curve fitting of these results can give a model which is not physically reasonable due to being noncausal, but this difficulty can be avoided by using an approach in which the soil is modeled as a set of springs, dashpots, and masses.

Finally, kinematic interaction will be shown to be somewhat more complicated than a simple filtering operation, even for the simplest known linear model. In particular, there is no linear transfer matrix which can characterize the kinematic interaction effect. In spite of this, kinematic interaction does have a filter-like effect of modifying the frequency content of the motion.

This paper will emphasize the filter-like frequency modification behavior of each aspect of soil-foundation-structure interaction, and how each affects the structural response.

2. Problem formulation

The division of the soil-foundation-structure interaction problem into three stages is represented schematically in Fig. 1. The output in this figure is the distortion of the structure, and the input is the "free-field" motion which the soil would experience in the absence of any foundation or structure, since this is the simplest way to separate the characteristics of the earthquake from the characteristics of the structure. It should be noted that the effects of any local geological features are considered to be included already in the free-field motion. In some studies the free-field motion is itself considered to be the output of a linear system excited by white noise. This is done either by considering the bedrock motion to be white noise or by considering the free-field motion to be some more general filtered white noise. If one so desired, the schematic diagram in Fig. 1 could be extended to the left to present the dynamics problem as one with white noise as input, the structural response as output, and four intermediate stages of linear dynamics. This study, though, focuses on the soil-foundation-structure interaction aspects of the problem, rather than the local geological effects influencing the free-field motion.

The last block in Fig. 1 represents the only aspect of the total problem which is purely a structural engineering problem. This is the determination of the structural distortions based on known properties of the true foundation motion. This aspect of the problem is the same whether or not one considers interaction between the structure and the soil. In particular, ignoring soil-structure interaction consists of assuming that the true foundation motion is the same as if the

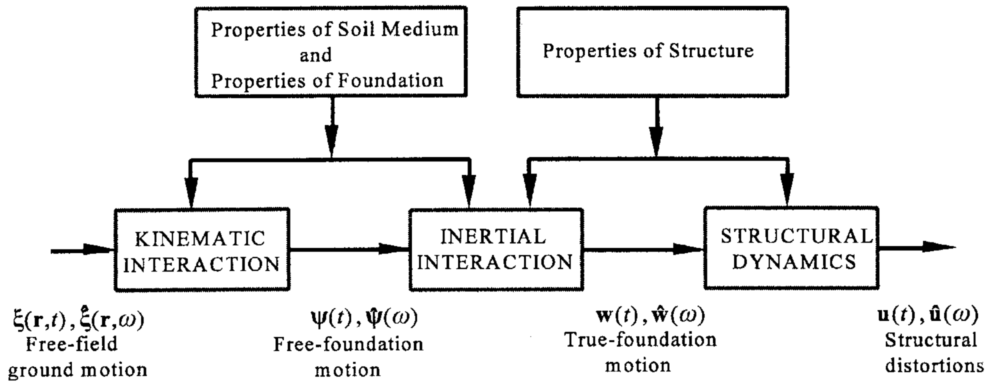

Fig. 1 Schematic diagram of soil-structure interaction

structure were absent. In fact, though, the true foundation motion differs from the free-field motion because of the effects represented by blocks to the left in Fig. 1. The block labeled "inertial interaction" represents the effect of forces that the superstructure applies to the foundation, so that the true foundation motion differs from the free-foundation motion, which is the hypothetical motion which would occur if the structure were replaced only by a massless foundation with the same geometry and stiffness. Finally, the free-foundation motion generally differs from the free-field motion due to "kinematic interaction". The only situation in which there is no kinematic interaction is when the free-field motion is identical at every point of attachment of the foundation to the soil.

It should be noted that the concept represented in Fig. 1 is the same whether the ground and foundation motions are considered to be accelerations or displacements. Similarly, one could study structural acceleration responses, rather than distortions, if desired. In a similar way, it makes no difference, in principle, whether any of the motions is represented in the time domain or the frequency domain. Thus, each of the motions in Fig. 1 is denoted both as a time history and as its Fourier transform, represented with a ^ over the appropriate symbol. Each of the motion quantities is shown in bold face, since each is generally a vector quantity. The use of the frequency domain is particularly convenient for linear problems, of course, since simple products in the frequency domain replace time domain convolution integrals.

For the sake of simplicity, the stochastic field $\xi(\mathbf{r}, t)$ and the stochastic processes $\boldsymbol{\psi}(t)$, $\mathbf{w}(t)$, and $\mathbf{u}(t)$ will be considered to be stationary. The spectral densities can then be taken as the Fourier transforms of the corresponding autocorrelation functions. For example, the spectral density matrix for the $\mathbf{u}(t)$ vector stochastic process is the Fourier transform of $E[\mathbf{u}(t+\tau)\mathbf{u}^T(t)]$, in which $E(\cdot)$ denotes mathematical expectation and the superscript T denotes transpose. One can also consider this to be a representation of an approximate relationship based on the Fourier transforms of the time histories. In particular, if $\hat{\mathbf{u}}(\omega)$ represents a truncated Fourier transform of $\mathbf{u}(t)$ based on an integration over an interval of length T, then

$$\mathbf{S}_{uu}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} E\,[\mathbf{u}(t+\tau)\mathbf{u}^T(t)]e^{-i\omega\tau}\,d\tau \approx \frac{2\pi}{T}\,E\,[\hat{\mathbf{u}}(\omega)\hat{\mathbf{u}}^{*T}(\omega)] \tag{1}$$

in which the superscript $*$ denotes the complex conjugate. Since numerical Fourier transforms are alway based on such truncated integrals, the final form in Eq. (1) is particularly useful in practics.

The definition of the spectral density matrix $\mathbf{S}_{\psi\psi}(\omega)$ for the free-foundation motion is exactly the same as Eq. (1), with only the symbols changed. For the free-field motion, though, one also must have a cross-spectral density matrix $\mathbf{S}_{\xi(\mathbf{r}_1)\xi(\mathbf{r}_2)}(\omega)$ in which the vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ denote different spatial locations. The form is still the same as in Eq. (1), but with $E[\mathbf{u}(t+\tau)\mathbf{u}^T(t)]$ replaced by $E[\boldsymbol{\xi}(\mathbf{r}_1, t+\tau)\boldsymbol{\xi}^T(\mathbf{r}_2, t)]$ and $E[\hat{\mathbf{u}}(\omega)\hat{\mathbf{u}}^{*T}(\omega)]$ replaced by $E[\hat{\boldsymbol{\xi}}(\mathbf{r}_1, \omega)\hat{\boldsymbol{\xi}}^{*T}(\mathbf{r}_2, \omega)]$.

It should be noted that the stationary spectral density results given here also provide other useful information, such as qualitative information for nonstationary problems in which the statistics of the excitation are not changing rapidly with time, and the mean squared value of any of the various processes. In addition, if the processes are Gaussian, then the spectral density contains all the necessary information regarding the first passage time or the statistics of the extreme response during a fixed interval of time.

The order of the presentation of the following dynamic analyses will be to work backward from the superstructure to the free-field motion. That is, we will first review the filter interpretation of the structural distortions resulting from known stochastic foundation motion, then we will show that the same type of model describes how the stochastic foundation motion has been affected by the mass of the structure and foundation. Finally we will look at the effect of the spatial variability of the free-field ground motion. One advantage of this approach is that it progresses from the simplest to the most complex aspect of the analysis. In addition, it parallels common structural engineering thinking, in which soil-structure interaction is usually ignored, and if considered at all is treated as a correction to the "standard" approach of considering only the superstructure to have dynamic behavior.

3. Dynamics of the superstructure

The relationship between structural distortion and true foundation motion is the same whether that foundation motion is obtained from a very elaborate or a very simplified procedure. This procedure will only be briefly reviewed here. For simplicity we will consider the foundation to be rigid, so that its true motion can be represented by a vector with only six components, representing translation in the direction of the (x, y, z) axes, and rotations about these axes:

$$\mathbf{w}(t) = [w_x(t), w_y(t), w_z(t), w_{\theta x}(t), w_{\theta y}(t), w_{\theta z}(t)]^T$$

For a flexible foundation it would be necessary to include more degrees of freedom in the $\mathbf{w}(t)$ vector, but the procedure would be the same. Let n be the number of degrees of freedom of the fixed-based structure and $\mathbf{X}(t)$ be an n-component vector sufficient to describe the displacement of every point of the structure away from its position of static equilibrium. There is then some rectangular matrix $\mathbf{B}$ such that the vector of pertinent distortions can be written as $\mathbf{u}(t)=\mathbf{B}\mathbf{X}(t)$.

For a linear structure it is always possible to find mass, damping, and stiffness matrices ($\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$, respectively) for the structure such that its dynamic behavior is described by $\mathbf{M}\ddot{\mathbf{X}}(t)+\mathbf{C}\dot{\mathbf{X}}(t)+\mathbf{K}\mathbf{X}(t)=\mathbf{G}\ddot{\mathbf{w}}(t)$. Taking the Fourier transform, then solving for $\hat{\mathbf{X}}(\omega)$ gives the relationship describing the motion of the superstructure in response to the true motion of the foundation. The Fourier transform of the distortion, $\hat{\mathbf{u}}(\omega)=\mathbf{B}\hat{\mathbf{X}}(\omega)$, can then be written as $\hat{\mathbf{u}}(\omega)=\mathbf{H}_d(\omega)\hat{\mathbf{w}}(\omega)$, which is precisely the standard form of the input-output relationship for a linear filter. The transfer function (or gain) $\mathbf{H}_d(\omega)$ of this filter for distortion is given by

$$\mathbf{H}_d(\omega) = \omega^2\mathbf{B}[\mathbf{K} + i\,\omega\,\mathbf{C} - \omega^2\mathbf{M}]^{-1}\mathbf{G} \tag{2}$$

Eq. (1) then shows that the spectral density matrix for the vector of structural distortions is

$$\mathbf{S}_{uu}(\omega) = \mathbf{H}_d(\omega)\mathbf{S}_{ww}(\omega)\mathbf{H}_d^{*T}(\omega) \tag{3}$$

Finding $\mathbf{H}_d(\omega)$ from Eq. (2) involves inverting a matrix of dimension $(n \times n)$, in which n is the number of degrees of freedom of the superstructure, and one usually needs to evaluate this inverse for many different frequencies. The calculation is efficiently performed by using a modal analysis based on the eigensolution for the fixed-base structure. The eigenanalysis, of course, must be done only once, not repeatedly for each of the many frequencies.

It should be noted that Eqs. (2) and (3), as presented, include more terms than are required when soil-structure interaction is ignored. In particular, without soil-structure interaction there is generally considered to be no rotational motion of the foundation, so that the final three components of $\mathbf{w}(t)$ could be omitted. The more general form presented here will be required, though, in the following sections. Representative plots of two components of building distortion are shown in Fig. 2 for a simple structural model. The structural model chosen is a shear-beam representation of a 21-story building with inextensible columns, for which the details are given in the Appendix. The symmetry of the model chosen uncouples the motion in the x and y directions, so that these can be handled with separate, simplified analyses. Furthermore, the vertical z motion of the foundation has no effect. Since Fig. 2 represents the results neglecting soil-structure interaction, the foundation motion has been taken to approximate a well-known earthquake record. In particular, the foundation has been given no rotation and the translation in the x direction has been chosen in the form of a Kanai-Tajimi approximation of the N-S motion of the 1940 El Centro record. The details of the spectral density are given in the Appendix.

The results in Fig. 2 are typical of the spectral density of response for a lightly damped structure subjected to a broadband excitation. Each of the sharp peaks in the curves corresponds to a resonant frequency of the fixed-base structure. Only the first few of the 21 resonant frequencies can be seen in the plots, since the seismic ground motion provides relatively little excitation of the higher modes of the structure.

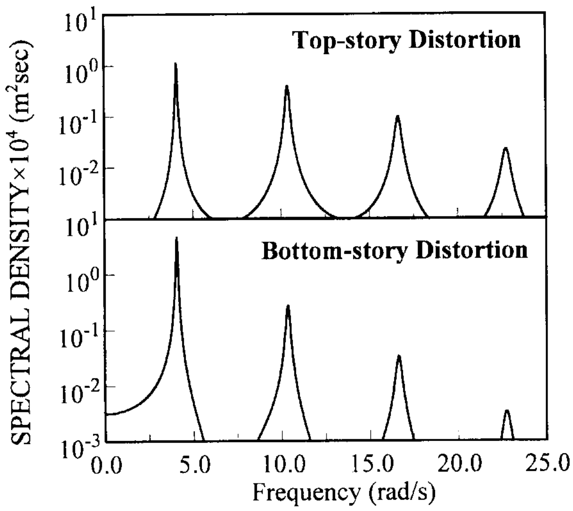

Fig. 2 Spectral density of the structural distortion neglecting soil-structure interaction

4. Inertial interaction

The problem in inertial interaction is to describe the true foundation motion as a function of the free-foundation motion which would occur with a massless foundation with no superstructure. It will be seen that inertial interaction constitutes a filtering effect quite similar to that observed in the preceding section, but determination of the transfer function is not as simple. Fig. 3 illustrates the dynamic situation being investigated. Letting $\mathbf{F}_f(\omega)$ and $\mathbf{F}_s(\omega)$, respectively, denote loading vectors of forces and moments applied to the foundation and the base of the structure, we presume for the moment that we know the transfer matrices $\mathbf{H}_f(\omega)$ for the foundation-soil combination and $\mathbf{H}_s(\omega)$ for the structure, so that the motions in the absence of any seismic excitation would be $\hat{\mathbf{w}}_0(\omega)=\mathbf{H}_f(\omega)\mathbf{F}_f(\omega)$ and $\hat{\mathbf{w}}(\omega)=\mathbf{H}_s(\omega)\mathbf{F}_s(\omega)$. From equilibrium it is known that $\mathbf{F}_f(\omega)=-\mathbf{F}_s(\omega)$, and superposition shows that $\hat{\mathbf{w}}(\omega)=\hat{\mathbf{w}}_0(\omega)+\hat{\boldsymbol{\psi}}(\omega)$ is the motion of the foundation in the presence of both forces from the structure and free-foundation motion $\hat{\boldsymbol{\psi}}(\omega)$. These equations can be combined to eliminate the $\mathbf{F}_s(\omega)$ and $\hat{\mathbf{w}}_0(\omega)$ terms. For example, $\mathbf{F}_s(\omega)=\mathbf{H}_s^{-1}(\omega)\hat{\mathbf{w}}(\omega)$, so that $\hat{\mathbf{w}}_0(\omega)=-\mathbf{H}_f(\omega)\mathbf{H}_s^{-1}(\omega)\hat{\mathbf{w}}(\omega)$, and putting this into the superposition equation gives a result of $\hat{\boldsymbol{\psi}}(\omega)=[\mathbf{H}_s(\omega)+\mathbf{H}_f(\omega)]\mathbf{H}_s^{-1}(\omega)\hat{\mathbf{w}}(\omega)$. The inverse of this relationship then gives an appropriate form relating the output $\hat{\mathbf{w}}(\omega)$ to the input $\hat{\boldsymbol{\psi}}(\omega)$ for the inertial interaction process so that we have $\hat{\mathbf{w}}(\omega)=\mathbf{H}_{II}(\omega)\hat{\boldsymbol{\psi}}(\omega)$ with

$$\mathrm{H}_{II}(\omega) = \mathrm{H}_s(\omega)[\mathrm{H}_s(\omega) + \mathrm{H}_f(\omega)]^{-1} \tag{4}$$

and

$$\mathrm{S}_{\mathrm{ww}}(\omega) = \mathrm{H}_{II}(\omega)\mathrm{S}_{\psi\psi}(\omega)\mathrm{H}_{II}^{*T}(\omega) \tag{5}$$

If one knows the $\mathbf{H}_s(\omega)$ and $\mathbf{H}_f(\omega)$ matrices, then it is relatively simple to evaluate the $\mathbf{H}_{II}(\omega)$ matrix describing inertial interaction. For a rigid foundation, the calculation only involves numerically inverting a matrix with dimension no greater than (6×6), followed by one matrix multiplication. In many situation the calculation is even simpler because of the inertial interaction equations reducing into several uncoupled sets. The greatest simplification results when the soil medium, the foundation, and the structure are all symmetric with respect to both the x and y axes.

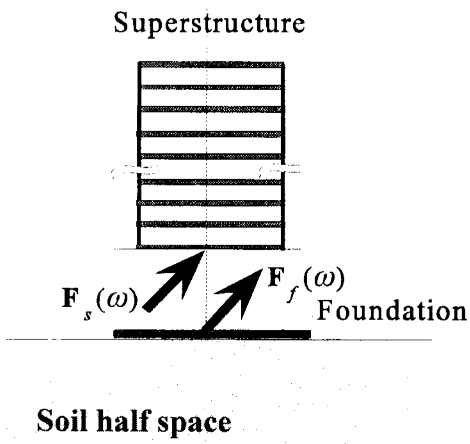

Fig. 3 Dynamic interaction between the superstructure and the soil-foundation system

In this situation a force in the z direction causes only displacements in the z direction, so that this inertial interaction is uncoupled from the other five components of foundation motion. Similarly the θ_z torsional rotation is also uncoupled from the other components of motion. There may be coupling in pairs within the remaining four components. In particular, the x and θ_y components of inertial interaction may be coupled, and the y and θ_x components may also be coupled. Thus, in practice, one may need to invert no matrix larger than (2×2) in finding $\mathbf{H}_{ll}(\omega)$ from Eq. (4).

Evaluating the $\mathbf{H}_s(\omega)$ and $\mathbf{H}_f(\omega)$ matrices is generally much more significant than the calculation in Eq. (4). Finding the $\mathbf{H}_f(\omega)$ matrix for the soil-foundation system from the mixed boundary value equations of elasticity for the soil medium is a particularly complex mathematical task. This problem has been extensively investigated numerically over the past decades, using uniform elastic, layered elastic, and viscoelastic models for the soil (e.g., Veletsos and Wei 1971, Luco and Mita 1987, Luco and Wong 1987, Wolf 1985). Based on these numerical results, one may able to interpolate or extrapolate to obtain the elements of the $\mathbf{H}_f(\omega)$ matrix for a particular soil-structure interaction problem. There is at least one significant hazard in doing this strictly as a curve fitting problem, though. In particular, inconsistencies in the interpolation or extrapolation can give a noncausal model for the soil dynamics. That is, if one obtains impulse response functions for the soil-foundation system by taking inverse Fourier transforms of elements of $\mathbf{H}_f(\omega)$, these impulse response functions may not be zero for negative times, implying that the response at time t depends on excitations after time t as well as on excitations prior to time t. This noncausal model will result if the interpolated $\mathbf{H}_f(\omega)$ happens to have poles with negative imaginary parts, within the space of complex ω values. The systems for which the numerical results are available, of course, are all causal, but errors in curve fitting approaches can lead to such noncausal approximations.

One way in which the issue of noncausality can be avoided is to fit the numerical $\mathbf{H}_f(\omega)$ results with similar curves for systems made up of a finite number of masses, springs, and dashpots. The application to other situations can then be done by interpolating the parameters of these "physical" models, which are of necessity causal models. This idea has been used very effectively in the past (Richart *et al.* 1970, Veletsos and Wei 1971, Wolf and Somaini 1986) and has been particularly explored by Wolf (1994).

Finding the $\mathbf{H}_s(\omega)$ matrix to describe the structure is generally a simpler problem, but may also involve significant computation. To illustrate this idea, let $\mathbf{X}_e(t)$ be an extended vector of dimension $(n+6)$ describing the displacement away from the position of static equilibrium of every coordinate of both the rigid foundation and the structure. One can now obtain extended mass, damping, and stiffness matrices for the structure-foundation system such that its dynamics are described by $\mathbf{M}_e\ddot{\mathbf{X}}_e(t)+\mathbf{C}_e\dot{\mathbf{X}}_e(t)+\mathbf{K}_e\mathbf{X}_e(t)=\mathbf{F}_e(t)$. Taking the Fourier transform of this equation, then solving for $\hat{\mathbf{X}}_e(\omega)$ gives $\hat{\mathbf{X}}_e(\omega)=[\mathbf{K}_e+i\omega\mathbf{C}_e-\omega^2\mathbf{M}_e]^{-1}\hat{\mathbf{F}}(\omega)$ as the relationship describing the response. For the problem of seismic response with $\mathbf{X}_e(t)=[\mathbf{X}^T(t), \mathbf{w}^T(t)]^T$, the force vector is $\mathbf{F}(t)=[\mathbf{0}^T, \mathbf{F}_s^T(t)]^T$, in which $\mathbf{F}_s(t)$ is the vector of forces and moments applied to the structure from the foundation. Thus, the $\mathbf{H}_s(\omega)$ matrix is simply the lower right (6×6) submatrix of $[\mathbf{K}_e+i\omega\mathbf{C}_e-\omega^2\mathbf{M}_e]^{-1}$.

It may be seen that finding the (6×6) $\mathbf{H}_s(\omega)$ matrix involves inverting a matrix of dimension $[(6+n), (6+n)]$, in which n is the number of degrees of freedom of the superstrucure, and 6 is the dimension of $\mathbf{w}(t)$. As in the preceding section, this calculation is performed efficiently by using a modal analysis for the structure-foundation system. This can be done by using the natural modes of either a free-base structure described by $\mathbf{X}_e(t)$ or the fixed-base structure described by $\mathbf{X}(t)$, although the former calculation is slightly simpler than the second (Lutes and Sarkani 1995). For either set of modes, the calculation will be easiest if the damping matrix has the classical form which leads to uncoupled real modes. It is common to choose the damping such that a structure does have such uncoupled modes in the fixed-base configuration, and this particular model will not give uncoupled free-base modes. Thus, it is usually desirable to use the fixed-base modes, as done by Wu and Smith (1995). If the structure does not have uncoupled modes in either configuration, it is still possible to perform an uncoupling operation, but it requires using the eigenvalues of a matrix of twice the usual dimension (Lutes and Sarkani 1995, 1997).

Fig. 4 shows examples of the inertial interaction transfer function obtained for the example structure attached to a rigid slab foundation resting on a soft soil. In addition to showing values for some elements of $\mathbf{H}_{II}(\omega)$, the figure includes information about the $\mathbf{H}_s(\omega)$ and $\mathbf{H}_f(\omega)$ matrices used in computing $\mathbf{H}_{II}(\omega)$. The foundation was modeled using a set of masses, dashpots and springs as suggested by Wolf and Somaini (1986). The details of the models are given in the Appendix. The results shown are for the (x, θ_y) motion of a symmetric model, for which these components are coupled with each other, but uncoupled from the other motions. Thus, the $\mathbf{H}_{II}(\omega)$ matrix can be considered to be of dimension (2×2). For convenience, both the input and output of $\mathbf{H}_{II}(\omega)$ have been taken as the (x, b_x, θ_y) pair, in which b_x denotes the x distance from the centroidal axis to the edge of foundation. Thus, the magnitude of each rocking term is being characterized by the magnitude of the corresponding $b_x\theta_y$ induced edge motion in the z direction. In this form the $\mathbf{H}_{II}(\omega)$ transfer function matrix is dimensionless.

Each peak of the $\mathbf{H}_{II}(\omega)$ transfer functions can be considered to represent a resonant frequency

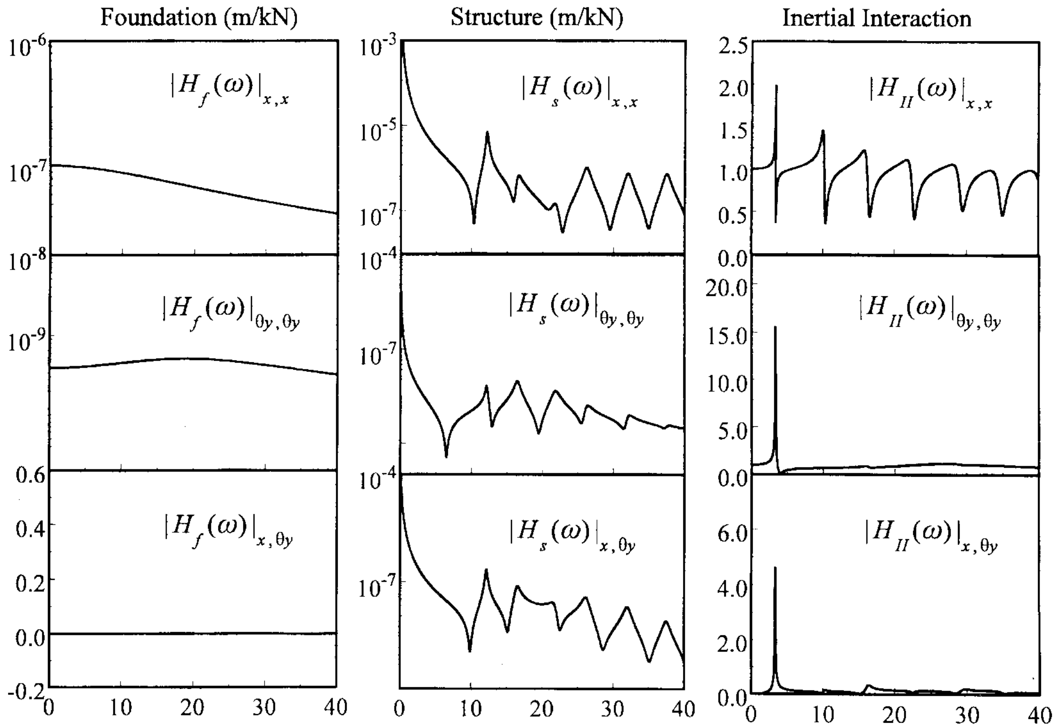

Fig. 4 Transfer functions for inertial interaction analysis (Square foundation, dimension=31.0 m, G=115 Mpa, v_s=244 m/s)

of the combined soil-foundation-structure system. It may be noted that these are not the same frequencies as the peaks in the structural distortion spectral density of Fig. 2, due to peaks of $\mathbf{H}_d(\omega)$. Also, they are not the same frequencies as the peaks in the $\mathbf{H}_s(\omega)$ transfer function in Fig. 4. Each of these three sets of peaks, though, can be considered to correspond to a set of resonant frequencies of the structure. The difference, of course, is in the support conditions at the base of the structure. The peaks of $\mathbf{H}_d(\omega)$ correspond to the fixed-base resonances, those of $\mathbf{H}_s(\omega)$ correspond to the free-base resonances, and those of $\mathbf{H}_{II}(\omega)$ correspond to the resonances of the combined soil-foundation-structure system. Caution is appropriate in attempting to find any direct relationship between the frequencies of "corresponding" modes in the three configurations. The number of degrees of freedom also depends on the base support condition, so it is difficult to identify corresponding modes in the different situations.

One can combine the results for structural distortion and inertial interaction to obtain the spectral density of the structural response for a model which ignores only the kinematic interaction. That is, Eqs. (3) and (5) give $\mathbf{S}_{uu}(\omega)=\mathbf{H}_d(\omega)\mathbf{H}_{II}(\omega)\mathbf{S}_{\psi\psi}(\omega)\mathbf{H}_{II}^{*T}(\omega)\mathbf{H}_d^{*T}(\omega)$ and ignoring kinematic interaction is the same as assuming that $\mathbf{S}_{\psi\psi}(\omega)$ is the spectral density of the free-field motion. Fig. 5 shows sample results based on this analysis for the example problem. The response components shown are the same as in Fig. 2, so that comparing the two figures directly shows the effect of inertial interaction. As in Fig. 2, only translational foundation motion is considered as the input, since the free-foundation motion would not include rotational components in the absence of kinematic interaction. This does not imply, of course, that the true foundation motion lacks rotational components when inertial interaction is included.

Comparing Figs. 2 and 5 shows that inertial interaction shifts the first, and largest, peak of the spectral density to a significantly lower frequency, as would be expected for a more compliant system. There is very little change, though, in the magnitude of this dominant component of the response. In addition, it is seen that inertial interaction somewhat reduces the magnitudes of the less important, higher-frequency, components of structural response. These changes in the modal response magnitudes may be interpreted in terms of a balance between two counteracting effects. The increased compliance due to inertial interaction tends to increase the response levels, while energy loss into the soil provides a damping which tends to reduce the structural response. For the

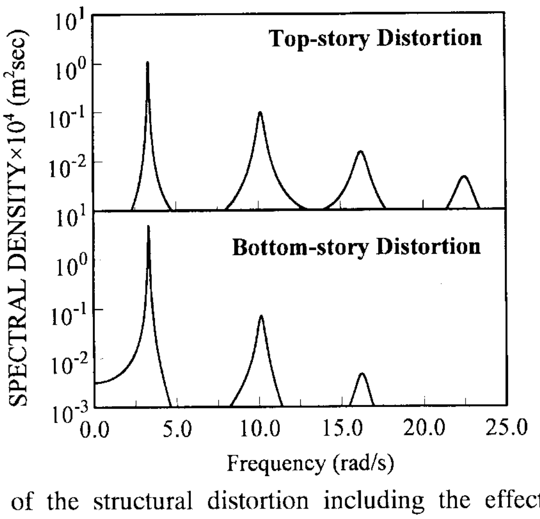

Fig. 5 Spectral density of the structural distortion including the effect of inertial interaction

fundamental mode the two effects seem to cancel each other, while for the higher-frequency modes the damping effect dominates.

5. Kinematic interaction

The output of the kinematic interaction box in Fig. 1 is the vector $\boldsymbol{\psi}(t)$ describing the free-foundation motion, and the input is the stochastic field $\boldsymbol{\xi}(\mathbf{r}, t)$ describing the free-field motion at every point on the interface between the soil and the foundation. That is, $\boldsymbol{\xi}(\mathbf{r}, t)$ is a vector stochastic process for each choice of the location vector $\mathbf{r}$. We will limit attention to the situation with a rigid foundation, so that the vector $\boldsymbol{\psi}(t)$ has no more than six components. Furthermore, the free-field motion at any location $\mathbf{r}$ has a dimension of only three, since it is motion at a point: $\boldsymbol{\xi}(\mathbf{r}, t)=[\xi_x(\mathbf{r}, t), \xi_y(\mathbf{r}, t), \xi_z(\mathbf{r}, t)]^T$. The complication in describing kinematic interaction is that the output depends on the free-field motion at all locations $\mathbf{r}$ where the soil is in contact with the foundation.

One precise formulation of the kinematic interaction relationship for a rigid foundation can be derived using a form of Betti's law (Luco and Mita 1987) as

$$\hat{\boldsymbol{\psi}}(\omega) = [\int_{S_f} \mathbf{T}^T(\omega, \mathbf{r})\mathbf{Q}(\mathbf{r})\, dA(\mathbf{r})]^{-1} \int_{S_f} \mathbf{T}^T(\omega, \mathbf{r})\hat{\boldsymbol{\xi}}(\omega, \mathbf{r})\, dA(\mathbf{r}) \tag{6}$$

in which $dA(\mathbf{r})$ represents an increment of area and S_f denotes the surface of contact between the foundation and the soil. The matrices $\mathbf{T}(\omega, \mathbf{r})$ and $\mathbf{Q}(\mathbf{r})$ and both of dimension (3×6) and are defined such that $\mathbf{T}(\omega, \mathbf{r})\hat{\boldsymbol{\psi}}(\omega)$ and $\mathbf{Q}(\mathbf{r})\hat{\boldsymbol{\psi}}(\omega)$, respectively, give the three components of traction and the three components of motion induced at location $\mathbf{r}$ due to a rigid body motion $\hat{\boldsymbol{\psi}}(\omega)$. The $\mathbf{Q}(\mathbf{r})$ matrix depends only on the geometry of the interface S_f, but the $\mathbf{T}(\omega, \mathbf{r})$ matrix can be a quite complicated function of the dynamic properties of the soil medium.

The problem is greatly simplified when one uses an approximation originally introduced by Scanlan (1976) in which each term of the $\mathbf{T}(\omega, \mathbf{r})$ matrix is considered either to be constant or to vary linearly with the components of $\mathbf{r}$. This approximation has frequently been adopted (e.g., Veletsos and Wei 1971, Luco and Wong 1987) and extensive numerical investigations of specific problems have shown that it generally gives quite accurate results (Luco and Mita 1987). In this approximation, one can take $\mathbf{T}(\omega, \mathbf{r})=\mathbf{Q}(\mathbf{r})\mathbf{k}(\omega)$ in which $\mathbf{k}(\omega)$ is a diagonal matrix representing the "stiffness" of the foundation. The form of $\mathbf{k}(\omega)$ is not important for the kinematic interaction problem, since the term cancels out of Eq. (6).

To illustrate the effect of kinematic interaction, we will consider the case of a rigid foundation resting on the surface of the soil. For this situation with $\mathbf{r}=[x, y]^T$ the $\mathbf{Q}(\mathbf{r})$ matrix is given by

$$\mathbf{Q}(\mathbf{r}) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -y \\ 0 & 1 & 0 & 0 & 0 & x \\ 0 & 0 & 1 & y & -x & 0 \end{bmatrix} \tag{7}$$

If the origin of the axis system (i.e., $\mathbf{r}=\mathbf{0}$) is taken as the centroid of the S_f contact area, and the foundation is symmetric with respect to the x and y axes, then considerable simplification takes place and Eq. (6) can be rewritten as an integral of $\boldsymbol{\Gamma}(\mathbf{r})\hat{\boldsymbol{\xi}}(\omega, \mathbf{r})$, in which

$$\boldsymbol{\Gamma}(\mathbf{r}) = \begin{bmatrix} 1/A & 0 & 0 & 0 & 0 & -y/I_z \\ 0 & 1/A & 0 & 0 & 0 & x/I_z \\ 0 & 0 & 1/A & y/I_x & -x/I_y & 0 \end{bmatrix}^T \tag{8}$$

with A being the area of S_f, and I_x, I_y, and I_z being the moments of inertia of S_f about the three axes. The spectral density of the free-foundation motion is then given by

$$S_{\psi\psi}(\omega) = \int_{S_f}\int_{S_f} \Gamma(\mathbf{r}_1)\mathbf{S}_{\xi(\mathbf{r}_1)\xi(\mathbf{r}_2)}(\omega)\Gamma^T(\mathbf{r}_2)\, dA\,(\mathbf{r}_1)\, dA\,(\mathbf{r}_2) \tag{9}$$

Note that this approximate formulation gives the effect of kinematic interaction as depending only on the geometry of the S_f interface between the soil and the foundation. In particular, this relationship does not depend on the properties such as stiffness or flexibility of the soil medium. Thus, this approximation of kinematic interaction is simpler than the more general situation illustrated in the schematic of Fig. 1. The properties of the soil medium now are used only in computing the inertial interaction, and they are not needed in analyzing kinematic interaction.

In many situations the free-field ground motion is considered to be homogeneous, such that the spectral density of $\boldsymbol{\xi}(\mathbf{r}, t)$ is the same for all $\mathbf{r}$ values. That is, the spectral density matrix can be written as $\mathbf{S}_{\xi(\mathbf{r})\xi(\mathbf{r})}(\omega)=\mathbf{S}^0(\omega)$ for all $\mathbf{r}$. In this situation it is reasonable to seek a relationship between the $\mathbf{S}_{\psi\psi}(\omega)$ spectral density matrix describing the output from kinematic interaction, and the $\mathbf{S}^0(\omega)$ matrix describing the input free-field motion. The relationship in Eq. (9), though, also depends on the cross-spectral density between the motion at the two locations $\mathbf{r}_1$ and $\mathbf{r}_2$. Thus, finding a relationship between the $\mathbf{S}_{\psi\psi}(\omega)$ and $\mathbf{S}^0(\omega)$ matrices requires an initial step of relating the cross-spectral density of the free-field motion to its autospectral density matrix $\mathbf{S}^0(\omega)$. One quite general form for this relationship can be written as

$$S_{\xi(\mathbf{r}_1)\xi(\mathbf{r}_2)}\,(\omega) = \mathbf{R}\,(\mathbf{r}_1 - \mathbf{r}_2,\,\omega)\;\mathbf{S}^0(\omega)\mathbf{R}^{*T}\,(\mathbf{r}_2 - \mathbf{r}_1,\,\omega) \tag{10}$$

in which $\mathbf{R}(\mathbf{r}_1 - \mathbf{r}_2,\ \omega)$ is any (3×3) complex matrix function of two arguments which tends to the identity matrix when $\mathbf{r}_1=\mathbf{r}_2$ for any ω value. The particular form of the matrix product in Eq. (10) has been chosen to ensure that $\mathbf{S}_{\xi(\mathbf{r}_2)\xi(\mathbf{r}_1)}(\omega)=\mathbf{S}^{*T}_{\xi(\mathbf{r}_2)\xi(\mathbf{r}_1)}(\omega)$, as required by Eq. (1).

Substituting Eq. (10) into Eq. (9) gives the spectral density for the free-foundation motion as

$$\mathbf{S}_{\psi\psi}(\omega) = \int_{S_f}\int_{S_f} \Gamma(\mathbf{r}_1)\mathbf{R}(\mathbf{r}_1 - \mathbf{r}_2,\,\omega)\mathbf{S}^0(\omega)\mathbf{R}^{*T}(\mathbf{r}_2 - \mathbf{r}_1,\,\omega)\;\Gamma^T(\mathbf{r}_2)\,dA\,(\mathbf{r}_1)\,dA\,(\mathbf{r}_2) \tag{11}$$

This, of course, gives a linear relationship between each component of $\mathbf{S}_{\psi\psi}(\omega)$ and the various components of the $\mathbf{S}^0(\omega)$ matrix. The matrix multiplications in this formula seem to imply that each of the 36 components of $\mathbf{S}_{\psi\psi}(\omega)$ is the sum of 81 terms, but this is not the case, because of the sparseness of the $\boldsymbol{\Gamma}(\mathbf{r})$ matrix. It is generally true that each of the components of $\mathbf{S}_{\psi\psi}(\omega)$ may depend on every one of the nine components of $\mathbf{S}^0(\omega)$, but this situation may also be simplified if the $\mathbf{R}(\mathbf{r},\ \omega)$ matrix is chosen to have some zero terms. It should also be noted that both $\mathbf{S}_{\psi\psi}(\omega)$ and $\mathbf{S}^0(\omega)$ are Hermitian (i.e., the matrix is equal to the transpose of its complex conjugate).

The linear relationship between the $\mathbf{S}_{\psi\psi}(\omega)$ and $\mathbf{S}^0(\omega)$ matrices, as given in Eq. (11), surely resembles the nature of a linear filter. By proper definition of a term $U_{jkl_2l_3}(\omega)$, the general component relationship for Eq. (11) can be rewritten as $S_{\psi_j\psi_k}(\omega)=\sum_{l_2}\sum_{l_3} S^0_{l_2l_3}(\omega)U_{jkl_2l_3}(\omega)$, which is very similar to the expression for a linear filter. In particular, if there were a linear filter with transfer function $\mathbf{H}(\omega)$ such that $\hat{\boldsymbol{\psi}}(\omega)=\mathbf{H}(\omega)\hat{\boldsymbol{\xi}}^0(\omega)$, with $\mathbf{S}^0(\omega)$ being the spectral density matrix for the vector $\hat{\boldsymbol{\xi}}^0(t)$, then the spectral density relationship for the free-foundation motion would be identical to that from Eq. (11) except that $U_{jkl_2l_3}(\omega)$ would be replaced by $H_{jl_2}(\omega)H^*_{kl_3}(\omega)$. In general, though, it is not possible to find a transfer function $\mathbf{H}(\omega)$ which gives such equivalence. The difference between Eq. (11) and a linear filter will be explored in more detail in the following section for an important special case of kinematic interaction.

6. Kinematic interaction for a simplified situation

In one sense the cross-spectral density model of Eq. (10) may be too general. In particular, the authors are unaware of any studies in which a model has been developed for the cross-correlation of each component of free-field motion at one location and each component of free-field motion at another location. That is, it is generally not clear how to choose the $\mathbf{R}(\mathbf{r}_1 - \mathbf{r}_2, \omega)$ matrix in Eq. (10). On the other hand, models have been suggested for the cross-correlation between one component of motion at one location and the corresponding component at another location. People have then used this idea in developing simplified models for the cross-spectral density. For one of the simpler models which has been used, the cross-spectral density between motions at two points is written as $\mathbf{S}^0(\omega)$ multiplied by a scalar function:

$$\mathbf{S}_{\xi(\mathbf{r}_1)\xi(\mathbf{r}_2)}(\omega) = g(\mathbf{r}_1 - \mathbf{r}_2, \omega)\mathbf{S}^0(\omega) \tag{12}$$

with the necessary limitation that $g(\mathbf{r}_2 - \mathbf{r}_1, \omega) = g^*(\mathbf{r}_1 - \mathbf{r}_2, \omega)$. This scalar function is then separated into two terms, one of which represents a phase variation due to wave propagation across the site, and the other of which is an empirical representation of the incoherence which is found between the motion at different locations. Probably the most common form of this model is

$$g(\mathbf{r}_1 - \mathbf{r}_2, \omega) = \exp\left[\frac{i\beta\omega|\mathbf{r}_1 - \mathbf{r}_2|}{v_s}\right]\exp\left[-\left(\frac{\gamma\omega|\mathbf{r}_1 - \mathbf{r}_2|}{v_s}\right)^{\mu}\right] \tag{13}$$

in which v_s is the velocity of propagation of shear waves through the earth and β is the cosine of the angle between the presumed direction of wave propagation (in three-dimensional space) and the vector $(\mathbf{r}_1 - \mathbf{r}_2)$. Models of this type have been used by a number of previous investigators. For example, Luco and Wong (1987), Luco and Mita (1987), and Veletsos and Prasad (1989) have used precisely this equation with $\mu=2$, and Kameda and Morikawa (1994) and Hoshiya (1995) have used it with $\mu=1$. Also, a similar model was used by Harichandran and Vanmarcke (1986), but with a different scalar incoherence function.

The limitations of the cross-spectral density given by Eqs. (12) and (13) should be noted (Jin 1995, Jin *et al.* 1997, Lutes *et al.* 1995). For example, the expression gives the phase difference between motion at different points as being due only to the propagation of one plane wave. Furthermore, this wave is limited to propagation at the velocity of a shear wave. The fact that the incoherence term is real means that no phase difference is introduced by that term. In addition to these limitations on the phase change, there is also another major limitation which follows directly from the fact that $\mathbf{S}_{\xi(\mathbf{r}_1)\xi(\mathbf{r}_2)}(\omega)$ is taken as $\mathbf{S}^0(\omega)$ multiplied by a scalar function. In fact, Eq. (12) is the special case of Eq. (10) with $\mathbf{R}(\mathbf{r}_1 - \mathbf{r}_2, \omega) = [g(\mathbf{r}_1 - \mathbf{r}_2, \omega)]^{1/2}\mathbf{I}$ so that the effect of location on cross-spectral density is the same for every term of $\mathbf{S}_{\xi(\mathbf{r}_1)\xi(\mathbf{r}_2)}(\omega)$. Among the implications of this form are the fact that the cross-spectral component relating $\xi_j(\mathbf{r}_1)$ and $\xi_k(\mathbf{r}_2)$ for any choice of (j, k) depends only on the (j, k) component of the $\mathbf{S}^0(\omega)$ matrix, whereas Eq. (11) shows that it is more generally possible for this cross-spectrum to involve all nine terms of $\mathbf{S}^0(\omega)$.

Substituting $\mathbf{R}(\mathbf{r}_1-\mathbf{r}_2,\omega)=[g(\mathbf{r}_1-\mathbf{r}_2,\omega)]^{1/2}\,\mathbf{I}$ into Eq. (11) gives the component form of

$$S_{\psi_j\psi_k}(\omega)=\sum_{l_2}\sum_{l_3}S^0_{l_2l_3}(\omega)\int_{S_f}\int_{S_f}\Gamma_{jl_2}(\mathbf{r}_1)g(\mathbf{r}_1-\mathbf{r}_2,\omega)\Gamma_{kl_3}(\mathbf{r}_2)\,dA(\mathbf{r}_1)\,dA(\mathbf{r}_2) \tag{14}$$

and the sparseness of the $\Gamma(\mathbf{r})$ matrix in Eq. (8) leads to considerable additional simplification. In addition to the fact that many of the terms in the (l_2, l_3) summation are zero, it is seen that there are only a very limited number of definite integrals which must be evaluated in order to find the complete $\mathbf{S}_{\psi\psi}(\omega)$ matrix. In particular, the nine components of $\mathbf{S}_{\psi\psi}(\omega)$ which relate only to translational motion of the foundation depend only on the double integral of $g(\mathbf{r}_1-\mathbf{r}_2,\omega)$ over the interface, the eighteen cross-spectra between translation and rotation depend on the double integral of $g(\mathbf{r}_1-\mathbf{r}_2,\omega)$ multiplied by either an x or y component of either $\mathbf{r}_1$ or $\mathbf{r}_2$ and the nine components related only to rotations depend on the double integrals of $g(\mathbf{r}_1-\mathbf{r}_2,\omega)$ multiplied by second-order terms like x_1x_2, x_1y_2, y_1x_2, and y_1y_2. In general these integrals must be evaluated numerically, but analytical solutions have been found for several special cases described by Eq. (13) with either $\gamma=0$ or $\beta=0$ and with S_f being either a circle or a rectangle (Veletsos and Prasad 1989, Scanlan 1976, Chan 1994).

The simplicity, and the meaning, of Eq. (14) become more clear when some of the terms are written out explicitly. In particular, Eqs. (8), (12) and (14) give the components of $\mathbf{S}_{\psi\psi}(\omega)$ involving only x and θ_y motion as

$$S_{\psi_x\psi_x}(\omega)=\frac{S^0_{xx}(\omega)}{A^2}\int_{S_f}\int_{S_f}g(\mathbf{r}_1-\mathbf{r}_2,\omega)\,dA(\mathbf{r}_1)\,dA(\mathbf{r}_2) \tag{15}$$

$$S_{\psi_{\theta_y}\psi_{\theta_y}}(\omega)=\frac{S^0_{zz}(\omega)}{I_y^2}\int_{S_f}\int_{S_f}x_1x_2g(\mathbf{r}_1-\mathbf{r}_2,\omega)\,dA(\mathbf{r}_1)\,dA(\mathbf{r}_2) \tag{16}$$

$$S_{\psi_x\psi_{\theta_y}}(\omega)=\frac{S^0_{xz}(\omega)}{AI_y}\int_{S_f}\int_{S_f}x_2g(\mathbf{r}_1-\mathbf{r}_2,\omega)\,dA(\mathbf{r}_1)\,dA(\mathbf{r}_2) \tag{17}$$

These expressions also provide a simple demonstration that $\mathbf{S}_{\psi\psi}(\omega)$ is not the same as $\mathbf{S}_{\hat{Z}\hat{Z}}(\omega)$ for any vector described by $\hat{\mathbf{Z}}(\omega)=\mathbf{H}(\omega)\boldsymbol{\xi}^0(\omega)$. In particular, $\mathbf{S}_{\hat{Z}\hat{Z}}(\omega)=\mathbf{H}(\omega)\mathbf{S}^0(\omega)\mathbf{H}^{*T}(\omega)$ gives

$$S_{Z_jZ_k}(\omega)=\sum_{l_2}\sum_{l_3}S^0_{l_2l_3}(\omega)H_{jl_2}(\omega)H^*_{kl_3}(\omega) \tag{18}$$

One can make Eqs. (15) and (16) agree with Eq. (18) by choosing $H_{xl}(\omega)=0$ for $l\neq x$, $H_{\theta_yl}(\omega)=0$ for $l\neq z$,

$$|H_{xx}(\omega)|^2=A^{-2}\int_{S_f}\int_{S_f}g(\mathbf{r}_1-\mathbf{r}_2,\omega)\,dA(\mathbf{r}_1)\,dA(\mathbf{r}_2) \tag{19}$$

and

$$|H_{\theta_yz}(\omega)|^2=I_y^{-2}\int_{S_f}\int_{S_f}x_1x_2g(\mathbf{r}_1-\mathbf{r}_2,\omega)\,dA(\mathbf{r}_1)\,dA(\mathbf{r}_2) \tag{20}$$

However, using these choices gives $S_{Z_xZ_{\theta_y}}(\omega)=S^0_{xz}(\omega)H_{xx}(\omega)H^*_{\theta_yz}(\omega)$ and this is generally incompatible with Eq. (17). This illustrates explicitly that the kinematic interaction model is not the same as an ordinary linear filter. Nonetheless, Eq. (14) does provide a quite simple method for evaluating the spectral densities of the free-foundation motion for the special case of a scalar cross-spectral relationship, and Eq. (11) provides a feasible computation method for more general cross-spectra.

Fig. 6 shows results obtained from Eqs. (15)-(17) for the x and θ_y motion of the example foundation, using the simplified model of Eqs. (12) and (13). The β and γ parameters have been chosen to represent situations with incoherence of the ground motion as well as wave propagation across the site. Even though this is not a simple filtering situation, each of the plots has been put in a form similar to the obsolute value of a transfer function. In particular, the results for x and θ_y motion correspond to the square roots of Eqs. (19) and (20), respectively. For the special case of β =0, the numerical results for the $S_{\psi_x\psi_{\theta_y}}(\omega)/S^0_{xz}(\omega)$ term from Eq. (17) are identically zero, due to symmetry. This further confirms that this term cannot be written as $H_{xx}(\omega)H^*_{\theta_y z}(\omega)$.

The results in Fig. 6 show that the free-foundation translation in the x direction resembles a low-pass filtered version of the free-field motion in that same direction. The rocking component about the y axis, on the other hand, is seen to resemble a band-pass filtered version of the free-field motion in the z direction. That is, the rocking contribution is small at both low and high frequencies, but it can be substantial at intermediate frequencies. Interestingly, the magnitude of this "transfer function" is sometimes seen to exceed unity, so that the vertical free-foundation motion at the edge of the foundation, $b_x\hat{\psi}_{\theta_y}(\omega)$, can be greater than the free-field motion in the same direction. Since earthquakes may include quite substantial vertical motions, this implies that the free-foundation motion may include correspondingly substantial rocking components, but only at intermediate frequencies.

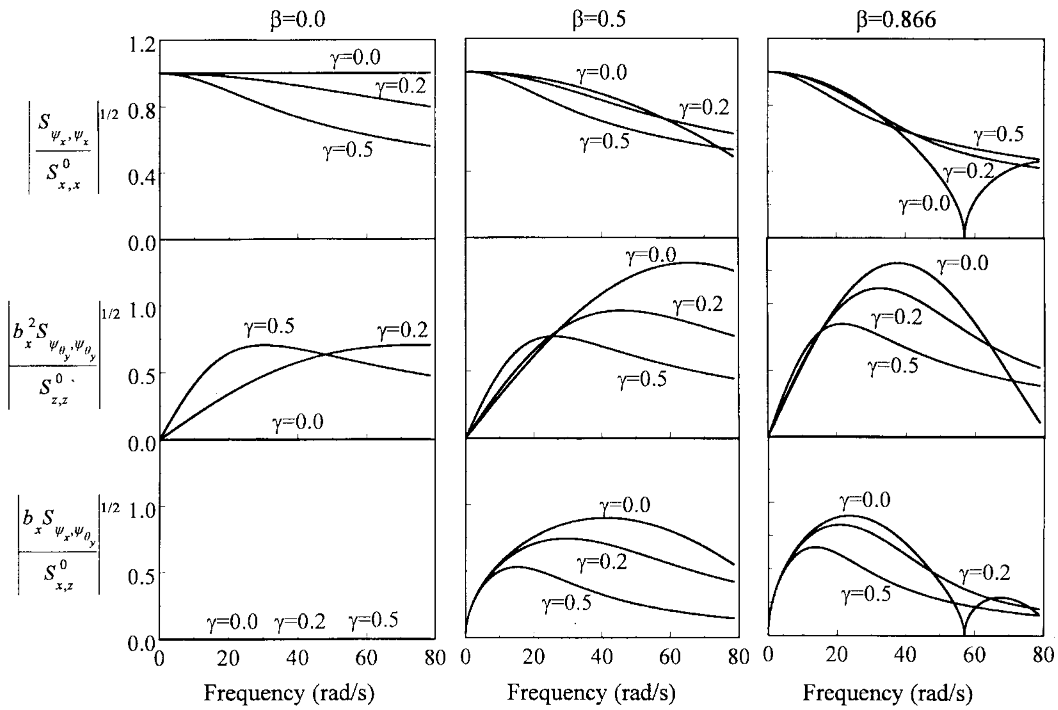

Fig. 6 Numerical results for the filter-like effects of kinematic interaction (square foundation b_x=15.5 m, γ=0 is seismic wave effect only; β=0 is seismic incoherence only)

7. Combined spectral density results

It is now possible to combine all the dynamic analyses to obtain the spectral density for the structural distortions including the effect of both types of soil-structure interaction, as well as the dynamics of the superstructure. Sample results of this type for the example problem are shown in Fig. 7. Note that kinematic interaction couples the motions so that each component of response (including the distortions in the superstructure) depends on more than one component of the excitation. In particular, the vibrations in the (x, z) plane depend on both the x and z components of the free-field motion. The spectral density of the vertical motion and the $S_{xz}^0(\omega)$ cross-spectral density, as well as $S_{xx}^0(\omega)$, have been chosen as Kanai-Tajimi approximations of El Centro 1940.

In addition to the complete soil-structure interaction result, Fig. 7 includes the curves from Figs. 2 and 5 representing the analyses ignoring soil-structure interaction, and including inertial interaction but neglecting kinematic interaction. Thus, the figure allows comparison of the structural distortions for three types of approximation.

For the example problem considered here, Fig. 7 shows that kinematic interaction has very little effect on the structural distortions. This may seem somewhat surprising, since Fig. 6 demonstrated that kinematic interaction could cause significant rocking of the foundation, and one might expect this to induce considerable distortion. This is an aspect of the problem, though, that can be explained by consideration of the filtering effect of the interaction and the degree of participation of the various structural modes. The dynamic excitation provided by rocking is much the same as forces having a triangular distribution over the height of the structure. Furthermore, modal analysis shows that this excitation is very similar to the shape of the fundamental mode of the structure. The excitation is not effective in exciting the higher modes of the structure because the force distribution is almost orthogonal to those mode shapes. The rocking motion could very effectively excite the fundamental structural mode except for one thing – the energy of the rocking is concentrated at frequencies for which the fundamental mode of this structure does not respond

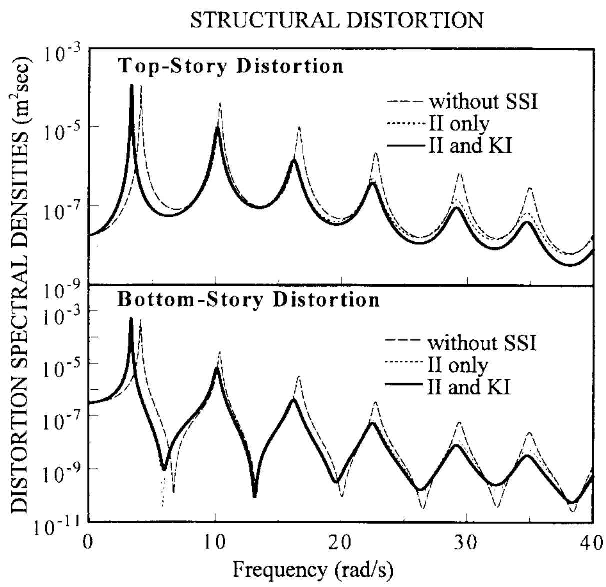

Fig. 7 Effect of soil-structure interaction on spectral densities of structural distortion

significantly. In particular, the large values of the spectral density of rocking occur at frequencies well above the fundamental resonance of the structure.

It is clear in Fig. 7 that kinematic interaction tends to reduce the high frequency components of the structural distortions. This can be attributed to the low-pass characteristic of the effect of kinematic interaction on foundation translation. Since translation, rather than rocking, is the excitation causing most structural distortion, the attenuation of high-frequency translation of the foundation results in a corresponding reduction in the structural distortions.

As noted previously, the most obvious effect of inertial interaction is the frequency shifting of the spectral density peaks. For a smooth excitation spectrum, such as the Kanai-Tajimi form used here, this results in relatively little change in the expected structural distortions. For any one particular time history of seismic excitation, though, there are apt to be very sharp peaks in the spectral density computed from Eq. (1), and by shifting the resonant frequency of the structure, inertial interaction could greatly affect the magnitude of the response. The resonance shift could cause the peak of the transfer function either to be in-tune or out-of-tune with a local peak in the excitation spectrum, so that the response might be either increased or decreased.

8. Conclusions

Stochastic modeling of linear soil-foundation-structure interaction has been reviewed, using the approach of considering kinematic interaction, inertial interaction, and structural distortion as three separate stages of the dynamic response to the free-field motion. The focus has been on the way in which each of the three stages of response modifies the frequency content of the motion, as measured by spectral density functions. Interpretation of the results has been emphasized, rather than the development of new analysis techniques. The following summarizes some of the more important features noted:

1) The standard form of analysis of structural distortion can be viewed precisely as a linear filtering operation, with the true foundation motion as the input and the distortion as the output. The common practice of ignoring soil-structure interaction amounts to using this stage of analysis with the free-field motion as the input to the filter.

2) Inertial interaction is also precisely modeled as a linear filtering operation, with the input being the free-foundation motion and the output being the true foundation motion. The transfer function for this filter depends on the dynamic characteristics of both the structure and soil-foundation system. Since accurate numerical analysis of the soil-foundation system is relatively difficult, it is common to use models based on interpolation of numerical data for similar situations. It is recommended that any such interpolation or extrapolation be performed within the constraints of physically based models, since simple numerical interpolation can lead to physically ambiguous, noncausal models.

3) Kinematic interaction has a filter-like effect on the frequency content of the motion. For example, the spectral density of a horizontal translation component of free-foundation motion is the same as the output of a low-pass filter with an input which has the spectral density of the horizontal component of the homogeneous free-field motion. Similarly, the free-foundation rocking component is the same as the output of a band-pass filter with an input of the vertical component of the homogeneous free-field motion. Nonetheless, kinematic interaction cannot be precisely modeled as a linear filtering operation. For example, the cross-spectral density of the translation and rocking of the foundation is inconsistent with any such linear filter model. This

difference between kinematic interaction and linear filtering has been shown to be present for a quite general stochastic field model of homogeneous free-field motion, as well as for a relatively common model in which the cross-spectral density of the free-field motion is significantly simplified to reduce numerical computations.

4) Based on the results for a sample 21-story building, the most important aspect of inertial interaction can be the lowering of the resonant frequencies of the structure, and the dominant effect of kinematic interaction is often the reduction in response due to the low-pass filtering of the translational motion. Even when kinematic interaction induces quite large free-foundation rocking, this motion generally does not induce significant distortion in the structure. The spectral density of the rocking motion is concentrated at intermediate frequencies, but the structural modes in this frequency range have mode shapes which are not sensitive to this type of excitation.

Appendix - example problem

Building structural model: Structural properties of the typical 21 - story building model (Blume 1961) are given in Table 1. The first ten non-zero modal frequencies of the undamped structural system are given in Table 2, for both the fixed-base and the free-base configurations. The damping is chosen to give a modal damping ratio of 1% in each fixed-base mode.

Foundation model: The building is considered to be supported on a 31.0 m square, rigid slab. The supporting soil is a Bay mud with shear stiffness=115 MPa, v_s=244 m/s and Poisson's ratio=1/3. A simple mass-spring-dashpot model (Wolf and Somaini 1986) is used to approximate the dynamic behavior of the linear soil-foundation system. This model is shown in Fig. 8.

Spectral density of free-field motion: Each component of the free-field spectral density has been approximated by using a Kanai-Tajimi form:

Table 1 Properties of a typical 21 - story building structural model

Floor level	Mass (NMsec^2/m) m_i	Spring constant (MN/m) k_i	Interstory height (m) $h_i - h_{i-1}$
base (0)	2.098		
1	2.098	7.530×10^3	4.572
2	1.977	7.425×10^3	3.658
3	1.855	7.180×10^3	3.658
4	1.741	6.970×10^3	3.658
5, 6	1.728	5.849×10^3	3.658
7	1.609	5.569×10^3	3.658
8	1.581	4.063×10^3	3.658
9	1.581	3.677×10^3	3.658
10	1.567	3.677×10^3	3.658
11	1.555	3.677×10^3	3.658
12	1.555	3.415×10^3	3.658
13	1.492	3.415×10^3	3.658
14	1.474	2.854×10^3	3.658
15, 16	1.474	2.469×10^3	3.658
17	1.455	2.329×10^3	3.658
18, 19	1.434	1.769×10^3	3.658
20	1.345	1.524×10^3	3.658
21	1.378	1.278×10^3	3.658

Table 2 First ten modal frequencies of the structural model

Mode No.	Fixed-base (rad/s)	Free-base (rad/s)
1	4.09	3.35
2	10.4	10.2
3	16.6	16.3
4	22.7	22.5
5	29.4	29.2
6	35.0	34.8
7	40.8	40.6
8	46.6	46.5
9	51.9	51.8
10	56.8	58.3

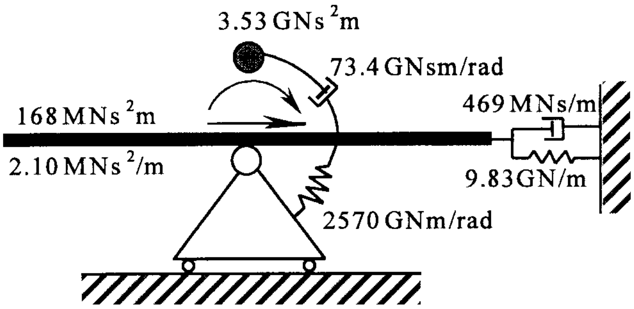

Fig. 8 Dynamic model of soil-foundation system (Wolf and Somaini 1986)

Table 3 Parameters for Kanai-Tajimi spectral density

Component (i, j)	ζ_{ij}	ω_{ij} (rad/s)	S^0_{ij} m^2/(rad·s^3)
(x, x)	0.6	12.0	0.0160
(x, z)	0.6	12.0	0.0092
(z, z)	0.4	60.0	0.0069

$$S^0_{\xi_i\xi_j} = \frac{[1 + (2\zeta_{ij}\,\omega/\omega_{ij})^2]\,S^0_{ij}}{[1 - (\omega/\omega_{ij})^2]^2 + (2\zeta_{ij}\,\omega/\omega_{ij})^2} \tag{21}$$

Table 3 gives the values used for the parameters, and Fig. 9 compares these formulas with smoothed spectral densities computed from Fourier transforms of the N-S and vertical components of the El Centro 1940 earthquake.

Acknowledgements

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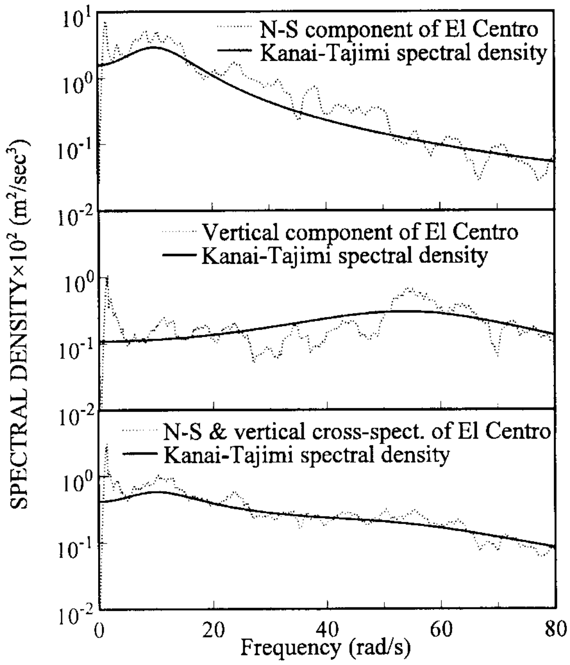

Fig. 9 Spectral density of free-field ground acceleration

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