Structural Engineering and Mechanics, Vol. 7, No. 4 (1999) 401-412
DOI: http://dx.doi.org/10.12989/sem.1999.7.4.401

Optimization of thin shell structures subjected to thermal loading

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Abstract. The purpose of this paper is to show how the Evolutionary Structural Optimization (ESO) algorithm developed by Xie and Steven can be extended to optimal design problems of thin shells subjected to thermal loading. This extension simply incorporates an evolutionary iterative process of thermoelastic thin shell finite element analysis. During the evolution process, lowly stressed material is gradually eliminated from the structure. This paper presents a number of examples to demonstrate the capabilities of the ESO algorithm for solving topology optimization and thickness distribution problems of thermoelastic thin shells.

Key words: topology and thickness design; thin shell; thermoelasticity; finite element analysis; evolutionary structural optimization.

1. Introduction

Shell structures are extensively used in the fields of aeronautical, civil, mechanical and marine engineering (Farshad 1992, Ramm and Mehlhorn 1991). Although there have been man-made shell structures since ancient times, the significant progress in research for topology and shape optimization has only been made recently in conjunction with the rapid development of powerful computers and numerical techniques such as finite element methods. In today's engineering world, shell structures, often under strict design regulations, must exhibit optimum or near optimum performance characteristics in an economical fashion.

There have been only few papers devoted to the optimization problems of thin shells. Marcelin (1988) presented a sensitive analysis algorithm by selecting appropriate design variables of axisymmetric shells, where the maximum stress was minimized. Rao and Hinton (1993) extended

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the shape design sensitivity analysis (SDSA) method to shell problems, where optimal geometric forms and thickness distributions were sought by monitoring either strain energy or structural weight. Tenek and Hagiwara (1994) developed a scheme for determining thickness distribution or the density of a repetitious microstructure using homogenization theory. Maute and Ramm (1996) solved the maximum stiffness problems for shell structures by combining the adaptive optimization technique with the homogenization concept.

All the above presented methods have involved complicated mathematical formulations. Furthermore, none of these have considered the effects of thermal stresses. The recent development of the Evolutionary Structural Optimization (ESO) method by Xie and Steven (1993, 1994, 1996 and 1997) has established a new and considerably simpler method for structural optimization. It has been successfully applied to many optimization problems, which include linear static (Xie and Steven 1993), multiple loads (Xie and Steven 1994), natural frequency (Xie and Steven 1996), stiffness (Chu *et al.* 1997), buckling (Manickarajah *et al.* 1995) and heat transfer (Li *et al.* 1997), etc. This paper aims at extending the versatile ESO algorithm to thin shell problems subjected to both mechanical and thermal loading. The extension incorporates an evolutionary iterative process of thermoelastic thin shell finite element analysis (FEA). It is able to easily deal with both uniform and non-uniform distributions of temperature change by carrying out a heat conduction FEA prior to the thermoelastic thin shell FEA and element elimination cycles. A number of examples presented in this paper verify this extension.

2. Evolutionary optimisation procedure for thin shells

Like most other structural optimization methods, the evolutionary structural optimization (ESO) method is iterative because of the highly nonlinear nature of structural optimization itself. The ESO method follows a very simple and robust concept of repeatedly modifying structural shape and topology on the basis of finite element analysis. In this method, a dense finite element mesh which fully covers the maximum design domain is set up. A heat conduction finite element analysis is firstly carried out to determine the temperature field. Subsequently, thermal and mechanical stress distribution throughout this domain can be found by a thermoelastic finite element analysis. It often happens that part of the material is not effectively used. The ESO method introduces a simple strategy of element. By gradually removing inefficient material from the structure, the resulting shape evolves towards an optimum. For example, those elements are removed whose von Mises stresses σ_e^{vm} are less than *Rejection Ratio* (*RR*) times the maximum (or mean) von Mises stress σ_{max}^{vm} (i.e., $\sigma_e^{vm} \leq RR_{SS} \times \sigma_{max}^{vm}$). Such a thermoelastic shell finite element analysis and element elimination cycle is repeated using the same value of *RR*, until there are no more elements being removed at current iteration. This means that a *Steady State* (*SS*) has been reached and the lowest stress within the structure has been greater than a certain percentage of maximum (or mean) stress value. For the optimization process to continue at this stage, an *Evolution Rate* (*ER*) is introduced and added to *RR* (i.e., $RR_{SS}=RR_{SS-1}+ER$). The iteration takes place again until a new steady state is attained.

The detailed description of this method can be found in references (Xie and Steven 1993, 1994, 1996, 1997). The principles and fundamental procedures that define the ESO algorithm for solving thermoelastic thin shell problems are given as follows:

Step 1. Set up a dense finite element mesh that fully covers the maximum design domain. Apply all thermal and mechanical boundary constraints, loads and material properties;

Step 2. Select the optimization criterion, e.g., maximum or mean von Mises stress (σ_{max}^{vm} or $\bar{\sigma}^{vm}$); Set the number of initial *Steady State*: SS=0 and specify the ESO driving parameters such as initial *Rejection Ratio*: RR_o=RR_{SS}=0 and *Evolutionary Rate*: ER (usually RR_o=0% and ER=0.1-1%);

Step 3. Carry out a finite element heat analysis to determine the temperature distribution, thereby, obtain the temperature change at each node;

Step 4. Carry out a thermoelastic finite element analysis for the current thin shell structure;

Step 5. Compare the von Mises stress σ_e^{vm} of each element with the σ_{max}^{vm} or $\bar{\sigma}^{vm}$ of the entire structure, then eliminate those elements which satisfy: $\sigma_e^{vm} \leq RR_{SS} \times \sigma_{max}^{vm}$;

Step 6. If there are elements eliminated in Step 5, repeat Steps 4-6 (for uniform temperature) or repeat Steps 3-6 (for non-uniform temperature);
If there is no element eliminated in Step 5, this means a steady state has been reached; set SS=SS+1 and increase RR by ER, i.e., RR_{SS}=RR_o+$ER \times SS$. Repeat Steps 5-6.

Since thin shell structures may experience transverse and membrane loads, the combination of bending and membrane load will produce varying stresses through the thickness ($-t/2 \leq z \leq +t/2$) of shell elements. In general, a standard FEA program can directly provide or easily calculate the stresses at three particular locations along the shell thickness direction, i.e., $z=-t/2$ (lower surface), z=0 (mid-surface) and $z=+t/2$ (upper surface). It can be shown the maximum magnitude of the stresses must occur in one of the three locations (Querin 1997). When determining the criterion for element elimination (e.g., von Mises stress), the one with the largest magnitude should be selected. In other words, the maximum magnitude of the varying stresses is viewed to as the reference value of the element elimination.

Due to the progressive elimination of under-utilized material, the structure gradually approaches a nearly uniform stressed design. The technique can minimize the structural volume while maintaining the stresses in the structure within an allowable limit. In practice, the evolutionary process may be terminated when a prescribed stress limit (e.g., σ^{*vm}) or volume constraint (e.g., V/Vo) has been reached.

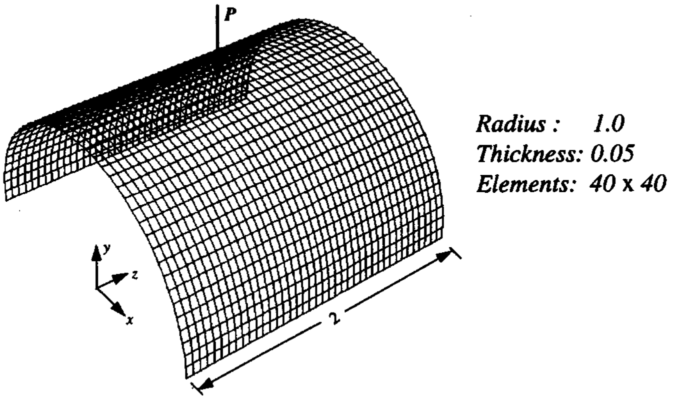


Fig. 1 FE model with mechanical load P=100N and thermal load ΔT=100°C

3. Example problems

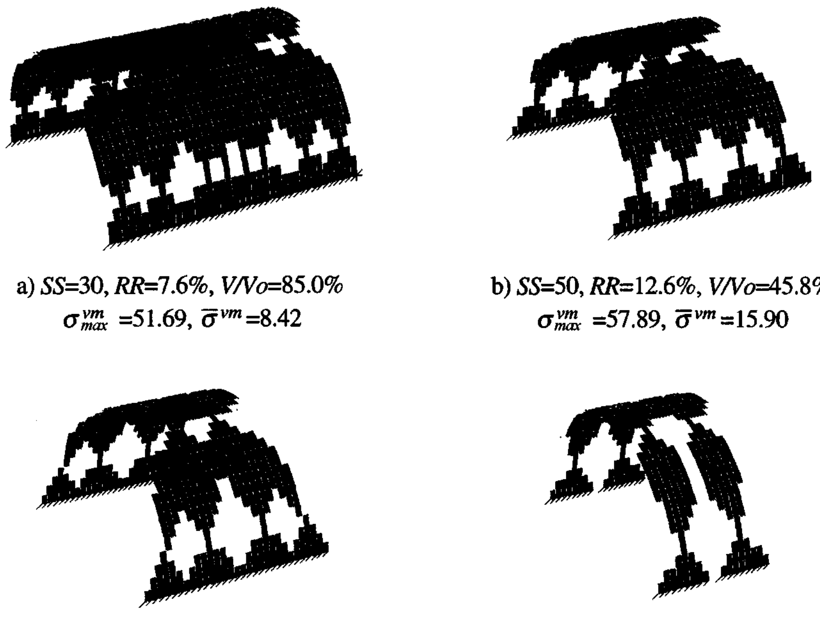
There are two types of problems frequently involved in shell optimization: topology design and thickness re-distribution. Using the above extended evolutionary structural optimization procedure which contains the effects of thermal loads, a number of examples are presented to illustrate the capabilities of ESO method for solving the two types of problems. Linear quadrilateral finite element meshes are used, and both thermal and mechanical loads are considered for all examples.

3.1. Examples of topology design

Cylindrical regions, as shown in Fig. 1, with different boundary conditions are taken as examples for topology optimization. The two kinematic boundary conditions studied are: (1) fully clamped along the straight longitudinal edges; (2) fully clamped at all four corners. A concentrated mechanical load of P=100N and a uniform temperature variation of ΔT=100°C are set for both cases. The evolution processes start with initial rejection ratio RR_o=0% and evolutionary rate ER=0.1%.

3.1.1. Fixed at longitudinal edges

Figs. 2a)-d) show four steady state results of evolutionary optimization process for the first set of kinematic boundary conditions. As the rejection ratio RR increases, more and more relatively inefficient material is removed from the region. The evolution histories of mean and maximum



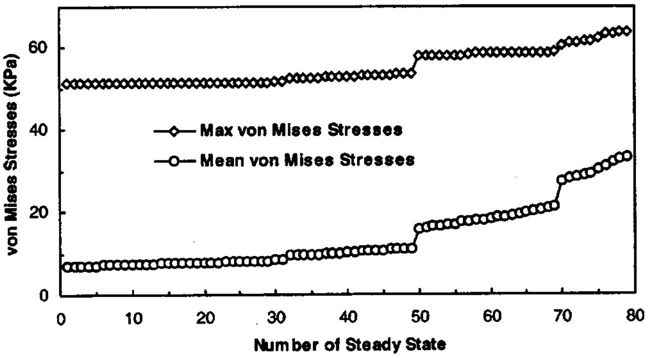
a) SS=30, RR=7.6%, V/Vo=85.0%
σ^{vm}_{max} =51.69, $\bar{\sigma}^{vm}$=8.42

b) SS=50, RR=12.6%, V/Vo=45.8%
σ^{vm}_{max} =57.89, $\bar{\sigma}^{vm}$=15.90

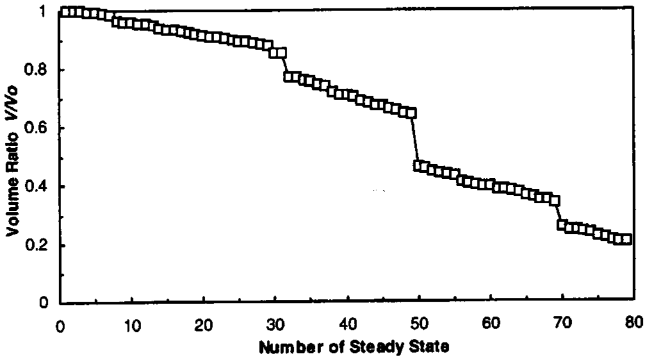
c) SS=60, RR=16.6%, V/Vo=38.5%
σ^{vm}_{max} =58.76, $\bar{\sigma}^{vm}$=18.54

d) SS=70, RR=22.4%, V/Vo=24.8%
σ^{vm}_{max} =61.31, $\bar{\sigma}^{vm}$=28.15

Fig. 2 Evolutionary process of a cylindrical shell fixed at the longitudinal edges



a) Evolution history of von Mises stresses



b) Volume reduction *vs* steady states

Fig. 3 Evolution histories for the shell clamped at longitudinal edges

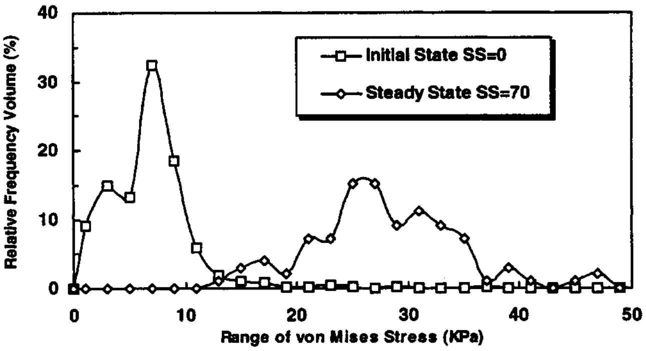
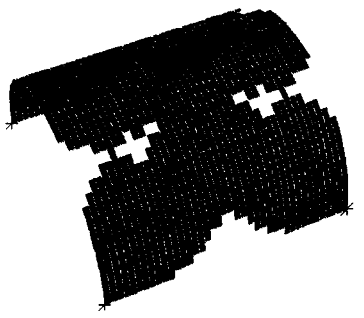


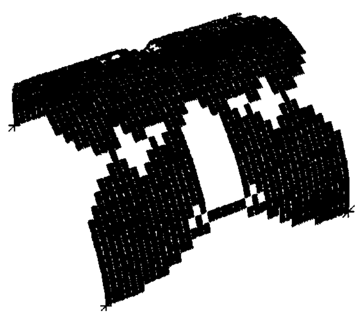
Fig. 4 Comparison of von Mises stress spectrums

von Mises stresses are plotted in Fig. 3a), and the history of volume reduction is shown in Fig. 3b). It can be observed that although there has been significant volume reduction, the increase in maximum von Mises stress is relatively small. For example, comparing *SS*=50 and *SS*=70 with the initial state, the volume reductions are about 55% and 75%, whereas the increases in maximum von Mises stresses are only 12% and 19% respectively.

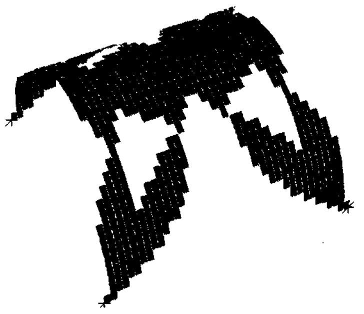
Fig. 4 illustrates the comparison of von Mises stress spectrums in the initial design model with that at steady state 70. It clearly demonstrates the improvement of the structural efficiency. At initial state, approximately 97% of material is very lowly stressed in a range between 0 and



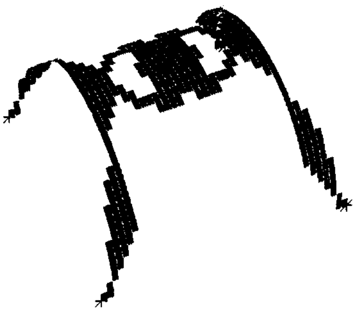
a) SS=20, RR=8.6%, V/Vo=87.3%
σ^{vm}_{max} =54.81, $\bar{\sigma}^{vm}$=10.52



b) SS=35, RR=11.6%, V/Vo=72.3%
σ^{vm}_{max} =56.90, $\bar{\sigma}^{vm}$=12.48



c) SS=55, RR=17.8%, V/Vo=40.5%
σ^{vm}_{max} =80.44, $\bar{\sigma}^{vm}$=22.40



d) SS=60, RR=23.3%, V/Vo=20.8%
σ^{vm}_{max} =134.84, $\bar{\sigma}^{vm}$=64.30

Fig. 5 Evolution process of a shell clamped at four corners

15KPa. When the structure has evolved to steady state 70, nearly 96% of the material is stressed to a level higher than 15KPa.

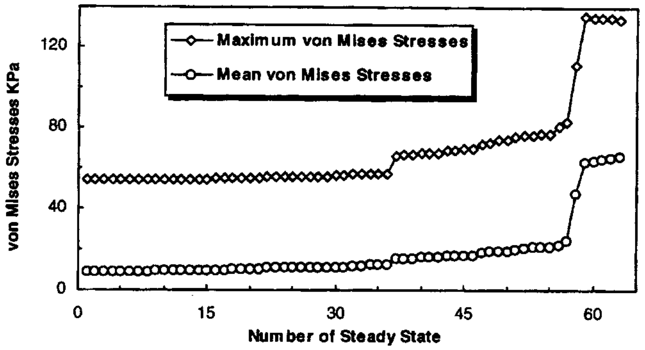
3.1.2. Fixed at four corners

Figs. 5a)-d) show four steady state results under fully clamped at four corners of the cylindrical thin shell. Similarly to the previous case, the fact that the percentage decrease in weight considerably overweighs the percentage increase in maximum von Mises stress can be again observed in Figs. 6a) and b).

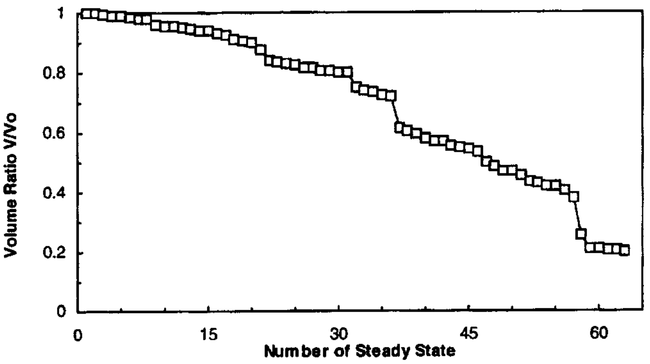
The improvement of structural efficiency can also be shown by plotting the stress spectrum curves as Fig. 7. For example, at initial state (*SS*=0), only 6% material is stressed between 15KPa and 50KPa, while at steady state *SS*=55, nearly 94% material stressed in the same range.

3.2. Examples of thickness design

Recently, mass minimization problems by varying the thickness distribution are attracting attention in shell optimization using the homogenization method (Rao and Hinton 1993, Tenek and Hagiwara 1994). If a continuous shell is required, for aesthetic reasons or simply to keep the rain



a) Evolution history of von Mises stresses



b) Volume reduction *vs* steady states

Fig. 6 Evolution histories for the shell clamped at four corners

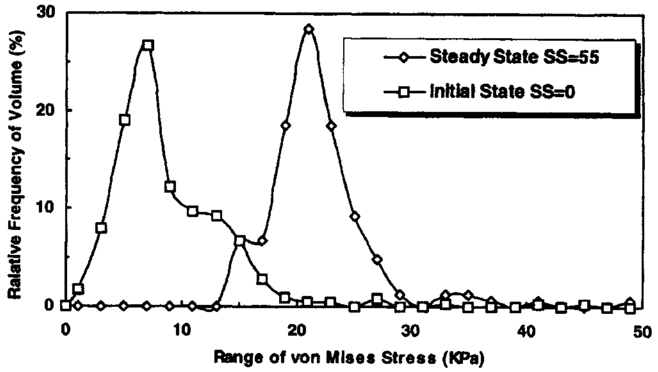


Fig. 7 Comparison of von Mises stress spectrums

out, then the only possibility to improve the structural efficiency is to vary the shell thickness distribution. In the ESO procedure, removing material from the region can be done by gradually reducing its thickness, which is called *Morphing*. In practice, the thickness of each element is assigned a value from a given set of thickness t_s with a step size Δt, which may be either uniform or non-uniform. When the lowly stressed elements are sorted out in Step 5 as mentioned in the previous section, their thickness will be reduced to the next lower value from the given set of thicknesses. When the thickness of an element has reached the minimum one, it will no longer

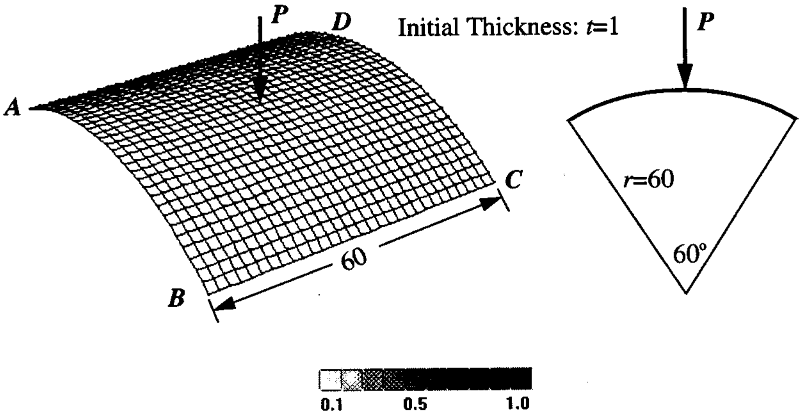
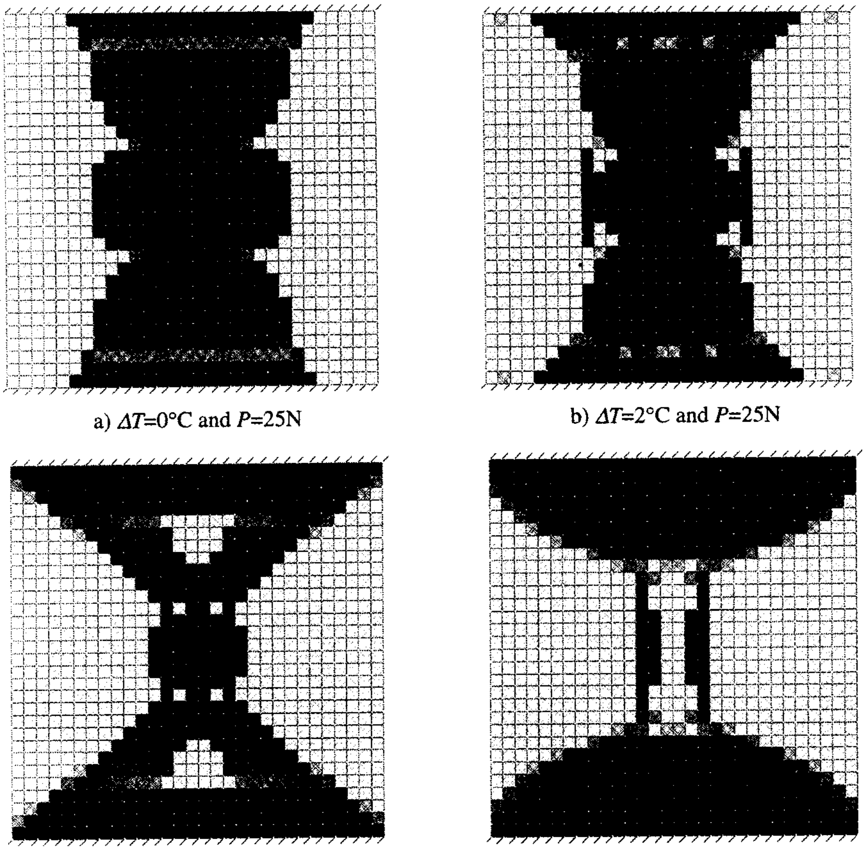


Fig. 8 Cylindrical shell FE model and the thickness distribution legend

change.

The cylindrical shell region, shown in Fig. 8, with different boundary conditions and temperature variations is studied to investigate the morphing capabilities of the ESO. The



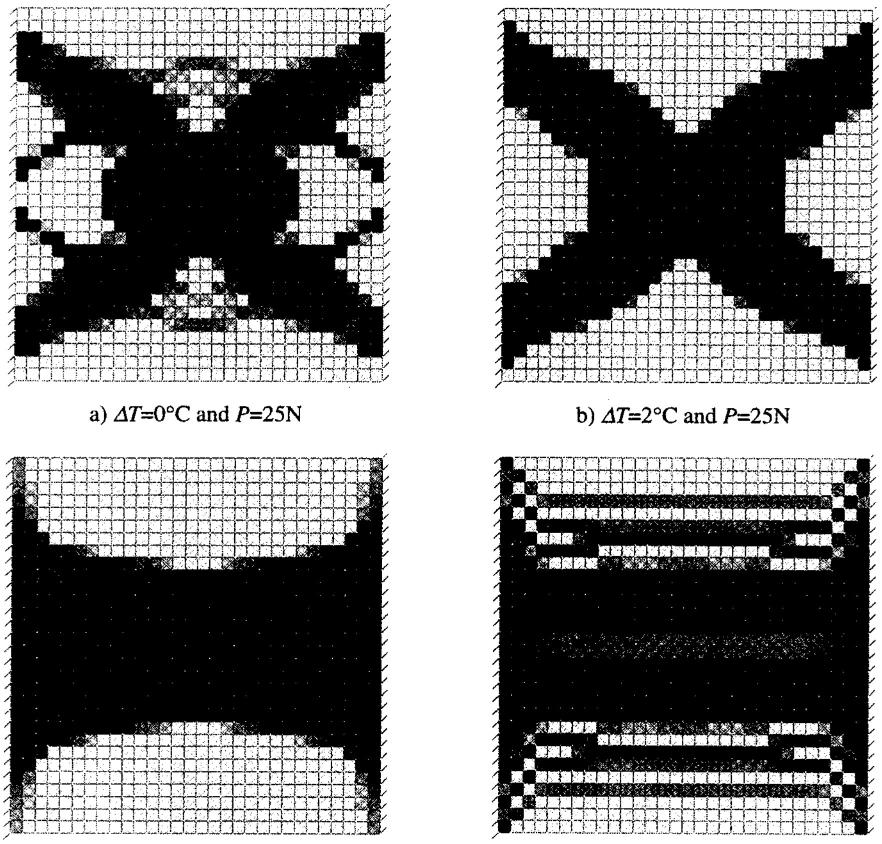
a) ΔT=0°C and P=25N

b) ΔT=2°C and P=25N

c) ΔT=10°C and P=25N

d) ΔT=10°C and P=0N

Fig. 9 Optimal thickness distribution of the shell fixed at longitudinal edges (V/Vo=51%)



a) ΔT=0°C and P=25N b) ΔT=2°C and P=25N

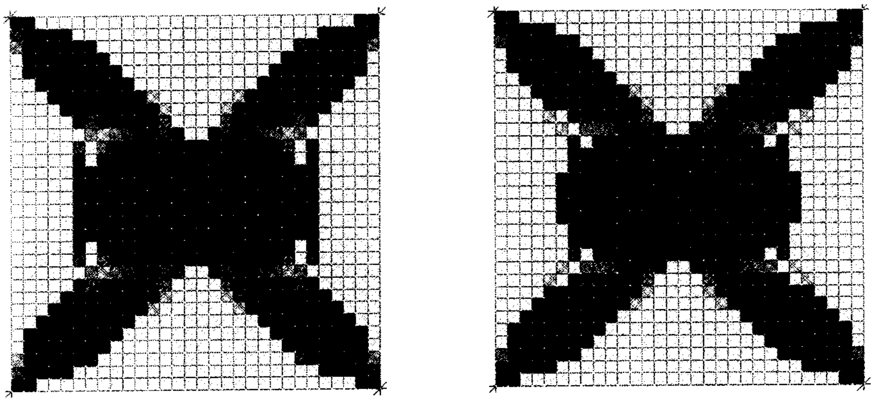
c) ΔT=10°C and P=25N d) ΔT=10°C and P=0N

Fig. 10 Optimal thickness distribution of the shell fixed at cross-section ends (V/Vo=52%)

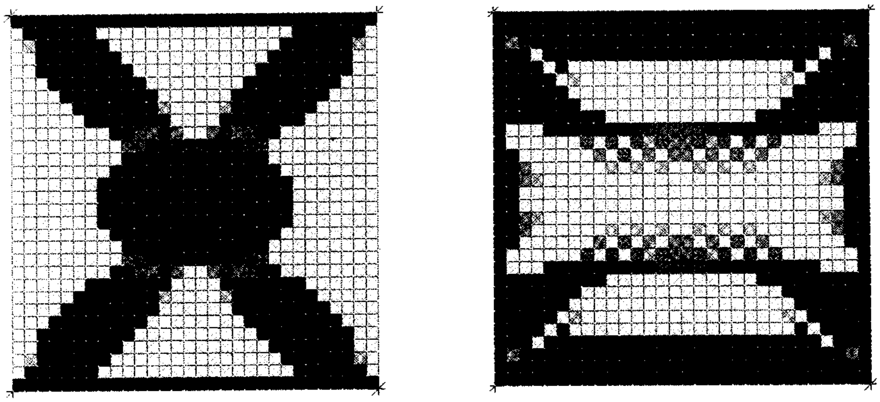
thicknesses are set in an equal interval: $t_s=\{1.0,\ 0.9,\ 0.8,\ \cdots,\ 0.1\}$. Initially, all elements are assigned the maximum thickness t=1.0. The initial rejection ratio RR_o=0% and evolutionary rate ER=1% are adopted. In Figures 9, 10, 11 and 12, the curved shell surfaces are developed to flat ones in order to show thickness variations.

3.2.1. Examples of uniform temperature changes

Figs. 9, 10 and 11 show the thickness distributions for different boundary conditions subjected to pure mechanical, lower thermal, higher thermal and pure thermal loading respectively. In these examples, the prescribed volume constrains are nearly equal (V/V_o=51%-52%). The effects of thermal loading on optimal thickness distributions can be clearly observed. The distinct trends to distribute the material for applying thermal or mechanical loading are revealed by comparing a) with d) in these figures. The thermal loading distributes the material in the vicinity of the clamped boundary in a continuous fashion, while mechanical loading distributes it in a classical frame like manner, similar to normal topology optimization. These figures reveal the potential improvement of shell thickness in terms of its weight and strength.

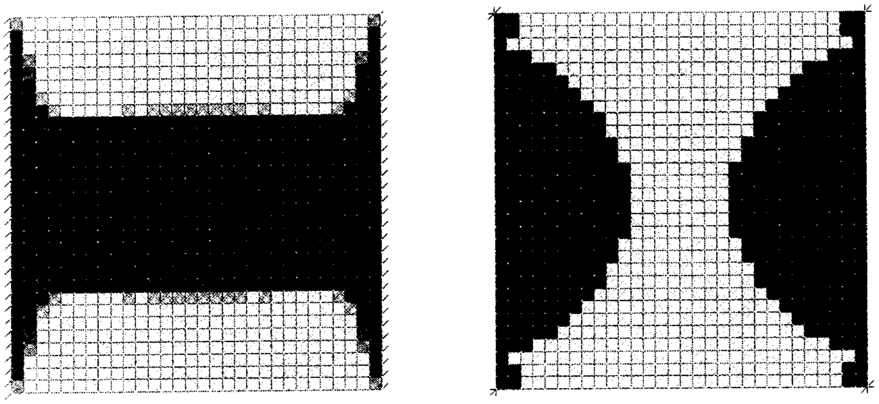


a) ΔT=0°C and P=25N   b) ΔT=2°C and P=25N



c) ΔT=10°C and P=25N   d) ΔT=10°C and P=0N

Fig. 11 Optimal thickness distribution of the shell fixed at corner points (V/Vo=52%)



a) Clamped at curved edges of shell   b) Clamped at the corner points

Fig. 12 Pure thermal loading subjected to non-uniform temperature variation (V/Vo=52%)

3.2.2. Examples of non-uniform temperature change

The thermal boundary conditions are set T_{AB}=T_{DC}=2°C and T_{AD}=T_{BC}=0 °C for the non-uniform temperature field. The pure thermal loads are applied to these examples. It is worth noting that, when the non-uniform temperature is experienced in the region to be optimized, a heat FEA must be carried out prior to the thermoelastic thin shell FEA at every iteration step, whereby the removal of material may change the temperature distributions. Comparing Figs. 12a) and b) with Figs. 10d) and 11d) respectively, the obvious difference in the solutions can be seen between uniform and non-uniform temperature variations.

4. Concluding remarks

This paper has demonstrated that the ESO algorithm can be easily extended to thermoelastic shell problems by combining the shell FEA with an evolution iteration. The generated FEA mesh can be used for both heat and thermoelastic shell solutions for all iterative steps. The method can deal with the topology and thickness optimization problems by a simply modification of the ESO procedure. The evolution process can yield a series of solutions with distinct improvement in structural efficiency. It offers the designers with many optimal feasible solutions, since the structure at each steady state may be chosen as an improved design.

The significant effects of thermal loading, heat boundary and structural boundary on shell design have been investigated in detail. The results show a strong solution dependence on the temperature changes and structural boundaries. This may be explained by the difference of thermal load contributing to the total structural response resulting from both thermal and mechanical loads.

Besides the basic capabilities presented here, the ESO method is readily amenable to other features and enhancements. For example, one development currently under investigation is the incorporation of the heat flux-based criterion and stress-based one into the ESO element elimination. As a result, the resulting topology will be of higher efficiency in both thermal and mechanical behavior.

The new features for ESO developed in this paper have been incorporated into the general ESO computer code EVOLVE (Querin *et al.* 1996).

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