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Cracking in reinforced concrete flexural members - A reliability model

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Abstract. Cracking of reinforced concrete flexural members is a highly random phenomenon. In this paper reliability models are presented to determine the probabilities of failure of flexural members against the limit states of first crack and maximum crackwidth. The models proposed take into account the mechanism of cracking. Based on the reliability models discussed, Eqs. (8) and (9) useful in the reliability-based design of flexural members are presented.

Key words: beam; flexural member; cracking; limit state; probability; reliability-based design.

1. Introduction

It is known that cracking of reinforced concrete (RC) flexural members is a highly random phenomenon. Due to variations in cross-sectional dimensions of beam and the modulus of rupture of concrete, the load at which first crack(s) appear on the surface of the beam is a random variable. Desayi and Rao (1989) carried out a probabilistic analysis of cracking moment of RC beams and proposed an equation for characteristic cracking moment of RC flexural beams. However, this study did not take into account the correlation of cracking resistance of various sections along the length of the beam. Recently, Rao and Rao (1995) proposed a reliability model for modelling the formation of first cracks in RC flexural members. The reliability model takes into account: (i) the fact that the flexure zone of the beam is divided into number of sections as soon as cracks form, and (ii) the correlation of cracking resistance of various sections. Based on the formulation and the results, they (Rao and Rao 1995) proposed an equation for determining the characteristic cracking moment of the beam. This equation can be used in the design of structures such as RC water tanks (design of sections at which bending effect is maximum) where first crack strength is important to ensure water tightness.

Crackwidth is one of the serviceability limit states to be considered in the design of RC members. Codes of practice (e.g., Wolfel 1995) recommend deemed to satisfy clauses for crackwidth limit state. However, in the case of severe exposure, to ensure adequate safety against corrosion of reinforcement, maximum crackwidth formed under service loads needs to be computed and checked against the allowable crackwidth. Hence, a reliable method of crackwidth estimation is required. Efforts have been made in the literature to predict the spacing and widths

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of cracks formed in flexural members (Gergely and Lutz 1968, Beeby 1971, Desayi and Ganesan 1985). Most of these studies are deterministic and propose equations for the estimation of crack spacing and crackwidth based on statistical analysis of test data. Oh and Kang (1987) proposed accurate formulas for estimating the maximum crackwidth and average crack spacing in RC flexural members. These formulas are derived based on the theory developed by Bazant and Oh (1983). They (Oh and Kang 1987) used strength criterion and energy criterion of fracture mechanics to predict the initiation and formation of fracture, respectively, in the flexural members. To propose an equation for characteristic crackwidth for a limit state code, information about the magnitude of scatter and the nature of probability distribution of crackwidth is needed. Desayi and Rao (1987) carried out probabilistic analyses of spacing and widths of cracks in RC beams and using these results, they proposed an equation for estimating the characteristic maximum crackwidth (1987). In the probabilistic analyses equations proposed by Desayi and Ganesan (1985) were used to predict the spacing and widths of cracks.

From the available literature it is found that the studies dealing with reliability analysis of crackwidths that take into account the mechanism of cracking are scanty. In this paper, a critical discussion on reliability modelling of cracking in RC flexural members is presented. Expressions for the estimation of probabilities of failure of RC flexural members against limit states of first crack and crackwidth, taking into account the mechanism of cracking of flexural members, are presented in this paper. It is hoped that the discussion presented will help in better understanding of the phenomenon of cracking in RC flexural members. An example problem is considered to demonstrate the efficiency of the proposed Eq. (19) in the estimation of probability of attaining the crackwidth limit state over the Monte Carlo simulation approach. In this paper failure probability implies probability of attaining a specified limit state and does not indicate probability of collapse of the member. The first crack strength is same as the cracking moment of the beam.

2. Research significance

The philosophy underlying the limit state design of reinforced concrete structures is to design a member for specified ultimate limit state(s) and check the design for serviceability limit state(s) (e.g., crackwidth and deflection.) Most of the limit state codes are semi-probabilistic. The recent trend in the design has been reliability-based design of reinforced concrete members. The main aim of reliability-based design is to design a member for specified reliability against collapse and/or serviceability limit state(s). Euro code (e.g., Wolfel 1995) recommends the use of reliability methods in the analysis/design of RC members. The code also recommends the values of target failure probability to be used in the design for both ultimate and serviceability limit states. Recently, Ditlevsen and Madsen (1996) suggested the type of probability distributions and the typical statistical properties of the basic variables that can be used in the reliability-based design. Due to the development of reliable stochastic models for action and response/resistance and, the use of computers in the designs reliability-based design of structural components has become a practical possibility. The Eqs. (8) and (19) presented in this paper will be useful in the reliability-based design of RC flexural members with respect to the limit states of first crack strength and crackwidth.

3. Mechanism of cracking

To develop a reliability model for cracking, it is important to know the mechanism of cracking in reinforced concrete flexural member. The mechanism of cracking is described in several references (for instance, Bresler 1974, Park and Paulay 1975, Nilson and Winter 1986) Only a brief description of the same will be presented here.

When an under reinforced concrete beam is subjected to monotonically increasing two point loading (Fig. 1) the following points can be noted:

(i) As long as the applied load is less than the first crack load of the beam, the tension forces are shared both by concrete and steel. The load-deflection curve will be essentially linear (portion A of Fig. 2). There will be internal micro-cracking of concrete present in the tension zone (Bresler 1974).

(ii) When the applied load is equal to the first crack load of the beam, visible crack(s) appears on the surface of the beam and the flexure zone of the beam will be divided into number of sections as shown in Fig. 3. The formation of first cracks will be characterised by a sudden drop in load (point B in Fig. 2). This occurs due to sudden loss of stiffness of the beam due to cracking. Typical variations of tensile stresses in concrete and in steel, and the bond stress in the flexure zone of the beam are shown in Fig. 3. The spacing of cracks a_m at this stage of loading (i. e., when the applied moment is equal to the cracking moment of the beam) can be obtained from (Desayi and Ganesan 1985)

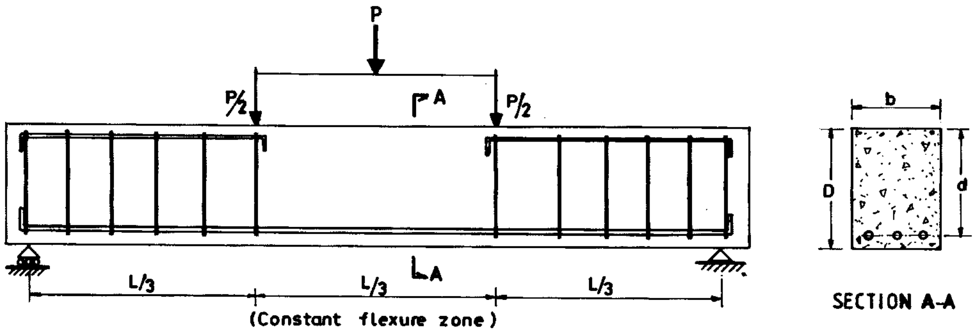

Fig. 1 Reinforced concrete beam subjected to two-point loading

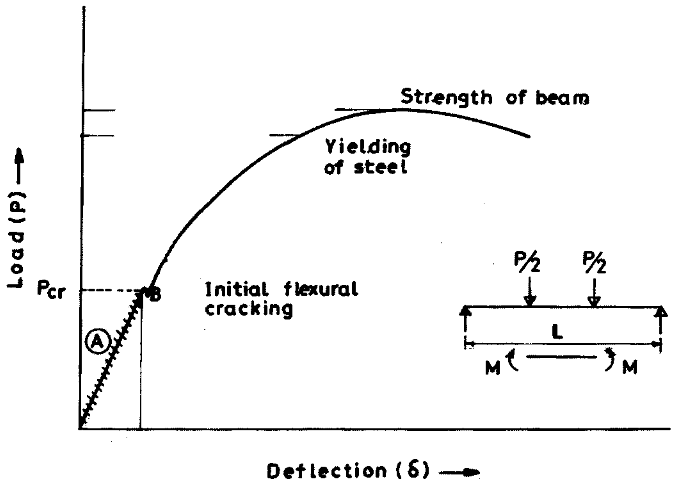

Fig. 2 Schematic load deflection diagram of a under reinforced concrete beam

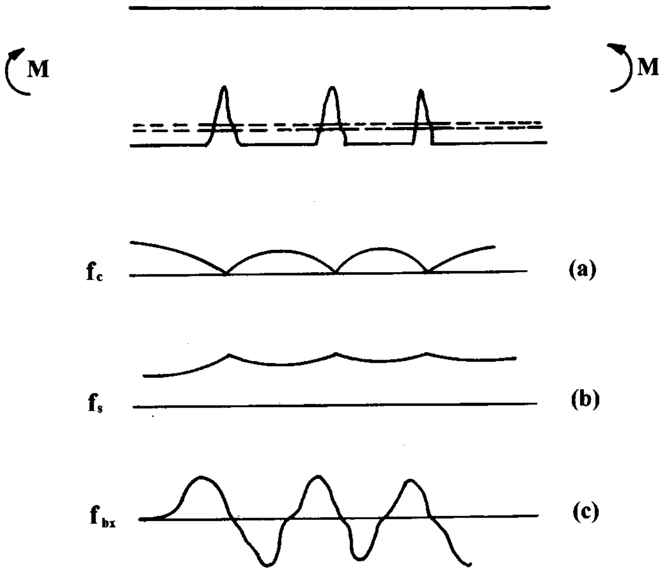

Fig. 3 Typical variation of: (a) Tensile stress in concrete, (b) Tensile stress in steel, (c) Bond stress in a cracked beam

$$a_m = \frac{k_t f_{ct} A_{ct}}{k_b f_{bu} \sum \pi\phi \left(\frac{M_{cr}}{M_u} \right)^{0.33}} \tag{1}$$

where A_{ct} is the effective area of concrete in the tension zone, f_{bu} and f_{ct} are the bond- and tensile-strengths of concrete, k_b and k_t are factors defining the average bond stress and average tensile stress; M_{cr} and M_u are cracking- and ultimate- moments of the beam; $\Sigma\pi\phi$ is the total perimeter of reinforcement in the tension zone. For further details reference (Desayi and Ganesan 1985) may be consulted. The number of sections n_c into which the flexure zone is divided just after cracking can be obtained from

$$n_c = \frac{l_f}{a_m} \tag{2}$$

where l_f is the length of flexure zone.

(iii) With the increase of load, beyond the first crack load, redistribution of bond stress takes place between the cracked sections. New cracks may form in between the existing cracks and also the existing cracks may widen/lengthen. The formation of new cracks results in reduction in crack spacing. The process of formation of new cracks will continue until the bottom fibre stress in concrete cannot reach a value equal to the modulus of rupture*. When this condition is reached, no more new cracks form and the existing cracks will widen/lengthen with the increase of load.

*In a controlled experiment the load - deflection curve of the member, beyond the first crack load contains several jumps. Each jump characterises the cracking instability of the beam. That is, at the point of each jump the beam can carry the specified load only with the increase in deflection. Normally in load controlled experiments, these jumps are not plotted and hence, the load - deflection curve is shown as a smooth curve beyond the first crack load.

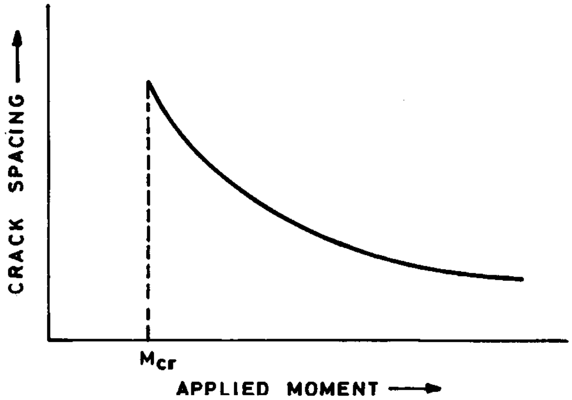

Fig. 4 Schematic diagram showing variation of crack spacing with applied moment

Thus, the spacing of cracks remain the same and the corresponding crack spacing is called stabilised crack spacing. Typical variation of average crack spacing with the applied load is shown in Fig. 4. The average crack spacing at any given stage of loading depends on various factors. Desayi and Ganesan (1985) proposed the following equation to predict average crack spacing, a_i, at any given i-th stage of loading

$$a_i = \frac{k_t f_{ct} A_{ct}}{k_b f_{bu} \sum \pi\phi \left(\frac{M_i}{M_u}\right)^{0.33}} \tag{3}$$

The notation in Eq. (3) is same as that in Eq. (1) except that in Eq. (3) M_i is the applied moment.

The number of sections, n_i, into which the flexure zone is divided at i-th stage of loading can be determined from,

$$n_i = \frac{l_f}{a_i} \tag{4}$$

It is noted that each of these n_i sections contain a crack. The value of n_i increases with the applied load and reaches a constant value (say, N) when spacing of cracks stabilizes. The maximum crackwidth at any i-th loading stage, at the level of steel, W_{mi} can be computed from (Desayi and Ganesan 1985),

$$W_{mi} = a_m \varepsilon_{si} \tag{5}$$

where ε_{si} is the strain in steel and can be obtained assuming linear variation of strain across the depth of beam (see for example Desayi and Rao 1987). The average crackwidth at any given stage of loading is given by (Desayi and Ganesan 1985)

$$W_{ai} = a_i \varepsilon_{si} \tag{6}$$

From Eqs. (1), (3), (5) and (6) it can be seen that crack spacing and crackwidths depend on several factors (viz., tensile- and ultimate bond- strength of concrete, total perimeter of steel bars and the way in which steel bars are distributed in the tension zone).

3.1. Deterministic models for estimation of average crack spacing and maximum crackwidth

For developing a reliability model for limit state of crackwidth, presented in Section 4.2, equations which predict the average crack spacing and maximum crackwidth satisfactorily are required. The average crack spacings and maximum crackwidths of three reinforced concrete beams tested in flexure (whose details are presented in Desayi and Rao 1987) are estimated, at various stages of loading, using equations proposed by Oh and Kang (1987) and Desayi and Ganesan (1985). The computed average crack spacings and maximum crackwidths are compared with the experimental values. From these comparisons it has been found that equations proposed by Oh and Kang (1987) and Desayi and Ganesan (1985) perform satisfactorily with respect to prediction of maximum crackwidths while the observed average crack spacings are consistently underestimated by the equation proposed by Oh and Kang (1987). The comparison of experimental average crack spacings with those predicted are shown in Fig. 5 typically for beam KB2 (of Desayi and Rao 1987). In this investigation, equations proposed by Desayi and Ganesan (1985) are used in developing the reliability model.

4. Development of reliability models

4.1. Reliability modelling for the limit state of first crack strength

Flexure tests on nominally similar RC beams show that the first crack strength of beams vary. The variation in cracking moment is due to variations in modulus of rupture of concrete and the cross-sectional dimensions of the beam. Hence, the cracking moment of a beam should be treated as a random variable. Desayi and Rao (1989) carried out a probabilistic analysis of cracking moment of RC beams in flexure treating the modulus of rupture of concrete and cross-sectional

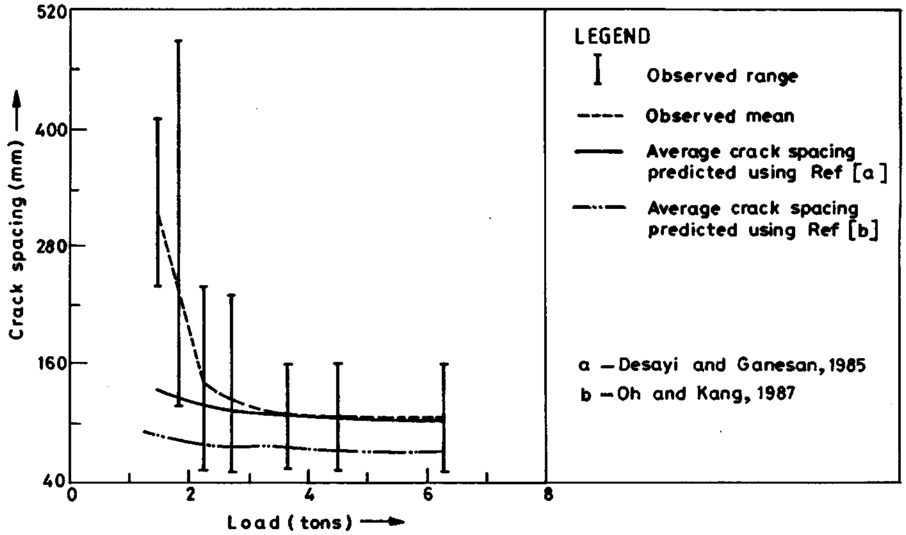

Fig. 5 Comparison of average crack spacing computed using equations proposed by Desayi and Ganesan (1985) and Oh and Kang (1987) with experimental values for Beam KB2

dimensions of the beams as random variables. It was found that cracking moment follows a normal distribution at 5% significance level. Based on this information they proposed an equation for determining characteristic cracking moment of the beam. Accordingly, the characteristic cracking moment, M_{cr}^*, is given by,

$$M_{cr}^* = 0.51\overline{M}_{cr} \tag{7}$$

where $\overline{M}_{cr}$ is the mean cracking moment of the beam and can be obtained by substituting the mean material properties and mean cross-sectional dimensions in the deterministic formula for cracking moment.

Recently, Rao and Rao (1995) extended the above study by considering the effect of correlation of first crack resistance of cross sections of beam, along the length of beam in the flexure zone, on the prediction of cracking moment of beams. They proposed a series reliability model for determination of probability of cracking of beam at any given stage of loading (the mechanism of internal microcracking of the beam is not considered separately). Eq. (2) was used to estimate the number of sections connected in series. From the reliability model developed it was noted that the reliability of the beam, against first-crack, increases with the decrease in n_c (i. e., larger a_m). Given the cross-sectional dimensions of the beam and strengths of concrete it can be seen from Eq. (1) that a_m increases with the decrease in $\Sigma\pi\phi$. This observation implies that, if the limit state considered is first-crack strength alone (in this case failure event is defined as the occurrence of first crack(s) on the beam), for the given area of tension steel, it is better to provide lesser number of bars of larger diameter. This conclusion is in contrast to the popular conclusion drawn from the crackwidth analysis studies (e.g., Nawy 1968, Beeby 1971, Desayi and Rao 1987): for a given area of tension steel, to control widths of cracks, it is better to provide more number of steel bars of smaller diameter in the tension zone. This should not be surprising because: (i) the load-deflection response of the beam is linear up to P_{cr} (which is typical of brittle material response) and for $P \geq P_{cr}$ the load-deflection response is nonlinear (Fig. 2), (ii) the failure events to be considered in the calculation of reliabilities against first crack and against maximum crackwidths are different. In view of (i), the reliability model that should be used in the estimation of reliability against limit state of crackwidth should be different from that to be used for limit state of first crack strength. Based on the reliability model suggested (Rao and Rao 1995) the probability of failure against first-crack, at i-th stage of loading was obtained from,

$$P_{fi} = 1 - \int_{-\infty}^{\infty} \left[\Phi\left(\frac{\beta_{ej} + \sqrt{\rho}t}{\sqrt{1-\rho}} \right) \right]^n \varphi(t)\, dt \tag{8}$$

where β_{ej}=reliability of section j against first-crack, ρ=correlation coefficient of first-crack strengths of any two adjacent sections. $\Phi(\ )$ and $\phi(\ )$ are cumulative distribution function and probability density function of the standard normal variate, respectively. From the study of effect of ρ on the prediction of P_{fi}, it was found that safe and satisfactory values of P_{fi} can be obtained assuming ρ=0.

From a reliability analysis of cracking moment of RC flexural members, Rao and Rao (1995) proposed expressions for bounds on cracking moment and the following equation for the calculation of characteristic cracking moment of the beam,

$$M_r^* = \overline{M}_{cr} \left\{ \frac{0.51+[1+0.30\Phi^{-1}[1-(0.95)^{\frac{a_m}{l_f}}]]}{2} \right\} \tag{9}$$

It can be noted from Eqs. (7) and (9) that while the former considers only the information about the probability distribution of cracking moment, the latter takes into account the information about the probability distribution of cracking moment and the mechanism of cracking of the beam. Comparing the predicted characteristic cracking moments (M_{cr}^*) with the experimental cracking moments, the authors (Rao and Rao 1995) concluded that it is important to consider the probabilistic modelling of mechanism of cracking of the beam in addition to the information about the probability distribution of the cracking moment of the beam, in the estimation of M_{cr}^*.

4.2. Reliability modelling for the limit state of crackwidth

The deterministic evolution of cracking in RC flexural members, as explained earlier, can be obtained using Eqs. (1)-(6). Flexure tests on nominally similar RC beams have indicated that crackwidth at a given stage of loading and at a specified location on the surface of beam shows large scatter (e.g., Nawy 1968, Desayi and Rao 1987). The observed scatter could be attributed to random variations in cross-sectional dimensions of the beam, strengths of steel and concrete. Due to these variations, as can be seen from Eqs. (1), (3), (5) and (6), the spacing and widths of cracks are random variables. In the following a reliability model that takes into account the mechanism of cracking is elucidated.

Using Eqs. (1), (3), (5) and (6) it can be shown that the maximum crackwidth at a given stage of loading is related to average crackwidth by the relationship

$$W_{mi} = \left(\frac{M_i}{M_{cr}} \right)^{0.33} W_{ai} \tag{10}$$

If W_{al} is the allowable maximum crackwidth, the beam is said to have failed in limit state of crackwidth if the maximum crackwidth at any given stage of loading equals or exceeds W_{al}. Therefore, the probability of failure, P_{fi} is given by

$$P_{fi} = P\,[W_{mi} \geq W_{al}] \tag{11}$$

Using Eq. (10), the above equation can be recast in terms of average crackwidth as

$$P_{fi} = P \left[W_{ai} \geq W_{al} \left(\frac{M_{cr}}{M_i} \right)^{0.33} \right] = P\,[W_{ai}(M_i)^{0.33} \geq X] \tag{12}$$

where $X = W_{al}(M_{cr})^{0.33}$. While W_{al} is a deterministic quantity, M_{cr} is a random variable. Hence, X is also a random variable. Since M_{cr} is the cracking moment of the beam, that is resistance against first crack, and W_{al} is allowable maximum crackwidth, X can be viewed as a resistance. Then, $[X-W_{ai}(M_i)^{0.33}]$ represents the safety margin. The probability of failure of beam against limit state of crackwidth can also be written as

$$P_{fi} = P\,[X - W_{ai}\,(M_i)^{0.33} \leq 0] \tag{13}$$

Substituting Eqs. (4) and (6) into Eq. (13), the probability of failure at any given stage of loading can be obtained from,

$$P_{fi} = P\ \{[n_i X - l_f\ \varepsilon_{si}(M_i)^{0.33}] \leq 0\} \tag{14}$$

From Eq. (14), it is clear that with the decrease in a_i, and hence an increase in n_i, the probability of failure of the beam against limit state of crackwidth at any given stage of loading, decreases.

Under the increasing load, the increasing tension strains (or stresses) are shared by the concrete and steel present below the neutral axis. Thus, at any stage of loading (applied moment > cracking moment) the cracked beam can be viewed as a system made-up of n_i components (or sections each containing a crack) connected in parallel. Since all the components of the system participate in resisting the strains the system is idealised as *active* parallel/redundant system. The term $n_i X$ represents the resistance of a system with n_i components connected in parallel, with resistance (X) of components identically distributed. It is known from reliability theory (Christensen and Baker 1982, Ang and Tang 1984) that for a parallel system, reliability increases with the number of components and/or with the decrease in correlation coefficient of strengths of components. Also, it is known that in the case of active redundant system the failure of components is sequential. If the resistance of the components are identically distributed and are perfectly correlated, the system reliability is same as the component reliability. It is noted from the cracking of reinforced concrete flexural members that not all cracks (present in the flexure zone) attain the maximum widths simultaneously. This observation implies that cracking resistance of various sections, while can be assumed to be identically distributed, are only partially correlated. However, it is very difficult to determine the values of correlation coefficients experimentally. It is shown by Rao and Rao (1995), that at higher stages of loading the cracking resistance can be assumed to be statistically independent. This assumption will be more correct when the density of cracking increases. Hence, it is desirable to distribute the steel bars in the tension zone in such a way that the crack density increases and thus the cracks can be assumed to be statistically independent of each other (i.e., the cracks are non-interacting). To increase the crack density, for a given area of tension steel, it is preferable to provide more number of lesser diameter bars than otherwise. Thus, the reliability model formulated above reinforces the experimental observation that it is better to have more number of cracks of smaller width than smaller number of cracks of larger crackwidths. And, the reliability model developed in Eq. (14) takes into account the mechanism of cracking.

Recently, from a study involving extensive computer experiments on cracking of two-dimensional matrix of general anisotropy Mauge and Kachanov (1994) showed that the approximation of non-interacting cracks remains accurate at high crack densities. This conclusion also supports the reliability model developed in Eq. (14) and thus the model is consistent with the numerical results of Mauge and Kachanov (1994).

4.2.1. Determination of failure probability

In order to determine the probability of failure of the beam against limit state of crackwidth using Eq. (13), the probability distributions of random variables X and W_{ai} should be known. From a probabilistic analysis of cracking moment of RC beams Desayi and Rao (1989) found that cracking moment follows a normal distribution at 95% confidence level with a coefficient of variation of 0.30. Knowing the probability density function of cracking moment, it can be shown that the probability density function of X is given by,

$$f_X(x) = \frac{1.21}{\sigma_1} \frac{x^{2.03}}{c_1^{3.03}} e^{-\frac{1}{2}\left[\frac{\left(\frac{x}{c_1}\right)^{3.03} - \mu_1}{\sigma_1}\right]^2} \qquad 0 \leq x \leq \infty \tag{15}$$

where $c_1 = W_{ai}$; μ_1 and σ_1 are the mean and standard deviation of cracking moment. The probability density function derived above is checked with the results of Monte Carlo simulation of the random variable X. In the simulation cracking moment is assumed to follow normal distribution and 2000 cycles are used. The comparison of cumulative distribution functions computed from Eq. (15) and that obtained from the results of simulation is shown in Fig. 6. As expected, the comparison shows good agreement. Assuming the external moment to be deterministic, and the average crackwidth to follow normal distribution at any loading stage, the probability density function of $Z = W_{ai}(M_i)^{0.33}$ is given by,

$$f_Z(z) = \frac{1}{\sqrt{2\pi}\sigma_2 c_2} e^{-\frac{1}{2}\left[\frac{\left(\frac{z}{c_2}\right) - \mu_2}{\sigma_2}\right]^2}, \qquad -\infty \leq z \leq \infty \tag{16}$$

where $c_2 = (M_i)^{0.33}$; μ_2 and σ_2 are the mean and standard deviation of average crackwidth at any i-th loading stage. It is reasonable to assume that X and Z are statistically independent and hence P_{fi} can be obtained from,

$$P_{fi} = \int_{-\infty}^{\infty} F_X(z) f_Z(z) dz \tag{17}$$

The above equation can also be written in the form,

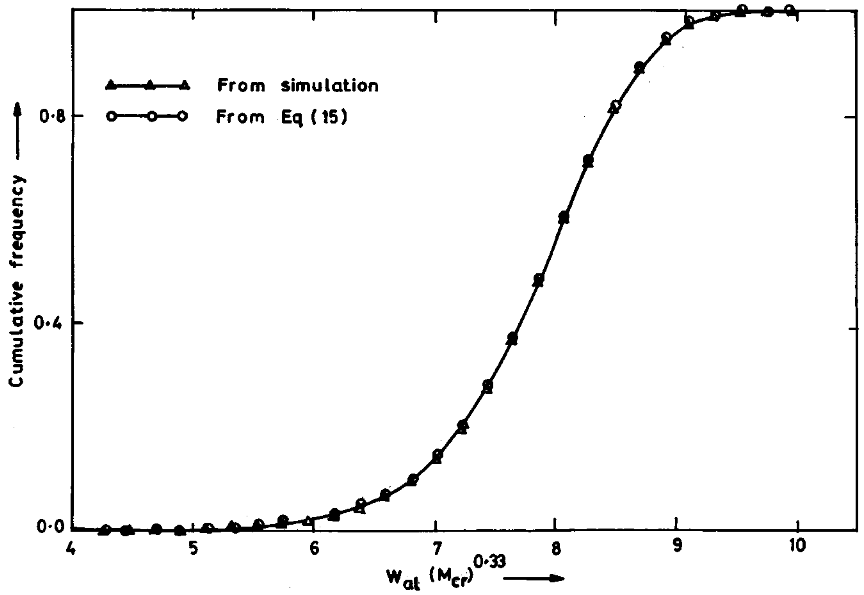

Fig. 6 Comparison of cumulative frequencies obtained using Eq. (15) and from Monte Carlo simulation

$$P_{fi} = 1 - \int_{-\infty}^{\infty} F_z(x)\, f_X(x)\, dx \tag{18}$$

Substituting the expressions for $F_Z(x)$ and $f_X(x)$ into Eq. (18) the following equation for probability of failure of the beam against the limit state of crackwidth is obtained.

$$P_{fi} = 1 - \int_{-\infty}^{\infty} \Phi\left[\frac{\left(\frac{x}{c_2}\right) - \mu_2}{\sigma_2}\right] \frac{1.21}{\sigma_1} \frac{x^{2.03}}{c_1^{3.03}} e^{-\frac{1}{2}\left[\frac{\left(\frac{x}{c_1}\right)^{3.03} - \mu_1}{\sigma_1}\right]^2} dx \tag{19}$$

Eq. (19) can be used to determine P_{fi} at any given stage of loading. It is noted that the application of Eq. (19) for the determination of P_{fi} is straightforward. The two parameters namely, the mean and standard deviation appearing in Eq. (19) can be estimated using first order approximations (Ang and Tang 1975 and 1984). In the following section, the applicability of Eq. (19) in the determination of probability of failure of a flexural member with respect to the limit state of crackwidth is demonstrated.

4.2.2. Use of the reliability model for design

A typical reliability-based design procedure of a flexural member would involve the following steps: (i) design the member for flexural limit state using the partial safety factors specified in the reliability-based design codes (ensuring factored resistance≥factored load effect), (ii) choose the combination of bars, from the available sizes, such that the required area of tension steel is provided, and (iii) check the design for crackwidth limit state (amongst other serviceability limit states).

The reliability model proposed in this paper can be used in step (iii), which consists of determining P_{fi} using Eq. (19) for service loads, specified in relevant codes of practice, and computing the reliability index from the equation

$$\beta_s = -\Phi^{-1}(P_{fi}) \tag{20}$$

If the computed value of β_s is greater than the target value (specified for example in Wolfel (1995)) then the design is safe for crackwidth limit state. Otherwise, for the same area of steel, computed based on ultimate moment requirements, the bars should be selected in such a way that the total perimeter of steel increases. These bars should be arranged such that the effective depth of the beam does not vary significantly from that used in the estimation of moment of resistance of cross-section. The value of β_s is recomputed and checked against the target value. This exercise is repeated till the crackwidth limit state is satisfied.

5. Example

Three simply supported, singly reinforced concrete beams subjected to the third point loading (Desayi and Rao 1987) are considered in this paper. These beams (namely KB1, KB2 and KB3) are reinforced with steel distributed in three different ways in the tension zone. Some of the test details of the three beams are presented in Table 1 (more details are available in Desayi and Rao, 1987). The main objectives of the example considered are: (i) to compare the failure probabilities computed using Eq. (19) with those obtained from Monte Carlo simulation technique at different applied load levels, (ii) to examine whether the values of P_{fi} computed using Eq. (19) are realistic, and (iii) to study the effect of distribution of steel bars in the tension zone on P_{fi}.

To compute the value of P_{fi} using Eq. (19) information regarding the statistical properties namely, mean and standard deviation of cracking moment and, the mean and standard deviation of average crackwidth are required. The mean cracking moment of the beam is calculated using first order approximation (Ang and Tang 1975 and 1984) of the equation $M_{cr}=f_r \cdot I_g/y_b$; where f_r is the modulus of rupture of concrete, I_g is the gross moment of inertia of the cross section and y_b is the distance of bottom-most tension fibre from the centre of gravity of the cross section. The standard deviation of M_{cr} is computed knowing that the coefficient of variation (cov) of M_{cr} is 0.30 (Desayi and Rao 1987). The mean average crackwidths at different stages of loading are computed for each beam using first order approximation of Eq. (6). From a probabilistic analysis of crackwidths (Desayi and Rao 1987) it was found that the cov of average crackwidth varies in the range 0.15-0.25. In the present study a value of 0.25 is assumed. Knowing the cov of crackwidth its standard deviation is computed at different applied load levels. To evaluate P_{fi} the value of allowable crackwidth is required. The allowable crackwidth depends on the exposure conditions. In this study a value of 0.30 mm (Desayi and Rao 1995) is used as allowable crackwidth.

To determine the value of P_{fi} at each stage loading the double integral appearing in Eq. (19) is solved by repeated application of Simpson's one-third rule. Also, the values of P_{fi} are computed from Monte Carlo simulation technique. In simulation Eq. (13) is used and 10,000 simulation cycles are used to estimate P_{fi}. The results of this study, for the three beams considered, are presented in Tables 2-4. The results of simulation are also shown in Fig. 7. From the results presented in Tables 2-4, it is noted that P_{fi} computed from Eq. (19) compare satisfactorily with those obtained from Monte Carlo simulation. The main advantage of Eq. (19) is its computational efficiency compared to the simulation. To examine whether the P_{fi} values computed using Eq. (19) represent satisfactorily the probability of reaching the crackwidth limit state, the following comparison is made (here the tested beam is considered as one of the samples of the ensemble). For instance, if beam KB1 is considered, for $M/M_{cr} \cong 3.42$ the value of P_{fi} computed using Eq.

Table 1 Some of the test details of three beams (Desayi and Rao 1987) considered in this study

Beam designation	Effective depth (mm)	Total perimeter of steel bars (mm)	Cube strength of concrete (N/mm^2)	Tensile strength of concrete (N/mm^2)	Modulus of rupture of concrete (N/mm^2)	Yield strength of steel (N/mm^2)	Modulus of elasticity of steel (N/mm^2)	Experimental cracking moment (kN-m)	Experimental ultimate moment (kN-m)
KB1	311.0	(2-16mm)* 100.42	33.08	2.56	4.03	490.00	2.066×10^5	16.48	70.41
KB2	305.4	(2-10mm; 2-12mm)* 148.35	40.42	2.84	3.58	468.00 516.00	2.093×10^5 2.054×10^5	15.15	78.59
KB3	303.5	(5-10mm)* 182.37	22.51	2.11	2.95	468.00	2.093×10^5	11.56	64.34

Note: All three beams were tested in 1/3-point loading over an effective span of 4200 mm and had a cross-sectional dimension of 200×350 mm.

*indicates the combination of bars used to get required area of steel.

Table 2 Values of probability of attaining the limit state of crackwidth computed from simulation and using Eq. (19) for beam KB1 (Desayi and Rao 1987) at different load levels

M/M_{cr}	${}^{(1)}P_{fi}^{\,sim.}$	${}^{(2)}P_{fi}^{\,The.}$
1.5259	0.0015	0.00223
1.9203	0.0813	0.07875
2.3733	0.3174	0.31993
2.7306	0.5885	0.59390
3.0831	0.7483	0.75240
3.4151	0.8389	0.84836

Note: (1) probability of attaining crackwidth limit state obtained from Monte Carlo simulation.
(2) probability of attaining crackwidth limit state computed using Eq. (19).

Table 3 Values of probability of attaining the limit state of crackwidth computed from simulation and using Eq. (19) for beam KB2 (Desayi and Rao 1987) at different load levels

M/M_{cr}	${}^{(1)}P_{fi}^{\,sim.}$	${}^{(2)}P_{fi}^{\,The.}$
1.0412	0.0000	0.00000
1.2108	0.0001	0.00000
1.4227	0.0006	0.00054
1.6332	0.0016	0.00238
2.0497	0.0224	0.02188
2.4661	0.1075	0.10670
3.0986	0.3777	0.3805

Note: (1) probability of attaining crackwidth limit state obtained from Monte Carlo simulation.
(2) probability of attaining crackwidth limit state computed using Eq. (19).

Table 4 Values of probability of attaining the limit state of crackwidth computed from simulation and using Eq. (19) for beam KB3 (Desayi and Rao 1987) at different load levels

M/M_{cr}	${}^{(1)}P_{fi}^{\,sim.}$	${}^{(2)}P_{fi}^{\,The.}$
1.4527	0.0000	0.00000
1.9593	0.0002	0.00003
2.7088	0.0040	0.00487
3.4626	0.0517	0.05092
4.4869	0.2893	0.28956
5.2522	0.5151	0.51653

Note: (1) probability of attaining crackwidth limit state obtained from Monte Carlo simulation.
(2) probability of attaining crackwidth limit state computed using Eq. (19).

(19) is 0.84836. This indicates that there is approximately 85% chance that a flexural member, with nominal dimensions and strengths of concrete and steel as that of beam KB1, will develop a maximum crackwidth of at least 0.30 mm at $M/M_{cr} \cong 3.42$. Interpreted in a relative frequency sense, the value of P_{fi} obtained indicates that if 100 nominally similar beams are tested 85 beams will develop a crackwidth of at least 0.30 mm at $M/M_{cr} \cong 3.42$. From the experimental data of

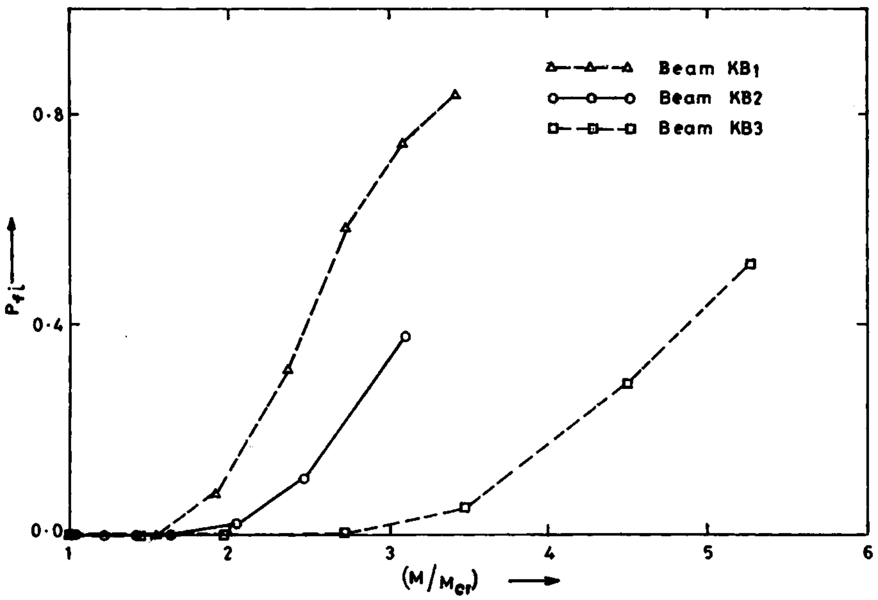

Fig. 7 Comparison of probabilities of attaining crackwidth limit state at different applied load levels for the three beams considered

crackwidths of beam KB1, it is found that the maximum crackwidth at this stage of loading is 0.375 mm. Similar comparisons were made for the other beams also. From these comparisons it is concluded that the values of P_{fi} computed from Eq. (19) are satisfactory. It can be seen from Fig. 5 that at a given stage of loading the failure probability is minimum for the beam with well distributed bars. This also shows that the reliability model developed in this paper is consistent with the fact that it is better to have more number of cracks of smaller width than smaller number of cracks of larger crackwidths.

From the discussions presented above it is noted that Eq. (19) can be used to obtain a satisfactory estimate of the failure probability of the flexural member for crackwidth limit state.

6. Conclusions

In this paper a critical discussion on reliability modelling of cracking in RC flexural members is presented. In presenting reliability models, for the limit states of first crack resistance and maximum crackwidth, the mechanism of cracking in flexural members is considered. To determine reliability against first crack, a series model is presented. This model is consistent with the fact that the load-deflection behaviour of beam is linear and the system behaviour is essentially brittle. However, to determine reliability against limit state of crackwidth a parallel system model is developed. This is consistent with the fact that system load-deflection behaviour is nonlinear beyond cracking load and that there will be redistribution of bond stresses between cracks. The reliability model for crackwidth reinforces the experimental observation that it is better to provide more number of steel bars of smaller diameter, for a given area of tension steel, to increase density of cracking and to control the crackwidth. With the increase in the density of cracking, the statistical correlation of cracking resistance among various sections reduces and they can be assumed to be statistically independent. This increases the system reliability against the limit state of crackwidth, which is desirable. For a flexural member the probabilities of attaining the limit states of first crack and crackwidth can be computed using Eqs.(8) and (19), respectively. One of the main advantages of Eq. (19) is its computational efficiency compared to Monte Carlo simulation technique. Using this equation a procedure for reliability-based design of reinforced concrete flexural member for limit state of crackwidth is presented in this paper.

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