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Analytical methodology for solving anisotropic materials of antiplane problems

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Abstract. An analytical methodology for solving antiplane problem of anisotropic materials is proposed and discussed in detail in this study. The material considered in this study possesses a symmetry plane at z=0. The relationship between the problems of anisotropic materials and the corresponding isotropic problems are established by Ma (1996) on the basis of the general solutions for the shear stresses and displacement in both the polar and Cartesian coordinate systems. This implies that any solution of an anisotropic problem can be obtained by solving a corresponding isotropic problem. In this study some examples and numerical results are presented as an explanation of how the complicated anisotropic problem could be solved by the associated simpler isotropic problem.

Key words: antiplane problem; anisotropic materials; shear stresses.

1. Introduction

The problem of finding the stress singularities at the apex of an isotropic elastic wedge was considered by Williams (1952) by using the eigenfunction-expansion method. Tranter (1948) used the Mellin transform in conjunction with the Airy stress function representation of plane elasticity to solve for the isotropic wedge problem. An eigenfunction approach was used by Williams (1959) for establishing the asymptotic nature of the dominant singular stresses of dissimilar materials with a semi-infinite crack. He found that the stresses share the inverse square root singularity of the crack in a homogeneous material and, in addition, exhibit an oscillatory behavior as the crack tip is approached. Following William's study on the bimaterial interfacial crack problems, Erdogan (1963), England (1965), Rice and Sih (1965), have attempted to obtain the solutions with the Muskhelishili's complex function theory in elasticity. The Mellin transform has been previously used by Bogy (1971) and Ma and Wu (1990) in treating the problem of two materially dissimilar isotropic elastic wedges of arbitrary angles. The stress field at the vertex of the edge or the corner of the elastic bimaterial wedge possesses a singularity, the nature of which is dependent on the composite parameters of the material combination. Two composite parameters have been derived from the elastic constants of the materials by Dundurs (1967). The stress field of a composite in a state of plane deformation has been shown by him to be dependent only on these two parameters.

Investigation of the associated wedge problems for anisotropic materials has initiated due to the fact that anisotropic materials have increasingly wider applications in modern technology. In real

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composite materials, each layer is a fiber reinforced laminated material and hence should be regarded as an anisotropic material. Stroh (1958) has obtained an analytical solution to the problem for the case of a plane crack in an anisotropic material of infinite extent. Following the approach of Stroh, Ting (1986) studied the stress distribution near the composite wedge of anisotropic materials. A complex function representation of a generalized Mellin transform has been previously employed by Bogy (1972) for analyzing stress singularities in an anisotropic wedge. The anisotropic media has been demonstrated by all of these studies to retain all those troublesome features of oscillation of the singular stress field observed in isotropic media. The antiplane problem of two dissimilar anisotropic wedges of arbitrary angles that are bonded together perfectly along a common edge has been recently considered by Ma and Hour (1989). The order of the stress singularity has been found to be always real for the antiplane dissimilar anisotropic wedge problems. That is quite a different characteristic from the in-plane case in which the complex type of stress singularity might exist.

A previous analysis of the dissimilar anisotropic antiplane wedge problem by Ma (1992) has shown that if an effective angle and effective material constant are introduced for the anisotropic case, then the order of singularity for the anisotropic material can be obtained from the result of the isotropic case. The results obtained in Ma (1992) have been extended by Ma (1996) and the correspondence relations of the full field solutions of stresses and displacement have been established for the anisotropic problem and that for the isotropic problem in antiplane deformation. The reduction in the number of elastic constants considerably simplifies the description of the stress and displacement state. The material is assumed to possess the material symmetry such that the inplane and the antiplane deformations are uncoupled. Through such a correspondence, the relationship of the stresses and displacement for anisotropic and the corresponding isotropic problem is established for both polar and Cartesian coordinate systems. Any solution of anisotropic material for antiplane problem can be obtained by solving a corresponding isotropic problem. Some examples of anisotropic material with defects of hole and crack in finite bodies are solved for providing the proof of the relationship established by Ma (1996).

2. Correspondence relations for anisotropic and isotropic problems

It is well-known that the only nonvanishing displacement component w^i is along z-axis for the antiplane deformation. For the absence of body force, the equilibrium equation for a homogeneous isotropic material is given by the following partial differential equation

$$\frac{\partial^2 w^i}{\partial r^2}+\frac{1}{r}\frac{\partial w^i}{\partial r}+\frac{1}{r^2}\frac{\partial^2 w^i}{\partial \theta^2}=0 \tag{1}$$

The nonvanishing stresses are

$$\tau^i_{rz}=\mu\frac{\partial w^i}{\partial r} \tag{2}$$

$$\tau^i_{\theta z}=\frac{\mu}{r}\frac{\partial w^i}{\partial \theta} \tag{3}$$

For the anisotropic case, if the plane of elastic symmetry is assumed to be normal to the z-axis, then there are only three relevant coefficients c_{44}, c_{45} and c_{55} to be considered. The stress components are related to the displacement as follows:

$$\tau^a_{yz} = c_{44}\frac{\partial w^a}{\partial y} + c_{45}\frac{\partial w^a}{\partial x} \tag{4}$$

$$\tau^a_{xz} = c_{45}\frac{\partial w^a}{\partial y} + c_{55}\frac{\partial w^a}{\partial x} \tag{5}$$

In the absence of body forces, the corresponding displacement equations of equilibrium for a homogeneous anisotropic material is given by

$$c_{55}\frac{\partial^2 w^a}{\partial x^2} + 2c_{45}\frac{\partial^2 w^a}{\partial x\,\partial y} + c_{44}\frac{\partial^2 w^a}{\partial y^2} = 0 \tag{6}$$

The Eq. (1) governing the anti-plane problems in linear, isotropic, elastic materials is the standard second order partial differential equation of elliptic type. The Eq. (6) is a general second order partial differential equation of elliptic type with constant coefficients. Such linear partial differential equation of elliptic type with constant coefficients can be changed into the standard partial differential equation of elliptic type by linear coordinate transformations.

For convenient, let the coordinate system associated with the anisotropic problem be denoted by r and θ (or x and y), and the isotropic problem be denoted by R and ϕ (or X and Y) in the following content. A very simple relationship for the anisotropic problem and the corresponding isotropic problem in the polar coordinate was established from Ma (1996) as follows

$$w^a(r,\ \theta) = \frac{\mu}{C}\, w^i\,(R,\ \phi) \tag{7}$$

$$r^a_{\theta z}(r,\ \theta) = \Psi(\theta)\,\tau^i_{\theta z}(R,\ \phi) \tag{8}$$

$$\tau^a_{rz}(r,\ \theta) = \frac{1}{\Psi(\theta)}\,[\Theta\tau^i_{\theta z}(R,\ \phi) + B\,\tau^i_{rz}(R,\ \phi)] \tag{9}$$

where

$$C = [c_{44}c_{55} - (c_{45})^2]^{1/2} \tag{10}$$

$$R = \Psi(\theta) r \tag{11}$$

$$\phi = \tan^{-1}\left\{\frac{\sqrt{c_{44}c_{55}-(c_{45})^2}\,\sin\theta}{c_{44}\cos\theta - c_{45}\sin\theta}\right\} \tag{12}$$

$$\Psi(\theta) = [\cos^2\theta - \frac{c_{45}}{c_{44}}\sin 2\theta + \frac{c_{55}}{c_{44}}\sin^2\theta]^{1/2} \tag{13}$$

$$\Theta = \sin\theta\cos\theta + \frac{c_{45}}{c_{44}}\cos 2\theta - \frac{c_{55}}{c_{44}}\sin\theta\cos\theta \tag{14}$$

$$B = \frac{\sqrt{c_{44}c_{55}-(c_{45})^2}}{c_{44}} \tag{15}$$

The solution for an anisotropic problem can be obtained by replacing the angle θ by ϕ and r by R in the isotropic solution. The geometrical changes of the boundary for the associated isotropic problem can be constructed from Eqs. (11) and (12). Hence, a material point in the anisotropic problem located at $(r,\ \theta)$ is then to be changed to $(R,\ \phi)$ in the corresponding isotropic problem. It is worthy to note that if θ=0° (or θ=180°), we have Ψ=1 and ϕ=0° (or ϕ=180°). The material points at θ=0 (or θ=180°) therefore do not change when transforming to the corresponding isotropic problem. From the relationship of the transformation from anisotropic to isotropic problem in polar coordinate system as shown in Eqs. (11) and (12), this relationship in Cartesian coordinate system can also be obtained here as follows

$$X = x - \frac{c_{45}}{c_{44}} y \tag{16}$$

$$Y = \frac{C}{c_{44}} y \tag{17}$$

where x-y and X-Y are the respective Cartesian coordinate systems in anisotropic and isotropic problems. The linear coordinate transformation as expressed in Eqs. (16) and (17) that transform the general second order partial differential equation of elliptic type with constant coefficients into the standard second order partial differential equation is not unique. It has some interesting features in the Cartesian coordinate transformation. For a straight line (x_1, y_o), (x_2, y_o) parallel to the x-axis will still be a straight line (X_1, Y_o), (X_2, Y_o) parallel to the X-axis and with the same length after transformation, i.e., $X_2 - X_1 = x_2 - x_1$. For a vertical line (x_o, y_1), (x_o, y_2) parallel to the y-axis will be a straight line (X_o, Y_1), (X_1, Y_2) but with a rotation angle γ with respect to the Y-axis, i.e., $\tan \gamma = -(X_1 - X_o)/(Y_2 - Y_1) = c_{45}/C$. Since $C>0$, hence c_{45} will control the character of the rotation. There will be no rotation if $c_{45}=0$ and will rotate counterclockwise for $c_{45}>0$, clockwise for $c_{45}<0$.

The solutions for anisotropic material can be obtained from the corresponding isotropic problem in Cartesian coordinate as follows (Ma 1996)

$$w^a(x, y) = \frac{\mu}{C} w^i(X, Y) \tag{18}$$

$$\tau^a_{yz}(x, y) = \tau^i_{YZ}(X, Y) \tag{19}$$

$$\tau^a_{xz}(x, y) = \frac{1}{c_{44}} [C \tau^i_{XZ}(X, Y) + c_{45} \tau^i_{YZ}(X, Y)] \tag{20}$$

3. Some examples

In this section, five examples are presented to show the validation of the published relationship established by Ma (1996) and explained in detail how the complicated anisotropic problem can be solved by the associated simpler isotropic problem. A pair of point loadings being applied on a semi-infinite anisotropic crack faces with distance d from the crack tip is the first problem considered here. The traction boundary conditions are given as follows

$$\tau^a_{\theta z}(r, \pi) = \delta(r-d), \quad \tau^a_{\theta z}(r, -\pi) = \delta(r-d) \tag{21}$$

We now transform the anisotropic problem to the corresponding isotropic case. Since $\Psi(\pi) = \Psi(-\pi) = 1$, $\phi(\pi) = \pi$ and $\phi(-\pi) = -\pi$, the boundary conditions for the associated isotropic case can therefore be obtained by using (8)

$$\tau^i_{\theta z}(R, \pi) = \delta(R-d), \quad \tau^i_{\theta z}(R, -\pi) = \delta(R-d) \tag{22}$$

The well known full field solutions of shear stresses and displacement for isotropic material with boundary condition, Eq. (22) are

$$w^i(R,\phi)=\frac{1}{2\mu\pi}\ln\left[\frac{1+2\sqrt{R/d}\,\sin\phi/2+r/d}{1-2\sqrt{R/d}\,\sin\phi/2+R/d}\right] \tag{23}$$

$$\tau^i_{\theta z}(R,\phi)=\frac{1}{\pi d}\sqrt{\frac{d}{R}}\frac{(1+R/d)\cos\phi/2}{(R/d)^2+2(R/d)\cos\phi+1} \tag{24}$$

$$\tau^i_{rz}(R,\phi)=\frac{1}{\pi d}\sqrt{\frac{d}{R}}\frac{(1-R/d)\sin\phi/2}{(R/d)^2+2(R/d)\cos\phi+1} \tag{25}$$

The full field solutions for the anisotropic material subjected to the boundary conditions Eq. (21) can finally be obtained from Eqs. (7)-(9) as follows

$$w^a(r,\theta)=\frac{1}{2C\pi}\ln\left[\frac{1+2\sqrt{r\,\Psi(\theta)/d}\,\sin\phi/2+r\,\Psi(\theta)/d}{1-2\sqrt{r\,\Psi(\theta)/d}\,\sin\phi/2+r\,\Psi(\theta)/d}\right] \tag{26}$$

$$\tau^a_{\theta z}(r,\theta)=\frac{\Psi(\theta)}{\pi d}\sqrt{\frac{d}{r\,\Psi(\theta)}}\frac{(1+r\,\Psi(\theta)/d)\cos\phi/2}{(r\,\Psi(\theta)/d)^2+2(r\,\Psi(\theta)/d)\cos\phi+1} \tag{27}$$

$$\tau^a_{rz}(r,\theta)=\frac{1}{\pi d\,\Psi(\theta)}\sqrt{\frac{d}{r\,\Psi(\theta)}}\frac{\Theta(1+r\,\Psi(\theta)/d)\cos\phi/2+B(1-r\,\Psi(\theta)/d)\sin\phi/2}{(r\,\Psi(\theta)/d)^2+2(r\,\Psi(\theta)/d)\cos\phi+1} \tag{28}$$

The results shown in Eqs. (26)-(28) are found to be the same as those obtained by Ma (1992). The second problem to be discussed here is a finite crack with crack length $2a$ in an infinite anisotropic medium and subjected to uniform distributed loading τ at crack faces. Since the crack faces are located in the x-axis and the traction is expressed by $\tau_{\theta z}$, the crack length and the traction do not change when we transform the anisotropic problem to the corresponding isotropic problem. The solutions for shear stresses of the associated isotropic problem are

$$\tau^i_{\theta z}(R,\phi)=-\tau\cos\phi+\tau\left\{\mathrm{Re}\left[\frac{1}{\sqrt{1-(ae^{-i\phi}/R)^2}}\right]\cos\phi-\mathrm{Im}\left[\frac{1}{\sqrt{1-(ae^{-i\phi}/R)^2}}\right]\sin\phi\right\} \tag{29}$$

$$\tau^i_{rz}(R,\phi)=-\tau\sin\phi+\tau\left\{\mathrm{Re}\left[\frac{1}{\sqrt{1-(ae^{-i\phi}/R)^2}}\right]\sin\phi+\mathrm{Im}\left[\frac{1}{\sqrt{1-(ae^{-i\phi}/R)^2}}\right]\cos\phi\right\} \tag{30}$$

The solutions of the anisotropic problem can be constructed by using the results (29) and (30) for the isotropic solutions. The results are

$$\tau^a_{\theta z}(r,\theta)=-\tau\Psi(\theta)\cos\phi+\tau\Psi(\theta)\left\{\mathrm{Re}\left[\frac{1}{\sqrt{1-(ae^{-i\phi}/r\,\Psi(\theta))^2}}\right]\cos\phi-\mathrm{Im}\left[\frac{1}{\sqrt{1-(ae^{-i\phi}/r\,\Psi(\theta))^2}}\right]\sin\phi\right\} \tag{31}$$

$$\tau^a_{rz}(r,\theta)=-\tau\left[\sin\theta+\frac{c_{45}}{c_{44}}\cos\theta\right]+\tau\left\{\left[\sin\theta+\frac{c_{45}}{c_{44}}\cos\theta\right]\mathrm{Re}\left[\frac{1}{\sqrt{1-(ae^{-i\phi}/r\,\Psi(\theta))^2}}\right]+B\cos\theta\,\mathrm{Im}\left[\frac{1}{\sqrt{1-(ae^{-i\phi}/r\,\Psi(\theta))^2}}\right]\right\} \tag{32}$$

The results shown in Eqs. (31) and (32) are in agreement with those previously obtained by Sih and Chen (1980). Next, we consider a cylinder which is made of isotropic material and with inner radius a and outer radius b as shown in Fig. 1a. The boundary conditions are chosen to be

$$w^i(b, \phi)=0 \tag{33}$$

$$\tau^i_{rz}(a, \phi)=-\sin\phi \tag{34}$$

The solutions for stresses and displacement are very easy to obtain as follows

$$w^i(R, \phi)=\frac{a^2}{\mu(a^2+b^2)}\left(\frac{b^2}{R^2}-1\right)R\sin\phi \tag{35}$$

$$\tau^i_{rz}(R, \phi)=-\frac{a^2}{(a^2+b^2)}\left(\frac{b^2}{R^2}+1\right)\sin\phi \tag{36}$$

$$\tau^i_{\theta z}(R, \phi)=\frac{a^2}{(a^2+b^2)}\left(\frac{b^2}{R^2}-1\right)\cos\phi \tag{37}$$

The corresponding boundary in an anisotropic case can be obtained by Eq. (11) and the circular boundary becomes an ellipse as shown in Fig. 1b, with boundary conditions given as follows

$$w^a\left(\frac{b}{\Psi}, \theta\right)=0 \tag{38}$$

$$\tau^a_n\left(\frac{a}{\Psi}, \theta\right)=-\frac{1}{B\sin\theta}\left\{\Psi^2+2\frac{c^2_{45}}{c^2_{44}}\sin^2\theta-\frac{c_{45}}{c_{44}}\sin 2\theta\right\}^{1/2} \tag{39}$$

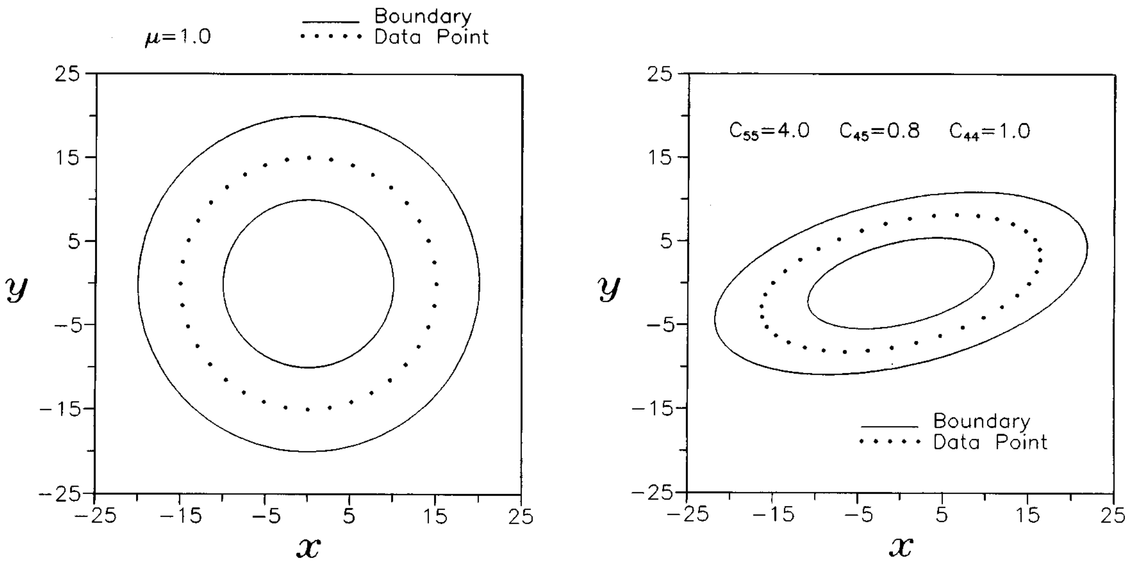

Fig. 1 (a) An isotropic cylinder with an inner radius a=10 and an outer radius b=20 (b) The elliptic boundary in an anisotropic problem

The solutions for anisotropic material subjected to boundary condition Eqs. (38) and (39) are obtained from Eqs. (7)-(9)

$$w^a(r,\theta)=\frac{a^2}{c_{44}(a^2+b^2)}\left(\frac{b^2}{r^2\Psi^2}-1\right)r\sin\theta \tag{40}$$

$$\tau^a_{rz}(r,\theta)=\frac{a^2}{(a^2+b^2)}\frac{1}{\Psi^2}\left\{\left(\frac{b^2}{r^2\Psi^2}-1\right)\left(\cos\theta-\frac{c_{45}}{c_{44}}\sin\theta\right)\Theta-B^2\left(\frac{b^2}{r^2\Psi^2}+1\right)\sin\theta\right\} \tag{41}$$

$$\tau^a_{\theta z}(r,\theta)=\frac{a^2}{(a^2+b^2)}\left(\frac{b^2}{r^2\Psi^2}-1\right)\left(\cos\theta-\frac{c_{45}}{c_{44}}\sin\theta\right) \tag{42}$$

The analytical solutions shown in Eqs. (40)-(42) are verified by using the numerical boundary element method. The results for displacement and stresses are displaced in Figs. 2-4. A perfect agreement of the analytical and numerical solutions are indicated in the figures.

A finite square with a circular hole made of anisotropic material (c_{55}/c_{44}=4, c_{45}/c_{44}=0.8) as shown in Fig. 5 is investigated by numerical boundary element method. A uniformly distributed antiplane loading of unit magnitude is applied at the boundary $y=\pm 20$ and the circular hole of radius 10 is fixed. After transformation to the corresponding isotropic problem, the square boundary becomes a parallelogram and the circular hole becomes an elliptic hole as shown in Fig. 5. The boundary conditions for the associated isotropic problem are uniformly distributed unit loading applied at $Y=\pm 36.66$ and the elliptic hole is held fixed. The data points at x=15 for anisotropic problem is evaluated by the boundary elememt method in two different ways. One directly solves the

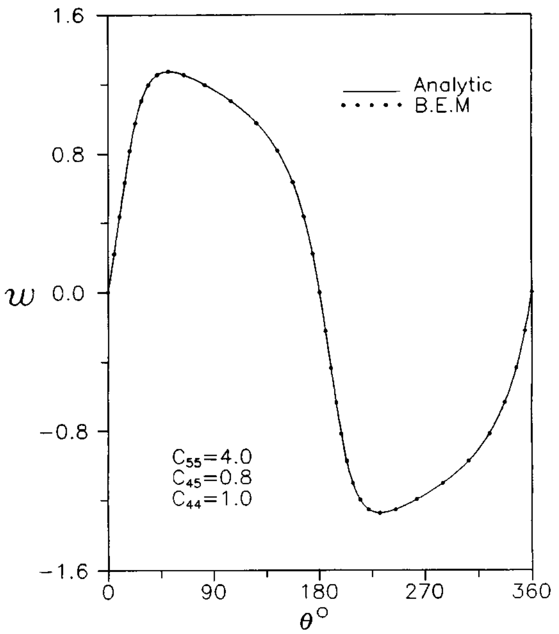

Fig. 2 Numerical results of displacement w for an anisotropic material with elliptic boundary

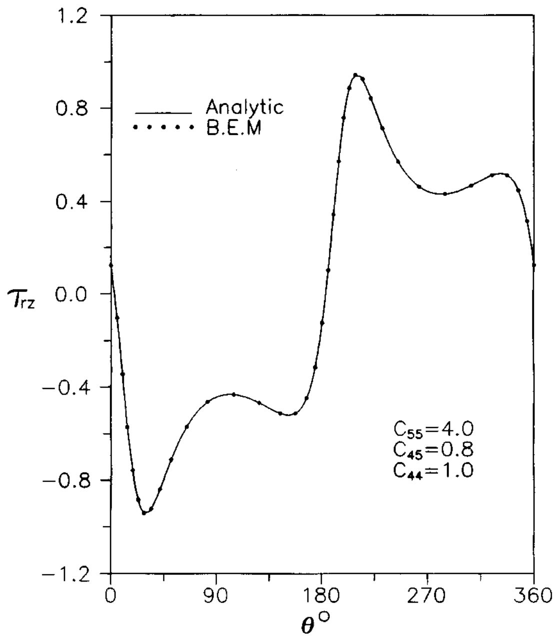

Fig. 3 Numerical results of shear stress τ_{rz} for an anisotropic material with elliptic boundary

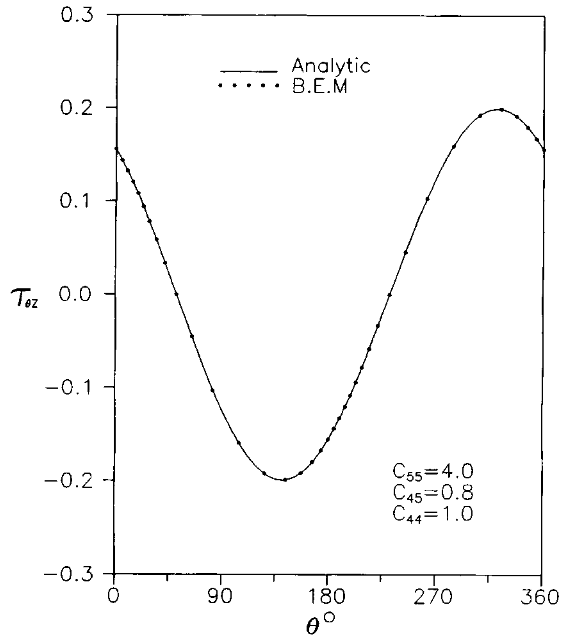

Fig. 4 Numerical results of shear stress $\tau_{\theta z}$ for an anisotropic material with elliptic boundary

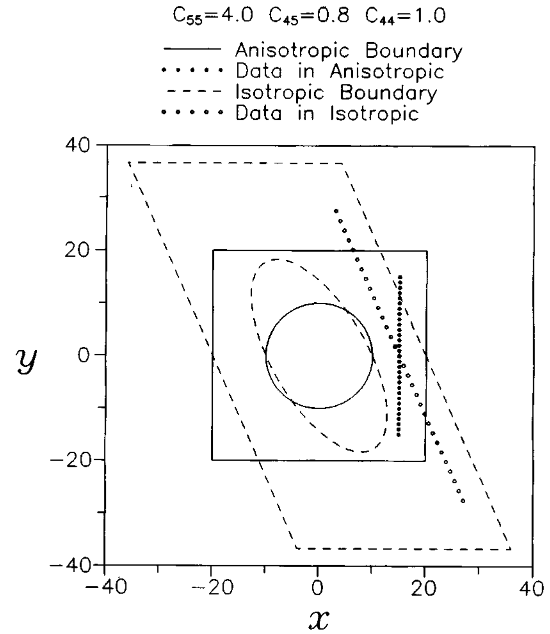

Fig. 5 A finite square with a circular hole for an anisotropic problem and the configuration for the corresponding isotropic problem

anisotropic problem, the other one calculates the solutions for the corresponding isotropic problem first and then uses the relationships presented in Eqs. (7)-(9) to obtain the solutions for the original anisotropic problem. The results of displacement and stresses obtained by these two ways

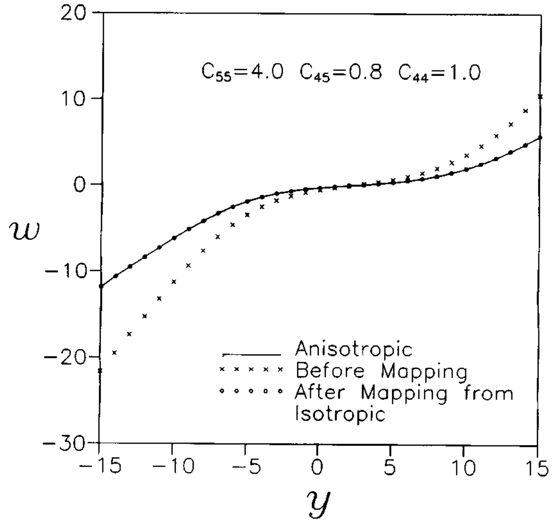

Fig. 6 Numerical results of displacement w for an anisotropic material for a finite square with a circular hole

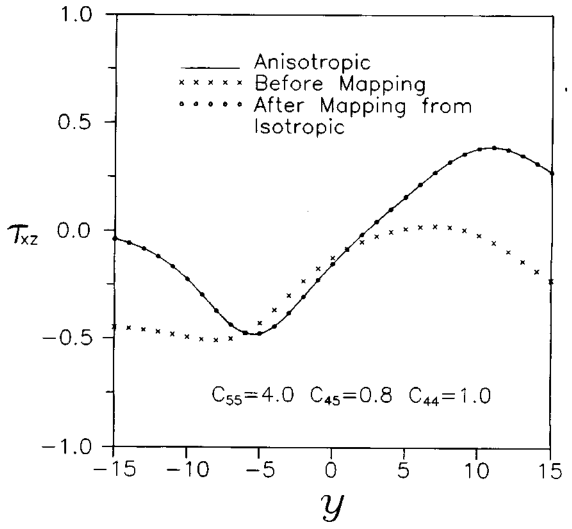

Fig. 7 Numerical results of shear stress τ_{xz} for an anisotropic material for a finite square with a circular hole

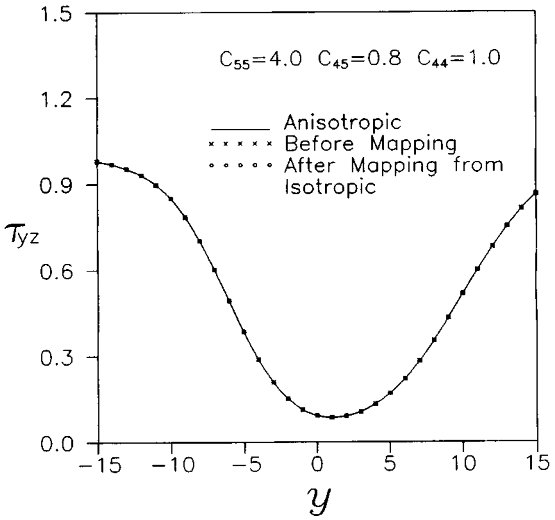

Fig. 8 Numerical results of shear stress τ_{yz} for an anisotropic material for a finite square with a circular hole

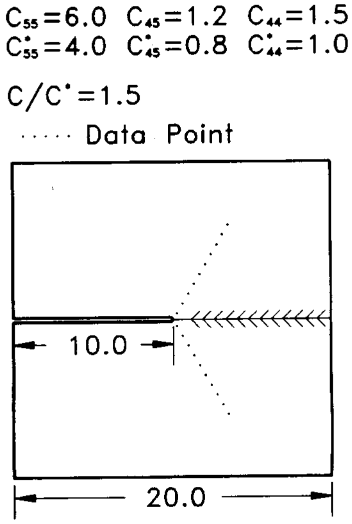

Fig. 9 An anisotropic structure with an interfacial crack

are shown in Figs. 6-8. We can see the perfect agreement of these two methods, the solutions for the isotropic problem before transformation are also indicated in the figures.

Finally, a composite structure made of two different anisotropic materials with material constants c_{44}=1.5, c_{45}=1.2 and c_{55}=6.0 in the upper part and with c_{44}*=1.0, c_{45}*=0.8 and c_{55}*= 4.0 in the lower part. An interfacial crack with a crack length of 10 is present in the interface of the composite as shown in Fig. 9. A uniformly distributed loading with unit magnitude is applied at $y=\pm 10$ of the boundary. The corresponding isotropic problem after transformation is shown in

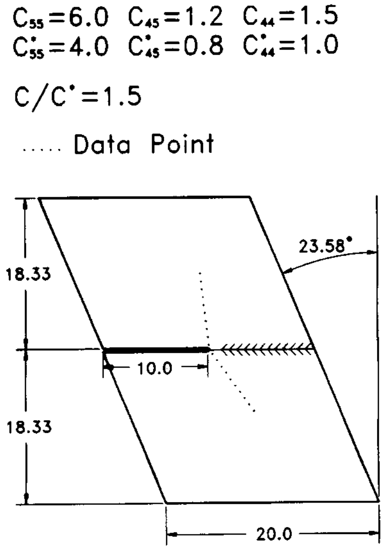

Fig. 10 The associated boundary in the corresponding isotropic problem

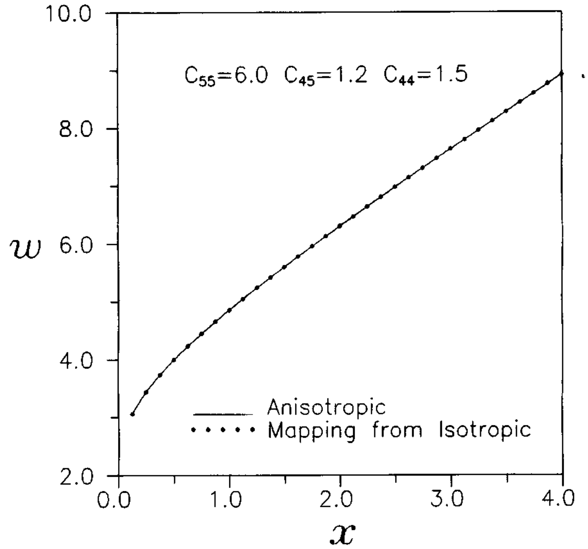

Fig. 11 Numerical results of displacement w for an anisotropic interfacial crack problem

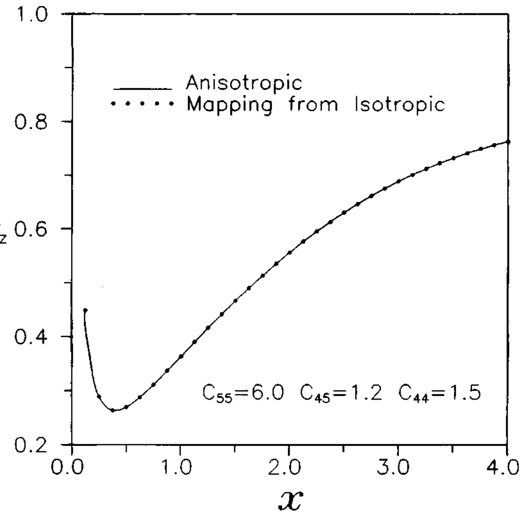

Fig. 12 Numerical results of shear stress τ_{rz} for an anisotropic interfacial crack problem

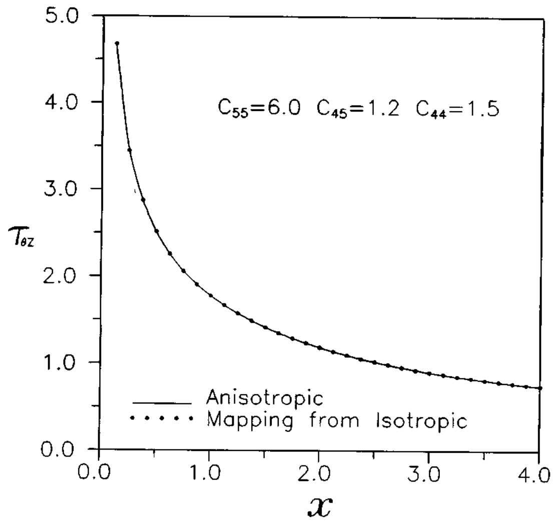

Fig. 13 Numerical results of shear stress $\tau_{\theta z}$ for an anisotropic interfacial crack problem

Fig. 10. We have μ=3 in the upper part and μ^*=2 in the lower part with uniformly distributed unit loading applied at the boundary of Y=18.33. As we have indicated previously that the linear coordinate transformation (i.e., Eq. (16) and (17)) is not unique and this study just choose one of those. The linear coordinate transformation provided in this study has a good feature that field quantities (i.e., displacement and stress) of two materials still continuous at the interface as shown in Fig. 10. This problem is solved here by two methods: one solves the problem directly by boundary element method and the other one solves the corresponding isotropic problem first and then obtains the original anisotropic problem by using the solutions for an isotropic case. The results in the upper material for these two methods are in excellent agreement and are shown in Figs. 11-13.

4. Conclusions

Solving an isotropic problem has always been easier than the corresponding anisotropic problem in both the analytical analysis and the numrical investigation. The antiplane problem of anisotropic materials has been investigated in detail in this study. The relationship for the stress and displacement between the anisotropic materials and the corresponding isotropic problems have been established by Ma (1996). These relations suggest an easier way to solve the anisotropic problem by the corresponding isotropic solutions. With this correspondence at hand, investigating the complicated antiplane anisotropic problem has become very convenient. The attention needs, therefore, only be focused on the problem of isotropic materials.

The anisotropic problem has been shown here to be able to be converted into the one involving isotropic material by properly changing the geometry of the body and the tractions on the boundary. Some examples have been provided for indicating how to solve the anisotropic problems by using the corresponding isotropic solutions. Each example has shown that this method has become a good way for solving anisotropic antiplane problems. However the coordinate transformation associated with the relationship will change the geometry and boundary conditions of an anti-plane problem in anisotropic materials. Such coordinate transformation may result in complicated geometry and boundary conditions for the corresponding problem in isotropic materials. For problems with simple geometry (i.e., first and second examples), analytical solutions for anisotropic materials are especially suitable by using the proposed transformation method. For the finite boundary case (i.e., fourth and fifth examples), the geometry of the corresponding anti-plane problem in isotropic materials is a little bit complexity than that in anisotropic materials, but it is still in the same order of complexity if the numerical method is used to solve the problem.

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