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Study on thermal buckling and post-buckling behaviors of FGM tubes resting on elastic foundations

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Abstract. This paper studies thermal buckling and post-buckling behaviors of functionally graded materials (FGM) tubes subjected to a uniform temperature rise and resting on elastic foundations via a refined beam model. Compared to the Timoshenko beam theory, the number of unknowns of this model are the same and no correction factors are required. The material properties of the FGM tube vary continuously in the radial direction according to a power function. Two ends of the tube are assumed to be simply supported and in-plane boundary conditions are immovable. Energy variation principle is employed to establish the governing equations. A two-step perturbation method is adopted to determine the critical thermal buckling loads and post-buckling paths of the tubes with arbitrary radial non-homogeneity. Through detailed parametric studies, it can be found that the tube has much higher buckling temperature and post-buckling strength when it is supported by an elastic foundation.

Keywords: functionally graded materials; tubes; thermal buckling; post-buckling; elastic foundation

1. Introduction

Tubes are widely encountered in the nature and engineering applications. For example, bamboo and straw are the stems of some plants which can be understood as a representative of tubes (Zhang and Fu 2013), animal's vessels and tracheae can also be regarded as tubes (Fu *et al.* 2015, Zhong *et al.* 2016, She *et al.* 2017a, b). In addition, in practical project application, tubes play an important role of conveying a variety of fluids or gas, which is always round in cross-section. Most of tubes in engineering are made by metals (such as steel, iron and copper), plastic and rubber. Due to superior performance of health, environmental protection, low cost, tubes have applied in many disciplines such as spaceflight, petrochemical engineering, mine and nuclear industry. On the other hand, functionally graded materials (FGM) is a new non-uniform composite materials. Many researches for FGM structures can be found in references (e.g., Kiani 2016, Sun *et al.* 2016, She *et al.* 2017, Kiani *et al.* 2010a, b, Wattanasakulpong *et al.* 2011, Tossapanon and Wattanasakulpong 2016, Wu *et al.* 2016, Shvartsman and Majak 2016, Hadji *et al*. 2016, Hadji *et al*. 2017, Hadji *et al*. 2016, Hadji and Bedia 2015, Mouaici *et al.* 2016, Ebrahimi and Javari 2016, Ebrahimi and Habibi 2016, Ebrahimi and Zia 2015, Lal *et al.* 2016, Heydari *et al.* 2016, Gan 2016, Rajasekaran and Khaniki 2017, Tuna and Kirca 2016, Nejad and Hadi 2016a, b, Nejad *et al*. 2016, Şimşek 2016, Huang *et al.* 2017, Nejad *et al.* 2017, Ji *et al.* 2017, Song *et al.* 2017, Wang *et al.* 2016, Zhu *et al.* 2017, Bousahla *et al.* 2016, Bouderba *et al.* 2016, Amar *et al.* 2017, Zouatnia *et al.* 2017, Zhao *et al.* 2016, Tu *et al.* 2017, El-Haina *et al.* 2017, Barati 2017a, b, Ebrahimi and Barati 2016, Ebrahimi *et al*. 2016, Shahverdi and Barati 2017, Elmossouess *et al.* 2017, Zidi *et al.* 2017, Ebrahimi and Daman 2017a, b, Ebrahimi *et al*. 2017, Ebrahimi *et al*. 2017, Karami *et al*. 2018, Karami *et al*. 2017, Chikh *et al.* 2016, Dai and Dai 2014, 2015, 2016, 2017).

Recently, Zhang and Fu (2013) advanced a refined beam theory for analysis of tubes. Compared to the Timoshenko beam theory, the number of unknowns are the same and no correction factors are required. Based on this model, Fu *et al*. (2015), She *et al*. (2017a), She *et al*. (2017) analyzed the thermal buckling and post-buckling of FGM tubes. Besides, a set of studies performed by Dehrouyeh-Semnani and his partners (Dehrouyeh-Semnani 2017, Dehrouyeh-Semnani 2018, Dehrouyeh-Semnani *et al*. 2017) are devoted to the thermal buckling and snap-through buckling of functionally graded beams or nanobeams.

However, previous works (Zhang and Fu 2013, Fu *et al.* 2015, Zhong *et al.* 2016, She *et al*. 2017a) only have focused on thermal buckling analysis of FGM tubes without any elastic foundation. This paper considers the effects of the elastic foundations and intends to study the nonlinear thermal stability of FGM tubes.

The novel contributions in this study may be summarized as follows:

(i) A higher-order shear deformation theory for tubes is used to formulate the mechanical model, and the nonlinear strain-displacement relationships is also considered.

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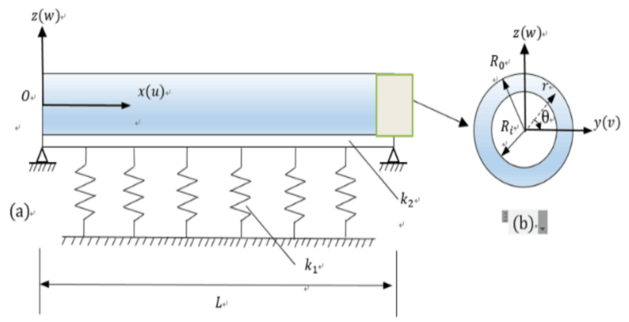

Fig. 1 An FGM tube with simply supported ends

(ii) To solve this problem, a two-step perturbation method is employed to obtain the expression of thermal post-buckling load-deflection relationship, and the solutions of Euler and Timoshenko beam models are also presented.

(iii) The effects of transverse shear deformation, volume fraction index and foundation stiffness on the thermal post-buckling response are discussed in detail.

2. Theoretical formulations

Consider an FGM tube with length L, inner radius R_i and outer radius R_0. The tube is referred to a coordinate system (x, y, z), and the corresponding displacements are designated by u_1, u_2, u_3. φ is the rotation, w is deflection, as shown in Fig. 1. The tube rests on a two-parameter elastic foundation, according to Shen (2013), the load-displacement relationship of the elastic foundations can be expressed by $p = k_1 w - k_2 (d^2 w / d^2 x)$. Assuming that the effective properties P_f are exponential function ($P_f = P_f(r)$) in the radial direction. According to the tube model, the displacement fields have the form (Zhang and Fu 2013, Fu *et al.* 2015, Zhong *et al.* 2016, She *et al.* 2018)

$$u_1(x,y,z) = u(x) + f(y,\ z)\frac{dw}{dx} + g(y,\ z)\varphi(x) \tag{1a}$$

$$u_2(x,y,z) = 0 \tag{1b}$$

$$u_3(x,y,z) = w(x) \tag{1c}$$

in which

$$f(y,z) = z\left(R_0^{\,2} R_i^2 r^{-2} - r^2/3\right)\left(R_0^{\,2} + R_i^{\,2}\right)^{-1} \tag{2a}$$

$$g(y,z) = z + z\left(R_0^{\,2} R_i^2 r^{-2} - r^2/3\right)\left(R_0^{\,2} + R_i^{\,2}\right)^{-1} \tag{2b}$$

In Eq. (2), $z = r\sin\theta$, $y = r\cos\theta$, it should be pointed out that, if taking $f(y,z) = 0$, Eq. (1) will have the same forms of Timoshenko beam, if taking $f(y,z) = -z$, Eq. (1) will have the same forms of Euler beam. Considering nonlinear strain-displacement relationships, the normal strain ε_{xx} and shear stress strains γ_{xy}, γ_{xz} are (She *et al.* 2017a)

$$\varepsilon_{xx} = \frac{du_1}{dx} + \frac{1}{2}\left(\frac{du_3}{dx}\right)^2 = \varepsilon_x^{(0)} + f\varepsilon_x^{(1)} + g\varepsilon_x^{(2)} \tag{3a}$$

$$\gamma_{xy} = \frac{\partial f}{\partial y}\gamma_{xz}^{(0)} \tag{3b}$$

$$\gamma_{xz} = \left(1 + \frac{\partial f}{\partial z}\right)\gamma_{xz}^{(0)} \tag{3c}$$

in which

$$\varepsilon_x^{(0)} = \frac{du}{dx} + \frac{1}{2}\left(\frac{dw}{dx}\right)^2 \tag{4a}$$

$$\varepsilon_x^{(1)} = \frac{d^2 w}{dx^2} \tag{4b}$$

$$\varepsilon_x^{(2)} = \frac{d\varphi}{dx} \tag{4c}$$

$$\gamma_{xz}^{(0)} = \left(\varphi + \frac{dw}{dx}\right) \tag{4d}$$

The constitutive relations can be expressed as

$$\sigma_{xx} = E_f(r,T)\left[\varepsilon_{xx} - \alpha(r,T)\Delta T\right] \tag{5a}$$

$$\tau_{xy} = G_f(r,T)\gamma_{xy} \tag{5b}$$

$$\tau_{xz} = G_f(r,T)\gamma_{xz} \tag{5c}$$

where $E_f(r,\ T)$ is Young's modulus, $G_f(r,\ T)$ the shear modulus, and $\nu_f(r,\ T)$ is Poisson ratio, ΔT is the temperature rise, N^T is the thermal force.

The stress resultants and couples can be defined by

$$N_x = \int_A \sigma_{xx} dA = A_0\varepsilon_x^{(0)} - N^T \tag{6a}$$

$$M_x = \int_A \sigma_{xx} f dA = A_1\varepsilon_x^{(1)} + A_2\varepsilon_x^{(2)} \tag{6b}$$

$$P_x = \int_A \sigma_{xx} g dA = A_2\varepsilon_x^{(1)} + A_3\varepsilon_x^{(2)} \tag{6c}$$

$$Q = \int_A\left[\tau_{xy}\frac{\partial f}{\partial y} + \tau_{xz}\left(1 + \frac{\partial f}{\partial z}\right)\right] dA = A_4\gamma_{xz}^{(0)} \tag{6d}$$

in which

$$A_0 = \int_A E_f(r,T) dA \tag{7a}$$

$$A_1 = \int_A E_f(r,T) f^2 dA \tag{7b}$$

$$A_2 = \int_A E_f(r,T)(zf + f^2)dA \tag{7c}$$

$$A_3 = \int_A E_f(r,T)(z^2 + 2zf + f^2)dA \tag{7d}$$

$$A_4 = \int_A \left\{ \frac{E_f(r,T)}{2[1+\upsilon_f(r,T)]} \left[\left(\frac{\partial f}{\partial y}\right)^2 + \left(\frac{\partial f}{\partial z}\right)^2 + 2\frac{\partial f}{\partial z} + 1 \right] \right\} dA \tag{7e}$$

$$N^T = \int_A E_f(r,T)\alpha_f(r,T)\Delta T dA \tag{7f}$$

The total energy can be expressed as

$$\Pi = \frac{1}{2}\left[\int_\Omega \left(\sigma_{xx}\varepsilon_{xx} + \tau_{xy}\gamma_{xy} + \tau_{xz}\gamma_{xz} - \sigma_{xx}\alpha_f \Delta T\right)\right] + \frac{k_1}{2}w^2 + \frac{k_2}{2}\left(\frac{dw}{dx}\right)^2 \tag{8}$$

By integrating Eq. (8), one can obtain the following governing equations

$$\delta u: \quad \frac{dN_x}{dx} = 0 \tag{9a}$$

$$\delta w: \quad \frac{d^2M_x}{dx^2} - N_x\frac{d^2w}{dx^2} - \frac{dQ}{dx} + \left(k_1 w - k_2\frac{d^2w}{dx^2}\right) = 0 \tag{9b}$$

$$\delta\varphi: \quad \frac{dP_x}{dx} - Q = 0 \tag{9c}$$

Eq. (9a) implies, for immovable supports,

$$N_x = A_0\left[\frac{du}{dx} + \frac{1}{2}\left(\frac{dw}{dx}\right)^2\right] - N^T = \frac{A_0}{2L}\int_L \left(\frac{dw}{dx}\right)^2 dx - N^T \tag{10}$$

Substituting Eqs. (3), (5), (6) and (10) back into Eq. (9) leads to the following governing equations

$$A_1\frac{d^4w}{dx^4} + A_2\frac{d^3\varphi}{dx^3} - N_x\frac{d^2w}{dx^2} - A_4\left(\frac{d^2w}{dx^2} + \frac{d\varphi}{dx}\right) + \left(k_1 w - k_2\frac{d^2w}{dx^2}\right) = 0 \tag{11a}$$

$$A_2\frac{d^3w}{dx^3} + A_3\frac{d^2\varphi}{dx^2} - A_4\left(\frac{dw}{dx} + \varphi\right) = 0 \tag{11b}$$

$$N_x = \frac{A_0}{2L}\int_L \left(\frac{dw}{dx}\right)^2 dx - N^T \tag{11c}$$

If we introduce

$$\xi = \frac{\pi x}{L} \tag{12a}$$

$$W = \frac{w}{L} \tag{12b}$$

$$\Phi = \frac{\varphi}{\pi} \tag{12c}$$

$$\gamma_0 = \frac{A_0 L^2}{\pi^2 D} \tag{12d}$$

$$\gamma_1 = \frac{A_1}{D} \tag{12e}$$

$$\gamma_2 = \frac{A_2}{D} \tag{12f}$$

$$\gamma_3 = \frac{A_3}{D} \tag{12g}$$

$$\gamma_4 = \frac{A_4 L^2}{\pi^2 D} \tag{12h}$$

$$\left(\overline{K}_1, K_1\right) = k_1\left(\frac{L^4}{\pi^4 D}, \frac{L^4}{E_m I}\right) \tag{12i}$$

$$\left(\overline{K}_2, K_2\right) = k_2\left(\frac{L^4}{\pi^4 D}, \frac{L^4}{E_m I}\right) \tag{12j}$$

$$\gamma_T = \frac{A_x^T L^2}{\pi^2 D} \tag{12k}$$

$$\lambda_T = \Delta T \tag{12l}$$

in which E_m denotes Young's modulus of metal at 300K, and

$$D = \int_A z^2 E_f(r)dz \tag{13a}$$

$$A_x^T = \int_A \alpha_f(r)E_f(r)dz \tag{13b}$$

$$I = \int_A z^2 dz \tag{13c}$$

The introduction of the dimensionless quantities in Eq. (12) enables Eq. (11) to be rewritten as

$$\gamma_1\frac{d^4W}{d\xi^4} + \gamma_2\frac{d^3\Phi}{d\xi^3} - \left[\frac{\pi\gamma_0}{2}\int_0^\pi \left(\frac{dW}{d\xi}\right)^2 d\xi\right]\frac{d^2W}{d\xi^2} + \gamma_T\lambda_T\frac{d^2W}{d\xi^2} - \gamma_4\left(\frac{d^2W}{d\xi^2} + \frac{d\Phi}{d\xi}\right) + \left(\overline{K}_1 W - \overline{K}_2\frac{d^2W}{d\xi^2}\right) = 0 \tag{14a}$$

$$\gamma_2 \frac{d^3W}{d\xi^3} + \gamma_3 \frac{d^2\Phi}{d\xi^2} - \gamma_4 \left(\frac{dW}{d\xi} + \Phi \right) = 0 \tag{14b}$$

3. Solution methodology

The solution of Eq. (14) will be solved by the perturbation method (Shen 2013, 2014, She *et al*. 2017, She *et al*. 2017a, b, c), and the displacements and thermal loading are to assume that

$$\lambda_T(\xi,\varepsilon) = \sum_{k=1} \varepsilon^k \lambda_k(\xi) \tag{15a}$$

$$W(\xi,\varepsilon) = \sum_{k=1} \varepsilon^k w_k(\xi) \tag{15b}$$

$$\Phi(\xi,\varepsilon) = \sum_{k=1} \varepsilon^k \varphi_k(\xi) \tag{15c}$$

Substituting Eq. (15) into Eq. (14), a series of perturbation equations can be obtained. The first-order equation is

$$\begin{aligned} O(\varepsilon^1): \gamma_1 \frac{d^4w_1}{d\xi^4} + \gamma_2 \frac{d^3\varphi_1}{d\xi^3} + \lambda_T^{(0)}\gamma_T \frac{d^2w_1}{d\xi^2} \\ -\gamma_4\left(\frac{d^2w_1}{d\xi^2} + \frac{d\varphi_1}{d\xi}\right) \\ +\left(\overline{K}_1 w_1 - \overline{K}_2 \frac{d^2w_1}{d\xi^2}\right) = 0 \end{aligned} \tag{16a}$$

$$\gamma_2 \frac{d^3w_1}{d\xi^3} + \gamma_3 \frac{d^2\varphi_1}{d\xi^2} - \gamma_4 \left(\frac{dw_1}{d\xi} + \varphi_1 \right) = 0 \tag{16b}$$

We assume the solution of the Eq. (16) for simply supported ends, are

$$w_1 = A_{10}^{(1)} \sin(m\xi) \tag{17a}$$

$$\varphi_1 = B_{10}^{(1)} \cos(m\xi) \tag{17b}$$

Putting Eq. (17) into Eq. (16) leads to

$$\begin{aligned} \gamma_1 m^4 A_{10}^{(1)} + \gamma_2 m^3 B_{10}^{(1)} - \gamma_T \lambda_T^{(0)} m^2 A_{10}^{(1)} + \gamma_4 m^2 A_{10}^{(1)} \\ + \gamma_4 m B_{10}^{(1)} + \overline{K}_1 A_{10}^{(1)} + m^2 \overline{K}_2 A_{10}^{(1)} = 0 \end{aligned} \tag{18}$$

$$-m^3\gamma_2 A_{10}^{(1)} - \gamma_3 m^2 B_{10}^{(1)} - \gamma_4\left(m A_{10}^{(1)} + B_{10}^{(1)}\right) = 0 \tag{19}$$

Then, we can obtain

$$\begin{aligned} \lambda_T^{(0)} = \frac{m^2}{\gamma_T}\left(\frac{\overline{K}_1}{m^4} + \frac{\overline{K}_2}{m^2}\right) \\ + \frac{m^2}{\gamma_T}\left(\gamma_1 - \gamma_2 \frac{m^2\gamma_2 + \gamma_4}{m^2\gamma_3 + \gamma_4} - \gamma_4 \frac{\gamma_2 - \gamma_3}{m^2\gamma_3 + \gamma_4}\right) \end{aligned} \tag{20}$$

The third-order equation is

$$\begin{aligned} O(\varepsilon^3): \gamma_1 \frac{d^4w_3}{d\xi^4} + \gamma_2 \frac{d^3\varphi_3}{d\xi^3} + \lambda_T^{(0)}\gamma_T \frac{d^2w_3}{d\xi^2} \\ -\gamma_4\left(\frac{d^2w_3}{d\xi^2} + \frac{d\varphi_3}{d\xi}\right) + \left(\overline{K}_1 w_3 - \overline{K}_2 \frac{d^2w_3}{d\xi^2}\right) \\ = -\lambda_T^{(2)}\gamma_T \frac{d^2w_1}{d\xi^2} + \frac{\gamma_0\pi}{2}\int_0^\pi \left(\frac{dw_1}{d\xi}\right)^2 d\xi \frac{d^2w_1}{d\xi^2} \end{aligned} \tag{21a}$$

$$\gamma_2 \frac{d^3w_3}{d\xi^3} + \gamma_3 \frac{d^2\varphi_3}{d\xi^2} - \gamma_4 \left(\frac{dw_3}{d\xi} + \varphi_3 \right) = 0 \tag{21b}$$

We assume the solution of the Eq. (21), is

$$w_3(\xi) = A_{30}^{(3)} \sin(3m\xi) \tag{22a}$$

$$\varphi_3(\xi) = B_{30}^{(3)} \cos(3m\xi) \tag{22b}$$

Substituting Eq. (22) into Eq. (21) leads to

$$\begin{aligned} \left[81m^4\gamma_1 - 81m^4\gamma_2\left(\frac{9m^2\gamma_2 + \gamma_4}{9m^2\gamma_3 + \gamma_4}\right)\right] A_{30}^{(3)} \sin(3m\xi) \\ -\left[81m^4\gamma_4\left(\frac{\gamma_2 - \gamma_3}{9m^2\gamma_3 + \gamma_4}\right)\right] A_{30}^{(3)} \sin(3m\xi) \\ -9m^2\gamma_T\lambda_T^{(0)} A_{30}^{(3)} \sin(3m\xi) \\ +\left[\overline{K}_1 + 9m^2\overline{K}_2\right] A_{30}^{(3)} \sin(3m\xi) \\ = -\frac{\pi^2\gamma_0}{4} m^4 \left(A_{10}^{(1)}\right)^3 \sin(m\xi) \\ + m^2\gamma_T\lambda_T^{(2)} A_{10}^{(1)} \sin(m\xi) \end{aligned} \tag{23a}$$

$$-27m^3\gamma_2 A_{30}^{(3)} - 9\gamma_3 m^2 B_{30}^{(3)} - \gamma_4\left(3m A_{30}^{(3)} + B_{30}^{(3)}\right) = 0 \tag{23b}$$

From Eq. (23), obtains

$$\lambda_T^{(2)} = \frac{m^2}{4\gamma_T}\left(\pi^2\gamma_0\right)\left(A_{10}^{(1)}\right)^2 \tag{24a}$$

$$A_{30}^{(3)} = 0 \tag{24b}$$

As a result, the asymptotic solutions can be obtained as

$$\begin{aligned} W(\xi,\varepsilon) = A_{10}^{(1)}\sin(m\xi) + A_{30}^{(3)}\sin(3m\xi) + O(\varepsilon^4) \\ = A_{10}^{(1)}\sin(m\xi) + O(\varepsilon^4) \end{aligned} \tag{25}$$

$$\begin{aligned} \Phi(\xi,\varepsilon) = B_{10}^{(1)}\cos(m\xi) + B_{30}^{(3)}\cos(3m\xi) + O(\varepsilon^4) \\ = -\frac{m A_{10}^{(1)}\left(m^2\gamma_2 + \gamma_4\right)}{m^2\gamma_3 + \gamma_4}\cos(m\xi) + O(\varepsilon^4) \end{aligned} \tag{26}$$

$$\lambda_T = \lambda_T^{(0)} + \lambda_T^{(2)} + O\left(\varepsilon^4\right) = \frac{m^2}{\gamma_T}\left(\gamma_1 - \gamma_2\frac{m^2\gamma_2+\gamma_4}{m^2\gamma_3+\gamma_4} - \gamma_4\frac{\gamma_2-\gamma_3}{m^2\gamma_3+\gamma_4}\right) + \frac{m^2}{\gamma_T}\left(\frac{\overline{K}_1}{m^4}+\frac{\overline{K}_2}{m^2}\right) + \frac{m^2}{4\gamma_T}\left(\pi^2\gamma_0\right)\left(A_{10}^{(1)}\right)^2 + O\left(\varepsilon^4\right) \tag{27}$$

Taking $\left[\xi = \pi/(2m)\right]$ in Eq. (25), we obtain

$$W\big|_{x=(\pi/2m)} = W_m = A_{10}^{(1)}\sin(\pi/2) + A_{30}^{(3)}\sin(3\pi/2) + O\left(\varepsilon^4\right) = A_{10}^{(1)} + O\left(\varepsilon^4\right) \tag{28}$$

Combining Eqs. (28), (27) and (12b), the thermal post-buckling load-deflection relationship for present model can be obtained as

$$W_m{}^2 = \frac{\lambda_T - \frac{m^2}{\gamma_T}\left(\frac{\overline{K}_1}{m^4}+\frac{\overline{K}_2}{m^2}\right)}{\frac{m^2}{4\gamma_T}\left(\pi^2\gamma_0\right)} + \frac{-\frac{m^2}{\gamma_T}\left(\gamma_1 - \gamma_2\frac{m^2\gamma_2+\gamma_4}{m^2\gamma_3+\gamma_4} - \gamma_4\frac{\gamma_2-\gamma_3}{m^2\gamma_3+\gamma_4}\right)}{\frac{m^2}{4\gamma_T}\left(\pi^2\gamma_0\right)} \tag{29}$$

As indicated by references (Shen 2013, Shen and Wang 2014, Shen 2014), the critical buckling loads can be obtained by setting $W_m = 0$, thus, the critical buckling temperature is given by $\Delta T_{cr}(\text{present}) = \frac{m^2}{\Upsilon_T}\left(\frac{\bar{K}_1}{m^4}+\frac{\bar{K}_2}{m^2}+\Upsilon_1-\Upsilon_2\frac{m^2\Upsilon_2+\Upsilon_4}{m^2\Upsilon_3+\Upsilon_4}-\Upsilon_4\frac{\Upsilon_2-\Upsilon_3}{m^2\Upsilon_3+\Upsilon_4}\right)$ for the present model. For Timoshenko beam, $\Delta T_{cr}(\text{Timoshenko}) = \frac{m^2}{\Upsilon_T}\left(\frac{\bar{K}_1}{m^4}+\frac{\bar{K}_2}{m^2}+\frac{\Upsilon_3\Upsilon_4\kappa_S}{m^2\Upsilon_3+\kappa_S\Upsilon_4}\right)$, the exact value of the shear correction factor κ_S can be obtained from She *et al.* (2017a). For Euler beam, $\Delta T_{cr}(\text{Euler}) = \frac{m^2}{\Upsilon_T}\left(\frac{\bar{K}_1}{m^4}+\frac{\bar{K}_2}{m^2}+\Upsilon_1\right)$, In the following analysis *m*=1.

4. Results and discussions

The following example of comparison are shown to verify the correctness and reliability of the research content, taking R_i = 0, for the tube without any foundation, the corresponding dimensionless buckling load $N_T L^2/(EI)$ for the present model, Timoshenko beam theory and Euler beam model are denoted by P_{cr}^P, P_{cr}^T and P_{cr}^E, and

Table 1 Comparisons of buckling loads for a simply supported homogeneous columns without any foundation

Source	*L/R*	HSDT	Timoshenko		Euler
			κ_1	κ_2	
Huang and Li (2010)	5	7.5956	7.6538	7.7266	9.8996
Present	5	7.6010	7.6538	7.7266	9.8996
Huang and Li (2010)	10	9.1824	9.2035	9.2296	9.8996
Present	10	9.1828	9.2035	9.2296	9.8996
Huang and Li (2010)	15	9.5519	9.5620	9.5745	9.8996
Present	15	9.5520	9.5620	9.5745	9.8996
Huang and Li (2010)	20	9.6883	9.6942	9.7014	9.8996
Present	20	9.6884	9.6942	9.7014	9.8996
Huang and Li (2010)	25	9.7528	9.7566	9.7613	9.8996
Present	25	9.7528	9.7566	9.7613	9.8996

Table 2 Material properties for Si_3N_4 (υ=0.24) and SUS304 (υ=0.3262), from Fu *et al*. (2015), Reddy and Chin (1998)

Materials	Proprieties	P_0	P_{-1}	P_1	P_2	P_3
Si_3N_4	E_c(Pa)	348.43e+9	0.0	-3.070e-4	2.160e-7	-8.964e-11
	α_c (1/K)	5.8723e-6	0.0	9.095e-4	0.0	0.0
SUS304	E_m(Pa)	201.04e+9	0.0	3.079e-4	-6.543e-7	0.0
	α_m (1/K)	12.33e-6	0.0	8.086e-4	0.0	0.0

$$P_{cr}^P = \pi^2\frac{m^2}{\Upsilon_T}\left(\Upsilon_1 - \Upsilon_2\frac{m^2\Upsilon_2+\Upsilon_4}{m^2\Upsilon_3+\Upsilon_4} - \Upsilon_4\frac{\Upsilon_2-\Upsilon_3}{m^2\Upsilon_3+\Upsilon_4}\right) \tag{30a}$$

$$P_{cr}^T = \pi^2\frac{m^2}{\Upsilon_T}\left(\frac{\Upsilon_3\Upsilon_4\kappa_S}{m^2\Upsilon_3+\kappa_S\Upsilon_4}\right) \tag{30b}$$

$$P_{cr}^E = \pi^2\frac{m^2}{\Upsilon_T}(\Upsilon_1) \tag{30c}$$

In Table 1, $\kappa_1 = 6\,(1+\upsilon)/(7+6\upsilon)$, $\kappa_2 = 6\,(1+\upsilon)^2/(7+12\upsilon+4\upsilon^2)$, where κ_1 and κ_2 are shear correction factor for Timoshenko beam theory, υ is Poisson ratio and equals 0.3. As a result, the critical buckling loads presented are almost the same as the results of Huang and Li (2010) by stress function method.

Numerical results are given out in following section, the effective material properties are

$$E_f(r) = E_m + (E_c - E_m)\left(\frac{r-R_i}{R_0-R_i}\right)^K \tag{31a}$$

$$G_f(r) = G_m + (G_c - G_m)\left(\frac{r-R_i}{R_0-R_i}\right)^K \tag{31b}$$

$$\alpha_f(r) = \alpha_m + (\alpha_c - \alpha_m)^K\left(\frac{r-R_i}{R_0-R_i}\right)^K \tag{31c}$$

$$\upsilon_f(r) = \upsilon_m + (\upsilon_c - \upsilon_m)\left(\frac{r-R_i}{R_0-R_i}\right)^K \tag{31d}$$

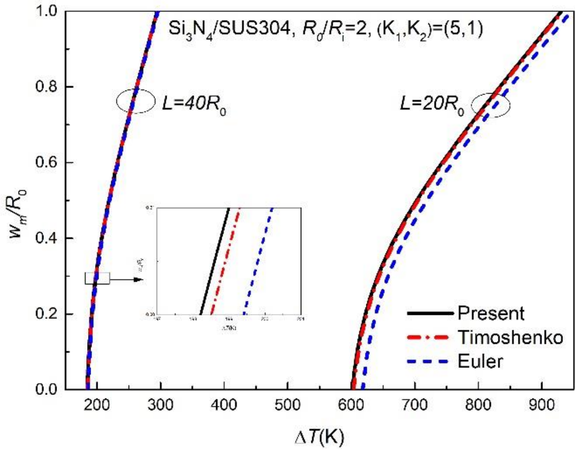

Fig. 2 Thermal post-buckling response of the FGM tubes under different beam models

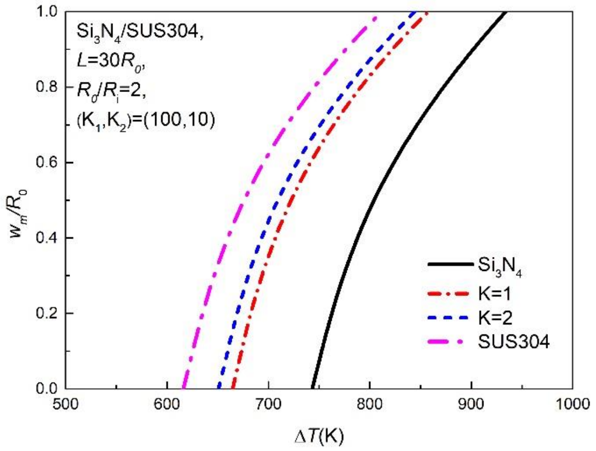

Fig. 3 Effect of volume fraction index K=(0,1,2,+∞) on the thermal post-buckling response for the FGM tubes

where K is the volume fraction index $(0 \leq K \leq +\infty)$, and the material properties are listed in Table 2.

The thermal buckling response for the FGM tubes supported on elastic foundations by adopting different beam theories are compared in Fig. 2. From this figure, we can find that the buckling temperatures and post-buckling response obtained by the Euler beam theory are overestimated than those by the present model and Timoshenko beam theory, especially when $L=20R_0$, and the results calculated by the present model are very close to those by using Timoshenko beam theory. When $L=40R_0$, the buckling temperature and post-buckling response presented by different model are very close. Which indicates the effect of shear deformation decreases gradually with the increase of the slenderness ratio.

Fig. 3 shows the effect of volume fraction index K=(0,1,2,+∞) on the thermal post-buckling response of FGM tubes. It can be found that, the buckling temperature and post-buckling strength decrease as the volume friction index K rises. As the volume friction index K rises, the volume fraction of the ceramic is reduced, so the buckling temperature and thermal post-buckling strength decrease.

Fig. 4 plots the effect of the Winkler foundation stiffness and the shearing layer stiffness of the foundation

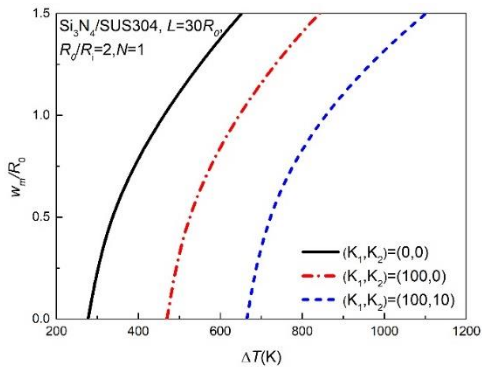

Fig. 4 Effect of the foundation stiffness on the post-buckling paths of FGM tubes

on the post-buckling load-deflection curves of FGM tubes. As a result, the tube has a higher buckling temperature and post-buckling strength when it is supported by a Winkler foundation, and has much higher buckling temperature and post-buckling strength when it is supported by a two-parameter elastic foundation.

5. Conclusions

The objective of this paper is to explore the thermal buckling and post-buckling behaviors of FGM tubes with immovable simply supported ends. The tubes are subjected to uniform temperature rise and rests on elastic foundations. Based on the two-step perturbation method, the expression of the critical buckling temperature and the post-buckling paths are obtained. It can be found that,

• The present model predicts lower values for the buckling temperature and thermal post-buckling strength, and the buckling temperature calculated by Euler beam theory is higher than those calculated by the present model and Timoshenko beam theory.

• With rising of the volume friction index, the volume fraction of the ceramic is reduced, as a result, the buckling temperature and thermal post-buckling strength decrease.

• The foundation stiffness has a significant effect on the thermal buckling and thermal post-buckling response of the FGM tubes, and the tube has a higher buckling temperature and post-buckling strength when it is supported by a Winkler foundation, and has much higher buckling temperature and post-buckling strength when it is supported by a two-parameter elastic foundation.

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