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Three dimensional finite elements modeling of FGM plate bending using UMAT

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Abstract. The purpose of the present paper is to study the bending and free vibration of Functionally Graded Material (FGM) plate using user-defined material subroutine on the finite element software ABAQUS. The FGM plate is simply supported and subjected to sinusoidal and uniform load. The Poisson's ratio is kept constant. The results obtained compared to those available in the literature show the convergence, the exactitude and the efficiency of the method used with various power index of the materials.

Keywords: FGM plate; power law; UMAT; finite element method; stress; displacement

1. Introduction

Nowadays, the development of new high-performance materials (hardness, corrosion resistance, optimum thermal conductivity, etc.) is a major industrial challenge, furthermore the advanced technical requirements have forced the research materials laboratories to delve into this discipline to develop new structure that bearing a suitable characteristic to meet demand while addressing the anomalies encountered (mechanical, thermal, acoustic...).

Generally, special materials like composite take many forms (plate, beam, shell) which have spread considerably in many sectors such as: automotive, construction, and aeronautic. On one hand, the composite materials have significant advantages over simple materials. While, on the other hand, conventional multilayer structures or layered composite materials present a problem on the interface due to the change of mechanical and thermal properties of the materials composing the structure.

To skip this barrier, in the eighties a research team from Japan proposed a new structure composed of many materials whose properties vary continuously and cannot contain the interface, this type was introduced under the name of Functionally Graded Materials (FGM). Usually these materials are associated with particulate composite in which particles' volume fraction varies in one or several directions.

The first goal to develop the FGMs was to serve as a thermal barrier Hirano *et al.* (1990). Today, there are several modern engineering applications of FGM, such as the spacecraft, rocking engine casings and packaging materials in the microelectronics industry, biomaterials (dental implants) and others (Bennoun *et al.* (2016), Bousahla *et al.* (2016), Hebali *et al.* (2014), Beldjelili *et al.* (2009, 2016), Mahi *et al.* (2015), Kar *et al.* (2016), Arani *et al.* (2016), Barati and Shahverdi (2016), Oonishi *et al.* (1994) and Watari *et al.* (1997)).

Many theories are used in literature to study bending performance of FGM plate. Classical Plate Theory (CPT) based on Kirchhoffs hypothesis is imprecise to obtain the distribution of the displacement and stresses in FGM plate, the theory done by Aydogdu (2008) to analyze the FGM plate, Chakraverty and Pradhan (2014) took CPT as a subject to discuss free vibration of FG rectangular plate with general boundary condition. Another theory used to treat the bending solicitation like First Order Shear deformation Theory (FOST) in which transverse shear strain is supposed to be constant in thickness direction and hence shear correction factor is needed. Four degrees of freedom for bending and free vibration analysis of FG plates was presented by Thai and Choi (2013). With a Local Meshless Petroves- Galerkin (MLPG) method and Higher-Order Shear and Normal Deformable Plate Theory (HOSNDPT), Gilhooley *et al.* (2007) treated the infinitesimal deformation of an FG thick elastic plate. Della Croce and Venini (2004) carried out a study to discuss the behavior of FG rectangular plate by using the simple power law and Reissner-Mindlin plate theory. The implementation of subroutine UMAT on abaqus has also been the subject of recent research work by Lavate and Shiyekar (2015) dealing with the effect flexure of power law governed FG plates using Abaqus UMAT his results are validated with Reddy (2000) Third Order Deformation Theory (TOT).

Other theories were mentioned by Lavate and Shiyekar

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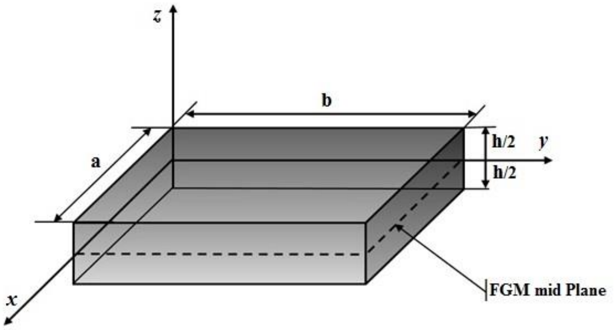

Fig. 1 Geometry of functionally graded plate

(2015) for calculating the stresses and displacement. Buckling and vibration of FGM plate is treated by Thai and Choi (2011). Where the theory used explain the quadratic variation of the transverse shear strains across the thickness, and satisfies the zero traction boundary conditions on the upper and lower surfaces of the plate without using shear correction factors. Gouasmi *et al.* (2015) has also used UMAT on his work whose aim of study was to focus on the variation of Stress Concentration Factor (SCF) in different directions around a notch on FGM plate. Analytical modeling two-dimensional (2D) Higher-Order Deformation Theory (HODT) is utilized by Matsunaga (2009) to evaluate the displacements and stresses in FGM plates subjected to thermal and mechanical loading. Many others simple refined theories are presented by (Ait Amar Meziane *et al.* 2014, Tounsi *et al.* 2013, Abualnour *et al.* 2018, Abdelaziz *et al.* 2017, Menasria *et al.* 2017, Bellifa *et al.* 2017, Belabed *et al.* 2014, Hamidi *et al.* 2015, Bousahla *et al.* 2014, Boukhari *et al.* 2016, Houari *et al.* 2016). These theories are used by Bouderba *et al.* (2016) to study thermal stability of functionally graded sandwich, by El-Haina *et al.* (2017) for thermal buckling of thick FG sandwich plates, by Bellifa *et al.* (2016) and (Saidi *et al.* 2016, Attia *et al.* 2015) for vibration, by Bouderba *et al.* (2013) for thermomechanical bending response of FGM thick plates resting on Winkler-Pasternak elastic foundations and by Zidi *et al.* (2014) for bending analysis of FGM plates under hygro-thermo-mechanical loading. Some of them are also used by Ait Yahia *et al.* (2015) for wave propagation in functionally graded plates. Number of presented theories are used for nanoscale structures (Zemri *et al.* 2015, Besseghier *et al.* 2017, Khetir *et al.* 2017, Bouafia *et al.* 2017, Bounouara *et al.* 2016, Mouffoki *et al.* 2017, Larbi Chaht *et al.* 2015, Belkorissat *et al.* 2015) and graded micro by Al-Basyouni *et al.* (2015).

Bourada *et al.* (2015) used a Simple and Refined Trigonometric Higher-Order Theory (SRTHOT) to study the bending and vibration of functionally graded beams, for which he added the displacement field with three unknown to its modeling as Timoshenko beam theory. A new hyperbolic displacement model (NHDM) was the subject of Benyoucef *et al.* (2010) to study the static response of simply supported functionally graded plates (FGP) under uniform and sinusoidal distributed load, in this case the transverse shear correction factors were not introduced because a correct representation of the transverse shear strain has been given. Zenkour (2006) had the same goal with Benyoucef *et al.* (2010), but he focused his study on rectangular FGM plate, generalized shear deformation theory was the key of studying based on enforcing traction-free boundary conditions at the plate faces.

Kar and Panda (2015a) present the nonlinear finite element solutions of bending responses of functionally graded spherical panels. Under a uniform thermal environment the geometrical nonlinear static behaviour is studied by Mehar and Panda (2017b) for a functionally graded carbon nanotube reinforced doubly curved shell panel. Kar and Panda (2016) examine the linear and Green-Lagrange type geometrical nonlinear deformation behaviour of functionally graded spherical shell panel. cylindrical, hyperbolic, and elliptical panel (Kar *et al.* 2015) subjected to thermomechanical load and under the influence of nonlinear thermal field Mahapatra *et al.* (2017). The FEM is also used to analyze composite, sandwich and carbon nanotube reinforced structure (Sahoo *et al.* 2015, Mahapatra *et al.* 2016a, b, c, Mehar and Panda 2017a, Sahoo *et al.* 2017, Mehar *et al.* 2017).

In the cited works, in general, the strain across the thickness is neglected and the field of displacement is a theory that relates the displacement of points out of the mid surface to those of the latter. In the present work, we use the 3D finite element method implemented in the FE software Abaqus (Dassault-Systemes 2011) and we write a user-defined material subroutine in FORTRAN for FGM structures. After a convergence study and validation and to test our subroutine we use it to find the displacement under sinusoidal distributed load and natural frequency for FGM plate, also to find the displacement under distributed load for FGM cylindrical panel. The simply supported boundaries conditions are used.

2. Formulation

Considering FGM square plate (Fig. 1) made of ceramic and metal subjected to the sinusoidal load of the form

$$q(x,y) = q_0 \sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right) \tag{1}$$

With q_0=100 *MPa*, or to a distributed load of a constant intensity.

Because of symmetry in loading and geometry, quarter of the plate has been modeled. The properties of different components of the FGM are listed in Table 1. The used power law is giving by Eq. (3).

$$E(z) = V(z)\,E_1 + \left(1 - V(z)\right)E_2 \tag{2}$$

$$V(z) = \left(\frac{z + h/2}{h/2}\right)^n \tag{3}$$

Where $V(z)$ is the volume fraction, n represent the power index and h is the thickness of the plate. E_1 and E_2 are the Young's modulus of the top and bottom of plate respectively. The variation of Young's modulus through the

Table 1 Materials proprieties

Structure analysis	Materials	$E\,(GPa)$	v	$\rho(kg\,/\,m^3)$
Plate in static	Metal (Al) Aluminum	72	0.3	2702
	Ceramic (AlO_3) Alumina	380	0.3	3800
Cylindrical panel in static	Metal (Al) Aluminum	70	0.3	-
	Ceramic (ZrO_2) Zirconia	151	0.3	-
Plate in vibration	Metal (Al) Aluminum	70	0.3	2702
	Ceramic (AlO_3) Alumina	380	0.3	3800

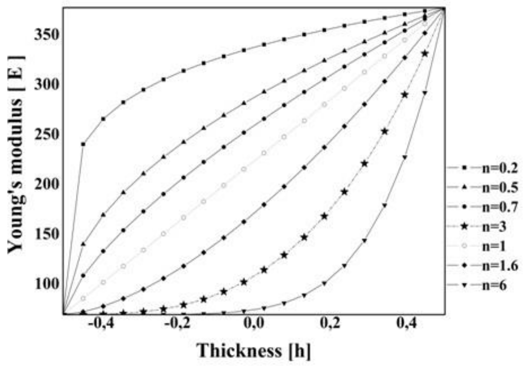

Fig. 2 Variation of young's modulus through the thickness of P-FGM Plate

thickness direction of P-FGM plate is shown in Fig. 2.

An ABAQUS user-defined material subroutine UMAT is used to define the mechanical constitutive behavior of a material. It will be called at all material calculation points of elements for which the material definition includes a user-defined material behavior. It must update the stresses and solution-dependent state variables to their values at the end of the increment for which it is called. It must provide the material Jacobian matrix, $\partial \Delta \sigma / \partial \Delta \varepsilon$, for the mechanical constitutive model (Dassault-Systemes 2011).

2.1 Convergence study

The convergence study of a simply supported plate is presented in Fig. 3. The mesh was refined in all directions. The used element is C3D20. According to Fig. 3, a mesh of 16000 elements provides a 10^3 precision and will be used in the next application. The convergence study is exposed here but it accompanies all results obtained in this work.

2.2 Validation

To verify the UMAT, the calculation of $\bar{W}$ given by Eq. (4) for an isotropic material representing the center normalized deflection of plate are mentioned in Table 2. The numerical results for non-dimensional transverse deflection $\bar{W}$ confirm the reliability of UMAT used.

$$\bar{W} = w\left(\frac{E_c}{q_0 + h}\right), \overline{\tau_{xz}} = \frac{\tau_{xz}}{q_0} \tag{4}$$

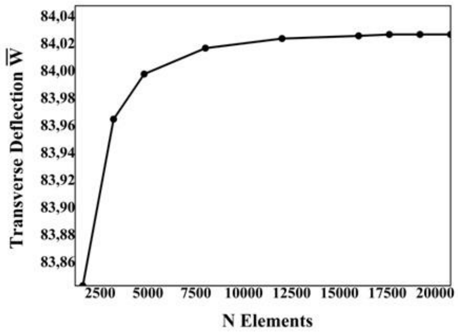

Fig. 3 Convergence study simply supported FGM plate under sinusoidal load $q_0 = 100MPa$

Table 2 Dimensionless transverse deflection of homogeneous simply supported plate $\bar{W}$

Materials	Number of elements	Standard ABAQUS UMAT ABAQUS	$\bar{W}$
Ceramic	16000	Standard	316.1393
		UMAT	316.1393
Cylindrical panel in static	16000	Standard	1716.1853
		UMAT	1716.1853

Table 3 Dimensionless transverse deflection for simply supported square FGM plate with a/h=10 and various power law index n

n	Reddy	Matsunaga	Lavate	Present	Error %
0	296.057	294.3	262.694	316.139	6.91
0.5	453.71	450.4	373.084	483.228	6.79
1	588.953	587.5	494.380	627.497	6.37
4	881.478	882.3	883.88	955.892	7.70
10	1008.7	1007.0	1210.3	1099.336	8.40

[%] - error w.r.t. Matsunaga

2.3 Results and discussions

2.3.1 First case

The results of this case are compared with Lavate and Shiyekar (2015), Reddy (2000) and Matsunaga (2009). Results for transverse displacement $\bar{W}$ and non-dimensional shear stress $\overline{\tau_{xz}}$ given by Eq. (4) for simply supported square FGM plate under sinusoidal normal load for various power index are shown in Tables 3 and 4. The contour plots for $\bar{W}$ and $\overline{\tau_{xz}}$ are shown in Figs. 4 and 5 respectively. Lavate and Shiyekar (2015) didn't mention the shear stress results. Numerical results obtained for the transverse deflection W has an average error of 7%. Shear stress $\overline{\tau_{xz}}$ obtained by the present method deviate within 2-3% as compared with Matsunaga (2009) results.

Stresses result as a function of z/h are presented in Fig. 6 and have the same curvature as Matsunaga (2009).

Table 4 Dimensionless shear stress of a simply supported FGM plate

n	Reddy	Matsunaga	Present	Error %
0	2.386	2.387	2.328	2.56
0.5	2.440	2.435	2.371	2.68
1	2.386	2.387	2.314	3.17
4	1.940	2.182	2.110	3.39
10	2.114	2.171	2.105	3.14

[%] - error w.r.t. Matsunaga

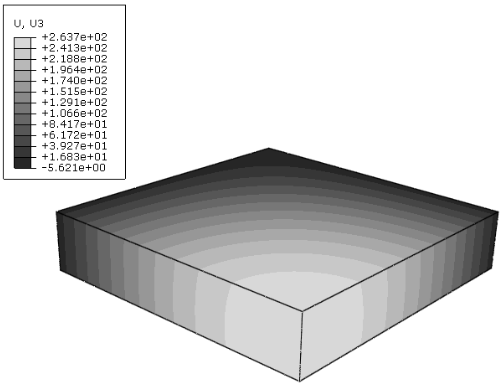

Fig. 4 Contour plots for dimensionless transverse displacement $\overline{W}$ of simply supported square FG plate having $a/h=10, n=5$

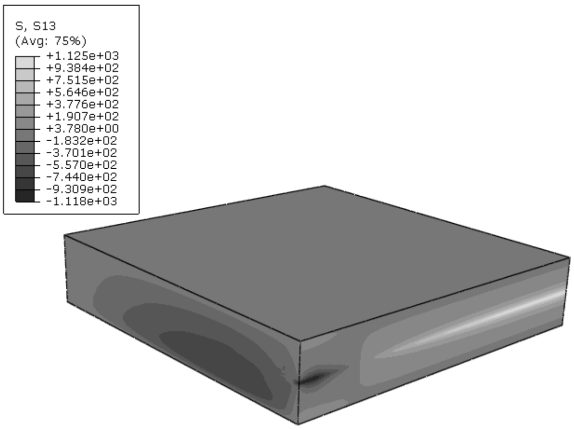

Fig. 5 Contour plots for non-dimensional shear stress $\overline{\tau_{xz}}$ of simply supported square FG plate having $a/h=10, n=5$

2.3.2 Second case

The same previous example is considered. Thai and Choi (2011) is the second reference to verify the accuracy of the present study. The dimensionless normal deflection and stresses are defined as follows

$$\overline{W}=\frac{10E_c h^3}{q_0 a^4}w\left(\frac{a}{2},\frac{b}{2}\right),\overline{\sigma_{xx}}=\frac{h}{q_0 a}\sigma_{xx}\left(\frac{a}{2},\frac{b}{2},\frac{h}{2}\right)$$
$$\overline{\sigma_{yy}}=\frac{h}{q_0 a}\sigma_{yy}\left(\frac{a}{2},\frac{b}{2},\frac{h}{3}\right),\overline{\sigma_{xy}}=\frac{h}{q_0 a}\sigma_{xy}\left(0,0,-\frac{h}{3}\right) \quad (5)$$

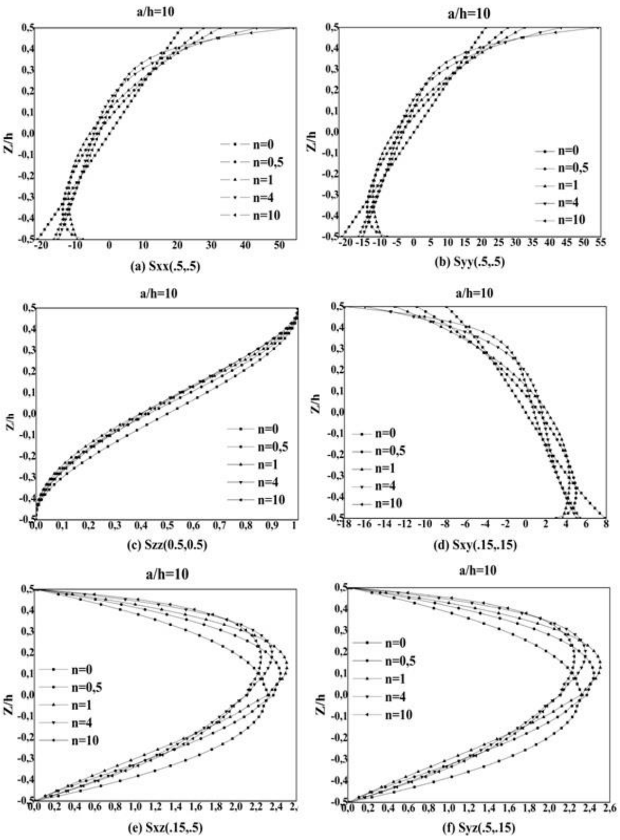

Fig. 6 Normal and shear stresses profile obtained by present UMAT

Table 5 Dimensionless transverse deflection for various power index

n	Benyoucef	Zenkour	Huu	Present	Error [%]
Ceramic	0.2960	0.2960	0.2961	0,3193	7,27
1	0.5889	0.5889	0.5890	0,6338	7,07
4	0.8810	0.8819	0.8815	0,9655	8,70
10	1.0083	1.0089	1.0087	1,1103	9,15
Material	1.6071	1.6070	1.6074	1,7333	7,26

[%] error w.r.t. Huu-Tai Thai

In this case, *a*/*h*=10 and various power index *n* are considered. The center deflection, normal and shear stresses results of the FGM plate are listed in Tables 5 and 6 respectively. It is concluded that the results of transverse deflection for various volume fractions are closed to Thai and Choi (2011) results, and are deviated within 7-8%. Further, normal stresses results are presented within 5-6% for various volume fractions. Shear stresses are deviate within 12,34% in isotropic case. For various volume fractions the shear stress is deviating within 14%. Important errors on the shear stress come from the fact that it is measured close to the boundary conditions, unlike the others constraints (see $\overline{\sigma_{xy}}$ equation). The Fig. 7 shows how the distribution of the shear stress changes near the boundary conditions. For more compatible values it is necessary to move away from the boundary conditions.

Table 6 Dimensionless stresses for various power index

n	Stresses	Benyoucef	Zenkour	Huu	Present	Error [%]
AL_2O_3	$\overline{\sigma_{xx}}$	1,9955	1,9955	1,9943	2,1100	5,48
	$\overline{\sigma_{yy}}$	1,3121	1,3121	1,3124	1,3931	5,79
	$\overline{\tau_{xy}}$	0,7065	0,7065	0,7067	0,6291	12,34
1	$\overline{\sigma_{xx}}$	3,087	3,087	3,085	3,2600	5,37
	$\overline{\sigma_{yy}}$	1,4894	1,4894	1,4898	1,5820	5,83
	$\overline{\tau_{xy}}$	0,611	0,611	0,6111	0,5525	10,61
4	$\overline{\sigma_{xx}}$	4,0693	4,0693	4,0655	4,3300	6,11
	$\overline{\sigma_{yy}}$	1,1783	1,1783	1,1794	1,1209	5,22
	$\overline{\tau_{xy}}$	0,5667	0,5667	0,5669	0,4894	15,84
10	$\overline{\sigma_{xx}}$	5,089	5,089	5,0849	5,4000	5,84
	$\overline{\sigma_{yy}}$	0,8775	0,8775	0,8785	0,9530	7,82
	$\overline{\tau_{xy}}$	0,5894	0,5894	0,5896	0,5026	17,31
Al	$\overline{\sigma_{xx}}$	1,9955	1,9955	1,9943	2,1071	5,35
	$\overline{\sigma_{yy}}$	1,3121	1,3121	1,3124	1,3931	5,80
	$\overline{\tau_{xy}}$	0,7065	0,7065	0,7067	0,6291	12,34

[%] error w.r.t. Huu Tai Thai

Fig. 7 Contour plots for σ_{xy} distribution in the plan $z=-h/3$

2.3.3 Third case

In this case we want to use our user-defined-material subroutine with a square cylindrical panel under various uniform loads (q). Geometry and material properties of the square cylindrical panel are given by: R/a=10, a/h=10, E_{Al}=70 *Gpa*, E_{ZrO2}=151 and v=0.3 through the thickness.

Obtained results for Non-dimensional central deflection $\bar{w}=w/h$ are compared with Kar and Panda (2015b) in Table 7. The obtained results show that the developed subroutine can be used with other geometries. In this comparison the accuracy of convergence are two decimal places. The small differences between the results come mainly from the strain through the thickness which is neglected by the reference in addition to the kinematics of the displacement field.

2.3.4 Fourth case

In this case we want to use our user-defined-material subroutine to analyze the free vibration of a simply supported square FG plate. Variation of density through the thickness is calculated automatically by ABAQUS in the center of elements using the analytic field.

Results for a plate with a/h=10 are compared with references Hosseini-Hashemi *et al.* (2011b), Hosseini-Hashemi *et al.* (2011a) and Thai and Kim (2013) and listed in Table 8.

Table 7 Non-dimensional central deflection with various loads for square simply-supported cylindrical FG panel

Method	Load ($q/10^8$)				
	3.5	7	10.5	14	17.5
Kar and Panda	1.6443	3.2886	4.9329	6.5772	8.2215
Present	1.6266	3.2520	4.8779	6.4759	8.0949

Table 8 Non-dimensional fundamental frequency $\bar{w}=wh\sqrt{\rho_c/E_c}$ of Al/Al_2O_3 square plate

a/h	Method	Power law index (n)				
		0	0.5	1	4	10
5	Hosseini-b	0.2113	0.1807	0.1631	0.1378	0.1301
	Hosseini-a	0.2112	0.1805	0.1631	0.1397	0.1324
	Thai	0.2113	0.1807	0.1631	0.1378	0.1301
	Present	0.1905	0.1658	0.1519	0.1260	0.1161
10	Hosseini-b	0.0577	0.0490	0.0442	0.0381	0.0364
	Hosseini-a	0.0577	0.0490	0.0442	0.0382	0.0366
	Thai	0.0577	0.0490	0.0442	0.0381	0.0364
	Present	0.0551	0.0477	0.0439	0.0379	0.0352
20	Hosseini-b	0.0148	0.0125	0.0113	0.0098	0.0094
	Hosseini-a	0.0148	0.0125	0.0113	0.0098	0.0094
	Thai	0.0148	0.0125	0.0113	0.0098	0.0094
	Present	0.0145	0.0125	0.0116	0.0101	0.0094

The differences between the obtained results and the references are small. The differences increase when the ratio (a/h) decrease. The deviations are due to the displacement field used and the neglected strain across the thickness. Since we are in dynamics, these differences are amplified by the kinetic energy.

3. Conclusions

The three-dimensional finite element method is coupled to a user-defined material subroutine for bending analysis of FGM cylindrical panels and plates under sinusoidal loading and distributed loading and also free vibration analysis of FGM plates. The obtained results are close to those of literature with rich displacement fields. Less small differences always exist because the deformation through the thickness is neglected by the references. In the vibration these small differences are amplified but the results remain comparable. The comparisons between the three-dimensional finite element method and the methods based on displacement field theories need some precautions in applying boundary conditions and extracting the desired values. With this method the advantages of ABAQUS are preserved for the treatment of complex structures, boundary conditions, analysis and loading (circular plate, panels, buckling, localized loading...)

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