Structural Engineering and Mechanics, *Vol. 66, No. 3 (2018) 343-351*
DOI: https://doi.org/10.12989/sem.2018.66.3.343

A fracture mechanics simulation of the pre-holed concrete Brazilian discs

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(Received August 18, 2017, Revised February 5, 2018, Accepted February 8, 2018)

Abstract. Brazilian disc test is one of the most widely used experiments in the literature of geo-mechanics. In this work, the pre-holed concrete Brazilian disc specimens are numerically modelled by a two-dimensional discrete element approach. The cracks initiations, propagations and coalescences in the numerically simulated Brazilian discs (each containing a single cylindrical hole and or multiple holes) are studied.

The pre-holed Brazilian discs are numerically tested under Brazilian test conditions. The single-holed Brazilian discs with different ratios of the diameter of the holes to that of the disc radius are modelled first. The breakage load in the ring type disc specimens containing an internal hole with varying diameters is measured and the crack propagation mechanism around the wall of the ring is investigated. The crack propagation and coalescence mechanisms are also studied for the case of multi-holes' concrete Brazilian discs. The numerical and experimental results show that the breaking mechanism of the pre-holed disc specimens is mainly due to the initiation of the radially induced tensile cracks which are growth from the surface of the central hole. Radially cracks propagated toward the direction of diametrical loading. It has been observed that for the case of disc specimens with multiple holes under diametrical compressive loading, the breaking process of the modelled specimens may occur due to the simultaneous cracks propagation and cracks coalescence phenomena. These results also show that as the hole diameter and the number of the holes increases both the failure stress and the crack initiation stress decreases. The experimental results already exist in the literature are quit agree with the proposed numerical simulation results which validates this simulation procedure.

Keywords: concrete rings; pre-holed Brazilian discs; crack analyses; cracks coalescence; discrete element method

1. Introduction

The recent engineering design of concretes involves both the strengths and the fracture mechanics concepts which consider the mechanical and failure behaviours of brittle materials. The mechanical behaviour of concrete is mainly controlled by the mechanisms of initiation, propagation and coalescence of cracks emanating from pores or pre-existing cracks.

Many analytical, numerical and experimental works have been carried out to investigate the fracturing mechanisms of the brittle materials specimens containing pores, holes and cracks (Lajtai 1975, Sammis 1986, Lin 2005, Jespersen 2010, Lin 2015). For example, Sammis and Ashby (1986) performed a vast experimental research on a set of uniaxial compression tests which carried out on the plate specimens containing a single hole of the same size or array of holes with different diameters. In their analyses, the cracks propagations and the interaction of cracks with the surfaces of the specimens were experimentally studied. Some other researchers focussed on the samples with multiple pre-existing cracks (Robert *et al.* 1979, Nemat-Nasser *et al.* 1982, Ashby *et al.* 1986, Wong 1998, Park 2009, Haeri *et al.* 2015, Fatehi Marji *et al.* 2006). However, among these tested specimens, those of Brazilian discs are the most widely used samples for determining the tensile strength of brittle materials such as concretes and rocks (Ozcebe, 2011, Yang *et al.* 2011, Zhang and Wong 2012, Zhang and Wong 2013, Yang 2015, Gerges *et al.* 2015, Zhao 2015, Haeri and Sarfarazi 2016a, 2016b, 2016c, Haeri *et al.* 2016d, Sarfarazi *et al*. 2016b, Sardemir 2016, Shemirani *et al*. 2016, Shemirani *et al*. 2017, Sarfarazi *et al*. 2017a, b, c, Shemirani *et al*. 2018). As an example, Al-Shayea (2005), carried out some experiments on the Central Straight through Crack Brazilian Disk (CSCBD) specimens of brittle limestone. These samples contained different crack inclination angles and loaded under mixed Mode I/II. The crack propagation process in these limestone specimens were studied. Some other tests conducted by Mellor and Hawkes (1971) investigated the mechanism of cracks initiation and propagation in the ring specimens under a diametrical compression. In another work carried out by Lin *et al.* (2015), the mechanisms of cracks coalescences within the granite specimens (each containing multiple holes) were studied.

As far as the numerical analyses are concerned, many numerical approaches and techniques have been developed in the last decades to simulate the mechanisms of cracks initiation, cracks propagation and cracks coalescence in various brittle materials (such as rocks, rock-like materials

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http://www.techno-press.com/journals/sem&subpage=7 ISSN: 1225-4568 (Print), 1598-6217 (Online)

and concretes). Among these numerical methods the most widely used methods includes; the finite element method (Roy 1999, Sukumar 2003, Lin 2014), the direct and indirect boundary element method (Altiero 1982, Aliabadi 1993, Tan 1998, Fatehi Marji *et al.* 2006, Fatehi Marji. 2014, Haeri *et al.* 2013, Haeri *et al.* 2014a, 2014b, 2015a, 2015b, 2015c, 2015d, 2015e, 2015f, Haeri 2015g, 2015h), the discontinuous deformation analysis (Shi 1988, Zheng 2009, Amadei 1996), the explicit discrete element modelling, (Cundall 1979, Ghazvinian 2012, Lin 2013, Sarfarazi 2014), 3D numerical manifold method (He 2010).

In the present research, discrete element analyses of the cracks propagation mechanism in the Brazilian disc type models containing either single hole or multi-holes are carried out. A two-dimensional particle flow code is used to numerically model these samples. The Brazilian disc specimens containing cylindrical holes (with different sizes) are modelled to measure the crack initiation and failure stresses. Also, study the mechanism of cracks propagation and cracks coalescence. The cracks propagation and coalescence mechanism in the bridge area (i.e., the area in between the two parallel holes within the samples) are studied and the effect of the number of holes (within each particular sample) on the crack initiation and failure stresses are also investigated.

2. The discrete element simulation procedure

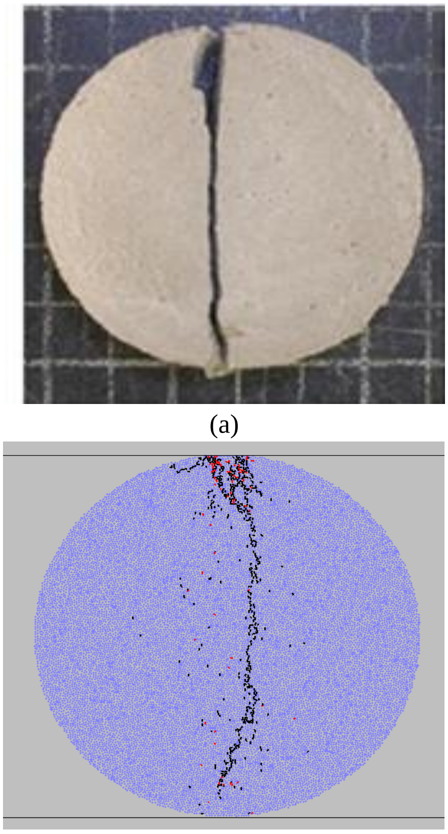
The concrete samples can be used to simulate the failure mechanism by a discrete element modelling procedure. In this simulation the rock material sample is considered as an assembly of discs bonded together at their contacts (points) and confined by the walls of the sample. Two types of bonding models are usually used in the literature i.e., (i) a contact bonded model and (ii) a parallel bonded model; Ghazvinian *et al.* (2012). Cundall (1979) generated a parallel bonded particle model for a two-dimensional particle flow code (PFC2D). He defined the following micro parameters in his work: modulus of the ball-to-ball contact; the stiffness ratio (i.e., k_n / k_s where k_n and k_s are the normal and shear stiffnesses, respectively); friction coefficient of the ball; strength of the contact normal bond; strength of the contact shear bond, ratio of the standard deviation to the mean bond strengths (both in normal and shear direction); and the minimum ball radius. The parallel-bonded particle model is used in the present study which also requires the three additional micro-parameters; the parallel-bond radius multiplier, the parallel-bond modulus, and the parallel-bond stiffness ratio.

2.1 Preparing and calibrating the numerical model

A discrete element approach implementing the parallel bond model is calibrated by measuring the tensile strength of concrete specimens using the standard Brazilian test. An assembly of the standard parallel bonding model is used to represent the test modelled specimen which involves establishment of the isotropic stress, (c) eliminating the floating particles, and (d) establishing the bonding contacts (points). Table 1. is listing the micro-properties used for the modelling procedure and the standard calibration approach proposed by Ghazvinian *et al.* (2012) are being applied to calibrate the bonding model particles assembly. The 54 mm diameter specimens of the Brazilian discs are simulated by the numerical modelling tests. The total number of 5,615 bonding particles are used to simulate each testing specimen. A low speed of 0.016 m/s is used to move the lateral walls of each testing disc. The failure mechanism

Table 1 Micro properties used to represent the concrete used in the analyses

Parameter	Value	Parameter	Value
Type of particle	disc	Parallel bond radius multiplier	1
density	3000	Young modulus of parallel bond (GPa)	40
Minimum radius	0.27	Parallel bond stiffness ratio	1.7
Size ratio	1.56	Particle friction coefficient	0.4
Porosity ratio	0.08	Parallel bond normal strength, mean (MPa)	7
Damping coefficient	0.7	Parallel bond normal strength, SD (MPa)	2
Contact young modulus (GPa)	40	Parallel bond shear strength, mean (MPa)	7
Stiffness ratio	1.7	Parallel bond shear strength, SD (MPa)	2



(a)

(b)

Fig. 1 failure pattern in (a) physical sample, (b) PFC2D model

Table 2 The Brazilian tensile strength of physical and numerical samples

Physical tensile strength (MPa)	1.5 and 1.7
Numerical tensile strength (MPa)	1.5

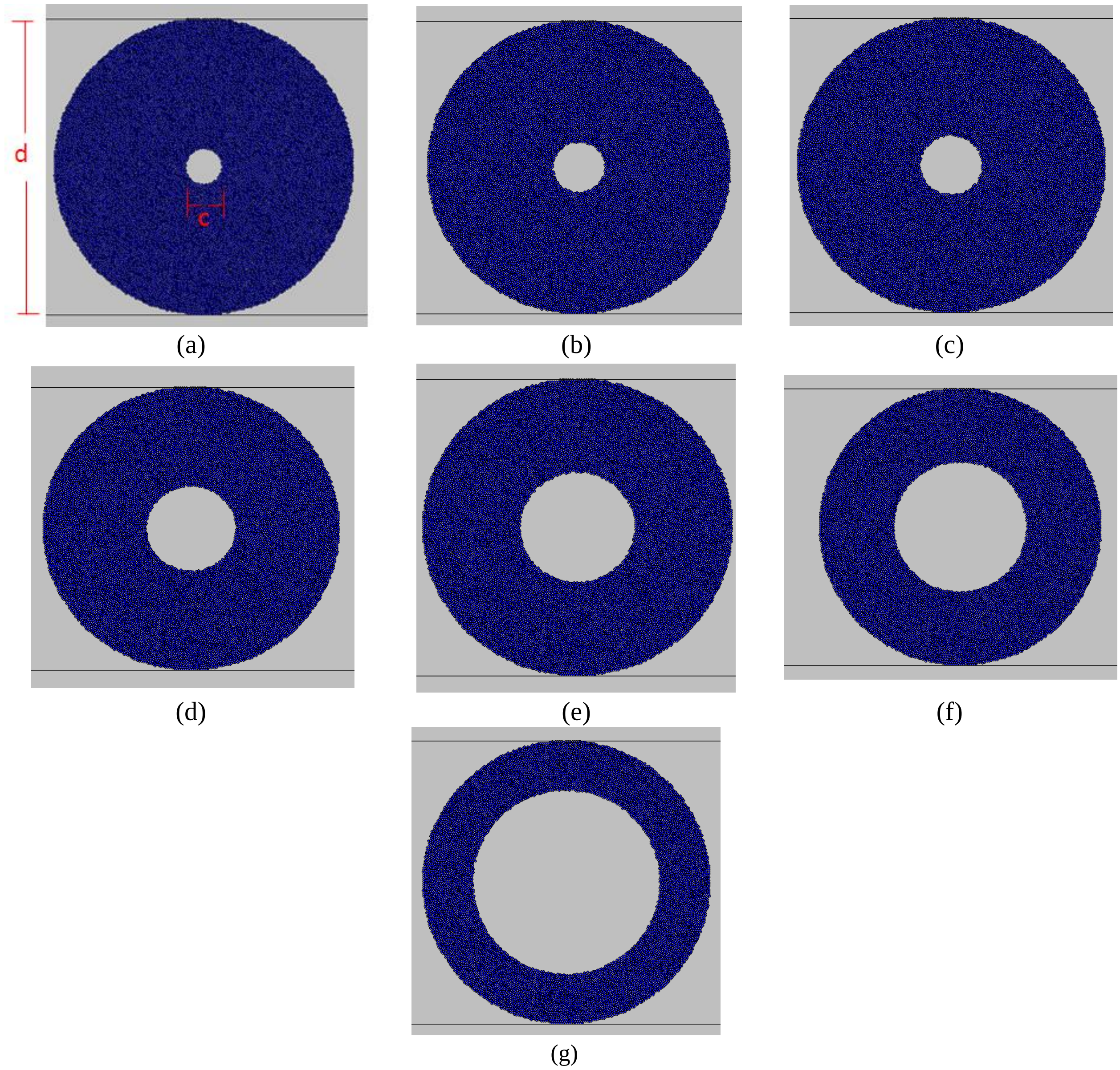


Fig. 2 The single-holed Brazilian disc samples under diametrical compression considering different c / d ratios; (a) c / d = 0.12, (b) c / d = 0.17, (c) c / d = 0.2, (d) c / d = 0.3, (e) c / d = 0.37, (f) c / d = 0.47 and (g) c / d = 0. 65

of the testing samples is illustrated in Figs. 1(a), (b) for the numerical and experimental cases, respectively. Fig. 1(b) also shows the displacement vector of particle and bond force distribution. These figures show that the numerical and laboratory tests for measuring the failure mechanism of the testing samples are well matching. Table 2 gives the tensile strength of the tested samples measured experimentally and predicted numerically. These results show that there is a good agreement between the numerical and experimental testing methods.

2.2 Model preparation for the discrete element simulation

The Brazilian disc specimens are numerically simulated considering the following two cases: (i) Specimens containing a single cylindrical hole (with different radii) in their middle part. and (ii) Specimens containing parallel multi-holes (with equal radii) in their central part.

2.2.1 Specimens containing a central hole (ring type disc specimens)

The mechanism of cracks initiation and crack propagation in the ring type Brazilian disc specimens are studied experimentally as shown in Figs. 2. These ring types disc specimens have different ratios of the hole diameter (c) to that of the disc itself (d) or the c/d ratios. The diameter of the Brazilian disc specimen models is kept as 10 cm where based on this constant diameter the ratios of c/d vary as 0.12, 0.17, 0.2, 0.3, 0.37, 0.47, 0.65 and are shown schematically in Figs. 2(a)-(g), respectively.

2.2.2 Brazilian discs containing multiple parallel cylindrical holes

The circular modelled samples which numerically simulating the Brazilian tensile test specimens each

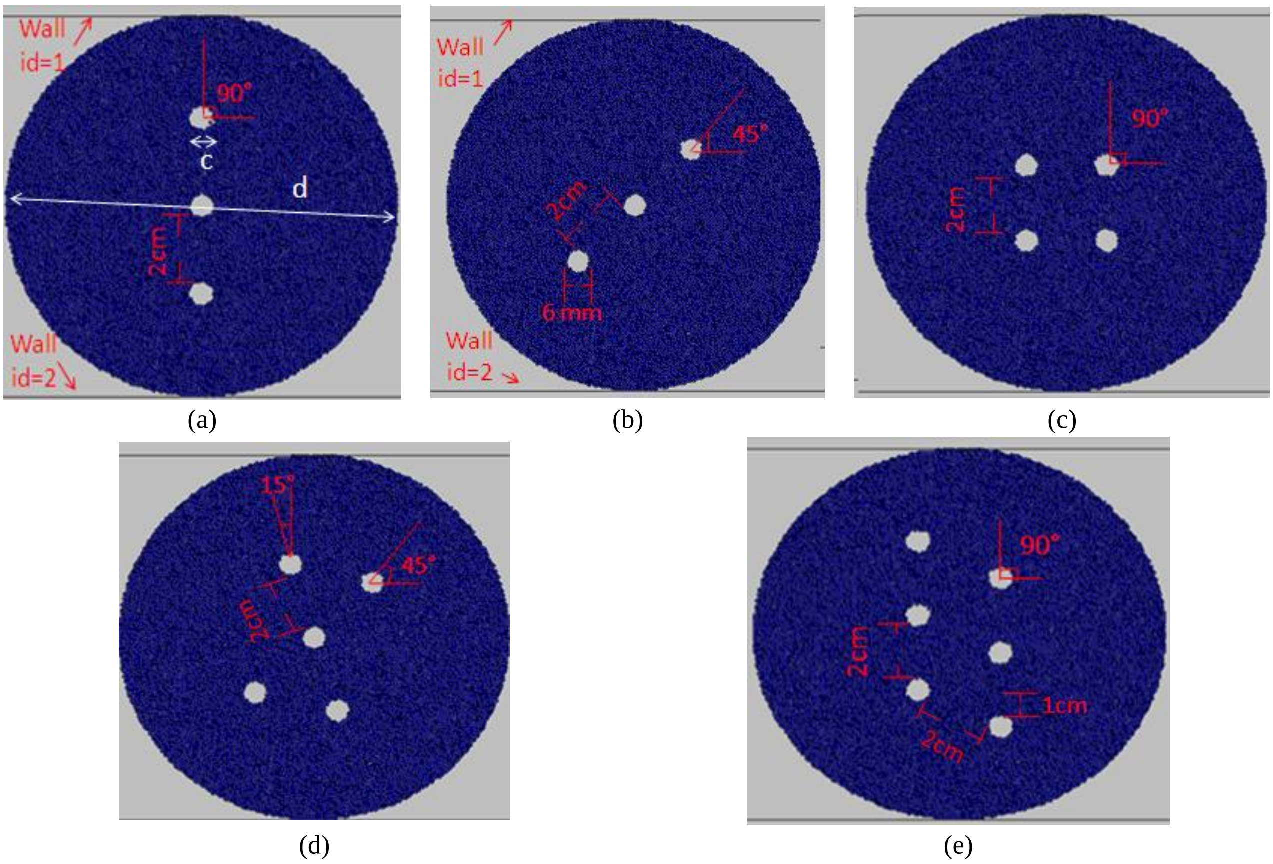


Fig. 3 The modelled Brazilian disc samples containing multi-cylindrical holes with a constant c/d ratio of 0.00 6 and loaded under diametrical compression; (a) the vertical arrangements of three holes in a disc specimen, (b) the diagonal arrangements of three holes in a disc specimen, (c) the two parallel arrangements of four hol es in a disc specimen, (d) the uniform arrangements of five holes in a disc specimen, (d) the two parallel arr angements of six holes in a disc specimen. (e) the two parallel arrangements of four holes in a disc specimen

containing multiple parallel cylindrical holes are shown in Figs. 3(a)-(e). Five different numerical models each having a different arrangement and numbers of the multiple holes within the specimens are being carried out to investigate the mechanism of cracks initiation and cracks propagation in the Brazilian disc type specimens containing multiple parallel cylindrical holes (Fig. 3). The modelled samples are designed to have a constant diameter d=10 cm and constant multi hole diameters c=0.6 mm (i.e., c/d = 0.006) but the number of holes and their arrangements may be different (as shown in Figs. 3 (a)-(e)). Therefore, Fig. 3(a) shows a vertical arrangement of three parallel holes in a disc specimen, Fig. 3(b) shows a diagonal arrangement of the three holes in a disc specimen, Fig. 3(c) shows two parallel arrangements of four holes in a disc specimen, Fig. 3(d) shows a uniform arrangement of five holes in a disc specimen, and, Fig. 3(e) shows two parallel arrangements of six holes in a disc specimen, respectively.

All these modelled samples are crushed by moving the lateral walls toward each other. The tensile and crack initiation forces are recorded by taking the reaction forces on the wall 1 shown in Fig. 3(a).

3. Results and discussions

The results and discussions for all of the numerically modelled concrete samples are given in this section. The mechanism of cracks initiation and cracks propagation and cracks coalescences are being discussed.

3.1 Effects of the central hole diameter on the cracks coalescence and interaction

As shown in Figs. 4(a)-(c), it is observed that for the ratios of r/ R = 0.12, 0.17 and 0.2, the tensile cracks initiate from the top and bottom of the modelled sample and then propagate toward the direction of the maximum compressive stress till interact with the internal ring. At this stage of fracturing process, some edge cracks in form of wedges are formed at the top of the modelled sample. At this stage due to the compression, some short shear bands may also occur in the sample. Figs. 4(d)-(g) show that the tensile cracks initiate from the top and bottom of the internal hole and propagate towards the direction of the applied compression till interact with the top and bottom of the sample and causing its failure on a single surface.

The experimental works on this modelling sample is already carried out by Haeri *et al.* (2015) and the results are shown in Figs. 5(a)-(g), Comparing Figs. 4 and 5, it may be visualized that there is a very good agreement in between the results of fracturing process of the modelled sample obtained experimentally and numerically. It’s to be note

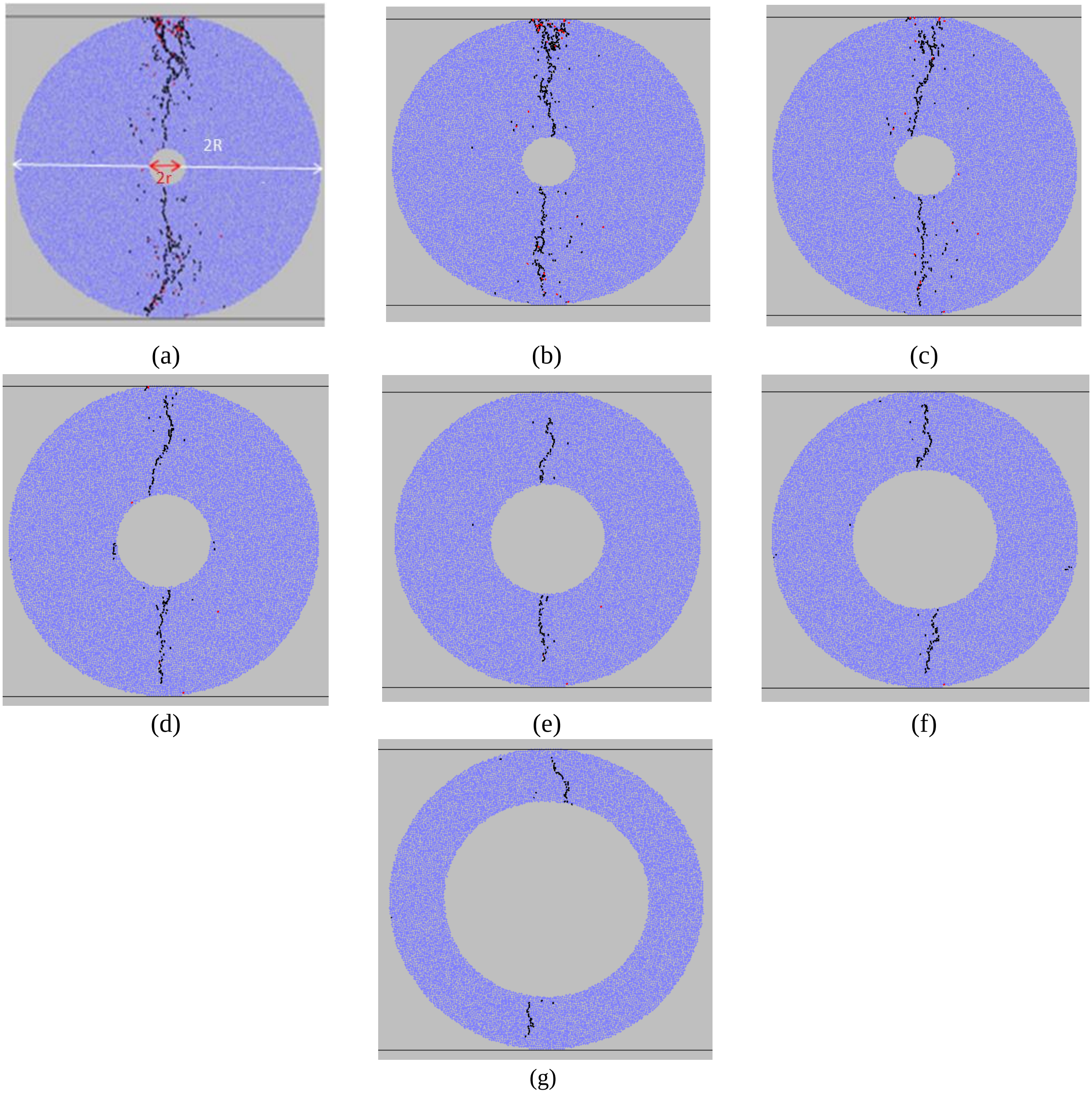


Fig. 4 Numerical tests showing the cracking patterns in the single-holed Brazilian discs under diametrical compression with different c/ d ratios: (a) c /d = 0.12, (b) c /d = 0.17, (c) c/d = 0.2, (d) c/d = 0.3, (e) c / d = 0.37, (f) c/d = 0.47 and (g) c /d = 0.65

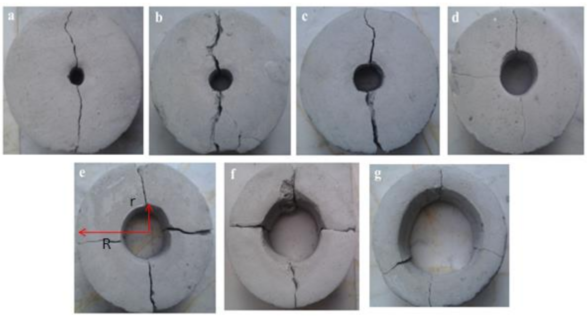


Fig. 5 Experimental tests showing the cracking patterns in the single-holed Brazilian discs under diametrical compression with different r / R ratios: (a) r / R = 0.12. (b) r/R = 0.17, (c) r/R = 0.2, (d) r /R = 0.3, (e) r /R = 0.37, (f) r / R = 0.47 and (g) r /R = 0.65 (Haeri *et al*. 2015)

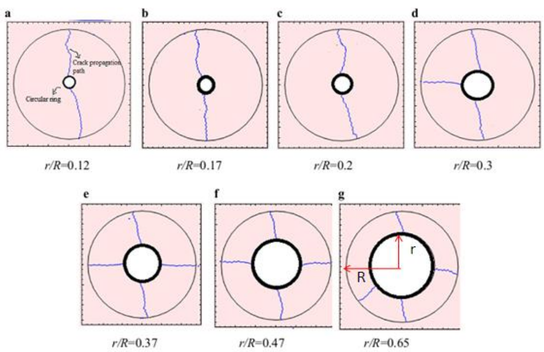
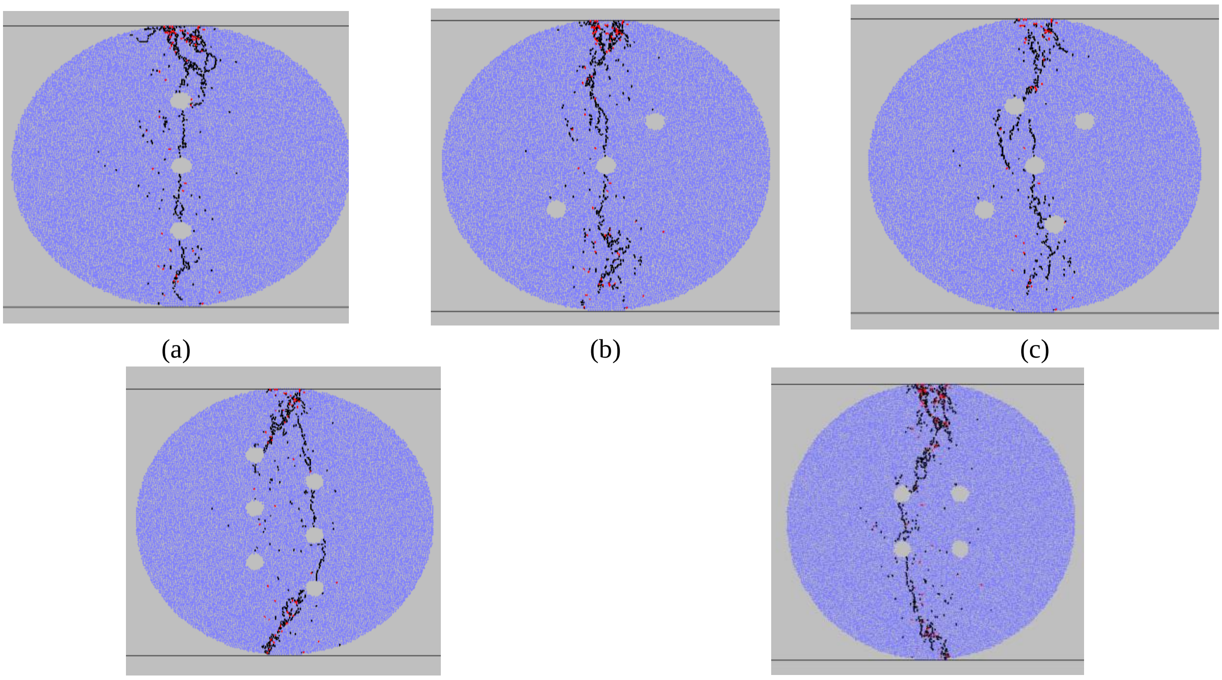


Fig. 6 Numerical simulation of the crack propagation path for cylindrical specimens with an axial hole (ring type disc specimen) under diametrical Compression (Haeri *et al*. 2015)



(a) (b) (c)

(d) (e)

Fig. 7 The discrete element numerical tests showing the cracking patterns in the disc specimens containing multi-cylindrical holes (c/d = 0.006) under diametrical compression. (a) Vertical arrangements of three holes, (b) Diagonal arrangements of three holes, (c) Uniform arrangements of five holes, (d) Two parallel arrangements of six holes, (e) Two parallel arrangements of four holes in disc specimens

that the failure patterns are different when the ratio of r/R is more than 0.37.

However, it is shown that the discrete element modelling (PFC2D) of the proposed concrete samples gives accurate and validates fracturing analyses results. It should be noted that the numerical modelling is cheaper, much faster, more flexible and easier to work as compared to that of the experimental work.

3.2 Effects of multiple cylindrical holes on the cracks coalescence and interaction

Fig. 7(a) shows a modelled sample with three vertically arranged parallel holes loaded under compression. The cracks are initiated from the top and bottom the modelled sample and propagated toward the holes till they coalesce with the internal holes and the initiated cracks in between them. The modelled sample with three diagonally arranged holes is shown in Fig. 7(b). In this case, the cracks are propagated from the central hole only in the direction of the applied compression. There may be no cracks to propagate from the other two holes. For the hole arrangement shown in Fig. 7(c) (the uniform arrangement of 5 holes), the cracks are propagated toward the direction of compression and also those three holes which are in approximately in line with the loading direction. Again, the other two holes are not affecting the failure process of the sample because they are not in range with the direction of loading (of course the size and distance of these holes are important but, in this study, to make the experiments more concise, they are kept constant). Fig. 7(d) illustrate the modelled sample containing six holes with a special hole arrangement. In this case, the cracks are initiated from the three central holes that were located at right side of specimen's central line and then these cracks coalesced with each other at the propagating crack tips. Finally, Fig. 7(e) illustrate a four holes' arrangement of the model. As shown in this Figure,

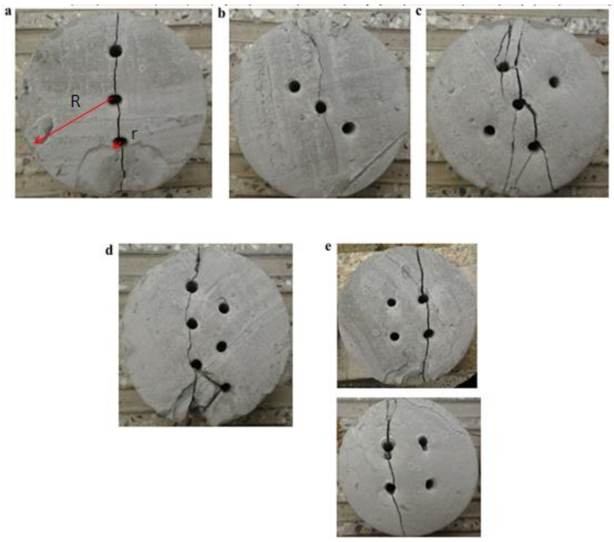


Fig. 8 The laboratory tests showing the cracking patterns in the disc specimens containing multi-cylindrical holes (c/d = 0.006) under diametrical compression. (a) Vertical arrangements of three holes, (b) Diagonal arrangements of three holes, (c) Uniform arrangements of five holes, (d) Two parallel arrangements of six holes, (e) Two parallel arrangements of four holes in disc specimens (Haeri e al. 2015)

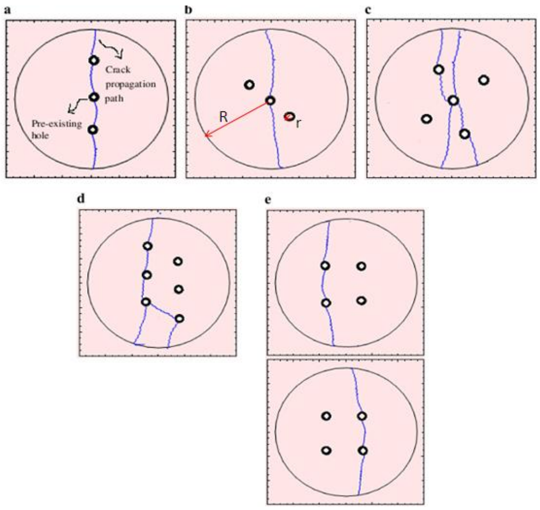


Fig. 9 The boundary element numerical tests showing the cracking patterns in the disc specimens containing multi-cylindrical holes (r/R = 0.006) under diametrical compression. (a) Vertical arrangements of three holes, (b) Diagonal arrangements of three holes, (c) Uniform arrangements of five holes, (d) Two parallel arrangements of six holes, (e) Two parallel arrangements of four holes in disc specimens (Haeri *et al*. 2015)

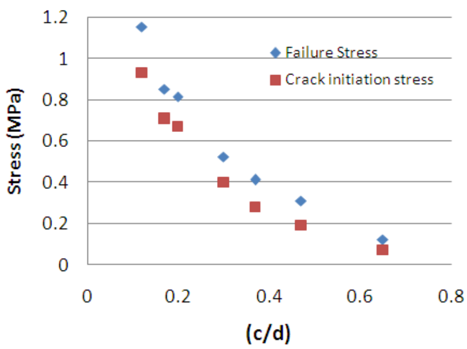


Fig. 10 The effect of hole diameter on the failure and crack initiation stresses

the cracks initiate from the two left holes and then propagates parallel to the loading direction. However, the modelled sample fails in the direction of the crack propagation paths shown in Figs. 7(a)-(e).

The crack propagation process in the Brazilian disc type samples is also studied experimentally (Fig. 8) and numerically (Fig. 9) by Haeri *et al*. (2015). A higher order displacement discontinuity method (a kind of indirect boundary element method) is used in their numerical work to simulate the experimental work. However, the present numerical study shown in Fig. 7 can be compared with those carried out by Haeri *et al*. (2015) in Figs. 8 and 9. These comparisons show that there is a very good agreement in between the present numerical simulation and the previous experimental and numerical results.

3.3 Effects of the internal hole diameter on the failure stress

Fig 10 shows the effects of hole diameter on the failure

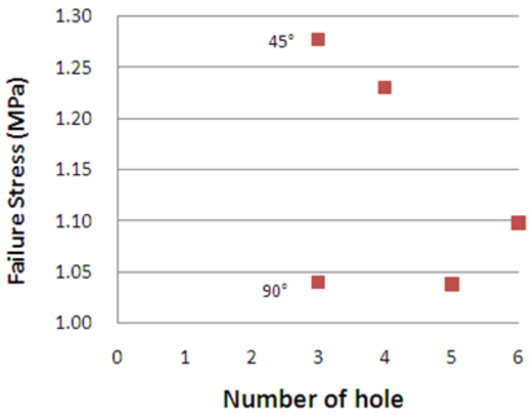


Fig. 11 The effect of number of holes on the failure stresses of model

and crack initiation stresses in the modelled samples. The purpose of failure stress is the final loading capacity of the model but crack initiation stress is the stress that 5% of total cracks develop within the model in this stress level; Ghazvinian *et al*. (2012). This figure shows that the failure and crack initiation stresses are decreased by increasing the hole diameter.

Effects of the number of holes on the failure stress of the modelled sample are graphically shown in Fig. 11. As shown in this Figure, the failure stress decreases by increasing the number of holes. For example, when three holes are arranged diagonally in the model, only one hole controls the failure process. In this case, the failure stress is high but for the vertically arranged configuration of three holes, the cracks initiates from all three holes therefore the failure stress is low. In the case of four holes' configuration, only two holes control the failure process therefore the failure stress is in between the two previous cases but less than previous case. For the two cases of five and six holes' configuration in the modelled samples, the cracks initiates from the three holes therefore, the failure stress in these configuration is similar to that of the modelled sample with three vertical holes' configuration.

4. Conclusions

The fracturing process of brittle materials such as concrete and rock is very significant in many engineering fields (like mining, civil and petroleum engineering) in recent years. The mechanism of crack initiation, propagation and cracks coalescence in rocks and rock like materials using the experimental and numerical tests are being carried out by many researches. In this numerical research, at the first stage, effects of the breaking load in the Brazilian disc type of pre-holed concrete modelled samples each containing a single hole of different size is studied. The failure stresses and the fracturing process analyses are numerically performed by a discrete element code and the results are compared and discussed with those of the previous works. At the second stage, the multi-holed Brazilian disc type of modelled concrete samples are tested under compression and these numerical results are compared with those already obtained by using the higher order displacement discontinuity method (a version of the indirect boundary element method).

• In the present study it is concluded that the mechanism of cracks initiation, propagation and coalescences the concrete samples can be numerically studied by using the discrete element modelling technique. Most of this crack fracturing mechanism and failure of the modelled concrete samples may occur due to the cracks coalescence phenomenon in the bridge area. These cracks extension phenomena are mainly caused by the propagation of tensile radial cracks emanating from the surface of the central holes.

• It has also been observed that the present numerical results are in good agreement with the experimental and numerical results recorded in the previous researches. For example, comparing the discrete element results with the corresponding results obtained by using the displacement discontinuity method shows the validity and accuracy of the present numerical method which illustrate that the tensile cracks and the cracks propagation paths are mainly produced by the coalescence phenomenon of the pre-existing multi-holes in the modelled concrete samples. Based on the present numerical analyses it may also be concluded that the tensile cracks may be initiated radially near the surface of the holes. The stress concentration can be released after the crack propagation phenomena and finally, the stresses in the modelled samples can be redistributed to attain a new equilibrium condition. The final breaking of the pre-holed disc type concrete specimens may be due to the propagation of radially induced tensile cracks initiated from the surface of the central hole. These cracks are propagating toward the direction of diametrical loading and/or perpendicular to it (for the case of modelled samples with larger holes). In the case of disc samples with multiple holes and in the breaking process of the modelled samples under the diametrical loading, the cracks propagation and cracks coalescence may occur simultaneously in the final stage of the breaking process.

• It has been shown that the failure and crack initiation stresses decrease by increasing the hole diameter, and also, the failure stress may be decreased by increasing the number of holes which are controlling the failure process of the modelled samples.

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