Structural Engineering and Mechanics, *Vol. 66, No. 3 (2018) 295-303*
DOI: https://doi.org/10.12989/sem.2018.66.3.295

Nonlinear dynamic analysis of spiral stiffened functionally graded cylindrical shells with damping and nonlinear elastic foundation under axial compression

Kamran Foroutan[a], Alireza Shaterzadeh* and Habib Ahmadi[b]

Faculty of Mechanical Engineering, Shahrood University of Technology, Shahrood, Iran

(Received June 11, 2017, Revised January 31, 2018, Accepted February 19, 2018)

Abstract. The semi-analytical method to study the nonlinear dynamic behavior of simply supported spiral stiffened functionally graded (FG) cylindrical shells subjected to an axial compression is presented. The FG shell is surrounded by damping and linear/nonlinear elastic foundation. The proposed linear model is based on the two-parameter elastic foundation (Winkler and Pasternak). A three-parameter elastic foundation with hardening/softening cubic nonlinearity is used for nonlinear model. The material properties of the shell and stiffeners are assumed to be FG. Based on the classical plate theory of shells and von Kármán nonlinear equations, smeared stiffeners technique and Galerkin method, this paper solves the nonlinear vibration problem. The fourth order Runge-Kutta method is used to find the nonlinear dynamic responses. Results are given to consider effects of spiral stiffeners with various angles, elastic foundation and damping coefficients on the nonlinear dynamic response of spiral stiffened simply supported FG cylindrical shells.

Keywords: FG cylindrical shells; nonlinear dynamic analysis; spiral stiffeners; damping and elastic foundation

1. Introduction

The stiffened functionally graded cylindrical shells have more application in a wide range of engineering applications, including submarines, aircrafts, bridges, ships, satellites and offshore structures. Study on nonlinear behavior of these structures is important of the practical. Then research on the dynamic analysis of these structures has been of interest of scientists from many years ago and great amount of studies have been done on nonlinear dynamic analysis of stiffened shell structures under mechanical loading.

The nonlinear vibration analysis of orthotropic FG cylindrical shells with an elastic foundation by using the first order shear deformation theory was investigated by Sofiyev *et al.* (2017). Also, Sofiyev *et al.* (2017) analyzed the nonlinear vibration response of composite orthotropic cylindrical shells with the nonlinear elastic foundations by using the shear deformation theory. Tang *et al.* (2017) studied the free and forced vibration behavior of multi-stepped cylindrical shells with arbitrary boundary conditions. They used the method of reverberation-ray matrix and Flügge thin shell theory. The stability behavior of compositionally graded ceramic–metal cylindrical shells under periodic axial compressive load was studied by sofiyev (2005). He used the Love’s shell theory and Lagrange-Hamilton type principle. Darabi *et al.* (2008) by using the large deflection theory, analyzed the dynamic stability of FG shells subjected to periodic axial loading. Sheng and Wang (2008) presented the vibration behavior of functionally graded cylindrical shell with flowing fluid and surrounded by elastic foundation under mechanical and thermal loads.

In the above-mentioned studies, the effects of stiffeners on the dynamic response of cylindrical shells have not been considered. Some studies have been done on dynamic analysis of stiffened cylindrical shells.

Chen *et al.* (2015) investigated the free and forced vibration behavior of ring-stiffened conical–cylindrical shells with arbitrary boundary conditions by using the Flügge thin shell theory. The nonlinear dynamic analysis of imperfect eccentrically stiffened FG cylindrical shells with an elastic foundation under mechanical and damping loads by using the first order shear deformation theory and Runge-Kutta method was studied by Duc and Thang (2015). Dung and Nam (2014) studied the nonlinear dynamic analysis of eccentrically stiffened FG cylindrical shells with an elastic foundation subjected to external pressure by using the classical plate theory of shells and Galerkin method. Bich *et al.* (2013) investigated the nonlinear static and dynamic analysis of FG cylindrical shells with eccentrically homogeneous stiffener system. They used the classical thin shell theory with the geometrical nonlinearity in von Kármán-Donnell sense and the smeared stiffeners technique to derive the governing equations of motion of cylindrical shells.

A review of studies shows that few studies have been done on the nonlinear dynamic analysis of simply supported stiffened FG cylindrical shells by semi-analytical approaches. In this study, the nonlinear dynamic behavior of stiffened FG cylindrical shells reinforced by spiral stiffeners with various angles, embedded in elastic media and linear damping and subjected to axial compression is

*Corresponding author
E-mail: a_shaterzadeh@shahroodut.ac.ir
[a]Ph.D. Student
[b]Ph.D.

http://www.techno-press.com/journals/sem&subpage=7
ISSN: 1225-4568 (Print), 1598-6217 (Online)

investigated using semi-analytic approaches. The elastic foundations are formulated based on the two-parameter elastic foundation (Winkler and Pasternak) and a nonlinear model, which is a three-parameter elastic foundation with hardening/softening cubic nonlinearity. The material properties of the shell and stiffeners are assumed to be continuously graded in the thickness direction according to a simple power law distribution in terms of volume fraction of constituents. The formulations of the governing equations are based on the classical shell theory and the smeared stiffeners technique in conjunction with the Galerkin method is used to solve the nonlinear problem. In continue the fourth-order Runge-Kutta method is used to find the nonlinear dynamic responses. In order to valid the formulations, comparisons are made with the previous researches. Results are presented to evaluate the effects of stiffener's angle, geometrical and material properties, elastic foundation, damping coefficient and axial loading on the nonlinear dynamic response of spiral stiffened simply supported FG cylindrical shells.

2. The basic formulation

2.1 FG material properties

Consider a stiffened FG cylindrical shell surrounded by elastic foundation and linear damping with radius R, thickness h and axial length L (see Fig. 1). The origin of the coordinate system (x, y, z) is shown in Fig. 1, where x and y are the axial and circumferential coordinate variables of the cylindrical shell and z is the inward radial coordinate variable. h_s, d and s are the thickness, width and spacing of the stiffeners, respectively. It is assumed that the cylindrical shell and the stiffeners are made of a mixture of ceramics and metals in two cases. In the first case with external stiffeners, the inner surface of the cylinder is metal-rich ($z = h/2$) and the outer surface ($z = -h/2$) is ceramic-rich and in order to keep material continuity, the lower surface of the stiffener is made of ceramic and the upper surface is made of metal. In the second case, a reverse order is used for internal stiffeners.

The FG material composition varies continuously by following simple power law in terms of the volume fractions of the constituents as be written as (Shaterzadeh *et al.* 2015)

$$V_i = V_i(z) = \left(\frac{2z+h}{2h}\right)^k$$
$$V_o = V_o(z) = 1 - V_i(z) \tag{1}$$

where $-h/2 \le z \le h/2$ and k is the material power law index of the FG shell, which takes values greater or equal to zero. In the first case with external stiffeners, $V_i = V_m$ and $V_o = V_c$, where the parameters V_m and V_c denote the metal and ceramic volume fractions and the subscripts m and c refer to the metal and ceramic constituents. For the second case with internal stiffeners, V_i and V_o represent the volume fraction of ceramic and metal, respectively. The effective properties P_{eff}, such as Young's modulus E and

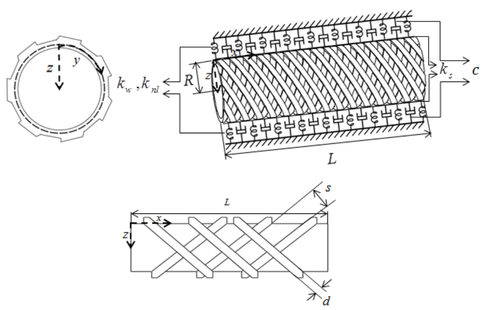


Fig. 1 Configuration of spiral stiffened FG cylindrical shell with damping and elastic foundation

mass density ρ, can be determined as (Dung and Nam 2014)

$$\mathrm{P}_{eff} = \mathrm{P}_o(z)V_o(z) + \mathrm{P}_i(z)V_i(z) \tag{2}$$

According to the mentioned law, the Young's modulus and mass density of the FG shell, external and internal stiffeners can be expressed in the following for

Shell

$$E(z) = E_o + (E_i - E_o)\left(\frac{2z+h}{2h}\right)^k, -\frac{h}{2} \le z \le \frac{h}{2}$$
$$\rho(z) = \rho_o + (\rho_i - \rho_o)\left(\frac{2z+h}{2h}\right)^k, -\frac{h}{2} \le z \le \frac{h}{2} \tag{3a}$$

External stiffeners

$$E_s(z) = E_i + (E_o - E_i)\left(\frac{2z+h}{2h_s}\right)^K, -\left(\frac{h}{2}+h_s\right) \le z \le -\frac{h}{2}$$
$$\rho_s(z) = \rho_i + (\rho_o - \rho_i)\left(\frac{2z+h}{2h_s}\right)^K, -\left(\frac{h}{2}+h_s\right) \le z \le -\frac{h}{2} \tag{3b}$$

Internal stiffeners

$$E_s(z) = E_i + (E_o - E_i)\left(\frac{2z-h}{2h_s}\right)^K, \frac{h}{2} \le z \le \left(\frac{h}{2}+h_s\right)$$
$$\rho_s(z) = \rho_i + (\rho_o - \rho_i)\left(\frac{2z-h}{2h_s}\right)^K, \frac{h}{2} \le z \le \left(\frac{h}{2}+h_s\right) \tag{3c}$$

where E, E_s and ρ, ρ_s are the Young's modulus and mass density of the FG shell and stiffeners, respectively. Also K is the material power law index of the FG stiffeners.

2.2 The theoretical formulation

2.2.1 Governing equations

According to the von Kármán nonlinear strain-displacement relations (Brush and Almrith 1975) the strain components on the middle plane of cylindrical shells can be expressed as

$$\varepsilon_x^0 = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2$$
$$\varepsilon_y^0 = \frac{\partial v}{\partial y} - \frac{w}{R} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2$$
$$\gamma_{xy}^0 = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} \tag{4}$$
$$\chi_x = \frac{\partial^2 w}{\partial x^2}, \ \chi_y = \frac{\partial^2 w}{\partial y^2}, \chi_{xy} = \frac{\partial^2 w}{\partial x \partial y}$$

where $u = u(x,y,t), v = v(x,y,t), w = w(x,y,t)$ are the displacement components along x,y,z axes, respectively. ε_x^0 and ε_y^0 are the normal strains, γ_{xy}^0 is the shear strain at the middle surface, $\chi_x, \chi_y, \chi_{xy}$ are the change of curvatures and twist of shell.

The strain components across the shell thickness at a distance z from the mid-surface are represented by (Darvizeh *et al.* 2010)

$$\varepsilon_x = \varepsilon_x^0 - z\,\chi_x \;,\varepsilon_y = \varepsilon_y^0 - z\,\chi_y \;,\gamma_{xy} = \gamma_{xy}^0 - 2z\,\chi_{xy} \tag{5}$$

According to Eq. (4), compatibility equation can be expressed in the following form

$$\frac{\partial^2 \varepsilon_x^0}{\partial y^2} + \frac{\partial^2 \varepsilon_y^0}{\partial x^2} - \frac{\partial^2 \gamma_{xy}^0}{\partial x \partial y} = -\frac{1}{R}\frac{\partial^2 w}{\partial x^2} + \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} \tag{6}$$

The stress-strain relationship for FG cylindrical shells can be written as follows

$$\begin{aligned} \sigma_x^{sh} &= \frac{E(z)}{1-\nu^2}\left(\varepsilon_x + \nu\varepsilon_y\right) \\ \sigma_y^{sh} &= \frac{E(z)}{1-\nu^2}\left(\varepsilon_y + \nu\varepsilon_x\right) \\ \tau_{xy}^{sh} &= \frac{E(z)}{2(1+\nu)}\gamma_{xy} \end{aligned} \tag{7}$$

where the Poisson's ratio ν is assumed to be constant $\sigma_x^{sh}, \sigma_y^{sh}$ and τ_{xy}^{sh} are normal stress in x,y direction and shearing stress of un-stiffened shell, respectively.

The stress-strain relations of the spiral stiffeners as follow (Shaterzadeh and Foroutan 2016)

$$\begin{aligned} \sigma_x^s = Z_1\Big[&\varepsilon_x\left(\cos^3\theta + \cos^3\beta\right) \\ &+2\gamma_{xy}\left(\sin\theta\cos^2\theta - \sin\beta\cos^2\beta\right) \\ &+\varepsilon_y\left(\sin^2\theta\cos\theta + \sin^2\beta\cos\beta\right)\Big] \end{aligned} \tag{8a}$$

$$\begin{aligned} \sigma_y^s = Z_2\Big[&\varepsilon_x\left(\sin\theta\cos^2\theta + \sin\beta\cos^2\beta\right) \\ &+2\gamma_{xy}\left(\sin^2\theta\cos\theta - \sin^2\beta\cos\beta\right) \\ &+\varepsilon_y\left(\sin^3\theta + \sin^3\beta\right)\Big] \end{aligned} \tag{8b}$$

$$\begin{aligned} \tau_{xy}^s = Z_3\Big[&\varepsilon_x\left(\cos^2\theta - \cos^2\beta\right) \\ &+2\gamma_{xy}\left(\sin\theta\cos\theta + \sin\beta\cos\beta\right) \\ &+\varepsilon_y\left(\sin^2\theta - \sin^2\beta\right)\Big] \end{aligned} \tag{8c}$$

where

$$\begin{aligned} Z_1 &= \frac{h_s d E_s}{s h_s}\frac{\sin(\theta+\beta)}{(\sin\theta + \sin\beta)} \\ Z_2 &= \frac{h_s d E_s}{s h_s}\frac{\sin(\theta+\beta)}{(\cos\theta + \cos\beta)} \\ Z_3 &= \frac{h_s d E_s}{2 s h_s}\sin(\theta+\beta) \end{aligned} \tag{9}$$

σ_x^s, σ_y^s and τ_{xy}^s are the normal and shear stress components of the stiffeners, respectively. To consider the effect of the stiffeners on the shell the smeared stiffeners technique is used. By integrating the stress-strain equations, the resultant forces and moments for stiffened FG cylindrical shells can be obtained as (Duc and Thang 2014)

Resultant forces

$$\begin{aligned} N_x &= J_{11}\varepsilon_x^0 + J_{12}\varepsilon_y^0 - J_{14}\chi_x - J_{15}\chi_y \\ N_y &= J_{21}\varepsilon_x^0 + J_{22}\varepsilon_y^0 - J_{24}\chi_x - J_{25}\chi_y \\ N_{xy} &= J_{33}\gamma_{xy}^0 - 2J_{36}\chi_{xy} \end{aligned} \tag{10}$$

Resultant moments

$$\begin{aligned} M_x &= J_{14}\varepsilon_x^0 + J_{15}\varepsilon_y^0 - J_{41}\chi_x - J_{42}\chi_y \\ M_y &= J_{24}\varepsilon_x^0 + J_{25}\varepsilon_y^0 - J_{51}\chi_x - J_{52}\chi_y \\ M_{xy} &= J_{36}\gamma_{xy}^0 - 2J_{63}\chi_{xy} \end{aligned} \tag{11}$$

where J_{ij} are the components of the extensional, bending and coupling stiffness of spiral stiffened FG cylindrical shells which are defined in Appendix A. By rearranging Eq. (10), the strain components can be defined as

$$\begin{aligned} \varepsilon_x^0 &= J_{22}^* N_x - J_{12}^* N_y + J_{11}^{**}\chi_x + J_{12}^{**}\chi_y \\ \varepsilon_y^0 &= J_{11}^* N_y - J_{21}^* N_x + J_{21}^{**}\chi_x + J_{22}^{**}\chi_y \\ \gamma_{xy}^0 &= J_{33}^* N_{xy} + 2J_{36}^{**}\chi_{xy} \end{aligned} \tag{12}$$

By substituting Eq. (12) into Eq. (11) the resultant moments can be expressed as

$$\begin{aligned} M_x &= A_{11}^* N_x + A_{21}^* N_y - A_{11}^{**}\chi_x - A_{12}^{**}\chi_y \\ M_y &= A_{12}^* N_x + A_{22}^* N_y - A_{21}^{**}\chi_x - A_{22}^{**}\chi_y \\ M_{xy} &= A_{36}^* N_{xy} - 2A_{36}^{**}\chi_{xy} \end{aligned} \tag{13}$$

The nonlinear equilibrium equations of thin circular cylindrical shell based on the classical shell theory are as follow (Bich *et al.* 2013, Ghiasian *et al.* 2013 and Volmir 1972)

$$\begin{aligned} &\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \\ &\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0 \\ &\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + N_x\frac{\partial^2 w}{\partial x^2} \\ &\quad + 2N_{xy}\frac{\partial^2 w}{\partial x \partial y} + N_y\left(\frac{\partial^2 w}{\partial y^2} + \frac{1}{R}\right) - k_w w \\ &\quad + k_s\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) + k_{nl}w^3 = \rho_1\frac{\partial^2 w}{\partial t^2} + 2\rho_1 c\frac{\partial w}{\partial t} \end{aligned} \tag{14}$$

where k_w, k_s and k_{nl} are the Winkler, Pasternak and nonlinear cubic constants of the elastic foundation, respectively. Also t is the time, c is damping coefficient and the mass density ρ_1 can be calculated as

$$\rho_1 = \left(\rho_{out} + \frac{\rho_{in} - \rho_{out}}{k+1}\right)h + 2\left(\rho_{in} + \frac{\rho_{out} - \rho_{in}}{K+1}\right)\frac{dh_s}{S} \tag{15}$$

According to the first two of Eq. (14), a stress function φ is defined as

$$N_x = \frac{\partial^2 \varphi}{\partial y^2}, N_y = \frac{\partial^2 \varphi}{\partial x^2}, N_{xy} = -\frac{\partial^2 \varphi}{\partial x \, \partial y} \tag{16}$$

By substituting Eq. (12) into the compatibility Eq. (6) and Eq. (13) into the third part of Eq. (14) and using the Eq. (4) and (16), the following system of equation can be obtained.

$$\begin{aligned} & J_{11}^{*} \frac{\partial^4 \varphi}{\partial x^4} + \left(J_{33}^{*} - J_{12}^{*} + J_{21}^{*}\right) \frac{\partial^4 \varphi}{\partial x^2 \partial y^2} + J_{22}^{*} \frac{\partial^4 \varphi}{\partial y^4} \\ & + J_{21}^{**} \frac{\partial^4 w}{\partial x^4} + \left(J_{11}^{**} + J_{22}^{**} - 2J_{36}^{**}\right) \frac{\partial^4 w}{\partial x^2 \partial y^2} \\ & + J_{12}^{**} \frac{\partial^4 w}{\partial y^4} + \frac{1}{R} \frac{\partial^2 w}{\partial x^2} + \left[\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2}\right] \\ & - 2\frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = 0 \end{aligned} \tag{17}$$

$$\begin{aligned} & \rho_1 \frac{\partial^2 w}{\partial t^2} + 2c\rho_1 \frac{\partial w}{\partial t} + A_{11}^{**} \frac{\partial^4 w}{\partial x^4} + \left(A_{12}^{**} + A_{21}^{**} + 4A_{36}^{**}\right) \frac{\partial^4 w}{\partial x^2 \partial y^2} \\ & + A_{22}^{**} \frac{\partial^4 w}{\partial y^4} - A_{21}^{*} \frac{\partial^4 \varphi}{\partial x^4} - \left(A_{11}^{*} + A_{22}^{*} - 2A_{36}^{*}\right) \frac{\partial^4 \varphi}{\partial x^2 \partial y^2} \\ & - A_{12}^{*} \frac{\partial^4 \varphi}{\partial y^4} - \frac{1}{R} \frac{\partial^2 \varphi}{\partial x^2} - \frac{\partial^2 \varphi}{\partial y^2} \frac{\partial^2 w}{\partial x^2} + 2\frac{\partial^2 \varphi}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} \\ & - \frac{\partial^2 \varphi}{\partial x^2} \frac{\partial^2 w}{\partial y^2} + k_w w - k_s \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) - k_{nl} w^3 = 0 \end{aligned} \tag{18}$$

Eqs. (17) and (18) are a system of nonlinear equation in terms of two unknown parameter φ and w.

2.2.2 Boundary conditions

A simply supported spiral stiffened FG cylindrical shell subjected to an axial compressive load $\bar{P}_X$ is considered. The compressive axial load is assumed to be positive and can be calculated as

$$\bar{P}_X = P_X h \tag{19}$$

where P_X is the average stress on the end sections of shell. The applied boundary conditions are of the following form

$$w = 0,\ M_x = 0,\ N_x = -P_X h, N_{xy} = 0,\ at\ x = 0; L \tag{20}$$

The deflection of the cylindrical shells is considered as (Volmir 1972 and Bich *et al.* 2012)

$$w = f(t) \sin\frac{m\pi x}{L} \sin\frac{ny}{R} \tag{21}$$

where $f(t)$ is the time dependent total unknown amplitude and m, n are the number of half wave and full wave in the axial and circumferential directions, respectively.

By substituting Eq. (21) in Eq. (17) and solving the system of equations, the unknown function φ can be obtained as

$$\varphi = \varphi_1 \cos\frac{2m\pi x}{L} + \varphi_2 \cos\frac{2ny}{R} - \varphi_3 \sin\frac{m\pi x}{L} \sin\frac{ny}{R} - P_X h \frac{y^2}{2} \tag{22}$$

where the coefficients $\varphi_i (i = 1,2,3)$ are as follows

$$\begin{aligned} \varphi_1 &= \frac{n^2 \lambda^2}{32 J_{11}^{*} m^2 \pi^2} f(t)\left(f(t) + 2\mu h\right) \\ \varphi_2 &= \frac{m^2 \pi^2}{32 J_{22}^{*} n^2 \lambda^2} f(t)\left(f(t) + 2\mu h\right) \\ \varphi_3 &= \frac{B}{A} f(t) \end{aligned} \tag{23}$$

Substituting Eqs. (21-23) into Eq. (18) and by applying the Galerkin method, the governing equation can be obtained as

$$\begin{aligned} & \left(\frac{d^2 f(t)}{dt^2} + 2\varepsilon \frac{df(t)}{dt}\right) + \left(a_1 + a_2 k_w + a_3 k_s\right) f(t) \\ & + \left(a_4 + a_5 k_{nl}\right) f(t)^3 - a_6 P_X f(t) = 0 \end{aligned} \tag{24}$$

where the coefficients A, B, B^*, D and G are defined in appendix B.

2.3 Nonlinear vibration analysis

2.3.1 Forced vibration analysis

Consider the spiral stiffened FG cylindrical shell subjected to axial compressive stress with the $P_X = Q \sin \Omega t$, Eq. (24) become

$$\begin{aligned} & \left(\frac{d^2 f(t)}{dt^2} + 2\varepsilon \frac{df(t)}{dt}\right) + \left(a_1 + a_2 k_w + a_3 k_s\right) f(t) \\ & + \left(a_4 + a_5 k_{nl}\right) f(t)^3 - a_6 Q sin\Omega t f(t) = 0 \end{aligned} \tag{25}$$

where Q is amplitude of excitation force and Ω is excitation frequency.

By using this equation, the nonlinear forced vibration response of spiral stiffened FG cylindrical shell is taken into account. The second order nonlinear governing differential equation (Eq. (25)) is solved by the fourth order Runge-Kutta method.

2.3.2 Free vibration analysis

In order to validate the present formulation, the free and linear vibration analysis of spiral stiffened FG cylindrical shells without damping, the Eq. (25) is obtained as

$$\frac{d^2 f(t)}{dt^2} + \left(a_1 + a_2 k_w + a_3 k_s\right) f(t) = 0 \tag{26}$$

The fundamental frequency of natural vibration of spiral stiffened FG cylindrical shells can be determined by

$$\omega_{mn} = \sqrt{\left(a_1 + a_2 k_w + a_3 k_s\right)} \tag{27}$$

where ω_{mn} is fundamental frequency of natural vibration of shell.

Table 1 Comparison on the natural frequency of un-stiffened cylindrical shells with Winkler foundation ($m = 1, L = 1$ m$, R = 0.5$ m$, E = 7 \times 10^{10}$ N/m$^2, \nu = 0.3, d = 0.0025$ m$, h_s = 0.01$ m)

n	present	Sofiyev *et al.* (2009)		Paliwal *et al.* (1996)	
			Error(%)		Error(%)
1	0.67480	0.67921	0.65	0.67882	0.60
2	0.36223	0.36463	0.66	0.36394	0.47
3	0.20670	0.20804	0.65	0.20526	0.70
4	0.13747	0.13824	0.56	0.12745	7.29

3. Numerical results

3.1 Validation of the present approach

To validate the present formulation, in Table 1, the obtained the natural frequencies of isotropic cylindrical shell surrounded by an elastic foundation are compared with those present by Sofiyev *et al.* (2009) and Paliwal *et al.* (1996). These comparisons show that the good agreements are obtained.

3.2 Nonlinear dynamic responses

In this section, nonlinear dynamic analysis of spiral stiffened FG cylindrical shells with damping and linear/nonlinear elastic foundation are investigated. The effects of different geometrical and material parameters such as angle of stiffeners, radius and thickness of the FG shell, volume fraction of FG material, damping coefficient and also elastic foundation parameters on nonlinear dynamic responses of spiral stiffened FG cylindrical shells are evaluated. Two different types of stiffened FG cylindrical shell are examined. In the first case, external stiffeners exist on the outer surface of the FG cylindrical shell and the exterior surface of the cylindrical shell is metal rich while the interior surface is ceramic rich. The second case, FG material direction is vice versa and the stiffeners are located on the interior surface of the FG cylindrical shell. In the present study, unless defined, the number of stiffeners is assumed to be thirty which are distributed uniformly along the length of the FG cylindrical shell. The FG cylindrical shell is assumed to be made of aluminum (Al) and alumina (Al_2O_3) with following material properties.

$$Al:\ E_m = 70\text{ GPa},\quad \rho_m = 2702\ \frac{\text{kg}}{\text{m}^3},\quad \nu_m = 0.3,$$

$$Al_2O_3:\ E_c = 380\text{ GPa},\quad \rho_c = 3800\ \frac{\text{kg}}{\text{m}^3},\quad \nu_c = 0.3.$$

Since the Poisson's ratio is assumed to be equal for ceramic and metal, thus in all solved examples the Poisson's ratio is defined by ν i.e. $\nu = \nu_m = \nu_c = 0.3$ also, number of half waves in axial direction (m) are assumed to be equal to 1.

The geometrical parameters of the FG shell and stiffeners are considered as follow

$$Shell:\ R = 0.5\text{ m},\ L = 0.75\text{ m},\ h = 0.002\text{ m},$$

$$\text{Stiffener: } h_s = 0.01\text{ m},\ d = 0.0025\text{ m},\ n_s = 30.$$

Also, the assumed elastic constants of the foundation are listed below

$$k_s = 2.5 \times 10^5\ \frac{\text{N}}{\text{m}}, k_w = 5 \times 10^6\ \frac{\text{N}}{\text{m}^3}, k_{nl} = 8 \times 10^{13}\ \frac{\text{N}}{\text{m}^5}$$

Table 2 Effect of R/h ratio and volume-fraction index k on the fundamental frequency of natural vibration (rad/s) of spiral stiffened FG cylindrical shells

R/h	k	Un-stiffened	Internal stiffeners	External stiffeners
150	0.2	1750.9 (7)	2720.6 (6)	2488.8 (5)
	1	21.38.3 (7)	2468.6 (6)	2797.7 (5)
	5	2455.6 (7)	2143.9 (5)	2829.4 (6)
200	0.2	1490.9 (7)	2495.5 (6)	2420.5 (5)
	1	1841.1 (7)	2365.6 (6)	2715.3 (5)
	5	2112.9 (7)	2109.7 (5)	2716.1 (6)
250	0.2	1353.6 (8)	2375.5 (6)	2376.5 (5)
	1	1654.1 (8)	2323.0 (6)	2665.4 (5)
	5	1899.5 (8)	2094.0 (5)	2675.4 (6)

*The numbers in the parenthesis denote the number of full wave (n)

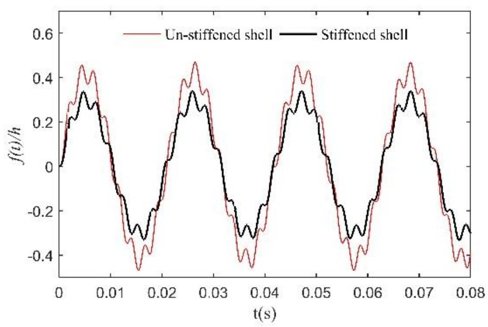
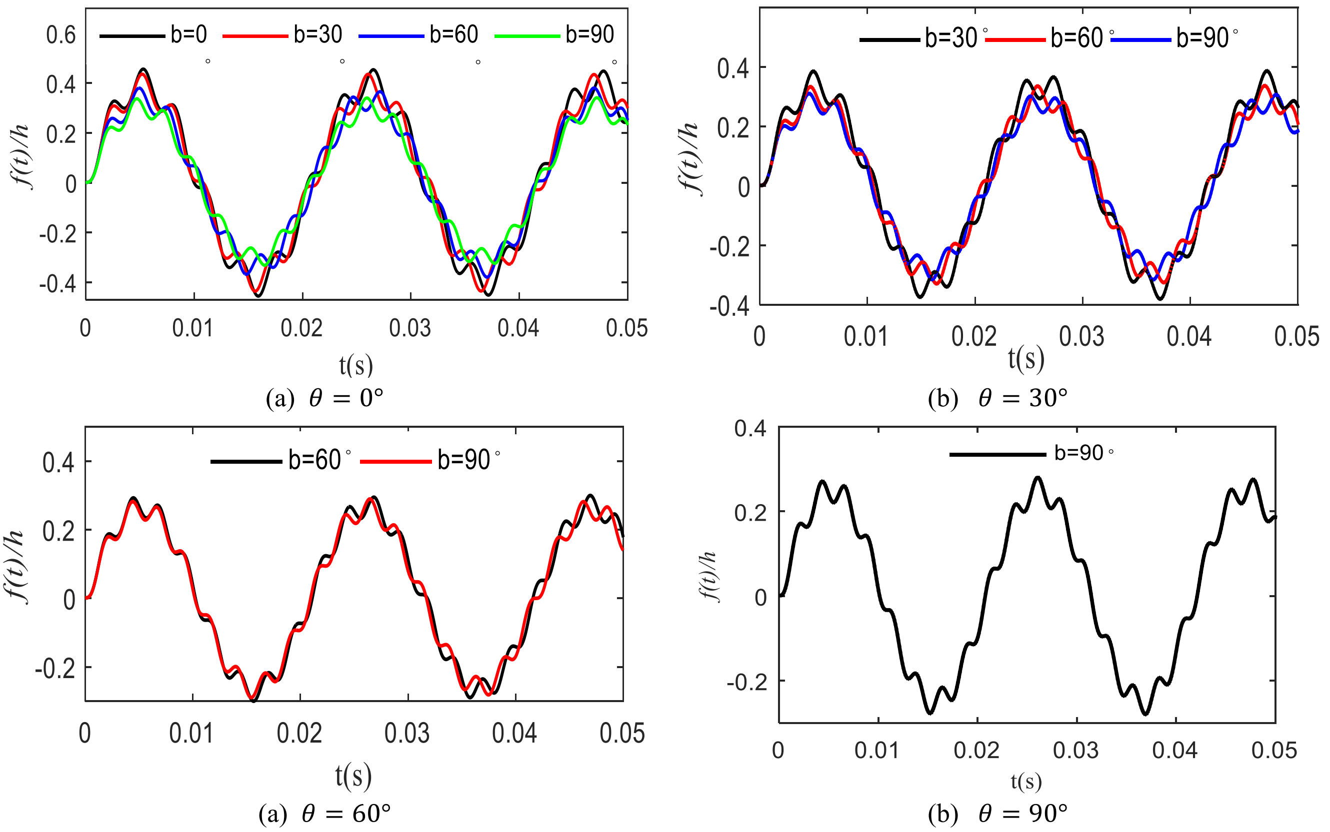


Fig. 2 Nonlinear responses of un-stiffened and internal stiffened FG cylindrical shells without elastic foundation ($P_X = 10^6 \sin(300t), \theta = 0°, \beta = 90°, K = k = 1$)

The effect of variation of radius-to-thickness ratio and volume-fraction index on the fundamental frequency of natural vibration of un-stiffened and stiffened FG cylindrical shell is demonstrated in Table 2. According to this table, by increasing the radius-to-thickness ratio, the value of natural frequency of shell decreases. The natural frequency increases when the proportion of metal decreases. Also, the natural frequency of FG cylindrical shells with external stiffeners is greater than one of shell with internal stiffeners and without stiffeners.

The nonlinear dynamic responses of stiffened and un-stiffened FG cylindrical shell are investigated in Fig. 2. The excitation frequencies equal to $P_X = 10^6 \sin(300t)$ are much smaller than fundamental frequencies of natural vibration. According to Fig. 2, can be show that the stiffeners strongly decreased the amplitude nonlinear vibration of the cylindrical shell when excitation frequency is far from natural frequency.

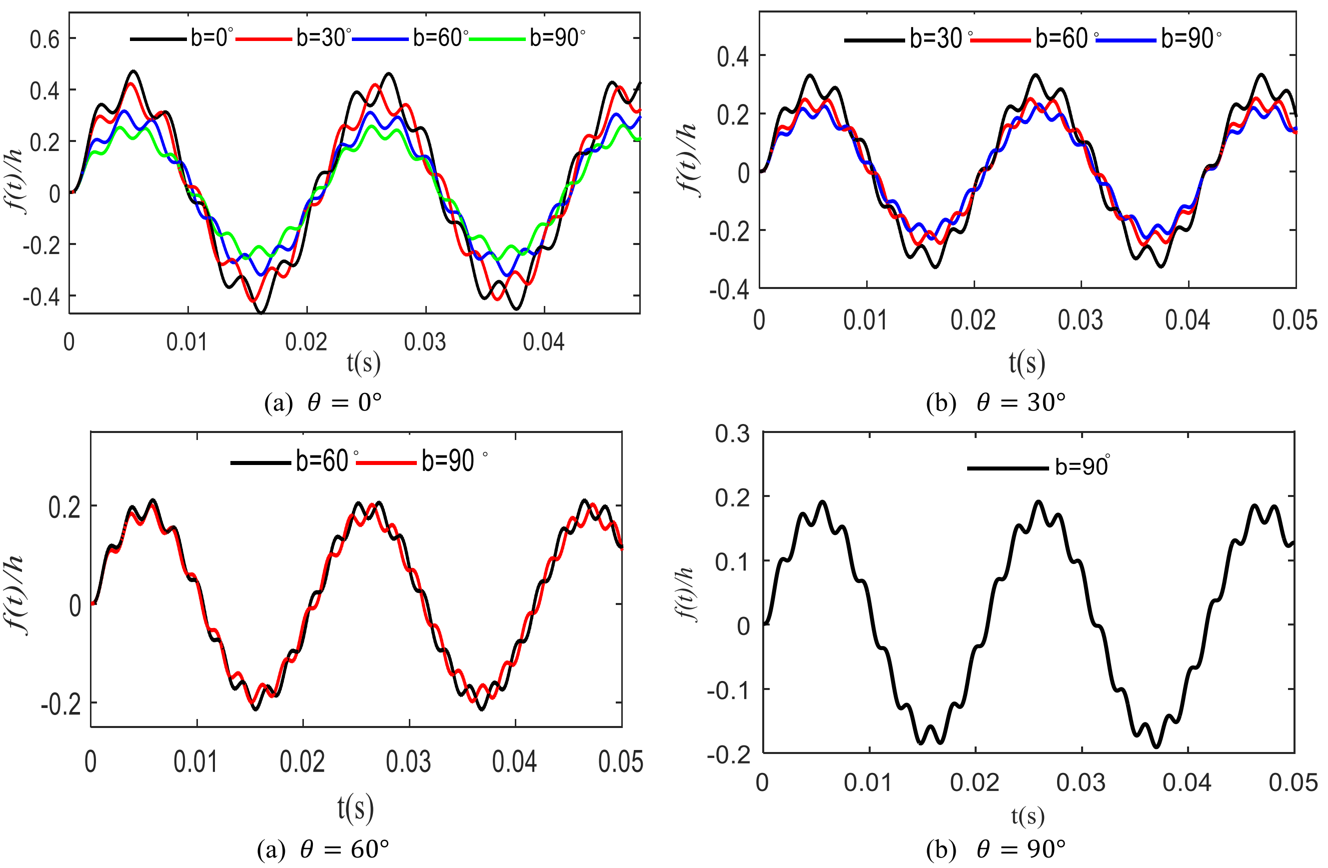
The effect of angle of stiffeners on nonlinear dynamic response of spiral stiffened FG cylindrical shell for different values of amplitude-to-thicknesses ratios with internal and external stiffeners are demonstrated in Figs. 3 and 4, respectively. In the present work the effects of different



(a) $\theta = 0°$ (b) $\theta = 30°$

(a) $\theta = 60°$ (b) $\theta = 90°$

Fig. 3 Nonlinear responses of internal spiral stiffened FG cylindrical shells without elastic foundation ($P_X = 10^6 \sin(300t)$, $K = k = 1$)



(a) $\theta = 0°$ (b) $\theta = 30°$

(a) $\theta = 60°$ (b) $\theta = 90°$

Fig. 4 Nonlinear responses of external spiral stiffened FG cylindrical shells without elastic foundation ($P_X = 10^6 \sin(300t)$, $K = k = 1$)

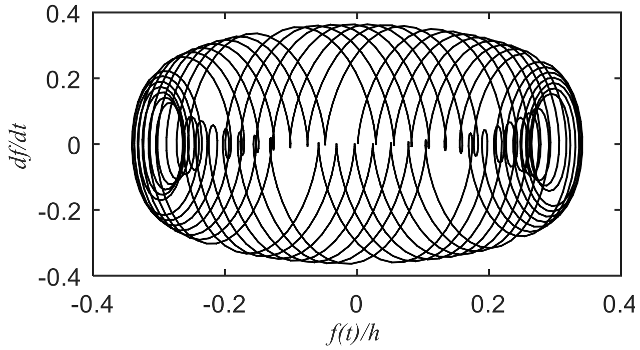


Fig. 5 Deflection-velocity relation of internal spiral stiffened FG cylindrical shell under $Q = 10^6$ N/m^2

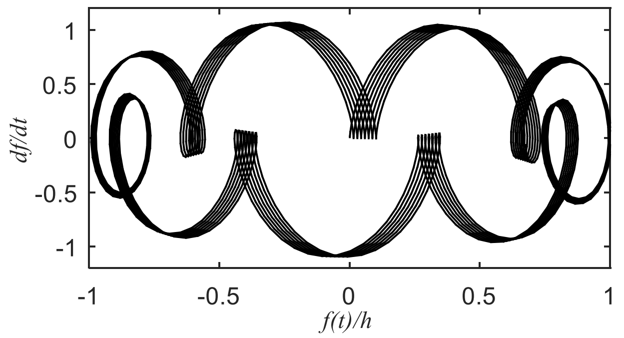


Fig. 6 Deflection-velocity relation of internal spiral stiffened FG cylindrical shell under $Q = 3 \times 10^6$ N/m^2

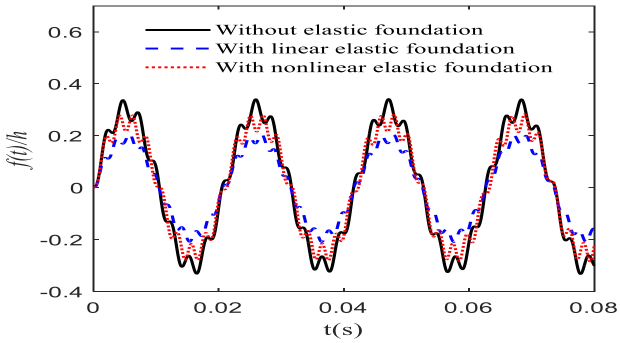


Fig. 7 Nonlinear vibration of internal spiral stiffened FG cylindrical shells ($P_X = 10^6 \sin(300t)$, $K = k = 1$)

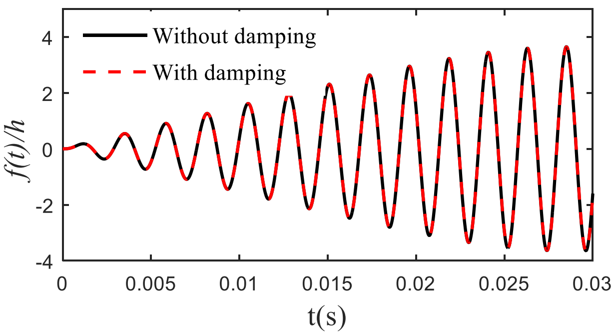


Fig. 8 Effect of damping on nonlinear vibration of spiral stiffened FG cylindrical shells in the first periods ($5 \times 10^5 \sin(2700t)$, $K = k = 1$)

values of stiffener's angle are examined. The amplitude nonlinear vibrations of FG stiffened cylindrical shell are

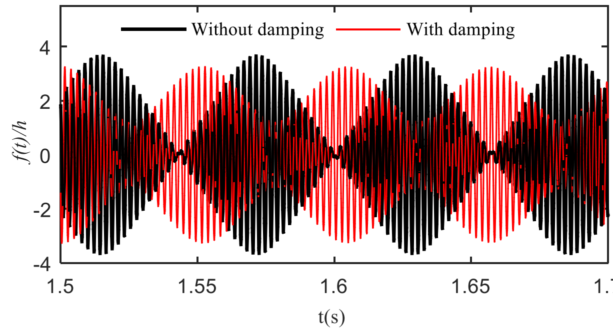


Fig. 9 Effect of damping on nonlinear vibration of spiral stiffened FG cylindrical shells in the far periods ($5 \times 10^5 \sin(2700t)$, $K = k = 1$)

less than other cases, when the angle between both series of stiffeners is 90° i.e., $\theta - \beta = 90°$.

Clearly, the maximum amplitude nonlinear vibrations of cylindrical shell happen when the angle between both series of stiffeners is 0°.

Fig. 5 shows the phase plane of spiral stiffened FG cylindrical shells. According to this figure, the deflection-velocity relation with has the closed curve, when the excitation force is small ($Q = 10^6$ N/m^2). Also, according to Fig. 6., the deflection-velocity relation becomes more disorderly, when the excitation force increases ($Q = 3 \times 10^6$ N/m^2).

The effect of linear and nonlinear elastic foundation on the nonlinear dynamic response of spiral stiffened FG cylindrical shells is shown in Fig. 7. It can be seen that elastic foundation decreases the amplitude of the nonlinear dynamic response.

The effect of linear damping on nonlinear dynamic responses is investigated in Figs. 8 and 9. As can be seen, in the first vibration periods, the damping influences on the nonlinear dynamic response are very small (Fig. 8) however, at the next far periods, it strongly decreases amplitude nonlinear dynamic (Fig. 9). Also, according to these figures, it can be shown that when the excitation frequencies are near to fundamental frequency of natural vibration, the interesting phenomenon like the harmonic beat phenomenon of a vibration is observed. The excitation frequency is 2700 rad/s which is near to fundamental frequency of natural vibration 2665.4 rad/s of external spiral stiffened FG cylindrical shell.

5. Conclusions

A semi-analytical method was used to study the nonlinear dynamic analysis of simply supported spiral stiffened FG cylindrical shells subjected to axial loading. Two different types of elastic foundation were assumed to encompass the FG cylindrical shell. The first model was formulated based on the two-parameter elastic foundation and the second model was a three parameter nonlinear elastic foundation. Also, it is assumed the cylindrical shell is surrounded by linear damping, too. The material properties of the shell and stiffeners were assumed to be continuously graded in the thickness direction. Based on the location of the stiffeners, two different models of stiffened FG cylindrical shells with internal and external stiffeners were formulated. The problem formulation was based on the classical shell theory with von Kármán nonlinear terms. The smeared stiffeners technique and Galerkin method were used to solve the nonlinear problem. Also, the fourth order Runge-Kutta method was used to find the nonlinear dynamic response of the FG cylindrical shells. The effects of different parameters such as material properties, geometrical dimensions, angle of stiffeners, damping coefficient and elastic foundation parameters on the nonlinear dynamic response of spiral stiffened FG cylindrical shells were examined and the following conclusions were obtained.

• The natural frequency of FG cylindrical shells with external stiffeners is greater than one of shell with internal stiffeners and without stiffeners.

• The minimum and maximum value of the amplitude nonlinear vibrations of stiffened FG cylindrical shells with internal and external stiffeners happen when the angle between both series of stiffeners is 90º and 0º, respectively.

• Linear and nonlinear elastic foundation decreases the value of the amplitude nonlinear vibration of FG cylindrical shells.

• At the next far periods, damping strongly decreases amplitude nonlinear vibration.

• When the excitation frequencies are near to fundamental frequency of natural vibration, the interesting phenomenon like the harmonic beat phenomenon of a vibration is observed.

References

Bich, D.H., Dung, D.V. and Nam, V.H. (2012), "Nonlinear dynamical analysis of eccentrically stiffened functionally graded cylindrical panels", *Compos. Struct.*, **94**(8), 2465-2473.

Bich, D.H., Dung, D.V., Nam, V.H. and Phuong, N.T. (2013), "Nonlinear static and dynamic buckling analysis of imperfect eccentrically stiffened functionally graded circular cylindrical thin shells under axial compression", *Int. J. Mech. Sci.*, **74**, 190-200.

Bich, D.H., Dung, D.V., Nam, V.H. and Phuong, N.T. (2013), "Nonlinear static and dynamic buckling analysis of imperfect eccentrically stiffened functionally graded circular cylindrical thin shells under axial compression", *Int. J. Mech. Sci.*, **74**, 190-200.

Brush, D.O. and Almroth, B.O. (1975), *Buckling of Bars, Plates and Shells*, Mc Graw-Hill. New York, U.S.A.

Chen, M., Xie, K., Jia, W. and Xu, K. (2015), "Free and forced vibration of ring-stiffened conical-cylindrical shells with arbitrary boundary conditions", *Ocean Eng.*, **108**, 241-256.

Darabi, M., Darvizeh, M. and Darvizeh, A. (2008), "Non-linear analysis of dynamic stability for functionally graded cylindrical shells under periodic axial loading", *Compos. Struct.*, **83**(2), 201-211.

Darvizeh, M., Darvizeh, A., Shaterzadeh, A.R. and Ansari, R. (2010), "Thermal buckling of spherical shells with cut-out", *J. Therm. Stress.*, **33**(5), 441-458.

Duc, N.D. and Thang, P.T. (2014), "Nonlinear buckling of imperfect eccentrically stiffened metal-ceramic-metal S-FGM thin cicular cylindrical shells with temperature-dependent properties in thermal environments", *Int. J. Mech. Sci.*, **81**, 17-25.

Duc, N.D. and Thang, P.T. (2015), "Nonlinear dynamic response and vibration of shear deformable imperfect eccentrically stiffened S-FGM circular cylindrical shells surrounded on elastic foundations", *Aerosp. Sci. Technol.*, **40**, 115-127.

Dung, D.V. and Nam, V.H. (2014), "Nonlinear dynamic analysis of eccentrically stiffened functionally graded circular cylindrical thin shells under external pressure and surrounded by an elastic medium", *Eur. J. Mech.-A/Sol.*, **46**, 42-53.

Ghiasian, S.E., Kiani, Y. and Eslami, M.R. (2013), "Dynamic buckling of suddenly heated or compressed FGM Beams resting on non-linear elastic foundation", *Compos. Struct.*, **106**, 225-234.

Paliwal, D.N., Pandey, R.K. and Nath, T. (1996), "Free vibration of circular cylindrical shell on Winkler and Pasternak foundation", *Int. J. Press. Vess. Pip.*, **69**(1), 79-89.

Shaterzadeh, A. and Foroutan, K. (2016), "Post-buckling of cylindrical shells with spiral stiffeners under elastic foundation", *Struct. Eng. Mech.*, **60**(4), 615-631.

Shaterzadeh, A.R., Rezaei, R. and Abolghasemi, S. (2015), "Thermal buckling analysis of perforated functionally graded plates", *J. Therm. Stress.*, **38**(11), 1248-1266.

Sheng, G.G. and Wang, X. (2008), "Thermomechanical vibration analysis of a functionally graded shell with flowing fluid", *Eur. J. Mech.-A/Sol.*, **27**(6), 1075-1087.

Sofiyev, A.H. (2005), "The stability of compositionally graded ceramic-metal cylindrical shells under aperiodic axial impulsive loading", *Compos. Struct.*, **69**(2), 247- 257.

Sofiyev, A.H. (2009), "The vibration and stability behavior of freely supported FGM conical shells subjected to external pressure", *Compos. Struct.*, **89**(3), 356-366.

Sofiyev, A.H., Hui, D., Haciyev, V.C., Erdem, H., Yuan, G.Q., Schnack E. and Guldal, V. (2017), "The nonlinear vibration of orthotropic functionally graded cylindrical shells surrounded by an elastic foundation within first order shear deformation theory", *Compos. Part B: Eng.*, **116**, 170-185.

Sofiyev, A.H., Karaca, Z. and Zerin, Z. (2017), "Non-linear vibration of composite orthotropic cylindrical shells on the nonlinear elastic foundations within the shear deformation theory", *Compos. Struct.*, **159**, 53-62.

Tang, D., Yao, X. Wu, G. and Peng, Y. (2017), "Free and forced vibration analysis of multi-stepped circular cylindrical shells with arbitrary boundary conditions by the method of reverberation-ray matrix", *Thin-Wall. Struct.*, **116**, 154-168.

Volmir, A.S. (1972), *Non-Linear Dynamics of Plates and Shells*, Science Edition M.

CC

Appendix A

$$
\begin{aligned}
J_{11} &= \frac{E_1}{1-\nu^2} + Z_1 E_{1s}\left(\cos^3\theta + \cos^3\beta\right) \\
J_{12} &= \frac{E_1\nu}{1-\nu^2} + Z_1 E_{1s}\left(\sin^2\theta\cos\theta + \sin^2\beta\cos\beta\right) \\
J_{14} &= \frac{E_2}{1-\nu^2} + Z_1 E_{2s}\left(\cos^3\theta + \cos^3\beta\right) \\
J_{15} &= \frac{E_2\nu}{1-\nu^2} + Z_1 E_{2s}\left(\sin^2\theta\cos\theta + \sin^2\beta\cos\beta\right) \\
J_{21} &= \frac{E_1\nu}{1-\nu^2} + Z_2 E_{1s}\left(\sin\theta\cos^2\theta + \sin\beta\cos^2\beta\right) \\
J_{22} &= \frac{E_1}{1-\nu^2} + Z_2 E_{1s}\left(\sin^3\theta + \sin^3\beta\right) \\
J_{24} &= \frac{E_2\nu}{1-\nu^2} + Z_2 E_{2s}\left(\sin\theta\cos^2\theta + \sin\beta\cos^2\beta\right) \\
J_{25} &= \frac{E_2}{1-\nu^2} + Z_2 E_{2s}\left(\sin^3\theta + \sin^3\beta\right) \\
J_{33} &= \frac{E_1}{2(1+\nu)} + 2Z_3 E_{1s}\left(\sin\theta\cos\theta + \sin\beta\cos\beta\right) \\
J_{36} &= \frac{E_2}{2(1+\nu)} + 2Z_3 E_{2s}\left(\sin\theta\cos\theta + \sin\beta\cos\beta\right) \\
J_{41} &= \frac{E_3}{1-\nu^2} + Z_1 E_{3s}\left(\cos^3\theta + \cos^3\beta\right) \\
J_{42} &= \frac{E_3\nu}{1-\nu^2} + Z_1 E_{3s}\left(\sin^2\theta\cos\theta + \sin^2\beta\cos\beta\right) \\
J_{51} &= \frac{E_3\nu}{1-\nu^2} + Z_2 E_{3s}\left(\sin\theta\cos^2\theta + \sin\beta\cos^2\beta\right) \\
J_{55} &= \frac{E_3}{1-\nu^2} + Z_2 E_{3s}\left(\sin^3\theta + \sin^3\beta\right) \\
J_{63} &= \frac{E_3}{2(1+\nu)} + 2Z_3 E_{3s}\left(\sin\theta\cos\theta + \sin\beta\cos\beta\right)
\end{aligned}
\tag{A.1}
$$

where

$$
\begin{aligned}
E_1 &= \int_{-h/2}^{h/2} E_{sh}(z)dz = \left(E_o + \frac{E_i - E_o}{k+1}\right)h \\
E_2 &= \int_{-h/2}^{h/2} zE_{sh}(z)dz = \frac{(E_i - E_o)kh^2}{2(k+1)(k+2)} \\
E_3 &= \int_{-h/2}^{h/2} z^2 E_{sh}(z)dz = \left[\frac{E_o}{12} + (E_i - E_o)\left(\frac{1}{k+3} + \frac{1}{k+2} + \frac{1}{4k+4}\right)\right]h^3
\end{aligned}
\tag{A.2}
$$

$$
\begin{aligned}
E_{1s} &= \int_{-(h/2+h_s)}^{-h/2} E_s(z)dz = \int_{h/2}^{h/2+h_s} E_s(z)dz = \left(E_i + \frac{E_o - E_i}{k_2+1}\right)h_s \\
E_{2s} &= \int_{-(h/2+h_s)}^{-h/2} zE_s(z)dz = \int_{h/2}^{h/2+h_s} zE_s(z)dz \\
&= \frac{E_i}{2}hh_s\left(\frac{h_s}{h}+1\right) + (E_o - E_i)hh_s\left(\frac{1}{k_2+2}\frac{h_s}{h} + \frac{1}{2k_2+2}\right) \\
E_{3s} &= \int_{-\left(\frac{h}{2}+h_s\right)}^{-\frac{h}{2}} z^2 E_s(z)dz = \int_{\frac{h}{2}}^{\frac{h}{2}+h_s} z^2 E_s(z)dz \\
&= \frac{E_i}{3}h_s^3\left(\frac{3}{4}\frac{h^2}{h_s^2} + \frac{3}{2}\frac{h}{h_s} + 1\right) \\
&\quad + (E_o - E_i)h_s^3\left[\frac{1}{k_2+3} + \frac{1}{k_2+2}\frac{h}{h_s} + \frac{1}{4(k_2+1)}\frac{h^2}{h_s^2}\right]
\end{aligned}
\tag{A.3}
$$

Appendix B

$$
\begin{aligned}
&\Delta = J_{11}J_{22} - J_{12}J_{21},\ J_{22}^* = \frac{J_{22}}{\Delta},\ J_{12}^* = \frac{J_{12}}{\Delta} \\
&J_{11}^* = \frac{J_{11}}{\Delta},\ J_{21}^* = \frac{J_{21}}{\Delta},\ J_{33}^* = \frac{1}{J_{33}},\ J_{36}^* = \frac{J_{36}}{J_{33}} \\
&J_{11}^{**} = J_{22}^*J_{14} - J_{12}^*J_{24},\ J_{12}^{**} = J_{22}^*J_{15} - J_{12}^*J_{25} \\
&J_{21}^{**} = J_{11}^*J_{24} - J_{21}^*J_{14},\ J_{22}^{**} = J_{11}^*J_{25} - J_{21}^*J_{15}
\end{aligned}
\tag{B.1}
$$

$$
\begin{aligned}
&A_{11}^* = J_{22}^*J_{14} - J_{21}^*J_{15},\ A_{21}^* = J_{11}^*J_{15} - J_{12}^*J_{14} \\
&A_{12}^* = J_{22}^*J_{24} - J_{21}^*J_{25},\ A_{22}^* = J_{11}^*J_{25} - J_{12}^*J_{24} \\
&A_{11}^{**} = J_{11}^{**}J_{14} - J_{21}^{**}J_{15} - J_{41} \\
&A_{12}^{**} = J_{12}^{**}J_{14} - J_{22}^{**}J_{15} - J_{42} \\
&A_{21}^{**} = J_{11}^{**}J_{24} - J_{21}^{**}J_{25} - J_{51} \\
&A_{22}^{**} = J_{12}^{**}J_{24} - J_{22}^{**}J_{25} - J_{52} \\
&A_{36}^{**} = J_{36}^{**}J_{36} - J_{63}
\end{aligned}
\tag{B.2}
$$

$$
\begin{aligned}
A &= J_{11}^*m^4\pi^4 + \left(J_{33}^* - J_{12}^* - J_{21}^*\right)m^2n^2\pi^2\lambda^2 + J_{22}^*n^4\lambda^4 \\
B &= J_{21}^{**}m^4\pi^4 + \left(J_{11}^* + J_{22}^* - 2J_{36}^{**}\right)m^2n^2\pi^2\lambda^2 \\
&\quad + J_{12}^{**}n^4\lambda^4 - \frac{L^2}{R}m^2n^2 \\
B^* &= A_{21}^*m^4\pi^4 + \left(A_{11}^* + A_{22}^* - 2J_{36}^{**}\right)m^2n^2\pi^2\lambda^2 \\
&\quad + J_{12}^{**}n^4\lambda^4 - \frac{L^2}{R}m^2n^2 \\
D &= A_{11}^{**}m^4\pi^4 + \left(A_{12}^{**} + A_{21}^{**} + 4A_{36}^{**}\right)m^2n^2\pi^2\lambda^2 + A_{22}^{**}n^4\lambda^4 \\
G &= \left(\frac{n^4\lambda^4}{16J_{11}^*} + \frac{m^4\pi^4}{16J_{22}^*}\right) \\
\lambda &= \frac{L}{R}
\end{aligned}
\tag{B.3}
$$

$$
\begin{aligned}
&a_1 = \frac{1}{L^2m^2\pi^2h},\ a_2 = \frac{G}{L^2m^2\pi^2h},\ a_3 = \frac{L^2}{m^2\pi^2h} \\
&a_4 = \frac{(\lambda n)^2 + (m\pi)^2}{m^2\pi^2h},\ a_5 = \frac{9L^2}{16m^2\pi^2h},\ a_6 = \frac{m^2\pi^2h}{L^2\rho_1}
\end{aligned}
\tag{B.4}
$$