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Reliability-based fragility analysis of nonlinear structures under the actions of random earthquake loads

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Abstract. This study presents the reliability-based analysis of nonlinear structures using the analytical fragility curves excited by random earthquake loads. The stochastic method of ground motion simulation is combined with the random vibration theory to compute structural failure probability. The formulation of structural failure probability using random vibration theory, based on only the frequency information of the excitation, provides an important basis for structural analysis in places where there is a lack of sufficient recorded ground motions. The importance of frequency content of ground motions on probability of structural failure is studied for different levels of the nonlinear behavior of structures. The set of simulated ground motion for this study is based on the results of probabilistic seismic hazard analysis. It is demonstrated that the scenario events identified by the seismic risk differ from those obtained by the disaggregation of seismic hazard. The validity of the presented procedure is evaluated by Monte-Carlo simulation.

Keywords: reliability; fragility curve; tail-equivalent linearization; failure probability; random vibration; point-source

1. Introduction

The main objective of structural engineering is safety assessment of structures subjected to stochastic dynamic processes such as earthquake ground motion, wind turbulence and ocean wave. The evaluation of accurate earthquake risk, requires a correct estimation of the seismic hazard and a good evaluation of the seismic vulnerability of structures through an appropriate earthquake damage model (Seyedi *et al*. 2010). In this framework, the seismic fragility curve is commonly utilized to estimate performance of structural and non-structural systems under seismic loads. Fragility curves represent the probability that the maximum response of structures or systems exceeds a threshold associated with a desired limit state, conditional on the seismic intensity measure (IM) (Radu and Grigoriu 2014).

In fact, the seismic demand of a structure due to uncertainties in ground motion and in structural properties needs to be properly characterized in structural engineering (Yazdani and Eftekhari 2012). In this case, the structural response will also be a stochastic process and is described in probabilistic terms (Yazdani and Takada 2011). Failure events unavoidably involve nonlinear response and accurate estimation of random responses for engineering structures subjected to the stochastic excitation is a crucial procedure in their design phase. Therefore, nonlinear random vibration is the most logical approach for studying the nonlinear structures under stochastic excitation. In general, very limited classes of nonlinear dynamic systems possess exact solutions and therefore various approximate methods were proposed for their solutions over the last four decades. Among these methods the implementation of classical methods such as Markov vector approach, perturbation, Fokker-Plank equation, stochastic averaging, moment closure, equivalent non-linearization and equivalent linearization method are inconvenient for nonlinear systems, let alone the arguments on their suitability and versatility especially in reliability analysis where the accurate distribution on the tails is needed (Crandall 2006). Furthermore, simulation-based methods such as Monte-Carlo simulation (MCS) and importance sampling have been widely used often together with variance reduction techniques for directly solving and/or checking approximate solutions of stochastic dynamics problems (Au and Beck 2003). However, some of these methods are computationally incompetent for large-scale reliability problems. In this regard, the tail-equivalent linearization method (TELM) is a recent alternative approach to solve this class of problems by improving the accuracy in the tail region. This method has been developed based on the first-order reliability method (FORM) and applied to time domain analysis for inelastic systems (Fujimura and Der Kiureghian 2007, Der Kiureghian and Fujimura 2009) and frequency domain analysis in the context of marine structures (Garrè and Der Kiureghian 2010). The general idea behind the method is to present a first-order approximation of the tail probability of the nonlinear system which is equal to the tail probability of the linear system. These studies have shown that the tail-equivalent linear system (TELS), which is characterized in terms of its frequency response function (FRF) or unit-impulse-response function (IRF) for a specified threshold can be used to compute various statistical quantities of interest for

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the nonlinear response. The presented procedures based on random vibration theory can be applied to fragility analysis when the ground motion is specified as a stochastic process. More recently, time- and frequency-domain TELMs were developed for analysis of stochastic dynamical systems (Alibrandi and Der Kiureghian 2012, Broccardo and Der Kiureghian 2013, 2015, Raoufi and Ghafory-Ashtiany 2016).

During the last decade, seismic fragility analyses have been investigated by numerous researchers and have been developed for a large number of structural and nonstructural systems (Schotanus *et al.* 2004, Sung Kwon and Elnashai 2006, Ellingwood *et al.* 2007, Mitropoulou and Papadrakakis 2011, liu *et al.* 2010, Ju *et al.* 2013, Mehani *et al.* 2013, Lallemant *et al.* 2015, Mandal *et al.* 2016, Khorami *et al.* 2017). Most of these researches are based on incremental dynamic analysis (IDA) (Vamvatsikos and Cornell 2002) that is developed to compute structural response through repeated time-history dynamic analysis and scaled ground motions. However, process of selecting and scaling of ground motion records requires many repetitive and time-consuming computations.

In addition, one of the main sources of uncertainty in the estimation of probability of exceeding various damage levels stems from alternative seismic IMs used for fragility analysis (Lee and Mosalam 2005). The most typical seismic IM used in fragility analysis is the spectral acceleration (*S*a), though other measures or additional measures can be used (Schotanus *et al.* 2004, Cimellaro *et al.* 2009, Silva *et al.* 2016, Kohrangi *et al.* 2017). However, most of the IMs used in fragility analyses are not a satisfactory measure for structural response. In this regard, Kafali and Grigoriu (2007) showed that peak ground acceleration and pseudo-spectral acceleration completely characterizes the maximum relative displacement for linear single degree of freedom (SDOF) systems, but it is not proper IM to use within the fragility analysis for nonlinear oscillators.

The objective of this study is to investigate the seismic reliability-based fragility curve of nonlinear systems based on random vibration theory by utilizing Fourier Amplitude Spectrum (FAS) of ground motion using information on the seismic source, seismic wave propagation through the earth, and geological site conditions that affect ground motion. The fragility analysis based on time-domain procedures are limited by the amount of available strong motion recorded data and by the fact that they are based on combined recorded data sets from different earthquakes recorded in different regions. As an alternative approach, in places where there is a lack of sufficient recorded ground motions, the stochastic ground motion simulation can be used in calculation of fragility curve. In addition, the probability of structural failure subjected to set of simulated ground motion based on different characteristic parameters, which has the same spectral acceleration, are compared for different levels of the nonlinear behavior of structures.

2. Seismic fragility analysis

From the viewpoint of performance-based earthquake engineering (PBEE), the seismic fragility assessment of structures is essential to prediction of the structural behaviors that are likely to occur during earthquakes. The seismic fragility of a structural system expresses the probability of occurrence of a certain level of structural failure due to earthquakes as a function of ground motion IM. Mathematically, the probability of structural failure corresponding to a specific threshold L should be defined as $P\left(L < \max X\left(t,\mathbf{u}\right) | IM = c\right)$, where $X\left(t,\mathbf{u}\right)_{\max}$ denotes the maximum structural response for the given intensity level *IM*=*c*. In reliability analysis of structural systems, the interest often lies in the occurrence of extreme and unpredictable events, which are associated with the tail part of probability distribution. One popular class of methods to estimate the tail of the response distribution for nonlinear systems is that of statistical linearization or equivalent linearization method. However, the solution of the response statistics other than the second moment, such as the auto-correlation, up-crossing rate and first-passage probability by the statistical linearization method may not be reliable because it is based on the minimization of the mean square error (Fujimura and Der Kiureghian 2007). In order to solve this issue, in this paper we used the TELM that is developed by Fujimura and Der Kiureghian (2007). The TELM is based on the previous works of Der Kiureghian *et al.* (Koo and Der Kiureghian 2003, Koo *et al.* 2005 and Haukaas and Der Kiureghian 2004, 2006) using the advantages of the first order reliability method (FORM). Recently, this method has been applied and expanded by several researchers for PBEE problems (Wang and Der Kiureghian 2016; Broccardo and Der Kiureghian 2017, Alibrandi and Mosalam 2017). This method is an appropriate approach to probabilistic structural analysis and reliability evaluation of structures in particular for softening systems under stochastic excitation (Broccardo 2014).

A stochastic analysis of failure requires study of the extreme values of the response process. For this purpose, in structural engineering the event of failure is usually described by the mean of up-crossing rate and the first-passage probability. In case of stationary-Gaussian processes the mean of up-crossing rate and first-passage probability (Vanmarcke 1975) can be defined as

$$\upsilon^{+}(L) = \frac{1}{2\pi}\left(\frac{\lambda_2(L)}{\lambda_0(L)}\right)^{0.5} \exp\left(\frac{L^2}{2\lambda_0(L)}\right) \tag{1}$$

$$P\left(L < \max_{0<t\leq T}\left|X\left(t,\mathbf{u}\right)\right|\right) \cong 1 - A\exp\left\{-\alpha T\right\} \tag{2}$$

In this case the period of process T is the earthquake ground motion duration. In Eq. (2), the parameters A and α are expressed as

$$A = 1 - \exp\left(-\frac{L^2}{2\lambda_0(L)}\right) \tag{3}$$

$$\alpha = \left\{ \frac{1}{\pi}\left(\frac{\lambda_2(L)}{\lambda_0(L)}\right)^{0.5} \times \frac{1-\exp\left[\left(\frac{\pi}{2}\right)^{0.5}\left(1-\frac{\lambda_1^2(L)}{\lambda_0(L)\lambda_2(L)}\right)^{1.2}\left(\frac{L}{\lambda_0(L)}\right)\right]}{\exp\left(\frac{L^2}{2\lambda_0(L)}\right)-1} \right\} \tag{4}$$

In Eqs. (1) and (4) $\lambda_n(L)$ is expressed as the n-th spectral moment defined by

$$\lambda_n(L) = 2\int_0^\infty \omega^n |H(\omega_m)|^2 S(\omega_m) d\omega \quad (5)$$

where ω_m are sequences of equally $\Delta\omega$-spaced frequency points and $S(\omega_m)$ denotes the two-sided power spectral density (PSD) of the stochastic excitation. It is noted that in structural engineering, the PSD of the excitation, which is a more fundamental description of the frequency content of ground motion, is calculated in most practical methods of simulation of earthquake ground motions. $|H(\omega_m)|$ represents the modulus of the frequency-response function of the linear system. In TELM with the definition of the TELS for a specified response threshold of the nonlinear system, FRF completely characterizes the system for the particular input-output pair $W(t)$ and $X(t)$. A considerable result in TELM is that the TELS is independent of any scaling of the excitation. That is, given a scaled excitation, i.e., $S_F.W(t)$, the IRF and FRF of the TELS for a specific threshold L are independent of the scale factor S_F .It is clear when the excitation is scaled by a factor S_F so that its PSD and all the spectral moments are equal to $S_F^2.S(\omega_m)$ and $S_F^2.\lambda_n(L)$, respectively. A broader and more in-depth treatment can be found in Fujimura and Der Kiureghian (2007). Hence, the fragility curve owing to the failure event during an interval of time (0, *T*) is computed by the approximate solution:

$$P\left(L < \max_{0<t\le T}|X(t,\mathbf{u})| \middle| S_F = s_f\right)$$
$$\cong 1-\left\{1-\exp\left(-\frac{L^2}{2s_f^2\lambda_0(L)}\right)\right\}\times\exp\left\{-\frac{T}{\pi}\left(\frac{\lambda_2(L)}{\lambda_0(L)}\right)^{0.5}\times\frac{1-\exp\left[\left(\frac{\pi}{2}\right)^{0.5}\left(1-\frac{\lambda_1^2(L)}{\lambda_0(L)\lambda_2(L)}\right)^{1.2}\left(\frac{L}{s_f^2\lambda_0(L)}\right)\right]}{\exp\left(\frac{L^2}{2s_f^2\lambda_0(L)}\right)-1}\right\} \quad (6)$$

As can be seen from Eq. (6), the superiority of this method over other methods such as IDA is that no extra computations are required for selecting and scaling excitations. However, this approach requires the identification of the TELS to estimate the tail of the response distribution for nonlinear systems under stochastic excitation.

In nonlinear dynamic analysis, input stochastic excitation is discretized in terms of a finite set of standard normal random variables. Several different kinds of stochastic discrete representation methods have been developed and are available for dynamic analysis purposes (Li and Der Kiureghian 1993, Zhang and Ellingwood 1994, Sudret and Der Kiureghian 2000, He 2015, Liu *et al.* 2017). In particular, in earthquake engineering, the following formulation developed by Der Kiureghian (2000) is practical

$$W(t) = \sum_{i=1}^{n} s_i(t)u_i = \mathbf{s}(t).\mathbf{u} \quad (7)$$

where $\mathbf{u}$ is a time-invariant vector of standard normal variables, $\mathbf{s}(t)$ is a time-variant row vector of basic functions related to the covariance structure of the excitation process, and *n* is a measure of the resolution of the representation. Owing to the random variables $\mathbf{u}$, the response of a dynamical system $X(t)$ subjected to $W(t)$ is stochastic and it can be represented as $X(t,\mathbf{u})$.

In TELM, to solve the tail probability $P[L \le X(t,\mathbf{u})] = P[G(L,t_n,\mathbf{u}) \le 0]$ for a specified threshold L, the first-order reliability method(FORM) is employed. The function $G(L,t,\mathbf{u}) = L - X(t,\mathbf{u})$ defines the limit-state function for response threshold value of *L* at time *t*. This function is approximated by a hyper plane at the nearest point to the origin of the standard normal random variables space, so-called design point $\mathbf{u}^*$. Design point plays an essential role in reliability analysis. In the simple case of a linear system that the limit state is a hyperplane with gradient $\mathbf{a}(t)$, the reliability index and corresponding design point are given as $\beta(L,t) = L/\|\mathbf{a}(t)\|$ and $\mathbf{u}^*(L,t) = L.\mathbf{a}(t)/\|\mathbf{a}(t)\|^2$. Furthermore, simple manipulation of design point $\mathbf{u}^*$ yields $\mathbf{a}(t) = L.\mathbf{u}^*(L,t)/\|\mathbf{u}^*(L,t)\|^2$ (Fujimura and Der Kiureghian 2007). The tail probability has the approximate solution

$$P[L \le X(t,\mathbf{u})] \cong \Phi[-\beta(L,t)], \quad (8)$$

where $\Phi[\bullet]$ is the standard normal cumulative probability function. In case of a nonlinear dynamic system, limit state is nonlinear and the design point is given as a solution of constrained optimization problem as follows

$$\mathbf{u}^*(L,t) = \operatorname{argmin}\left[\|\mathbf{u}\| \middle| G(L,t,\mathbf{u}) = 0\right] \quad (9)$$

Gradient-based algorithm is perhaps the most popular algorithm used to solve the constrained optimization problem of Eq. (9) in structural reliability analysis (Haukaas and Der Kiureghian 2004, 2006). In TELM for a specified response threshold of the nonlinear system, the equivalent linear system, so called TELS, is defined as the linear system that has the same tail probability as the first-order approximation of the tail probability of the nonlinear system in the space of the standard normal random variables under the stochastic excitation. To identify TELS, the nonlinear limit-state function is expanded in Taylor series at the design point and by keeping the linear terms of series, the first-order approximation of $P[L \le X(t,\mathbf{u})]$ is obtained.

In order to introduce the idea of TELM in frequency domain, we first consider excitation as stationary stochastic processes so that $W(t)$ is written in the following form (Garrè and Der Kiureghian 2010)

$$W(t) = \sum_{m=1}^{M} \sqrt{2S(\omega_m)\Delta\omega}[u_m \sin(\omega_m t) + u_{M+m} \cos(\omega_m t)] \quad m \in [1,..., M] \tag{10}$$

Thus, having determined the response of the linear system in the frequency domain, we can determine the steady-state response of system in the time domain by the inverse Fourier transform

$$\begin{aligned} X(t,\mathbf{u}) &= \sum_{m=1}^{M} \sqrt{2S(\omega_m)\Delta\omega}\left|H(\omega_m)\right| \\ &\times\left[u_m \sin(\omega_m t - \varphi_m) + u_{M+m} \cos(\omega_m t - \varphi_m)\right] \\ &= \mathbf{a}(t).\mathbf{u}, \qquad m \in [1,..., M] \end{aligned} \tag{11}$$

where $\mathbf{a}(t) = [a_1(t),..., a_M(t); a_{M+1}(t),..., a_{2M}(t)]$ and for a specific time t_n

$$a_m(t_n) = \sqrt{2S(\omega_m)\Delta\omega} \times \left|H(\omega_m)\right| \sin(\omega_m t - \varphi_m), \quad m \in [1,..., M] \tag{12}$$

$$a_{M+m}(t_n) = \sqrt{2S(\omega_m)\Delta\omega} \times \left|H(\omega_m)\right| \cos(\omega_m t - \varphi_m), \quad m \in [1,..., M]] \tag{13}$$

where $|H(\omega_m)|$ and φ_m represent the modulus and the phase angle of the frequency-response function of the linear system, respectively. Using the above Eqs. (12) and (13), the modulus of $H(\omega_m)$ and phase angle φ_m are obtained for each ω_m as

$$\left|H(\omega_m)\right| = \sqrt{\frac{a_m^2 + a_{M+m}^2}{2S(\omega_m)\Delta\omega}} \quad m \in [1,..., M] \tag{14}$$

$$\varphi_m = \omega_m t - \tan^{-1}\left(\frac{a_m}{a_{M+m}}\right) \quad m \in [1,..., M] \tag{15}$$

Finally, response of the linear system is obtained by basic principles of frequency-domain analysis.

3. Stochastic modeling of power spectral density (PSD) of ground motions

The main goals of engineering seismology in interpretation of ground motions are to improve the understanding of the physical processes that control ground motions and to develop reliable estimates of ground motions for use in engineering analyses (Yazdani and Salimi 2015). There are two options in selecting ground motions to assess the seismic reliability of structures; the use of either observed records or synthetic ground motions. The well-known simple Kanai-Tajimi filter (Kanai 1957, Tajimi 1960) and stochastic models (Brune 1970, 1971, Hanks and McGuire 1981, Boore and Atkinson 1987, Boore 2003) apply a band-pass filter in the frequency domain to mold the FAS and the corresponding PSD of the earthquake ground motion. The stochastic methods that model ground motions as a random process and band-limited white noise are suitable for engineering applications for low- to intermediate-period structures.

Seismological methods use stochastic models of the seismic source and wave propagation to simulate ground motions. The widely used point-source methods, which do not require information about the fault geometry, can predict high-frequency ground motions with acceptable accuracy (Boore 2003, 2009). To overcome the limitations of point-source models, stochastic finite-fault models should be used to simulate ground motions in the frequency range of engineering interest (Beresnev and Atkinson 1998, Motazedian and Atkinson 2005). In finite-fault simulations, the fault is subdivided into a number of sub-faults, each of which is modeled using a point-source model. The point-source approach offers the advantages of simplicity and stability whereas the finite-fault model involves more parameters and requires to average simulations over many azimuths and slip distributions. Atkinson and Silva (2000) postulated that the use of a point-source model with a two-corner source spectrum is equivalent to the use of a finite-fault model comprised of point-source sub-faults. They indicated that two-corner point-source and finite-fault stochastic models will generate similar median ground motions, when averaged over all azimuths. Seismic ground acceleration at a site can be modeled using a Gaussian process with a spectral density $S(\omega)$ as follows

$$S(\omega) = \left|Y(\omega)\right|^2 / T_w \tag{16}$$

where $Y(\omega)$ is the FAS of the strong ground motion at the site and T_w is the earthquake ground motion duration. In seismological simulation techniques, the ground motion duration is the summation of source rupture duration which is proportional to the inverse corner frequency, and the propagating time of the radiated waves from source to the station. A simplified form of the distance-dependent term (0.05R) is adopted in this study, and the rupture duration part is assumed to be predicted by π / ω_a (Boatwright and Choy 1992) where ω_a is the lower corner frequency. In a seismological model, the FAS can be expressed as the product of a number of functions (Boore 2003)

$$Y(\omega) = E(M_0, \omega)G(\omega)A(\omega)\exp(-\gamma(\omega)R)\exp(-\frac{\omega\kappa}{2}) \tag{17}$$

where ω and M_0 are the angular frequency and the seismic moment, respectively. R is equal to $R = \sqrt{d^2 + h^2}$ with d being closest distance to the fault plane and the equivalent point-source depth, h, is a function of fault size, and hence earthquake magnitude (Atkinson and Silva 2000). The terms $E(M_0, \omega)$, $G(\omega)$ and $A(\omega)$ are the earthquake source spectrum, the geometric spreading function and the upper crust amplification factor, respectively. The anelastic attenuation, $\gamma(\omega)$, is determined from the regional wave transmission quality factor, namely, the Q factor. The high-frequency amplitudes are reduced by near-surface attenuation, which is assumed to be independent of distance, through the *kappa* factor. The two-corner source spectrums can be described using the following functional form (Atkinson 1993)

$$E(M_0,\omega) = C\omega^2 M_0((1-\delta)/[1+(\omega/\omega_a)^2]+\varepsilon/[1+(\omega/\omega_b)^2]) \tag{18}$$

The constant C indicates the effect of the radiation pattern, the partition of total shear wave energy into horizontal components, the effect of the free surface, and the density and shear-wave velocity in the vicinity of the earthquake source. In this equation, the lower corner frequency, ω_a, is related to the size of the finite fault and is determined by the source duration, and the higher corner frequency, ω_b, is related to the sub-fault size and is the frequency at which the spectrum attains 1/2 of the high-frequency amplitude level (Atkinson 1993). The parameter δ is a relative weighting parameter whose value lies between 0 and 1. These two corner frequencies and δ can be derived by regression analysis using recorded data, after correcting for path and site effects in different regions.

4. Hysteretic SDOF system excited by a stationary Gaussian process

This section presents the importance of frequency content of ground motions on structural failure probability. This assertion is illustrated, numerically, for a SDOF system with inelastic material behavior under stationery stochastic excitation. For instance, for an inelastic SDOF system (Fig. 1) the equation of motion is expressed as follows

$$m\ddot{x} + 2\zeta m\omega_n \dot{x} + f(x,\dot{x}) = W(t) \tag{19}$$

where x is the displacement and a dot indicate the derivative with respect to time. $W(t)$ is input excitation and $f(x,\dot{x})$ is the restoring force. In this equation, m, ω_n and ζ are the mass, the natural circular frequency and the damping ratio within the linear range, respectively. In this work, we consider the hysteretic Bouc-Wen model to describe the inelastic material behavior (Bouc 1967, Wen 1976). In this case, Eq. (19) can be rewritten as

$$m\ddot{x} + 2\zeta m\omega_n \dot{x} + k[\alpha\, x + (1-\alpha)\, z(t)] = W(t) \tag{20}$$

where k, α and $z(t)$ are the initial elastic stiffness, the post- to pre-yielding stiffness ratio and the hysteretic force, respectively. The restoring force is obtained from combining the linear component and a hysteresis component $z(t)$. The hysteretic component $z(t)$ can often be modeled by a first order differential equation as

$$\dot{z}(t) = -\gamma|\dot{x}||z(t)|^{n-1} z(t) - \eta|z(t)|^n \dot{x} + A\dot{x} \tag{21}$$

In which γ, η and n determine the hysteresis shape, and A determines the tangent stiffness. Table 1 shows the selected value set for the system and hysteretic component variables. To describe the input excitation at the specific site, we applied the stochastic method of ground-motion simulation based on the seismological methods. In this study the stochastic point-source model with a two-corner source spectrum is used.

Table 2 shows the chosen mean value set for the earthquake ground motion variables for the calculation of the ground motion's FAS, which is used to obtain the structural failure probability. In Table 1, $u_y = \sigma_0$ is the

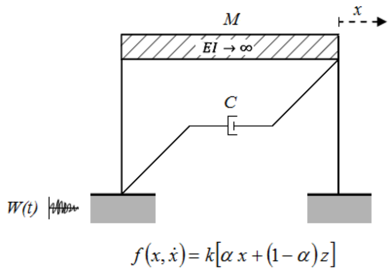


Fig. 1 Inelastic SDOF system subjected to stochastic excitation (T=1.0 sec)

Table 1 System and hysteretic component properties

System	$m[kg]$	5.32e5
	$k[kN]$	2.10e4
	$\zeta[\%]$	2
Hysteretic component	α	0.1
	$\gamma\left[\frac{1}{m^n}\right]$	$1/2u_y^n$
	$\eta\left[\frac{1}{m^n}\right]$	$1/2u_y^n$
	A	1
	n	3

Table 2 Set of earthquake ground motion variables

Variable	Mean value
Density, ρ_s	2.8 (gr/cm^3)
Shear-wave velocity, β_s	3.5 (km/cm)
Quality factor, $Q(\omega)$	$78.72\omega^{0.45}$
lower corner frequency, ω_a	$\log\omega_a = 2.979 - 0.496M_w$
Upper corner frequency, ω_b	$\log\omega_b = 3.208 - 0.408M_w$
Weighting parameter, δ	$\log\delta = 0.605 - 0.255M_w$
Seismic moment, M_0	$\log M_0 = 1.5M_w + 16.05$
Duration, $T_w(s)$	$\pi/\omega_a + 0.05R$
High-frequency attenuation parameter, $\kappa(s)$	0.03
Geometrical attenuation	$R^{-1}(\le 40km)$ $R^{-0.5}(> 40km)$
Amplification factor, $V(\omega)$	NEHRP class C $(\bar{V}_{30} = 520m/s)$

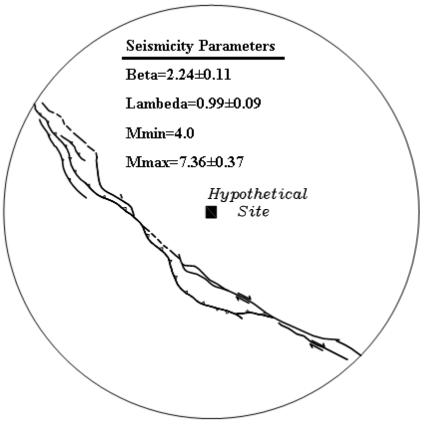


Fig. 2 Seismic sources and Location of hypothetical site used in this study

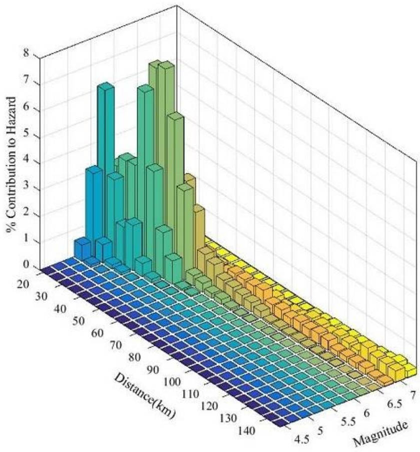


Fig. 3 Seismic hazard disaggregation in terms of magnitude and distance of the hazard for a hypothetical site, corresponding to a 10% probability of exceedance of 1 Hz spectral acceleration Sa (1.0 sec) = 0.08 g in 50 years

yield displacement, where $\sigma_0 = 0.02m$ is the estimated standard deviation of structural response $x(t)$ at $T = 10s$ to the stationary Gaussian excitation process. As mentioned in the previous section, FAS is a function of the earthquake moment magnitude *M*, the source-to site distance *R* and the site soil type. This is indeed a challenge, particularly for identification of a critical scenario for assessing the failure of structures. Therefore, the results of probabilistic seismic hazard analysis are needed to tackle the problem of generating and selecting earthquake ground motion. Probabilistic seismic hazard disaggregation techniques could be an answer to these technical problems. Disaggregation is a process that allows the identification of individual earthquake scenarios that contribute to a hazard for a given ground motion parameter at a selected annual frequency of exceedance at the specific site. The assumed regional seismotectonic in this study is shown in Fig. 2, which also demonstrates seismic source model and seismic parameters. Each bar in the disaggregation plot represents the contribution of each combination of (*M*, *R*) to exceed spectral acceleration $Sa(\omega_n)$ at the natural frequency ω_n. However, the magnitude-distance disaggregation for

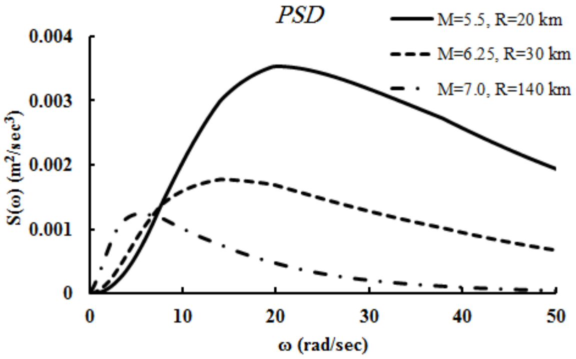


Fig. 4 Power spectra density for different scenario events

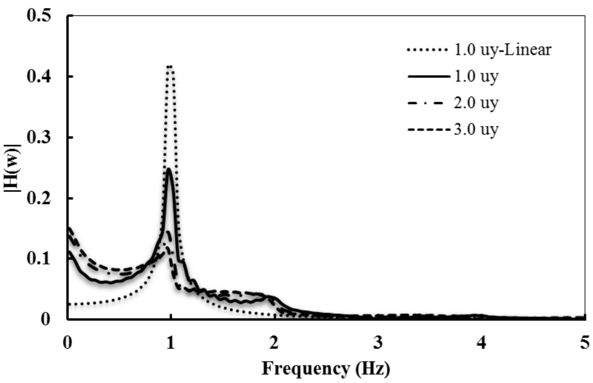


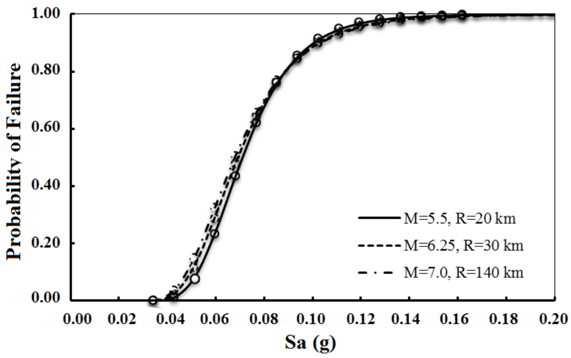
Fig. 5 FRFs of the TELS for different response thresholds of hysteretic SDOF

intended site associated with a 10% probability of exceedance of 1 Hz spectral acceleration in 50 years is shown from Fig. 3. The obtained disaggregation result is plotted in terms of *M* and *R*. Based on Eq. (16) the ground motion's PSD can be generated for each scenario (M and R bins). For example, we considered three different earthquake scenarios to assess the effect of frequency content on structural failure probability; M=5.5 at R=20 km, M=7.0 at R=140 km and magnitude and distance corresponding to modal value M=6.25 at R=30 km. Fig. 4 show the spectral density for three different magnitude–distance scenarios.

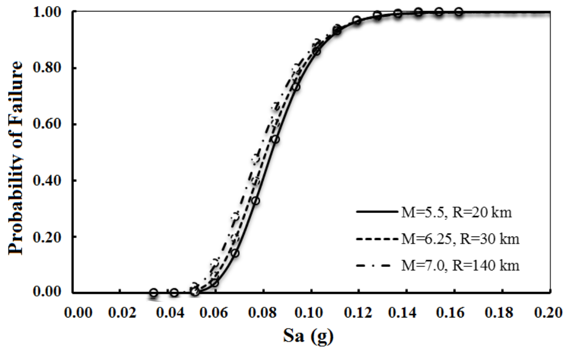
In addition, the FRFs of TELS could be obtained for different response threshold levels. Finally, fragility curves for each scenario can be calculated for different response threshold levels.

Fig. 5 indicates the modulus of the FRFs from simulated ground motion with M=6.25 and R=30 km for three distinct response thresholds, $L = 1.0u_y$, $L = 2.0u_y$ and $L = 3.0u_y$ in the range of frequencies [0, 5 Hz].

The dotted line in Fig. 5 represent the modulus of the FRFs for the linear system ($\alpha = 1.0$) at the response threshold $L = 1.0u_y$. As can be seen in Fig. 5, with the increasing response threshold, the dominant peak of the FRF shifts to the lower frequencies and decreases in intensity. However, there is always a local maximum at the natural frequency. The results indicate, the TELS strongly depends on the threshold of the nonlinear response. In addition, the structural responses of two ground motion



(a)

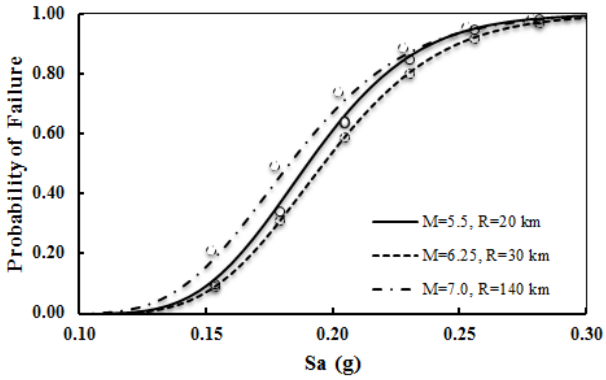


(b)

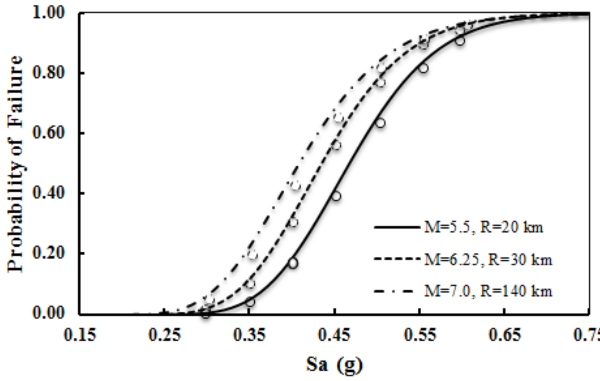
Fig. 6 Fragility curves for (a) linear and (b) nonlinear SDOF system under different scenario events at response threshold level $1.0u_y$. Circles are MCS results based on 30 000 simulations

processes can be very different even though the two ground motions have same spectral response. Hence, investigation of various scenarios to assess structural failure probability is necessary.

Figs. 6 and 7 show the calculated fragility curves based on Vanmarcke's formula for different threshold levels and compared with their corresponding MCS results. It can be seen in Figs. 6 and 7 that the fragility curves based on Vanmarcke's formula closely agree with those estimated from the MCS method. Traces (a) and (b) in Fig. 6 show fragility curves for the linear ($\alpha = 1.0$) and nonlinear SDOF system at the response threshold $L = 1.0u_y$, respectively. Moreover, these curves are plotted for moderate ($L = 2.0u_y$) and high response ($L = 5.0u_y$) threshold levels as shown in traces (a) and (b) in Fig. 7, respectively. Fig. 6 indicate that the probability of failure for the linear and nonlinear system for low response threshold level is independent of the particular value of M and R. In contrast, in the nonlinear system, failure probability depends strongly on characteristic pairs (M, R) when the response threshold is increased, as illustrated in Fig. 7. The traces in this figure demonstrate the wide variation that can exist between fragility curves at specific threshold level based on discrepancy between frequency content of ground motions. For example, in the case of inelastic SDOF system, for a fixed value of the $Sa(\omega_n) = 0.40g$ in Fig. (7b), the probabilities of failure for three different scenario events M=5.5 at R=20 km, M=6.25 at R=30 km and M=7.0 at R=140 km are 20.5%, 34.8% and 50.6%, respectively. Accordingly, we have a maximum percentage difference equal to 85% between different scenarios. Furthermore, comparison of the results shown in Fig. 7 indicates that the modal event (most-likely M, R pair) does not necessarily have the highest probability of failure. Therefore, greater attention needs to select an appropriate set of earthquake records that can be used for fragility analysis of nonlinear system.



(a)



(b)

Fig. 7 Fragility curves for nonlinear SDOF system under different scenario events and different response threshold levels (a) moderate threshold level $L = 2.0u_y$ (b) high threshold level $L = 5.0u_y$. Circles are MCS results based on 30 000 simulations

5. Conclusions

From an engineering point of view, safety assessment of structural and nonstructural systems, such as nuclear power plants, hospitals, bridges, or lifelines in an earthquake-prone zone, implies nonlinear dynamic analysis. Many different methods and procedures have been developed for assessing seismic structural performance. Among these methods commonly used in recent years, are incremental dynamic analysis and MCS methods where extra repetitive and time-consuming computations must be performed for each scaled time history. Given these conditions, one of the most appropriate approaches for effectively assessing the performance of structural systems is through a reliability analysis. In this regard, this study focuses on reliability based seismic performance analysis of nonlinear SDOF systems using the analytical fragility curves under simulated earthquake ground motions. The formulation of structural failure probability using random vibration theory, based on only the frequency information of the excitation, provides an important basis for structural analysis in places where there is a lack of sufficient recorded ground motions. The validity of the presented procedure was evaluated by MCS.

The stochastic point-source model with a two-corner source spectrum used in this study, despite possessing some theoretical deficiencies, yields results similar to those obtained using finite-fault methods for the ground motion frequencies at moderate and large distances from the fault that are of most interest to engineers (Beresnev and Atkinson 1999, Atkinson and Sillva 2000, Boore 2009). It is noted that although a stochastic process of earthquake ground motion is non-stationary throughout motion both in time and frequency domain, it can be taken approximately as stationary during the time of ground motion of a typical earthquake, and good estimates of the response can be obtained by the random vibration theory (Key 1988). We have investigated also the issue of importance of frequency content on structural failure probability for different selected scenario earthquake events based on the seismic hazard disaggregation. Most developed methods to select input ground motions for seismic structural analysis often use modal causal earthquake magnitude and distance from disaggregation as scenario earthquake (Baker and Cornell 2006, Haselton *et al.* 2009, Katsanos *et al.* 2010, Burks *et al.* 2015, Baker and Lee 2016). Results of this study indicate the probabilities of failure for linear and nonlinear system for low response threshold level are independent of the particular value of (M,R). On the other hand, when the response threshold is increased, failure probability depends strongly on characteristic (M,R) pairs. Hence, the scenario events identified by the seismic risk differ from those obtained by the disaggregation of seismic hazard. Therefore, various scenario events often need to be considered for purposes of seismic risk analysis and failure probability of structures. The results obtained from this study are similar to those obtained by Hong and Goda (2006) and Lin *et al.* (2013). It is worth noting that although the investigations carried out in this study were on an inelastic SDOF system; its merit will be largely enhanced if a group of structures or a set of structures located in a particular region is considered.

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