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Averaged strain energy density to assess mixed mode I/III fracture of U-notched GPPS samples

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Abstract. In the present contribution, fracture resistance of U-notched GPPS members under mixed mode I/III loading conditions is assessed by using the Averaged Strain Energy Density (ASED) criterion. This criterion has been founded based on the ASED parameter averaged over a well-defined control volume embracing the notch edge. The validation of the theoretical criterion predictions is evaluated through comparing with the results of a series of mixed mode I/III fracture tests conducted on rectangular-shaped GPPS specimens weakened by a single edge U-notch. A recently developed apparatus for mixed mode I/III fracture experiments is employed for measuring the fracture loads of the specimens. The test samples are fabricated with different notch tip radii with the aim of evaluating the influence of this major feature of the U-notched components on the mixed mode I/III fracture behavior. It is shown that the onset of brittle fracture in U-notched GPPS specimens under various combinations of tension and out-of-plane shear can well be predicted by means of the ASED criterion.

Keywords: brittle fracture; mixed-mode I/III; U-notch; averaged strain energy density (ASED); notch tip radius

1. Introduction

Various notch types, especially V- and U-shaped notches, are used in engineering parts and structures. Existence of these artificial stress concentrators makes the notched components vulnerable to the applied mechanical loads and decreases the load sustainability of the components significantly.

Two main mechanical failure mechanisms in notched components are ductile rupture and brittle fracture. Ductile rupture often takes place in ductile materials and commonly leads to moderate or large-scale yielding around the notch tip. Conversely, brittle fracture is a sudden failure type occurring in brittle and quasi-brittle materials with no noticeable plastic deformation and no precaution. Accordingly, in the case of dealing with brittle or quasi-brittle materials like ceramics, rocks, high-strength alloys, many types of polymers etc., paying much more attention to the design process is needed. Because of disastrous results, brittle fracture in notched members has been an interesting research topic for many years. The design and analysis of notched elements fabricated from brittle materials requires reliable failure criteria to anticipate their fracture resistance when they are subjected to different loading cases.

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Several failure models are found in the literature which have been proposed for brittle fracture prediction of notched components. The mentioned models are the mean-stress (MS) and point-stress (PS) (Ayatollahi and Torabi 2010a, b, Torabi 2013, Torabi 2014, Torabi and Amininejad 2015, Torabi and Abedinasab 2015, Torabi *et al.* 2013), the strain energy density (SED) (Lazzarin *et al.* 2009, Lazzarin and Zambardi 2001, Yosibash *et al.* 2004, Torabi and Berto 2013, Torabi and Berto 2014c, Berto and Barati 2011), the cohesive zone model (CZM) (Gómez and Elices 2003, Gómez *et al.* 2000, Cendón *et al.* 2015), the finite fracture mechanics (FFM) (Carpinteri *et al.* 2008, Sapora *et al.* 2013, Sapora and Firrao 2017, Leguillon 2002), and the generalized J-integral (Barati and Alizadeh 2011, Becker *et al.* 2012, Livieri 2003, Matvienko and Morozov 2004) criteria. The SED fracture criterion was first presented by Lazzarin and Zambardi (2001) for notch problems and it has been founded based on the SED parameter averaged over a well-defined control volume which depends on the used material. The remarkable advantage of the mean SED approach is its mesh independency (Lazzarin *et al.* 2008, Lazzarin *et al.* 2010). Recently, an alternative rapid method to estimate the averaged SED with coarse meshes has been proposed for different mixed mode loading conditions (Meneghetti *et al.* 2015, Campagnolo *et al.* 2016, Meneghetti *et al.* 2016). The concept of averaged SED has also been utilized for fatigue assessment of un-notched and notched components (Berto *et al.* 2014, Ferro *et al.* 2012). Validity of the estimations of the SED criterion has been proven by comparing with the experimental results of the tests performed with different specimens, materials and

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notch shapes (V-, U-, O- and VO-notches). Loading conditions applied to the notched components in the majority of the mentioned fracture assessments are mode I (opening), mode II (in-plane sliding) and mixed mode I/II. For instance, one can mention to the research works studying fracture of U-notched specimens under mixed mode I/II loading (Gómez *et al.* 2007, 2008, Torabi and Berto 2014a), V-notched specimens under mixed mode I/II loading (Berto and Ayatollahi 2011, Gómez *et al.* 2009, Priel *et al.* 2008) or under pure mode II (Torabi and Berto 2014b; Sapora *et al.* 2014), VO-notched specimens under pure mode II (Torabi *et al.* 2015b), specimens weakened by key-hole notches under mixed mode I/II loading (Torabi *et al.* 2015a) or under pure mode II (Torabi *et al.* 2014). On the other side, brittle fracture of notched members under the loading conditions including mode III (out-of-plane tearing) like mixed mode I/III loading case has been dealt with very little. For example, some researchers implemented the local SED criterion to the U- and V-notched specimens subjected to the torsion loading (Berto *et al.* 2013, Berto *et al.* 2012, Berto *et al.* 2012). The fracture behavior of PMMA (Polymethyl-methacrylate) round bars weakened by U-, V- and semicircular notches when they are under torsion at -60˚C (Berto *et al.* 2013) and at room temperature (Berto *et al.* 2012) has also been studied by the criterion. Theoretical predictions of the SED criterion have been verified in comparison with the experimental results of the V-notched graphite samples broken under out-of-plane loading (Berto *et al.* 2012). Efficiency of the SED model in assessing the brittle fracture of V-notches under combination of the tension and torsion loadings has also been confirmed well (Berto *et al.* 2015).

Present research investigates fracture resistance of the GPPS-made members containing a U-notch when they are under various combinations of the tension and out-of-plane shear loadings by means of the averaged SED (ASED) criterion. Because of high brittleness and fragility, it is necessary to know the fracture behavior of notched elements made of GPPS material. The accuracy of the theoretical predictions is evaluated by comparing with the results of a set of mixed mode I/III fracture experiments carried out by employing a recently developed test apparatus. In order to exhibit the performance of the ASED criterion in predicting the fracture loads of the U-notches having different notch tip radii, the test specimens are fabricated with three notch radii of 0.5, 1 and 2 mm.

2. Fracture tests

A series of fracture experiments are performed on GPPS-made U-notched samples under mixed mode loading conditions. GPPS polymer with commercial name of Crystal has a low elongation percentage and represents a fragile and brittle behavior. Table 1 includes some of the material characteristics of the GPPS polymer used in the experiments.

The fracture tests are conducted by means of the test rig displayed in Fig. 1. Recently, this test apparatus has successfully been utilized for experimental investigation of cracked GPPS samples (Safaei *et al.* 2017) and notched samples made of PMMA (Saboori *et al.* 2017). Fig. 2 depicts the U-notched test specimen employed in the test configuration of Fig. 1. The specimen thickness is the same as that considered for cracked specimen in the work done by Safaei *et al.* (2017). In designing the loading fixture in that work, most dimensions of the cracked specimen specially its thickness have been selected according to the recommendations of the ASTM-D5045-99 standard. This standard presents standard test methods for mode I plane-strain fracture toughness and strain energy release rate of plastic materials.

Table 1 Material properties of the tested GPPS (Safaei *et al.* 2017)

Material Property	Value
Elastic Modulus, E [GPa]	2.95
Poisson's Ratio, ν	0.34
Ultimate Tensile Strength [MPa]	30
Plane-strain mode I fracture toughness [MPa√m]	1.42
Mode III fracture toughness [MPa√m]	1.96

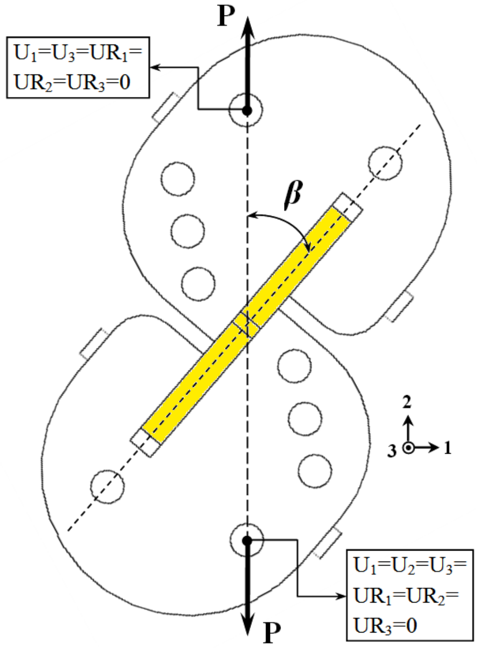

Fig. 1 Loading apparatus of mixed mode I/III fracture experiments

Two pairs of bolts and nuts connect the test specimen to the segments of loading fixture. Five holes devised on each half of the loading fixture make it possible to apply five combinations of mode I and mode III loadings to the test samples. By defining the β angle as the angle between the loading direction and the specimen longitudinal direction, the loading modes available by the test apparatus are pure mode I (β = 0˚), pure mode III (β = 90˚) and three intermediate mixed mode I/III loading cases corresponding to β = 40˚, 65˚, 72˚.

U-notched specimens are prepared with three notch tip radii of ρ = 0.5, 1 and 2 mm by cutting from a GPPS sheet by means of a high-precision water jet cutting machine.

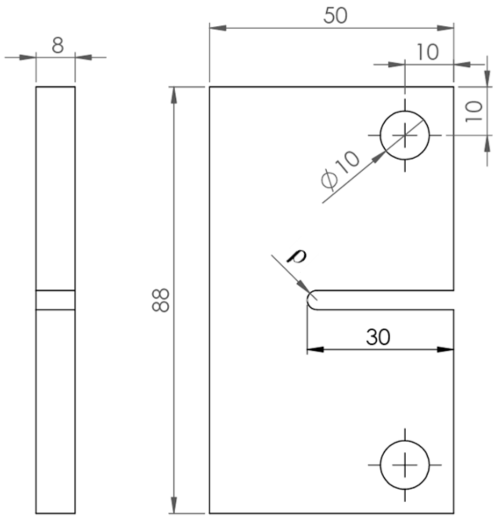

Fig. 2 U-notched specimen (dimensions in mm)

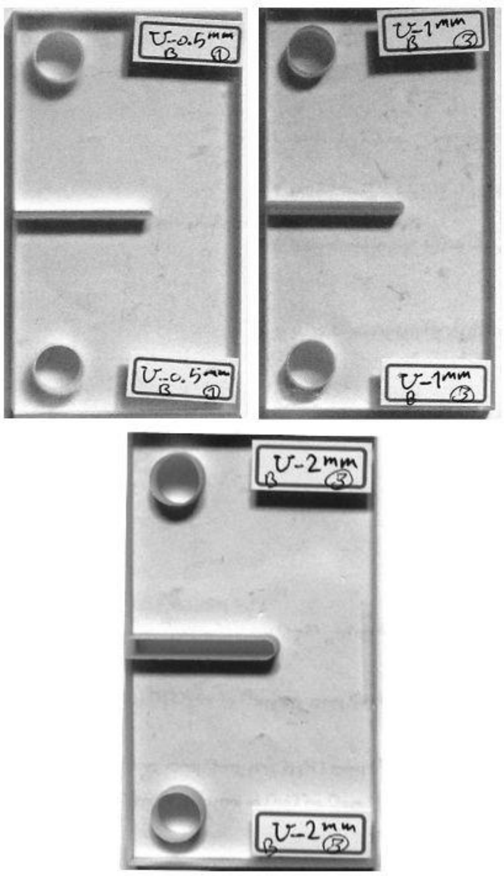

Fig. 3 U-notched GPPS samples with different notch tip radii of 0.5, 1 and 2 mm

Samples of prepared GPPS specimens are seen in Fig. 3. Pure mode I and some mixed mode I/III fracture tests are carried out for each notch tip radius. Each test is repeated three times and therefore, 39 fracture tests are performed in total.

Each test specimen installed within the introduced loading fixture is loaded up to its final fracture by means of

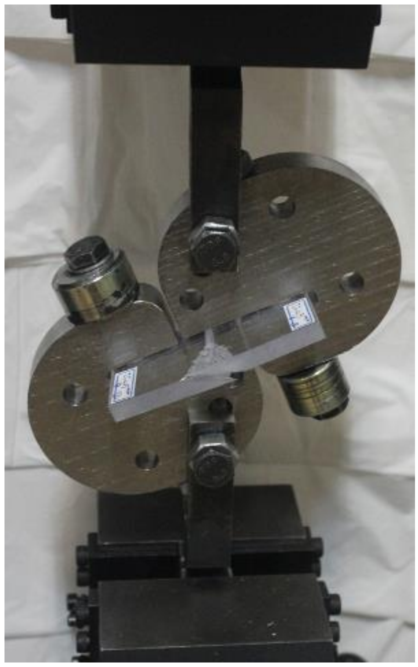

Fig. 4 The test configuration image showing the specimen fractured under the mixed mode loading case of $\beta = 65^{\circ}$

a uniaxial tension-compression test machine. The selected loading rate is 1 mm/min to ensure that monotonic loading condition is established during the experiments. Fig. 4 illustrates the test configuration under the mixed mode case corresponding to $\beta = 65^{\circ}$. The load-displacement curves obtained based on the data recorded by the loading machine computer indicate that fracture of all the specimens occurs suddenly and therefore, they confirm brittle behaviour of the tested GPPS. All the fracture loads obtained from the mixed-mode I/III experiments are listed in Table 2.

In forthcoming sections, the ASED criterion is described and utilized to predict the experimentally obtained fracture loads of the U-notched GPPS specimens summarized in Table 2.

3. Averaged strain energy density (ASED) criterion

The strain energy density is considered as the key parameter in several fracture criteria available in the open literature. For the first time, a point method based on strain energy density was proposed by Sih (1974). According to his fracture criterion, brittle fracture occurs when the strain energy density factor reaches a critical value. Afterwards, several scholars tried to improve the accuracy of the predictions by considering higher order terms of Williams' series in formulation of the strain energy density factor (Ayatollahi *et al.* 2015, Ayatollahi *et al.* 2016, Rashidi Moghaddam *et al.* 2017).

Lazzarin and Zambardi (2001) proposed a volumetric method based on the strain energy density, namely the ASED criterion. While the point methods consider the strain energy density in a specific distance from the notch root for fracture assessment, the volumetric method considers the strain energy density averaged in a control

Table 2 Measured fracture loads of the tested specimens

Notch radius, ρ (mm)	Loading mode, β (°)	Fracture load (N)
0.5	0 (Mode I)	257.5
		246.5
		250.2
	40	305.3
		264.8
		272.2
	65	309.0
		316.3
		309.0
	72	287.0
		261.2
		302.9
1	0 (Mode I)	279.6
		290.6
		272.2
	40	286.9
		316.4
		312.7
	65	327.4
		345.8
		320.0
	72	327.4
		312.7
		348.2
2	0 (Mode I)	331.1
		287.0
		323.7
	40	353.1
		345.8
		360.5
	65	386.2
		437.7
		389.9

volume. The ASED criterion states that brittle fracture occurs when the averaged value of the strain energy density over a control volume $\bar{W}$ is equal to a critical value of W_c which is material-dependent. The ASED criterion was successfully used for fracture assessment of various notch geometries under different combination of loadings to predict the fracture behaviour of different materials (Razavi *et al.* 2017, Razavi *et al.* 2017, Torabi *et al.* 2017a, b, Campagnolo *et al.* 2016). In this paper, an extended version of ASED criterion is presented for fracture assessment of U-notched GPPS samples under mixed mode I/III loading conditions.

A schematic view of the crescent shape control volume for U-notch geometry under tensile and out-of-plane shear loadings is illustrated in Fig. 5. The external radius of the

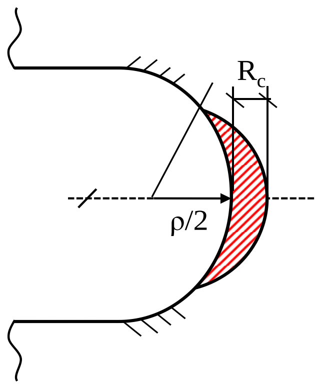

Fig. 5 Control volume for U-notch under mode I loading condition

crescent shape volume is equal to R_c+ρ/2 where R_c is the depth measured along the notch bisector line (can be obtained from Eqs. (1) and (2)) and ρ is the notch tip radius. Dealing with the plane-strain condition, R_c values for mode I and mode III loadings can be obtain from the below expressions (Yosibash *et al.* 2004)

$$R_{1c} = \frac{(1+\upsilon)(5-8\upsilon)}{4\pi}\left(\frac{K_{Ic}}{\sigma_u}\right)^2 \tag{1}$$

$$R_{3c} = \frac{e_3}{(1+\upsilon)}\left(\frac{K_{IIIc}}{\tau_u}\right)^2 \tag{2}$$

in which v is the Poisson's ratio, K_{Ic} and K_{IIIc} are the mode I and mode III fracture toughness, σ_u and τ_u are the ultimate tensile strength and ultimate torsional strength of material and e_3 is the parameter that quantifies the influence of all stresses/strains over the control volume. In the case of a sharp crack or U-notch (2α= 0) e_3 is equal to 0.4138 (Berto *et al.* 2012, Berto *et al.* 2012, Berto *et al.* 2013).

Lazzarin and Berto (2005) reported a formulation to calculate the mode I portion of average strain energy density, $\bar{W}_I$, using maximum tensile stress values at the notch root under tensile loading, $\sigma_{\max}$. On the other hand, Zappalorto *et al.* (2008) proposed an expression for the mode III portion of average strain energy density, $\bar{W}_{III}$, as a function of maximum torsional stress values at the notch root under out-of-plane shear loading, $\tau_{\max}$. According to their proposed equations, $\bar{W}_I$ and $\bar{W}_{III}$ can be expressed using the following expressions

$$\bar{W}_I = F(2\alpha)H(2\alpha, \frac{R_c}{\rho})\frac{\sigma_{\max}^2}{E} \tag{3}$$

$$\bar{W}_{III} = h_3(2\alpha, \frac{R_c}{\rho})\frac{\tau_{\max}^2}{2G} \tag{4}$$

Considering Eqs. (3) and (4), three unknown functions are required to calculate $\bar{W}_I$ and $\bar{W}_{III}$. Function F(2α) is

0.785 for 2α = 0. Function $H(2\alpha, R_c/\rho)$ is equal to 0.4180, 0.4653 and 0.5085 for ρ = 0.5, 1 and 2, respectively (Lazzarin and Berto 2005). Function $h_3(2\alpha, R_c/\rho)$ is equal to 0.5924, 0.7385 and 0.8507 for ρ = 0.5, 1 and 2, respectively (Zappalorto *et al.* 2008).

4. Finite element model

In order to obtain the averaged values of strain energy density, linear elastic 3D analyses were conducted on the single-edge U-notched tension (SENT) models using ABAQUS v6.12 commercial software. As the outcome of the finite element analysis, notch root stresses (σ_{max}, τ_{max}) were obtained to be used in Eqs. (3) and (4). According to the high stiffness of fixture material compared to the GPPS samples, the fixture parts were considered to be rigid. A mesh convergence study was also undertaken to ensure that a proper number of elements was used in finite element modeling. As illustrated in Fig. 6, higher mesh density was used near the notch tip to improve the accuracy of the results. Minimum size of the elements utilized at the notch tip in both circumferential and thickness directions is 0.2 mm. Second-order elements (20-node bricks) with the reduced integration scheme were used for meshing the test specimen whose each node has three translational degrees of freedom (DOFs). The loading angle was changed as β = 0, 40°, 65° and 72° resulting in pure mode I (β = 0°) and mixed mode I/III. The boundary conditions used for the finite element analyses are seen in Fig. 1 in which U_i and UR_i symbols indicate translational and rotational DOFs, respectively. According to Fig. 1, all DOFs were restricted for the bottom loading hole; however, the top loading hole was able to move along the loading axis.

If one obtains the mode I and mode III notch stress intensity factors (NSIFs) $K_I^{U,\rho}$ and $K_{III}^{U,\rho}$ from the notch root stresses σ_{max} and τ_{max}, the mixed loading modes provided by the loading fixture can be stated in terms of the mode mixity parameter M_U^e which is defined as

$$M_U^e = \frac{2}{\pi}\tan^{-1}\left(\frac{K_I^{U,\rho}}{K_{III}^{U,\rho}}\right) \tag{5}$$

M_U^e varies from zero for pure mode III loading to one for pure mode I loading.

The equations required for calculating mode I and mode III NSIFs from the notch root stress values can be found in (Saboori *et al.* 2016). For instance, if sample load 300 N is exerted to a U-notched specimen with ρ = 1 mm, the mixed mode loading conditions due to the loading angles β = 40°, 65° and 72° correspond to the M_U^e values 0.84, 0.62 and 0.49, respectively.

5. Fracture analysis of the tested GPPS specimens

Fracture behaviour of the tested U-notched specimens made of GPPS polymer is assessed in this section using the ASED criterion. According to the ASED criterion, when the averaged value of the strain energy density over a control volume, $\overline{W}$, reaches a critical value W_c, brittle fracture occurs. It is worth mentioning that the critical strain energy density depends on the material properties but not on the notch geometry. The critical strain energy density under mode I and mode III loading conditions can be calculated using the Beltrami's expression for the un-notched material

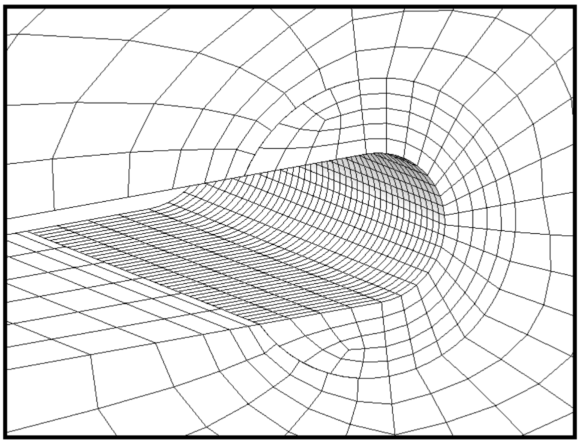

Fig. 6 Typical mesh pattern of the finite element model near the notch tip

$$W_{Ic} = \frac{\sigma_u^2}{2E} \tag{6}$$

$$W_{IIIc} = \frac{\tau_u^2}{2G} \tag{7}$$

Unlike the linear trend of applied load versus notch mouth opening displacement curves, the true stress-strain curve of quasi brittle materials such as GPPS exhibits a non-linear behaviour (Lazzarin and Berto 2005). Dealing with these cases, Lazzarin and Berto (2005) suggested to use "critical tensile stress, σ_u^*" instead of ultimate tensile strength of un-notched material, σ_u for calculating the critical strain energy density and also control volume radius. In order to obtain the critical tensile and torsional stresses at notch root under mode I and mode III, the experimental failure loads were applied to the linear elastic finite element model of the tested specimens with the maximum notch radius of ρ = 2 mm. According to the finite element analyses, the critical tensile stress, σ_u^* and critical torsional stress, τ_u^* were 55 MPa and 31 MPa, respectively. According to the material properties in Table 1 and substituting the critical stresses in Eqs. (6) and (7), the critical strain energy densities under mode I and mode III for the tested GPPS material are W_{Ic} = 0.504 MJ/m^3 and W_{IIIc} = 0.438 MJ/m^3, respectively. On the other hand, the resulting values of control radii are R_{1c} = 0.165 mm and R_{3c} = 0.133 mm, under pure mode I and pure mode III loadings, respectively.

A generalized form of the ASED criterion is used in this

Table 3 The overview of the analyzed data by use of the ASED criterion

Notch radius, ρ (mm)	Loading angle, β (°)	Fracture load (N)	$\sigma_{\theta\theta}\|_{max}$ (MPa)	$\sigma_{\theta z}\|_{max}$ (MPa)	$\bar{W}_I$ (MJ/m^3)	$\bar{W}_{III}$ (MJ/m^3)	$\bar{W}_I/W_{Ic}+\bar{W}_{III}/W_{IIIc}$	$(\bar{W}_I/W_{Ic}+\bar{W}_{III}/W_{IIIc})^{0.5}$
0.5	0 (Mode I)	257.5	84.29	0	0.667	0	1.324	1.151
		246.5	80.69	0	0.612	0	1.213	1.101
		250.2	81.9	0	0.630	0	1.250	1.118
	40	305.3	90.72	22.17	0.773	0.066	1.684	1.298
		264.8	78.68	19.23	0.581	0.050	1.267	1.125
		272.2	80.88	19.77	0.614	0.052	1.338	1.157
	65	309	55.91	39.77	0.294	0.212	1.067	1.033
		316.3	57.24	40.71	0.308	0.222	1.118	1.057
		309	55.91	39.77	0.294	0.212	1.067	1.033
	72	287	36.72	40.34	0.127	0.218	0.749	0.866
		261.2	33.42	36.71	0.105	0.181	0.621	0.788
		302.9	38.75	42.57	0.141	0.243	0.835	0.914
1	0 (Mode I)	279.6	66.49	0	0.482	0	0.956	0.978
		290.6	69.11	0	0.521	0	1.033	1.016
		272.2	64.73	0	0.457	0	0.906	0.952
	40	286.9	62.93	15.69	0.432	0.050	0.971	0.985
		316.4	69.41	17.31	0.525	0.061	1.181	1.087
		312.7	68.59	17.11	0.513	0.060	1.154	1.074
	65	327.4	47.2	31.97	0.243	0.208	0.957	0.978
		345.8	49.85	33.77	0.271	0.232	1.067	1.033
		320	46.13	31.25	0.232	0.199	0.914	0.956
	72	327.4	34.01	35.37	0.126	0.254	0.831	0.912
		312.7	32.48	33.78	0.115	0.232	0.758	0.871
		348.2	36.17	37.62	0.143	0.288	0.941	0.970
2	0 (Mode I)	331.1	57.52	0	0.398	0	0.789	0.888
		287	49.86	0	0.299	0	0.593	0.770
		323.7	56.24	0	0.380	0	0.755	0.869
	40	353.1	57.71	14.78	0.401	0.060	0.932	0.965
		345.8	56.51	14.48	0.384	0.058	0.894	0.945
		360.5	58.92	15.09	0.418	0.063	0.971	0.986
	65	386.2	43.25	29.62	0.225	0.242	0.998	0.999
		437.7	49.01	33.57	0.289	0.310	1.282	1.132
		389.9	43.66	29.9	0.229	0.246	1.017	1.009

paper to summarize the mixed mode I/III behaviour of U-notched GPPS samples. Lazzarin *et al.* (2004) firstly proposed an extension of ASED criterion for failure assessment of V-notched specimens under combined tension and torsion fatigue loading. In agreement with the methodology proposed by Lazzarin *et al.* (2004), Berto *et al.* (2017) proposed the generalized form of ASED method for fracture assessment of brittle materials under static mixed mode I/III loading using the following elliptic expression

$$\frac{\bar{W}_I}{W_{Ic}}+\frac{\bar{W}_{III}}{W_{IIIc}}=1 \qquad (8)$$

where $\bar{W}_I$ and $\bar{W}_{III}$ are the mode I and mode III portions of averaged strain energy density in control volume and W_{Ic} and W_{IIIc} are the critical values of strain energy density under pure mode I and pure mode III loadings.

The same elliptic expression is used in this paper for fracture analyses. According to Eqs. (3) and (4), the values of $\bar{W}_I$ and $\bar{W}_{III}$ are calculated as a function of the maximum stress values at the notch root. When the summation of the weighted contributions of mode I and mode III is equal to 1, brittle fracture of the U-notched specimens occurs. The details of numerical calculations employing the elliptic form of ASED criterion are reported in Table 3.

General synthesis of the ASED predictions using the square root values of the sum of the weighted energy contributions related to mode I and mode III loading conditions is shown in Fig. 7 as a function of the loading angle, β (°). According to ASED results, almost all of the analysed data are placed inside a scatter band ranging from 0.8 to 1.2. Having the critical strain energy density values of W_{Ic} and W_{IIIc}, the fracture behavior of U-notched specimens can be predicted by the elliptic expression of the ASED. The analyzed fracture model in this paper can be employed to analyze the onset of brittle fracture in U-notched specimens under a combination of tension and torsion loadings.

6. Conclusions

A new methodology based on the ASED criterion was proposed in this study to assess the brittle fracture of U-notched GPPS specimens under mixed mode I/III loading conditions. Different notch tip radii were considered to fabricate the single edge notched specimens. By taking advantage of the experimental data ranging from pure tension to close to pure torsion, a criterion was proposed and validated for the specific case of mixed mode I/III loading conditions. An elliptic formulation of the ASED criterion was presented in this paper and the fracture behavior of the U-notched GPPS plates was successfully predicted using the mentioned criterion. The theoretical predictions based on ASED criterion were found to be in good agreement with the experimental results.

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