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# Friction tuned mass damper optimization for structure under harmonic force excitation

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**Abstract.** In this work, an optimization method of Friction Tuned Mass Damper (FTMD) parameters is presented. Friction tuned mass dampers (FTMD) are attached to mechanical structures to reduce their vibrations with dissipating the vibratory energy through friction between both bodies. In order to exploit the performances of FTMD, the determination of the optimum parameters is recommended. However, the presence of Coulomb's friction force requires the resolution of a non-linear stick-slip problem. First, this work aims at determining the responses of the vibratory system. The responses of the main mass and of the FTMD are determined analytically in the sticking and sliding phase using the equivalent damping method. Second, this work aims to optimize the FTMD parameters; the friction coefficient and the tuned frequency. The optimization formulation based on the Ricciardelli and Vickery method at the resonance frequencies, this method is reformulated for a system with a viscous damping. The inverse problem of finding the FTMD parameters given the magnitude of the force and the maximum acceptable displacement of the primary system is also considered; the optimization of parameters leads to conclude on the favorable FTMD giving significant vibration decrease, and to advance design recommendations.

**Keywords:** tuned mass damper; coulomb friction; optimization parameters; vibration reduction

## 1. Introduction

Since Frahm (1909) invented the tuned mass damper, it is widely used as a passive device to control vibration level. It is an auxiliary component introduced to the mechanical structures to attenuate the vibrations of the main structure by dissipating the vibratory energy. To improve its efficiency, Ormondroyd and Den Hartog (1928) proposed the first theoretical approach on the TMD. Brock (1946) has extended this optimality to an analytical solution for the optimal damping rate. In addition, various recent studies have been carried out to increase the efficiency of absorbers (Liu and Liu 2005, Lee et *al.* 2006). In order to improve the efficiency of this device, a friction element is integrated into the TMD based on the reliability of the energy dissipation friction (Hartung et *al.* 2001, Lopez *et al*. 2004). Louroza et *al.* (2005) studied the effect of the mass ratio and the friction force on the frequency response of a vibratory system with a FTMD; their works show that the friction absorber is more effective in reducing vibration than the mass absorber.

The vibration reduction by FTMD needs the understanding of the excited system. Systems with FTMD are classified as non-linear system due to discontinuous behavior of the motion; this behavior is nonlinear relatively to velocity and to friction force. The motion balance between two cases; stick motion and slip motion. In the stick motion case, the main system and the FTMD are stick and the steady-state amplitudes are determined by the resolution of a system with one degree of freedom. While in the slip motion case, the system is linear and the steady-state amplitudes can be determined by the equivalent viscous damping method. The equivalent viscous damping method is an approximation method of friction. It consists on replacing the Coulomb friction damping by a linear viscous damping such as the latter which dissipated the same quantity of energy dissipated by the dry friction. Jacobson (1930), determined the equivalent viscous damping of friction damping, its results are compared by Den Hartog's work (Den Hartog 1931), and they are valid at resonance frequencies. Tan and Rogers (1995) used this method to study a system with several degrees of freedom in the slide regime. Recently, Fang et al. (2012) studied the effect of friction damping on the optimal parameters of TMD with a harmonic ground excitation system.

The identification of the optimum parameters leads to the best design of the TMD and increases its efficiency to industrial applications. The optimization consists on finding the optimum parameters of the TMD as a function of the main mass parameters. The researchers proposed several methods to find these parameters. Den Hartog (1956) proposed a method for optimizing the parameters of a mass absorber and reducing the sensitivity of the main mass response to the variation of the excitation frequency such that the two resonance peaks of the frequency response are equal. Pennestri (1998) used a method to minimize the maximum of the frequency amplitude based on an objective function with six equations and seven unknowns. For a system with several degrees of freedom, Son et *al.* (2015) used a pendulum and spring-mass absorbers to reduce the

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vibration of 2 degrees of freedom system. They used fixed point and genetic algorithms to determine the optimal absorber parameters. Ghosh and Basu (2007) used the ‘fixed-point’ theory to develop a closed-form expression for the optimum tuning ratio. The expression for the optimal tuning ratio is a function of the structural damping and the mass ratio. Lu et *al.* (2017) proposed an approach to design a multiple tuned mass dampers by using the transfer function to obtain the optimum stiffness and damping for each TMD.

However, the most of the previous studies FTMD optimization represent parametric studies constructed from a numerical resolution of the differential equations of motion. Louroza et *al.* (2005) studied the effect of the mass ratio and the friction force on the frequency response of a vibratory system with a FTMD. Gewei and Basu (2011) studied the effect of the increase of the friction coefficient on the amplitude of the vibrations of the main mass with seismic excitation. Pisal and Jangid (2016) are investigated the response of the single-degree-of-freedom (SDOF) structure with TMFD under harmonic and seismic ground excitations. They found that the displacement of the principal structure attainedits minimum value at a given level of excitation, an optimum value of mass ratio, tuning frequency ratio and damper slip force. Chung et al. (2012) used a friction pendulum tuned mass damper to reduce the vibrations of Taipei 101 under white-noise wind force. They combined a tuned mass damper with viscous damping and isolation systems with friction pendulums. The most interesting approach has been proposed by Ricciardelli and Vickery (1999). They proposed a closed form expression for optimal FTMD parameters; tuning frequency and coefficient of friction at the resonant frequencies of the systems. Their works don’t take into account the viscous damping of the mass absorber.

First, this paper examines the stick-slip behavior of one degree of freedom system with a friction tuned mass damper. In this section, the steady-state amplitudes in sticking case and sliding case of motion are analytically computed. Second, an optimization formulation of the FTMD parameters is presented for better design and industrial application.

## 2. Vibratory model

The main system that will be studied, illustrated in Fig. 1, is a one degree of freedom linear system, modeled by the mass $m_1$, the linear stiffness $k_1$ and the viscous damping $c_1$. The FTMD of mass $m_2$ is attached to the main system, with $k_2$ and $c_2$ which are respectively the linear stiffness and the viscous damping of the absorber. The contact between the two masses is modeled by the dry friction force of Coulomb $F_{nl}$. The main mass is excited by a harmonic force $F$ where $F_0$ is the amplitude and $\Omega$ is the pulsation.

The system is nonlinear due to the presence of the Coulomb friction force. This force is proportional to the sign of the relative velocity $\dot{x}_1 - \dot{x}_2$and the motion of the system balance between two states of the motion; the sliding motion and the sticking motion.

The equations that describe the sliding case are as follows

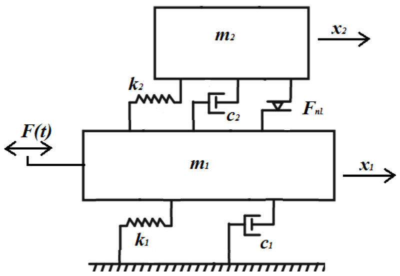



Fig. 1 One degree of freedom system with FTMD

$$m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 + c_2(\dot{x}_1 - \dot{x}_2) + k_2(x_1 - x_2) + F_{nl} = F(t) \quad (1a)$$

$$m_2\ddot{x}_2 - c_2(\dot{x}_1 - \dot{x}_2) - k_2(x_1 - x_2) - F_{nl} = 0 \quad (1b)$$

The friction force $F_{nl}$ as a constant force with varying signum function is written as follows

$$F_{nl} = \mu N sign(\dot{x}_1 - \dot{x}_2) \quad (2)$$

Where $N$ is the normal load applied by the mass FTMD and $\mu$ is the friction coefficient of Coulomb.

Furthermore, the sticking occurs if the condition in Eq. (3) is satisfied

$$\left| m_2\ddot{x}_2 - c_2(\dot{x}_1 - \dot{x}_2) - k_2(x_1 - x_2) \right| \leq \mu N \quad (3)$$

In this case, the equation of motion can be written as follows

$$(m_1 + m_2)\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 = F_0\cos(\Omega t) \quad (4)$$

To transform the equations of motion in the sliding case and in the sticking case to the non-dimensional form, the following quantities are used

$$y_1 = \frac{x_1}{x_s}, y_2 = \frac{x_2}{x_s}, x_s = \frac{N}{k_1}, \omega_1 = \sqrt{\frac{k_1}{m_1}}, \omega_2 = \sqrt{\frac{k_2}{m_2}}, \omega_a = \frac{\omega_2}{\omega_1}, \xi_1 = \frac{c_1}{2m_1\omega_1},$$
$$\xi_2 = \frac{c_2}{2m_2\omega_2}, r = \frac{m_2}{m_1}, \tau = \omega_1 t, f_e = \frac{F_0}{N}, \omega = \frac{\Omega}{\omega_1}$$

The non-dimensional equations Eqs. (1a)-(1b) of motion in the sliding state are written along these lines

$$\ddot{y}_1 + 2\xi_1\dot{y}_1 + y_1 + 2\xi_2 r\omega_a(\dot{y}_1 - \dot{y}_2) + r\omega_a^2(y_1 - y_2) + f_{nl} = f_e\cos(\omega\tau) \quad (5a)$$

$$\ddot{y}_2 - 2\xi_2\omega_a(\dot{y}_1 - \dot{y}_2) - \omega_a^2(y_1 - y_2) - \frac{f_{nl}}{r} = 0 \quad (5b)$$

Where the non-dimensional friction force is written as follows

$$f_{nl} = \mu sign(\dot{y}_1 - \dot{y}_2) \quad (6)$$

The condition of the sticking motion is written hence

$$\left| \ddot{y}_2 - 2\xi_2\omega_a(\dot{y}_1 - \dot{y}_2) - \omega_a^2(y_1 - y_2) \right| \leq \frac{\mu}{r} \quad (7)$$

If this condition is verified, the two masses stick together and the system have one degree of freedom. The non-

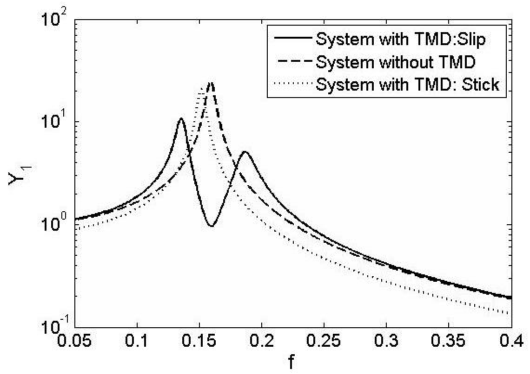



Fig. 2 Frequency response $Y_1$; with FTMD in sliding case, without FTMD, and with FTMD in sticking case

dimensional equation of motion is written thusly

$$\ddot{y}_1 + \frac{2\xi_1}{1+r}\dot{y}_1 + \frac{1}{1+r}y_1 = \frac{f_e}{1+r}\cos(\omega\tau) \tag{8}$$

## 3. Stick-slip behavior and steady-state responses

We started by investigating the stick-slip behavior of the two degrees of freedom system. Fig. 2 shows changes in the frequency response of the main mass by the addition of the FTMD. The parameters used in this example are; $\omega_a$=1, $r$=0.1 and $\xi_2$=$\xi_1$=0.01.

Initially, the one degree of freedom system without FTMD has a single resonant frequency $f$=0.16. Then, the integration of the absorber modifies the frequency response of the main mass and the resonant frequency decreases to $f$=0.15. In this so-called sticking regime, the friction force is greater than the maximum inertia force acting on the absorber. However, in sliding regime, the friction force is less than the maximum inertia force acting on the absorber. The two masses are in pure sliding and the resonant frequencies; $f_1$ =0.135 and $f_2$ =0.19, are the frequencies of a linear system with two degrees of freedom.

In the slip phase of motion, the equivalent viscous damping method is used to determine the responses of the main mass and of the FTMD. The idea is to replace the non-linear friction damping by a linear viscous damping that dissipates the same amount of energy during a single cycle of motion.

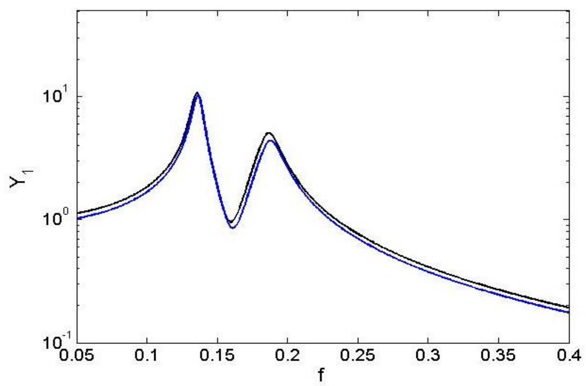
This amount of energy equals

$$E_d = \int_{cycle} \xi_e(\dot{y}_1 - \dot{y}_2)^2 d\tau \tag{9}$$

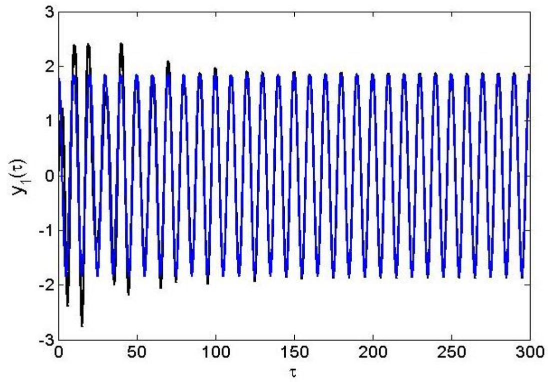
For a harmonic excitation, we propose that the solutions sought are harmonic solutions. Where $y_1(\tau)$ and $y_2(\tau)$ are written in the flowing form

$$\begin{aligned} y_1(\tau) &= Y_1 e^{i\omega\tau} \\ y_2(\tau) &= Y_2 e^{i\omega\tau} \end{aligned} \tag{10}$$

Depending on $Y_1$ and $Y_2$ and the non-dimensional parameters of the system, the equivalent viscous damping rate




(a) Frequency response




(b) Temporal response at f=0.1

Fig. 3 omparison between the numerical method(-) and the analytical method(-) (equivalent viscous damping) in the sliding regime

is defined as

$$\xi_e = \frac{4\mu}{\pi\omega(Y_1 - Y_2)} \tag{11}$$

The equivalent force to the friction force as a function of the equivalent damping and the relative velocity is written as

$$F_d = \xi_e(\dot{y}_1 - \dot{y}_2) \tag{12}$$

The frequency response of the relative displacement $Y_1$-$Y_2$ is written as follows

$$(Y_1 - Y_2) = \frac{f_e}{\sqrt{\left(A + 2\xi_1\frac{\xi_e}{r}\right)^2 + (B + C\xi_e)^2}} \tag{13}$$

Where

$$A = 1 + \omega_a^2(1+r) - \omega^2 - \frac{\omega_a^2}{\omega^2} - 4\xi_1\xi_2\omega_a,$$

$$B = 2\xi_1\left(\omega - \frac{\omega_a^2}{\omega}\right) + 2\xi_2\omega_a\omega\left(1 + r - \frac{1}{\omega^2}\right) \text{ and}$$

$$C = \omega\left(1 + \frac{1}{r} - \frac{1}{r\omega^2}\right)$$

Substituting the Eq. (11) of the equivalent viscous damping rate in Eq. (13), and then resolving it with respecting

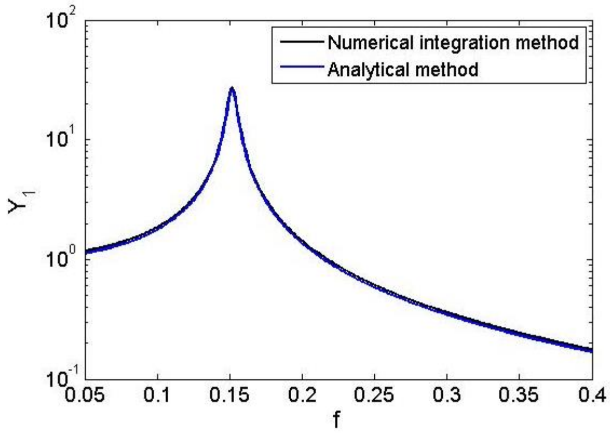



Fig. 4 Frequency responses $Y_1$: The numerical method and the analytical method in the sticking case of motion

$Y_1$-$Y_2$. The Eq. (13) has two solutions. Their nature depends on the system parameters. We take into account the real and positive solution.

To obtain the responses of the main mass and of the FTMD, we substitute the real and the positive solution of $Y_1$-$Y_2$ in Eq. (11) of the equivalent damping and from the equations of motion Eqs. (1a)-(1b), we can calculate the frequencies reponses $Y_1$ and $Y_2$.

The equivalent damping method is based on linearization of the non- linear friction force in slip regime. From the example shown in Figs. 3(a)-(b), this method gives a good approximation to represent the responses of the vibratory system.

On the other hand, the frequency response of the system in the sticking state is determined by Eq. (8), the frequency response is written thus

$$Y_1 = Y_2 = \frac{f_e}{\sqrt{\left(1-(1+r)\omega^2\right)^2 + (2\xi_1\omega)^2}} \tag{14}$$

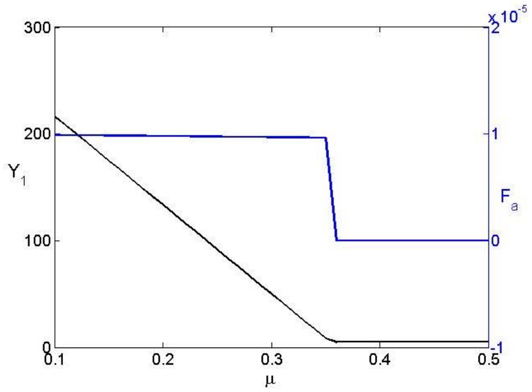
The advantage of the analytical determination of the frequency response in both cases of motions is the reduction of the resolution time since the numerical integration is slow (Fig. 4).

For determining the coefficient of friction which separates the sliding zone from of the sticking zone. The difference between the friction force and the maximum inertia force acting on the absorber $F_a$can be computed by the following equation

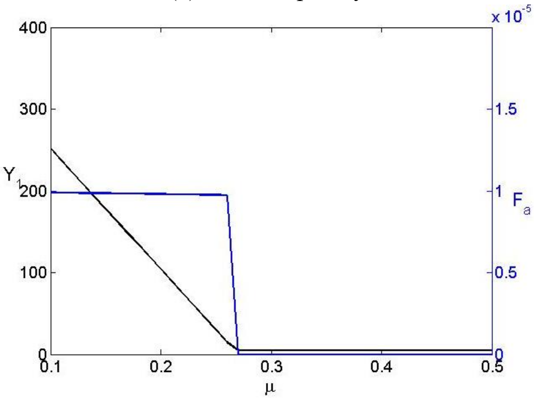
$$F_a = \left|\ddot{y}_2 - 2\xi_2\omega_a(\dot{y}_1 - \dot{y}_2) - \omega_a^2(y_1 - y_2)\right| - \frac{\mu}{r} \tag{15}$$

The effect of the increase of the friction coefficient on the difference $F_a$ and on the amplitude at the frequencies of the resonances is shown in Figs. 5(a)-(b). The amplitude $Y_1$ of the main mass decreases and the difference between the inertia force and the friction force is positive up to a limiting friction coefficient, where the two masses stick and the difference $F_a$ becomes negative.

The limit friction coefficient can be determined by the substitution of Eq. (14) on Eq. (7). At this limit value, the velocities of the main mass and of the FTMD are equals and




(a) First frequency




(b) First frequency

Fig. 5 Response of the main mass(-) and the difference $F_a$(-)as a function of the friction coefficient at the resonant frequencies

the amplitudes are also.

The limit friction coefficient is written so

$$\mu_{\lim} = \frac{r f_e \omega^2}{\sqrt{\left(1-(1+r)\omega^2\right)^2 + (2\xi_1\omega)^2}} \tag{16}$$

## 4. Optimum design of friction tuned mass damper

The Optimization consists of determining the optimum FTMD parameters; the friction coefficient $\mu$, the tuned frequency $\omega_a$ and the mass ratio $r$ at the frequencies of the resonance as a function of the main mass parameters as a function of the primary mass parameters and of the parameters excitation.

### *4.1 System without viscous damping*

For the two frequencies of the resonance, the minimum vibration amplitude corresponds to the limit friction coefficient. Without viscous damping, the system has the two following pulsations of resonance

$$\theta_1 = \sqrt{\frac{1+(1+r)\omega_a^2 - \sqrt{\left(1+(1+r)\omega_a^2\right)^2 - 4\omega_a^2}}{2}} \tag{17}$$

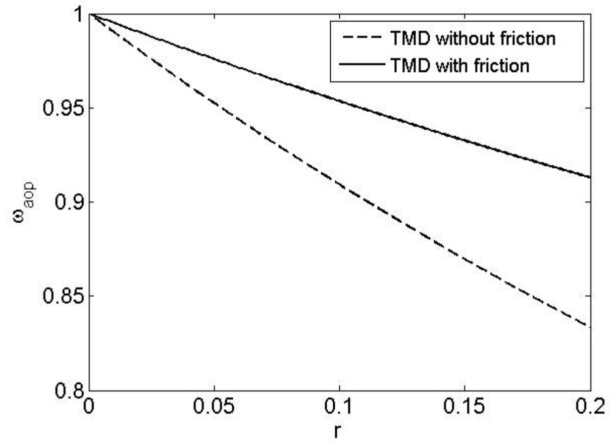



Fig. 6 Optimal tuned frequency as a function of the mass ratio

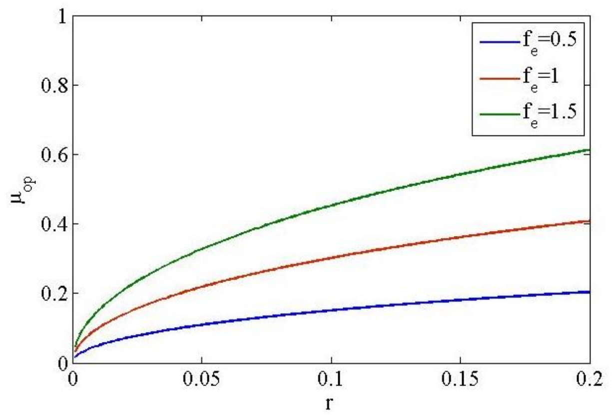



Fig. 7 Optimal friction coefficient as a function of the mass ratio

$$\theta_2 = \sqrt{\frac{1+(1+r)\omega_a^2 + \sqrt{\left(1+(1+r)\omega_a^2\right)^2 - 4\omega_a^2}}{2}} \tag{18}$$

The optimal friction coefficient can be determined by substituting the Eq. (17) and Eq. (18) in the Eq. (16) of the limit friction coefficient, hence the optimal coefficients for each pulse are written accordingly

$$\mu_1 = \frac{rf_e\left(1+(1+r)\omega_a^2 - \sqrt{\left(1+(1+r)\omega_a^2\right)^2 - 4\omega_a^2}\right)}{1-r+(1+r)^2\omega_a^2 - (1+r)\sqrt{\left(1+(1+r)\omega_a^2\right)^2 - 4\omega_a^2}} \tag{19}$$

$$\mu_2 = \frac{rf_e\left(1+(1+r)\omega_a^2 + \sqrt{\left(1+(1+r)\omega_a^2\right)^2 - 4\omega_a^2}\right)}{1-r+(1+r)^2\omega_a^2 - (1+r)\sqrt{\left(1+(1+r)\omega_a^2\right)^2 - 4\omega_a^2}} \tag{20}$$

The two coefficients are not equal. The equality of these two coefficients gives the optimal tuned frequency

$$\omega_{aop} = \frac{1}{\sqrt{1+r}} \tag{21}$$

We note that this result is coinciding with the result determined by Ricciardelli and Vickery when the vibratory system is subjected to an excitation with varying frequency. This tuned optimal frequency is used to validate the optimization criterion of the friction coefficient.

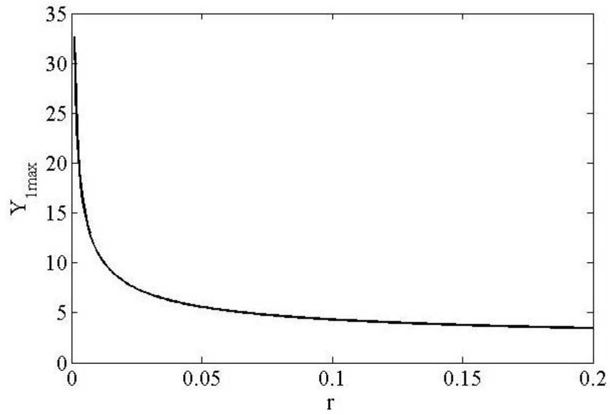



Fig. 8 Maximum response of the main mass as a function of the mass ratio

Then, the optimal coefficient of friction is obtained by replacing Eq. (21) in Eq. (19) or Eq. (20) and recalculating the coefficient of friction, the optimum coefficient of friction is obtained as follows

$$\mu_{op} = rf_e \frac{\sqrt{1+r} - \sqrt{r}}{r\sqrt{1+r} + (1+r)\sqrt{r}} \tag{22}$$

Eq. (21) shows that the best performance of the FTMD is obtained when this is tuned to the frequency of the stick system (Eq. (8)). Moreover, in Fig. 6, the optimum tuned frequency of a FTMD is higher than that of a viscous TMD. This frequency decreases by increasing the mass ratio. If the main mass $m_1$ and the stiffness $k_1$ are constant, it can be concluded that the increase in the mass of the FTMD favors the decrease of the stiffness $k_2$.

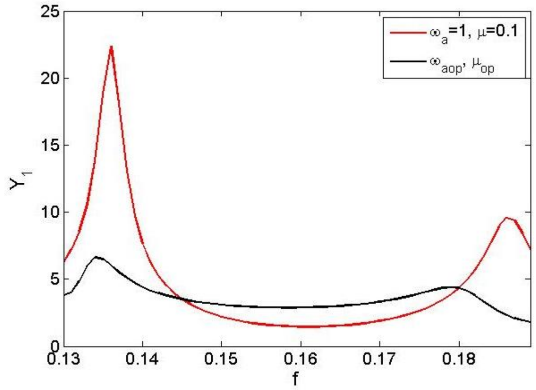
Fig. 7 shows that the optimum friction coefficient depends on the mass ratio. This optimal coefficient increases by increasing the mass ratio. Additionally, the amplitude of the excitation force $f_e$ favors the increase of the optimal coefficient at a constant mass ratio. These results also show that the zone of slip is proportional to the mass ratio and to the excitation force.

Further, the effect of the mass ratio on the maximum response of the main mass is shown in Fig. 8. The augmentation of the mass of the FTMD favors the decrease of the maximum displacement of the main mass.

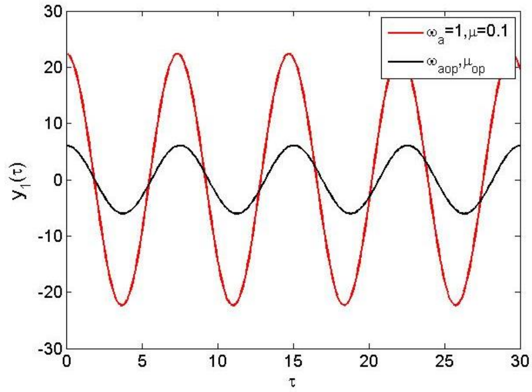
Figs. 9(a)-(b) shows the frequency and the temporal responses of the primary mass without optimization and with optimization. This method gives a signification reduction of the vibration response of primary mass.

However, it is recommended to optimizer the FTMD parameters for a desired primary mass displacement. It is posited that the amplitude of the primary mass equals to a desired displacement $Y_a$, the optimal friction coefficient can be written using Eq. (5b) and Eq. (14)

$$\mu_{op} = r\frac{Y_a - f_e}{1+r} \tag{23}$$




(a) Frequency responses




(b) temporal Frequency responses

Fig. 9 Frequency and temporal responses of the primary mass at the optimal parameters

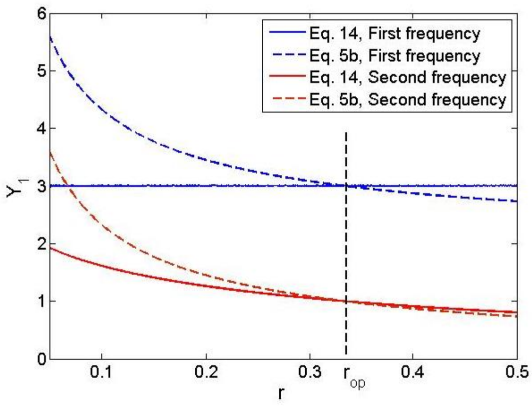



Fig. 10 $Y_1$ as a function of the mass ratio for the two resonant frequencies

Then, we substitute the resonant frequencies $\theta_1$ and $\theta_2$ into equations Eq. (5b) and Eq. (14) and used the optimal tuned frequency expression from Eq. (21). The resolution of the equality of the two equations for $r$ gives the optimal mass ratio.

Finally, the optimal mass ratio is written as

$$r_{op} = \frac{f_e^2}{Y_a(Y_a - 2f_e)} \tag{24}$$

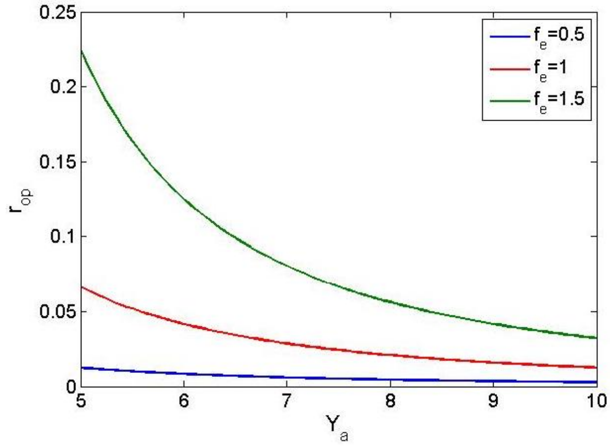



Fig. 11 Optimal mass ratio as a function of the desired displacement $Y_a$

For an undamped system, Fig. 11 shows that the optimal mass ratio increase if the amplitude of the excitation force augmented for a constant main mass displacement and this ratio decreases if the desired displacement increases.

On the contrary, the tuned frequency decreases when the amplitude force decreases (Fig. 12(a)). That is to say that if the mass and the stiffness of the primary system are constant, it is necessary to increase the mass of the FTMD and to decrease the elasticity of the FTMD to have weak amplitude.

On the other hand, to obtain a low displacement amplitude, the using of height friction coefficient is recommended, and it augments if the force amplitude increases (Fig. 12(b)).

### *4.2 Systems with viscous damping*

If the viscous dampers of the main mass and absorber are taken into account, the frequencies of the resonance are determined by the use of Rayleigh damping. The notion of Rayleigh makes it possible to define the non-dimensional damping matrix $C'$as a linear combination of the matrices of stiffness $K'$and of mass $M'$

$$C' = \alpha M' + \beta K' \tag{25}$$

Where

$$K' = \begin{bmatrix} 1 + r\omega_a^2 & -r\omega_a^2 \\ -\omega_a^2 & \omega_a^2 \end{bmatrix} \quad C' = \begin{bmatrix} 2\xi_1 + 2r\xi_2\omega_a & -2r\xi_2\omega_a \\ -2\xi_2\omega_a & 2\xi_2\omega_a \end{bmatrix}$$

$$M' = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

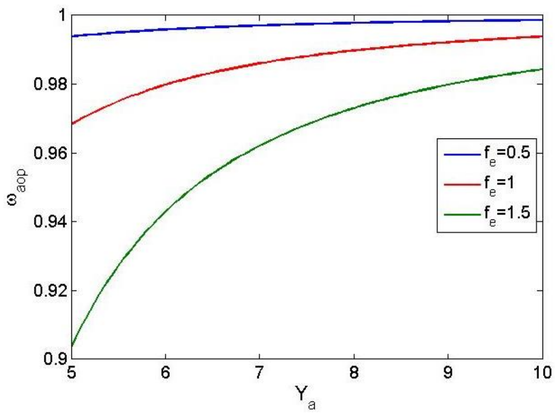
From Eq. (25), the coefficients $\alpha$ and $\beta$ are written as

$$\alpha = 0$$
$$\beta = \frac{2\xi_2}{\omega_a} \tag{26}$$

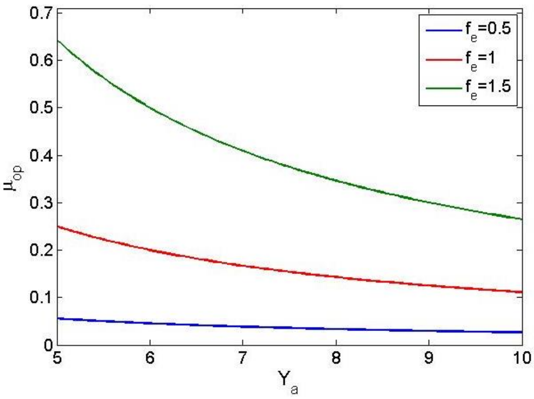
The combination is mathematically valid if the following condition is validated

$$\xi_2 = \xi_1\omega_a \tag{27}$$

The pulsations of the resonance are written hence

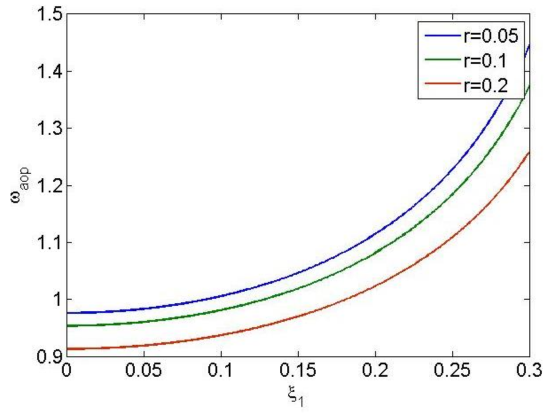



(a) Optimal tuned frequency

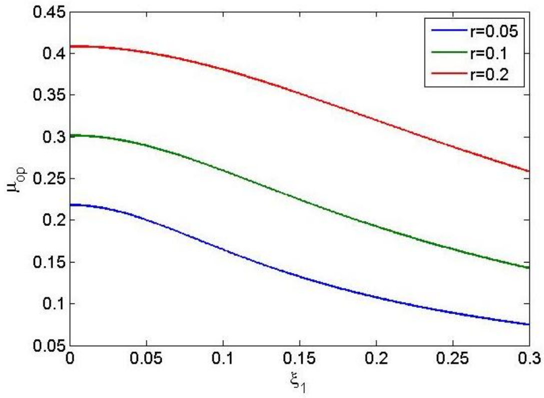



(b) Optimal friction coefficient

Fig. 12 Optimal parameters system without viscous damping as a function of the desired displacement $Y_a$




(a) Optimal tuned frequency




(b) Optimal friction coefficient

Fig. 13 Optimal parameters system with viscous damping as a function of the damping rate of the main mass

$$\theta_{d1}=\theta_1\sqrt{1-\left(\frac{\beta\theta_1}{2}\right)^2}$$
$$\theta_{d2}=\theta_2\sqrt{1-\left(\frac{\beta\theta_2}{2}\right)^2} \tag{28}$$

The approach consists first on substituting Eqs. (28) in Eq. (16). The two coefficients obtained are not equal. Second, the equalization of these two coefficients gives the optimal value of the tuned frequency; it can be determined by solving the following equation

$$\omega_a^4\left(\left(4\xi_1^2-2(1+r)\right)\left(\xi_1^4-\xi_1^2(1+r)\right)-\xi_1^2(1+r)^2\right)+$$
$$\omega_a^2\left(\left(4\xi_1^2-2(1+r)\right)\left(1-\xi_1^2\right)+r\left(1-2\xi_1^2\right)+1\right)+1-\xi_1^2=0 \tag{29}$$

Then the substitution of this frequency in Eq. (16) gives the optimal friction coefficient. This approach is applied by varying $\xi_1$ and by respecting the condition of Eq. (27).

The results in the Fig. 13(a) show that the increase in the damping rate $\xi_1$ favors the increase of the optimal tuned frequency for a constant mass ratio. On the other hand, at a constant damping rate, if the mass ratio increases the optimal tuned frequency decreases. Hence, we can conclude that if the main system damping increases, the FTMD must be more rigid. For a high damping rate the optimal tuned frequency is supeieur to 1, it is on the contrary of TMD where it does not exceed 1. This increase preserves the uniqueness of the coefficient of optimal friction.

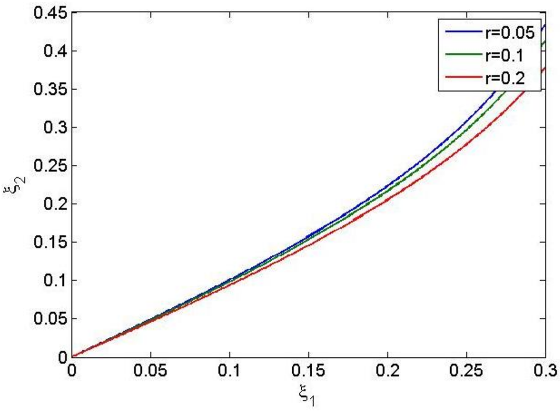



Fig. 14 Damping rate of the FTMD as a function of the damping rate of the main mass

The optimum friction coefficient represents the limit of the sliding zone. (Fig. 13(b)).This optimum coefficient of friction decreases by increasing the damping rate $\xi_1$ and the sliding area get reduced .

From Eq. (27), the damping $\xi_1$ and $\xi_2$ are proportional. This relation validates the use of the Rayleigh damping approximation. Fig. 14 shows that the FTMD damping rate increases respectively with the increase of the main mass damping.

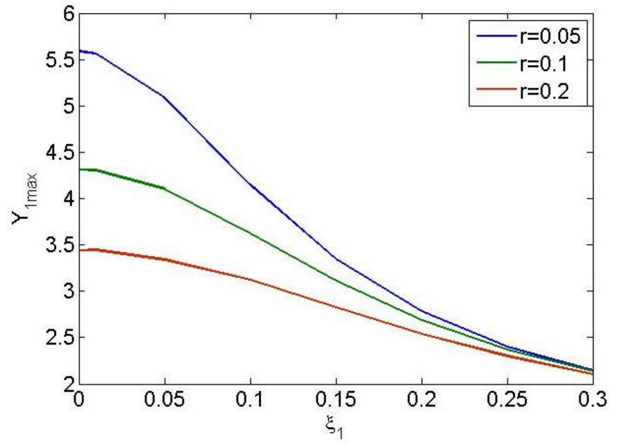



Fig. 15 Maximum response of the main mass as a function of the damping rate

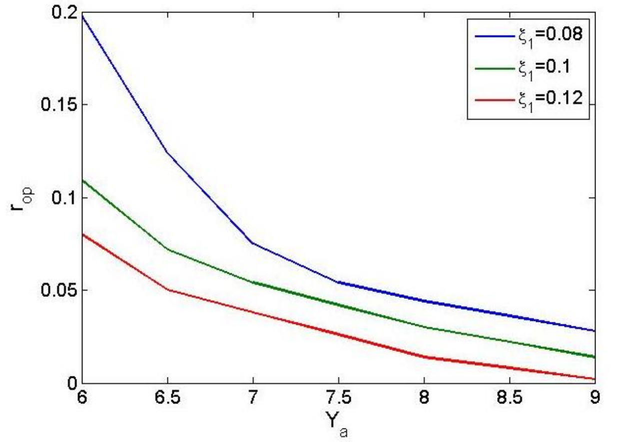



Fig. 16 Optimal mass ratio as a function of desired displacement $Y_a$

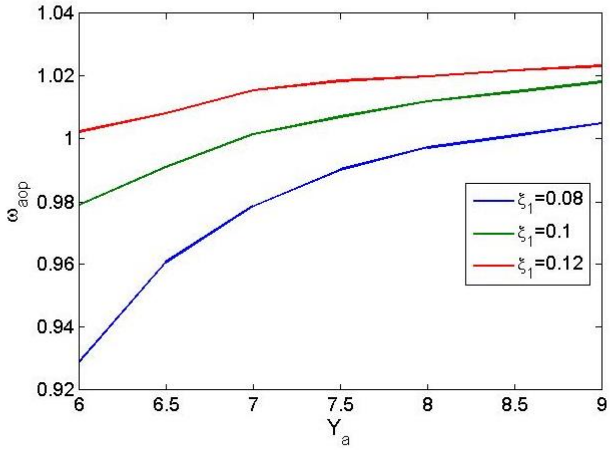
The damping rate effect on the maximum amplitude of the main mass vibrations shown in Fig. 15 demonstrates that the use of a high damping rate is recommended to reduce the vibrations of the main mass.

For a desired main mass displacement and if the viscous dampers of the main mass and FTMD are taken into account, the resonant frequencies are the same of the first section of optimization. Using the same method to determine the friction coefficient, it is written as

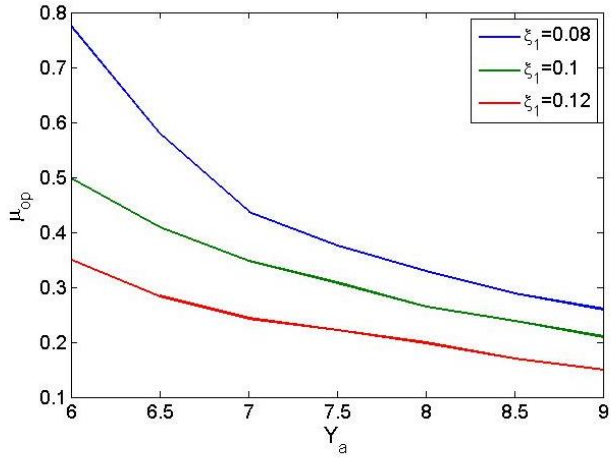
$$\mu = r\frac{(1+r)Y_a - \sqrt{f_e^2\left((1+r)^2 + 4\xi_1^2\right) - 16Y_a^2\xi_1^2}}{(1+r)^2 + 4\xi_1^2} \tag{30}$$

The analytical determination of the optimal parameters is complicated in this case, which requires the use of an algorithm implanted on Matlab. The tuned frequency can determine using Eq. (28). This method is only valid for low viscous damping rate. The effect of the viscous damping is very clear in Figs. 16, 17(a) and 17(b), the optimal values $r_{op}$, $\omega_{aop}$ and $\mu_{op}$, decreases with the increase of the damping of the primary mass.

Fig. 18 shows that the damping $\xi_1$ and $\xi_2$ are proportional and the increase in $\xi_1$ requires an increase in FTMD viscous damping $\xi_2$.




(a) Optimal tuned frequency




(b) Optimal friction coefficient

Fig. 17 Optimal parameters system with viscous damping as a function of desired displacement $Y_a$

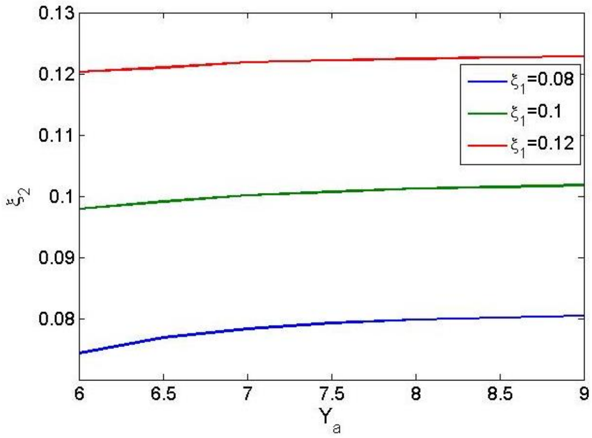



Fig. 18 Damping rate of the FTMD as a function of desired displacement $Y_a$

## 5. Conclusions

In this paper, the response of a single degree of freedom system under a harmonic excitation to which an FTMD is attached, has been determined. In the sticking phase, the two masses stick together and form a single-degree-of-freedom system and the response is set by solving an equation of motion of a single-degree of freedom system. In the slip phase, the equivalent damping method represents a good approximation method for settling the main mass and the FTMD responses.

Based on the method of Ricciardelli and Vickery, a formulation of the optimal parameters is established. For a system without viscous damping, the results found coincide with those of Ricciardelli and Vickery. The optimal coefficient of friction and the tuned frequency are determined with closed-form expressions in this case. For a system with viscous damping, the study illustrates the sensitivity of the optimal design of FTMD to the variation of the viscous damping of the main mass.

The inverse problem of finding the TMD parameters given the magnitude of the force and the maximum acceptable displacement of the primary system is also considered, this study presents for a better design for industrial applications of the FTMD.

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