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Simultaneous analysis, design and optimization of trusses via force method

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Abstract. In this paper, the Colliding Bodies Optimization (CBO), Enhanced Colliding Bodies Optimization (ECBO) and Vibrating Particles System (VPS) algorithms and the force method are used for the simultaneous analysis and design of truss structures. The presented technique is applied to the design and analysis of some planer and spatial trusses. An efficient method is introduced using the CBO, ECBO and VPS to design trusses having members of prescribed stress ratios. Finally, the minimum weight design of truss structures is formulated using the CBO, ECBO and VPS algorithms and applied to some benchmark problems from literature. These problems have been designed by using displacement method as analyzer, and here these are solved for the first time using the force method. The accuracy and efficiency of the presented method is examined by comparing the resulting design parameters and structural weight with those of other existing methods.

Keywords: force method; metaheuristic algorithms; analysis; design; optimization; truss; stress ratio; energy

1. Introduction

In advanced engineering problems of a multi-physics nature, developing methods of higher computation efficiency is an important issue. The analysis of structures having larger numbers of members requires a large memory size and high computation time. This costly computation has to be repeated many times (typically over 5,000 times) since the cross-section size of the members is not determined in the early stages of designing these structures. Thus, reducing the size of structural matrices and eliminating undue repetitions in the analysis and design of structures can lead to a considerable reduction in computational efficiency, Kaveh (2006, 2013). In this paper, this goal is achieved utilizing meta-heuristics algorithms that minimize the energy function indirectly. Besides, the design procedure and minimizing the weight of the structure is added to the analysis procedure.

One of the most reliable metaheuristic methods recently developed is the vibrating particles system (VPS). The VPS is a population-based optimization algorithm which is inspired by free vibration of single degree of freedom systems with viscous damping and introduced by Kaveh and Ilchi Ghazaan (2016). In this method, the solution candidates are considered as agents that gradually approach to their equilibrium positions. In order to have a proper balance between exploration and exploitation, equilibrium positions are attained from current population and historically best position.

Meta-heuristic algorithms are shown to be powerful optimization techniques. These algorithms exhibit global search capabilities and are suitable for complex, nonlinear and non-convex search spaces, especially when near-global optimum solutions are sought after using limited computational effort. Some of the examples of meta-heuristic algorithms include Genetic Algorithms (GA) (Holland 1975), Particle Swarm Optimization (PSO) (Eberhart and Kennedy 1995), Ant Colony Optimization (ACO) (Dorigo *et al.* 1996), Harmony Search (HS) (Geem *et al.* 2001), Big Bang-Big Crunch (BB-BC) (Erol and Eksin 2006), Firefly Algorithm (FA) (Yang 2010a), Charged System Search (CSS) (Kaveh and Talatahari 2010), Bat Algorithm (BA) (Yang 2010b, Teaching Learning Based Optimization (TLBO) (Rao *et al*. 2012), Colliding Bodies Optimization (CBO) (Kaveh and Mahdavi 2014), Water Cycle, Mine Blast and Improved Mine Blast algorithms (WC-MB-IMB) (Sadollah *et al.* 2015), Search Group Algorithm (SGA) (Gonçalves *et al.* 2015), the Ant Lion Optimizer (ALO) (Mirjalali 2015), the whale optimization (Mirjalili and Lewis 2015), Grasshopper optimization algorithm (Saremi *et al*. 2017) and Vibrating Particles System (VPS) (Kaveh and Ilchi Ghazaan 2017a, b). Metaheuristic algorithms have found many applications, some of which can be found in the work of Kaveh *et al.* (2015), Kaveh and Zolghadr (2014), Gholizadeh and Poorhoseini (2015), and Kaveh and Sharafi (2011). Other combinatorial optimization can be found in the work of Sharafi *et al.* (2004a, 2014b, 2017).

Minimum weight structural design can be achieved using the minimization of energy for analysis, Kaveh and Rahami (2006), instead of the direct solution of classic equations. This not only results in avoiding repetitive computations in the design and analysis, but also will not require the computation of the inverse of the large matrices. Therefore, one needs to formulate the equations based on the minimum energy principle, and use them in an efficient optimization algorithm. In this paper, the metaheuristic algorithms and the force method are combined using CBO,

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ECBO, and VPS. This combination has provided a suitable means for this purpose, since the former provides the optimization algorithm and the latter can be utilized for derivation of the energy equations.

The rest of this paper is organized as follows: in section 2 energy formulation based on the force method is presented and the CBO, ECBO and VPS algorithms are applied to the analysis procedure. In section 3, using these metaheuristic algorithms and prescribed stress ratios, structures are analyzed and designed, and finally, in section 4, weight minimization is performed by imposing the analysis procedure as a constraint in the CBO, ECBO and VPS methods and four structural benchmarks, previously solved using direct displacement method, are studied.

2. Analysis by force method and metaheuristic algorithms

Here, the main aim is to formulate the energy function of a structure and minimize the latter using the metaheuristic algorithms, while satisfying all compatibility conditions. The formulation is based on the minimum complementary work principle, Kaveh (2006).

Suppose $p=\{p_1, p_2,..., p_n\}^t$ is the vector of nodal forces, $q=\{q_1,q_2,...,q_r\}^t$ contains r redundant forces, and $R=\{s_1,s_2,...,s_m\}^t$ comprises of the internal forces of the members, Kaveh (2006), Kaveh and Rahami (2006). From equilibrium

$$\{R\}=[B_0]\{p\}+[B_1]\{q\} \tag{1}$$

And

$$U_c=\frac{1}{2}\{R\}^t[F_m]\{R\} \tag{2}$$

where $[F_m]$ is the unassembled flexibility matrix of the structure. Now $\{q\}$ should be calculated such that U_c becomes minimum. Substituting $\{R\}$ from Eq. (1) in Eq. (2) leads to

$$U_c=\frac{1}{2}[p\ \ q]^t[H]\begin{bmatrix}p\\q\end{bmatrix} \quad \text{where} \quad [H]=[B_0\ \ B_1]^t[F_m][B_0\ \ B_1] \tag{3}$$

Decomposing the matrix $[H]$ into four submatrices $[H_{pp}]$, $[H_{pq}]$, $[H_{qp}]$ and $[H_{qq}]$, U_c is obtained as

$$U_c=\frac{1}{2}\left(\{p\}^t\left[H_{pp}\right]\{p\}+\{p\}^t\left[H_{pq}\right]\{q\}+\{q\}^t\left[H_{qp}\right]\{p\}+\{q\}^t\left[H_{qq}\right]\{q\}\right) \tag{4}$$

In the classical method, the derivative of U_c with respect to $\{q\}$ is found and equated to zero, leading to

$$\{q\}=-\left[H_{qq}\right]^{-1}\left[H_{qp}\right]\{p\} \tag{5}$$

Since $[H]$ is symmetric, therefore $[H_{qp}]^t=[H_{pq}]$.

In the present method, formation of the inverse of $[H_{qq}]$ is not required. Instead, U_c from Eq. (3) is minimized by metaheuristic algorithms. The first term of U_c is constant and the second and third terms are equal. It can easily be shown that the third and fourth terms of U_c are symmetric. Therefore,

$$F_U=\{q\}^t\left[H_{qp}\right]\{p\} \tag{6}$$

should be minimized, Kaveh and Rahami (2006).

In order to minimize F_u, the CBO, ECBO and VPS algorithms are used which are based mainly on the algorithms by Kaveh and Mahdavi (2014), Kaveh and Ilchi Ghazaan (2014) and Kaveh and Ilchi Ghazaan (2016), respectively. One example is presented to state the accuracy of the analysis by the present approach.

A simple truss as shown in Fig. 1 is considered. This structure has four degrees of statical indeterminacy. Thus, F_u should be formed in terms of four unknowns.

The exact calculation of U_c, the obtained values of U_c using the present approach (F_u is added to the first term) and $\{q\}$ are presented in Table 1.

The population size for this example in three algorithms is selected as 20. Fig. 2 shows the variation of the U_c versus the number of iterations.

Table 1 Comparison of the values of U_c

	CBO	ECBO	VPS	Exact
q1	16.8597	16.8495	16.8597	16.8597
q2	9.5789	9.5208	9.5789	9.5789
q3	2.2753	2.2220	2.2753	2.2753
q4	-6.4376	-6.6395	-6.4376	-6.4376
Uc	1230.212488	1230.214436	1230.212488	1230.212488

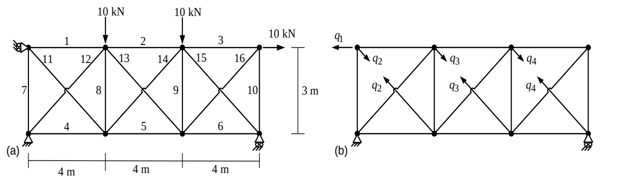


Fig. 1 A simple truss and the selected basic structure: (a) a planar truss; and (b) the selected basic structure

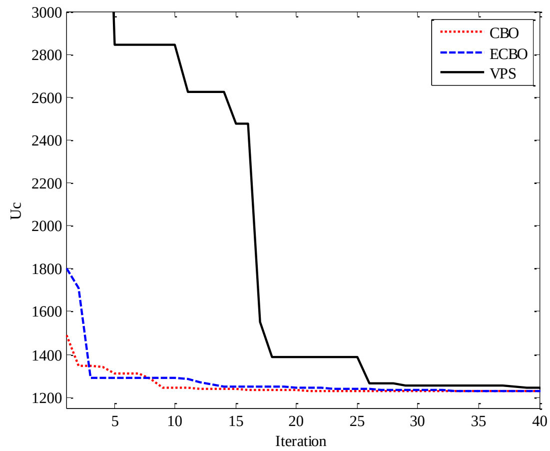


Fig. 2 Variation of U_c versus the iteration in the analysis of the 16-member truss

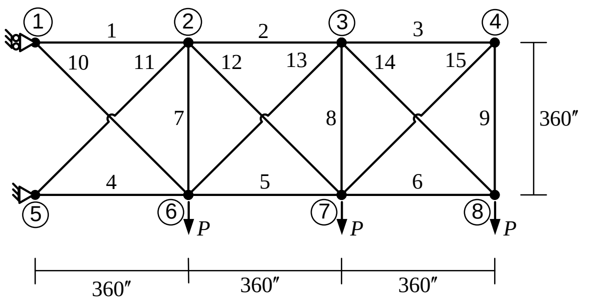


Fig. 3 Schematic of a 15-bar planar truss example

3. Structural design using force method and metaheuristic algorithms

In this section, design and optimization processes are added to the analysis presented in the previous section. Formulation of the objective function in the simultaneous analysis and design of an optimal structure is carried out by two following approaches:

1) Fully stress design.

2) Minimizing the weight of the structure.

3.1 Fully stress design for statically indeterminate structures

In this part, the metaheuristic algorithms and force method are applied to an Optimality Criteria Method (OCM), developed by Kirsch (1981), namely, Fully Stressed Design (FSD). FSD leads to an exact optimal weight for statically determinate structures under a single loading condition. In the FSD, all the members are supposed to be subjected to their maximal allowable stresses, Kaveh and Rahami (2006). Achieving such a design for an indeterminate structure with fixed geometry is not always possible. Even by changing the geometry, a FSD may not be achieved. Here, a formulation presented by Kaveh and Rahami (2006) is used for indirect analysis in the process of optimization. This formulation can be applied to all types of structure, however, for truss type structures the following strain energy is considered

$$U = \sum \frac{P^2 L}{EA} = \sum \frac{\gamma P^2 LA}{\gamma E A^2} = \frac{1}{\gamma E} \sum \sigma_i^2 w_i \qquad (7)$$

It should be noted that for constant E and γ, defined as the modulus of elasticity and mass density of the material, respectively, the minimum weight can be achieved only when the stresses in all the members are identical. In Eq. (7), P is the member axial force, A is the cross-sectional area of the member, L is the member length, σ is the member stress and w is the member weight. Therefore, in Eq. (7), the term corresponding to the stresses, i.e., σ_i^2, may be moved out of the summation. On the other hand, in the design procedure, one can consider the fully stress constraint instead of minimum weight. This is because the minimum weight corresponds to a structure whose members are all subjected to their maximum allowable stresses.

As an example, consider the planar truss shown in Fig. 3. The design and member size constraints are provided in Table 2. These constraints lead to a design for which not all the members are fully stressed.

Table 2 Design data for the 15-bar planar truss

Loading			
Node	Px: kips (kN)	Py: kips (kN)	Pz: kips (kN)
6	0	0	-100 (-444.8)
7	0	0	-100 (-444.8)
8	0	0	-100 (-444.8)
Design variable			
Variables:	$q_1; q_2; q_3$ (and $A_1; A_2; ...; A_{15}$ in case 3)		
Material property and constraint data			
Young's modulus:	$E = 1e7$ psi $= 6.895e7$ MPa		
Density of the material:	$\rho = 0.1$ lb/in^3 $= 0.00277$ kg/cm^3		
For all members:	$A_i \geq 0.1$ in^2; $i = 1, 2, ..., 15$		
Stress constraints			
(a) FSD			
Case 1:	$\lvert\sigma_i\rvert \leq 25$ ksi (172.375 MPa); $i = 1, ..., 15$		
Case 2:	$\lvert\sigma_i\rvert \leq 25$ ksi ; $i = 1, ..., 11, 13, 15$ and $\lvert\sigma_j\rvert \leq 50$ ksi (344.75 MPa); j = 12,14		
(b) weight minimization			
Case 3:	$\lvert\sigma_i\rvert \leq 25$ ksi ; $i = 1, ..., 11, 13, 15$ and $\lvert\sigma_j\rvert \leq 50$ ksi (344.75 MPa); j = 12,14		

Table 3 Results for the 15-bar planar truss (Cases 1-3)

Area (in^2)	Case 1 (FSD)			Case 2 (FSD)			Case 3 (weight minimization)		
	CBO	ECBO	VPS	CBO	ECBO	VPS	CBO	ECBO	VPS
A1	16.0621	16.1222	17.0544	12.1061	12.1061	12.2788	15.752	16.7175	16.1198
A2	7.9379	7.8632	7.1367	11.8939	11.8939	11.744	8.2512	7.2927	7.8883
A3	0.1	0.1383	0.5853	0.2478	0.4958	0.3799	0.2621	0.8196	0.2279
A4	19.9379	19.8778	18.9456	23.8939	23.8939	23.7212	20.2482	19.2981	19.8802
A5	8.0621	8.1368	8.8633	4.1061	4.1061	4.256	7.7504	8.7109	8.1195
A6	3.9379	3.8617	3.4147	3.7522	3.5042	3.6201	3.7423	3.187	3.7748
A7	0.1	0.1	0.191	0.1	0.1	0.1	0.1	0.1004	0.1041
A8	0.1	0.1	0.278	4.1417	4.3898	4.124	0.5119	0.1046	0.1058
A9	0.1	0.1383	0.5853	0.2478	0.4958	0.3799	0.2607	0.8149	0.2258
A10	11.2258	11.1409	9.8226	16.8206	16.8206	16.5763	11.6647	10.3088	11.1496
A11	5.7447	5.8297	7.148	0.15	0.15	0.3942	5.3858	6.6685	5.8421
A12	5.7447	5.8504	6.8778	0.1	0.1	0.181	3.4995	4.4403	3.8804
A13	5.569	5.4634	4.4359	11.1637	11.1637	10.9517	6.0121	4.6524	5.4878
A14	5.569	5.4613	4.8291	2.6532	2.4778	2.5598	2.6473	2.2592	2.6811
A15	0.1	0.1955	0.8278	0.3505	0.7012	0.5373	0.3687	1.1559	0.3192
Weight (lb)	3756.8	3756.2	3782	3768	3794.8	3770.2	3553.5	3554.8	3526.8

In this case, the internal forces in members 10, 12 and 14 are taken as redundants, forming the initial population of the considered metaheuristic algorithms. Twenty agents are selected in these methods. The objective function is the complementary energy as introduced before. Table 3 contains the results.

3.2 Minimum weight

Now if we want to perform a fully stressed design of the structure with the least possible weight, the problem

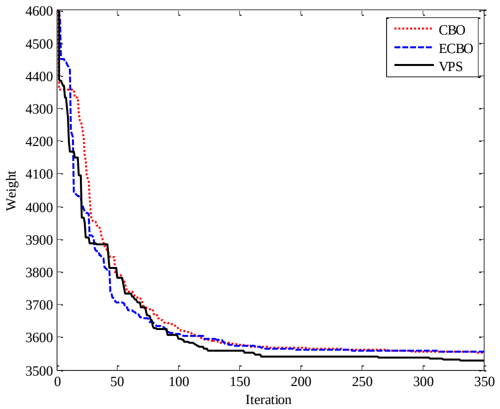


Fig. 4 Time histories of the CBO, ECBO and VPS algorithms for Case 3 (weight minimization)

becomes more involved, since different cases may arise with the condition of FS design, and one naturally wants the one with smallest weight. Choosing a function in the form of $f=W+\alpha U$ does not help since the penalty functions are commonly selected as

$$f = A + \alpha B \tag{8}$$

for which ultimately f converges to A, and B approaches to zero. Therefore, α is often selected as a big number. The main difficulty arises when for α, the minimum value of f does not correspond to the minimum U_c. In this case, W is minimum while the corresponding U_c is not minimum, i.e., the structure is not analysed yet. This leads to incorrect solution. Small α will not guarantee the minimality of U_c and a big α does not lead to minimum W. Therefore, a new formulation becomes necessary.

In this formulation, the second term of Eq. (8) is altered such that its minimum value becomes zero. Then one can make formulation similar to the common penalty function. For this purpose, the idea of structure being in equilibrium and compatible state when the sum of complementary energy and strain energy is zero is employed. Therefore, instead of the complementary energy, the sum of the complementary energy and the strain energy is used as the analysis criteria and is incorporated in the CBO, ECBO and VPS as a constraint.

U_c is previously introduced. If B_0 and B_1 matrices corresponding to all the DOFs are constructed, then the displacements can be calculated as (Kaveh and Rahami 2006)

$$\{d\} = \left[B_0^t\right]\left[F_m\right]\left(\left[B_0\right]\{p\} + \left[B_1\right]\{q\}\right) \tag{9}$$

and

$$U = \frac{1}{2}\{d\}^t\left[K\right]\{d\} - \{d\}^t\{F\} \tag{10}$$

where $[K]$ is the stiffness matrix and $\{F\}$ is the nodal force vector. For equilibrium, U is negative and $U+U$ is equal to zero. In metaheuristic algorithms the objective function f is selected as $f=W(1+\alpha(U+U_c)^2)$, where the first term corresponds to the optimization and the second term belongs to the analysis. Now α can be selected as a big number. Obviously, f will ultimately approach to W, since $(U+U_c)^2$ will become zero. This formulation is used for the 15-bar truss example for Case 3. The results are shown in Table 3. Twenty particles are utilized in the algorithms. Similar to the other cases, CBO, ECBO and VPS have shown a good performance. Fig. 4 shows the comparison of convergence histories for the CBO, ECBO and VPS methods in weight minimization case.

For large-scale structures, since no solution or inverse for large flexibility or stiffness matrices is needed, the present method is more efficient. The selection of the energy function considers the minimization of energy for analysis, in place of using a direct analysis.

4. An alternative formulation of optimal design

Apart from the above approach, one can perform optimal design using different formulation, Kaveh and Rahami (2006). In this method, one does not need U and optimization can be performed employing only U_c. Here, U_c can be written in the form given in Eq. (4). Differentiating this equation leads to

$$\frac{\partial U_c}{\partial q} = 0 \;\Rightarrow \left[H_{qp}\right]\{p\} + \left[H_{qq}\right]\{q\} = \{0\} \tag{11}$$

Thus $\{q\}$ should be selected such that Eq. (11) holds (in place of minimizing Eq. (6)). The left-hand side of this equation is a zero vector, and should be changed to a scalar. The best is to find its norm. If this norm is zero, all the entries should be zero. In the previous method, the scalar was taken as the complementary energy and to make I zero we had to combine with U. Here we use the equilibrium itself. In this case one can write

$$F(q,A) = W(A)\left(1 + \alpha \,\mathrm{norm}\left(\left[H_{qp}\right]\{p\} + \left[H_{qq}\right]\{q\}\right)\right) \tag{12}$$

Here, the input is $\{q\}$. Having $\{q\}$ from Eq. (12), the magnitude of F can be calculated and its minimum for a large value of α corresponds to minimum W. If a structure contains other constraints, then these should be normalized and added to the above function with a penalty coefficient. Therefore, the final formulation of the problem for the two cases of discrete and continuous cross sections will be as follows

s.t

$$\begin{aligned} &\text{find} \quad q, A;\; A \in \{S_d \text{ or } S_c\} \\ &\text{Min} \quad F(q,A) = \sum_{i=1}^{ne} A_i L_i \rho_i \left(1 + \alpha\left(\mathrm{norm}\left(\left[H_{qp}\right]\{p\} + \left[H_{qq}\right]\{q\}\right) + \sum_{m=1}^{nc} \max\left(0, g_m(A)\right)\right)\right) \end{aligned} \tag{13}$$

where S_d and S_c are the discrete and continuous cross sections, respectively. Here, $g_m(A)$ correspond to violations of the constraints, which include stress constraints, displacement constraints and buckling constraints. Their magnitudes can be written in the form of the absolute value of the existing value to permissible value minus one.

In the following, four examples are presented and the results are compared to those of the existing literature.

Table 4 Design data for the 15-bar planar truss

Material property and constraint data
Young's modulus: $E = 1\text{e}7 \text{ psi} = 6.895\text{e}7 \text{ MPa}$
Density of the material: $\rho = 0.1 \text{ lb/in}^3 = 0.00277 \text{ kg/cm}^3$
Stress constraints
$\lvert\sigma_i\rvert \le 25 \text{ ksi } (172.375 \text{ MPa}); i = 1,...,15$
Nodal displacement constraint in all directions of the co-ordinate system
$\lvert\Delta_i\rvert \le 3 \text{ in } (5.08 \text{ cm}) \; ; \; i = 1,...,8$
List of the available profiles
$A_i \ge 0.1 \text{ in}^2 (0.6452 \text{ cm}^2) \; ; \; i = 1,...,15$

Table 5 Comparison of optimal designs for the 15-bar planar truss (continuous)

Area (in^2)	CBO	ECBO	VPS
A1	63.4437	61.1288	65.9963
A2	40.4207	45.2588	42.7157
A3	0.1125	0.1	0.111
A4	81.5059	83.0437	84.3088
A5	33.6656	32.602	31.8039
A6	22.146	22.6973	22.5589
A7	0.1	0.1	0.1004
A8	0.1028	0.1	0.1052
A9	0.1	0.1142	0.1385
A10	42.6768	43.5579	38.8158
A11	11.4281	12.0779	11.0342
A12	11.7017	11.3511	11.9605
A13	30.0509	27.6849	31.5457
A14	32.6623	31.5182	30.9249
A15	0.1	0.1579	0.1062
Weight (lb)	15246	15258	15255

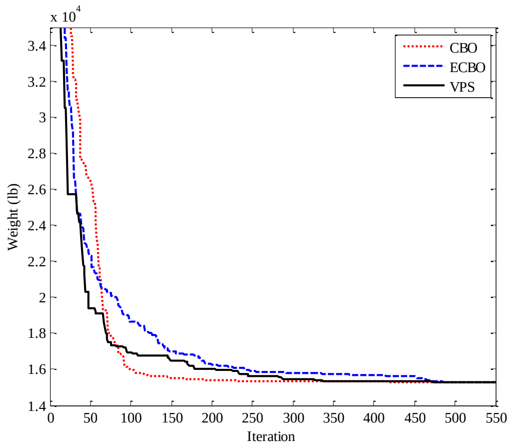
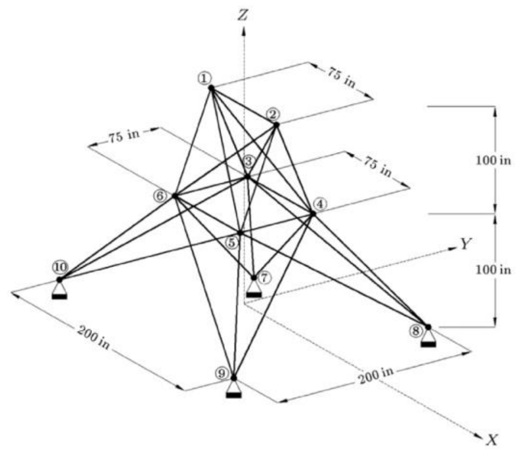


Fig. 5 Convergence curves obtained for the 15-bar planar truss (continuous)

4.1 Example 1: A 15-bar planar truss

Optimal design of a 15-bar planar truss, shown in Fig. 3, is considered. Table 4 contains the data for design of this



Group number	Members
1	1-2
2	1-4,2-3,1-5,2-6
3	2-5,2-4,1-3,1-6
4	3-6,4-5
5	3-4,5-6
6	3-10,6-7,4-9,5-8
7	3-8,4-7,6-9,5-10
8	3-7,4-8,5-9,6-10

Fig. 6 Schematic of a 25-bar spatial truss and grouping of the members

Table 6 Data for design of the 25-bar spatial truss

Design variables
Size variables: $A_1; A_2; A_3; A_4; A_5; A_6; A_7; A_8; q_1; q_2; q_3; q_4; q_5; q_6; q_7$
Material property and constraint data
Young's modulus: $E = 1\text{e}7 \text{ psi} = 6.895\text{e}7 \text{ MPa}$
Density of the material: $\rho = 0.1 \text{ lb/in}^3 = 0.00277 \text{ kg/cm}^3$
Stress constraints
$\lvert\sigma_i\rvert \le 40 \text{ ksi } (275.8 \text{ MPa}); i = 1,...,25$
Displacement constraint in the directions of X and Y in the co-ordinate System
$\lvert\Delta_i\rvert \le 0.35 \text{ in } (0.8890 \text{ cm}) \; ; \; i = 1,2$
List of the available profiles
Case 1: (Discrete sections)
$A_i = \{0.1, 0.5 \times I \, (I = 1,2,...,76), 39.81, 40\} \text{ in}^2$
$A_i = \{0.6452, 3.2258 \times I \, (I = 1,2,...,76), 256.8382, 258.0640\} \text{ cm}^2$
Case 2: (Continuous sections)
$A_i \ge 0.1 \text{ in}^2 (0.6452 \text{ cm}^2)$
Loading data

Node	Px: kips (kN)	Py: kips (kN)	Pz: kips (kN)
1	1(4.448)	-10 (-44.48)	-10(-44.48)
2	0	-10 (-44.48)	-10 (-44.48)
3	0.5(2.224)	0	0
6	0.6(2.6688)	0	0

Table 7 Comparison of optimal designs for the 25-bar spatial truss (Discrete)

Area (in^2)	CBO	ECBO	VPS	GA, Kaveh and Rahami (2006)	CSS, Kaveh and Ahmadi (2013)
A1	0.1	0.1	0.1	0.1	0.1
A2	0.5	0.5	0.5	0.5	0.5
A3	3	3	3	3	3
A4	0.1	0.1	0.1	0.1	0.1
A5	2	2	2	2	2
A6	1	1	1	1	1
A7	0.1	0.1	0.1	0.1	0.1
A8	4	4	4	4	4
Weight (lb)	479.755	479.755	479.755	479.755	479.75

Table 8 Comparison of optimal designs for the 25-bar spatial truss (Continuous)

6	CBO	ECBO	VPS	GA, Kaveh and Rahami (2006)	CSS, Kaveh and Ahmadi (2013)
A1	0.1	0.1	0.1169	0.1	0.1
A2	0.1029	0.1	0.1038	0.1	0.1001
A3	3.5539	3.5683	3.6151	3.7598	3.7449
A4	0.1056	0.1	0.1012	0.1	0.1005
A5	1.9539	1.9592	1.9546	1.8552	1.874
A6	0.7876	0.7893	0.7857	0.7755	0.7836
A7	0.1499	0.1461	0.1325	0.1408	0.1425
A8	3.9437	3.9354	3.9202	3.846	3.8414
Weight (lb)	467.3038	467.164	467.3819	467.6293	467.7457

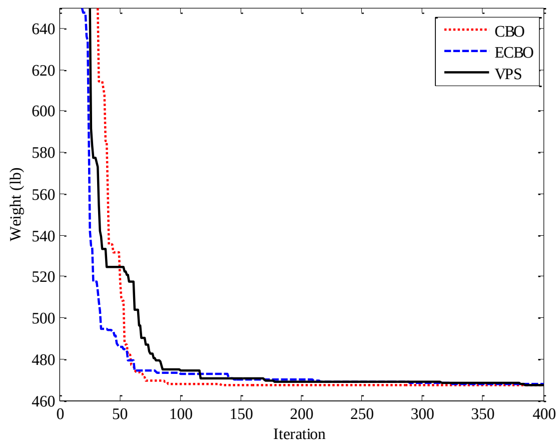


Fig. 7 Convergence curves obtained for the 25-bar spatial truss (continuous)

truss. Design variables are *A* and *q*. The results shown in Table 5 are obtained by the minimization of Eq. (13). Fig. 5 shows the comparison of convergence histories for the CBO, ECBO and VPS methods.

4.2 Example 2: A 25-bar spatial truss

The schematic of a spatial truss and grouping of its members are shown in Fig. 6. Table 6 contains the data for design of this truss. The optimal values of the eight size variables for two discrete and continuous cases and their comparison with other existing results are shown in Table 7 and Table 8, respectively. Redundant forces are selected as the internal forces in members: 12, 13, 21, 22, 23, 24 and 25. Fig. 7 shows the comparison of convergence histories for the CBO, ECBO and VPS methods in continuous case. In this example, the ECBO algorithm obtained the best weight (467.164 lb) in continuous case.

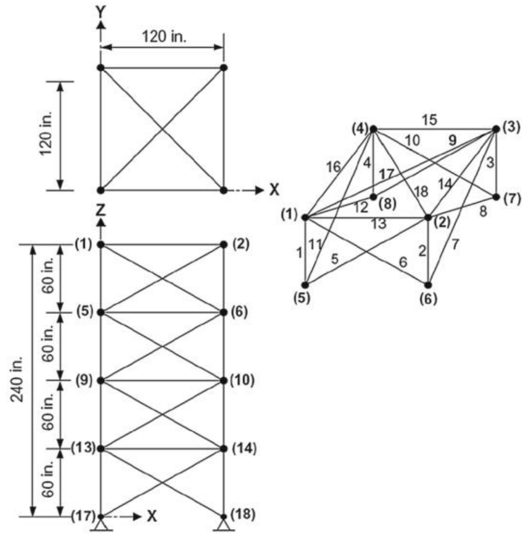


Fig. 8 Schematic of a 72-bar spatial truss

Table 9 Design data for the 72-bar spatial truss

Design variables	
Size variables:	$A_1; A_2; \ldots; A_{16}; q_1; q_2; \ldots; q_{24}$
Material property and constraint data	
Young's modulus:	$E = 1e7 \text{ psi} = 6.895e7 \text{ MPa}$
Density of the material:	$\rho = 0.1 \text{ lb/in}^3 = 0.00277 \text{ kg/cm}^3$
Stress constraints	
$\lvert\sigma_i\rvert \le 25 \text{ ksi } (172.37 \text{ MPa}); i = 1, \ldots, 72$	
Displacement constraint in the directions of X and Y in the co-ordinate System	
$\lvert\Delta_i\rvert \le 0.25 \text{ in } (0.635 \text{ cm}) ;\ i = 1, 2, 3, 4$	
List of the available profiles	
$A_i \ge 0.1 \text{ in}^2 \left(0.6452 \text{ cm}^2\right)$	

Table 10 Loading conditions for the 72-bar spatial truss

Nodes	Load Case 1			Load Case 2		
	Px(kips)	Py(kips)	Pz(kips)	Px(kips)	Py(kips)	Pz(kips)
1	5	5	-5	0	0	-5
2	0	0	0	0	0	-5
3	0	0	0	0	0	-5
4	0	0	0	0	0	-5

4.3 Example 3: A 72-bar spatial truss

The schematic of a 72-bar spatial truss is shown in Fig. 8 as the third design example. The necessary data for the

Table 11 Comparison of optimal designs for the 72-bar spatial truss

Area(in^2)	CBO	ECBO	VPS	ICA, Kaveh and Talatahari (2010)
A1	1.9	2.0364	1.9501	1.99
A2	0.5125	0.51	0.4888	0.442
A3	0.1	0.1	0.1001	0.111
A4	0.1	0.1	0.1168	0.141
A5	1.2155	1.4082	1.3009	1.228
A6	0.5303	0.505	0.5048	0.602
A7	0.1	0.1003	0.1113	0.111
A8	0.1054	0.1	0.115	0.141
A9	0.5168	0.5404	0.5015	0.563
A10	0.5063	0.4598	0.5184	0.563
A11	0.1	0.1	0.1112	0.111
A12	0.1095	0.1	0.1034	0.111
A13	0.169	0.1544	0.1552	0.196
A14	0.5567	0.5369	0.5545	0.563
A15	0.4301	0.4365	0.4203	0.307
A16	0.5561	0.6062	0.5854	0.602
Weight(lb)	381.8569	381.3952	382.4935	392.8483

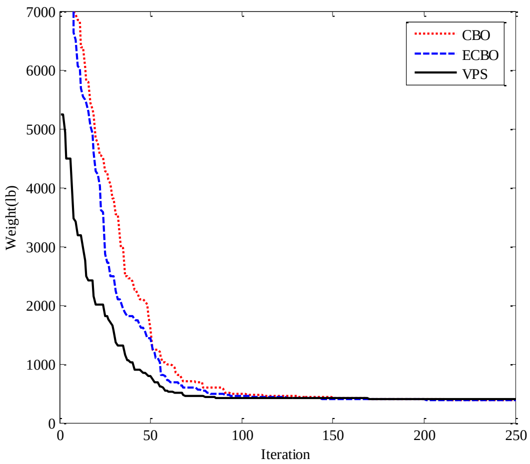


Fig. 9 Convergence curves obtained for the 72-bar spatial truss

design and the constraints are provided in Table 9. The elements are divided into sixteen groups using symmetry as follow

(1) $A_1 – A_4$, (2) $A_5 – A_{12}$, (3) $A_{13} – A_{16}$, (4) $A_{17} – A_{18}$, (5) $A_{19} – A_{22}$, (6) $A_{20} – A_{30}$, (7) $A_{31} – A_{34}$, (8) $A_{35} – A_{36}$, (9) $A_{37} – A_{40}$, (10) $A_{41} – A_{48}$, (11) $A_{49} – A_{52}$, (12) $A_{53} – A_{54}$, (13) $A_{55} – A_{58}$, (14) $A_{59} – A_{62}$, (15) $A_{63} – A_{70}$, (16) $A_{71} – A_{72}$.

The structure is subjected to the two load cases given in Table 10. Table 11 compares the results obtained by the CBO, ECBO and VPS algorithms and other optimization methods. The corresponding convergence curves are compared in Fig. 9. In this example, the CBO, ECBO and VPS algorithms as the best weight obtained 381.8569 lb, 381.3952 lb and 382.4935 lb, respectively. For these algorithms the maximum nodal displacements were 0.2499 in, 0.2496 in and 0.2495 in, respectively.

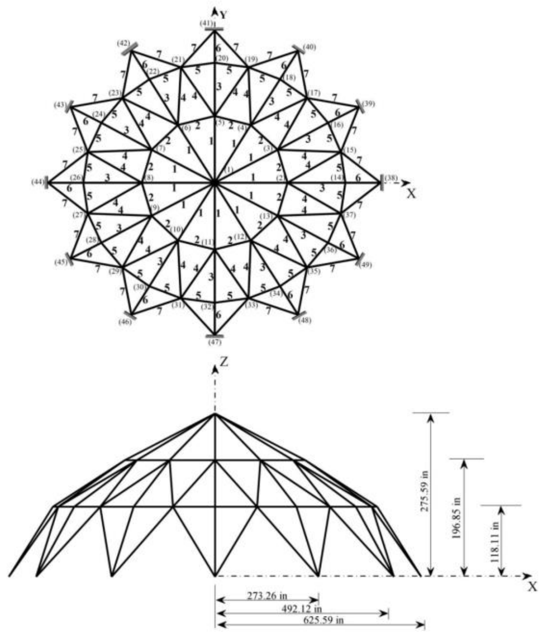


Fig. 10 Schematic of a 120-bar dome truss

Table 12 Design data for the 120-bar spatial truss

Design variables
Size variables: $A_1; A_2; A_3; A_4; A_5; A_6; A_7; q_1; q_2; q_3; q_4; q_5; q_6; q_7; q_8; q_9$
Material property and constraint data
Young's modulus: $E = 30450$ ksi $= 210000$ MPa Density of the material: $\rho = 0.288$ lb/in$^3 = 7971.810$ kg/m^3 For all members: $0.775 \le A_i \le 20$ in^2 $i = 1,...,120$
Constraints
$\lambda_i = \frac{L_i}{r}$ $r = \sqrt{0.4 \times A}$ $C_c = \sqrt{\frac{2\pi^2 E}{F_y}}$
For tensile members $\lambda_i \le 300$ $F_a \le 0.6F_y$ For compressive members $\lambda_i \le 200$ $F_a = \frac{\left[\left(1 - \frac{\lambda_i}{2C_c^2}\right)F_y\right]}{\left(\frac{5}{3} + \frac{3\lambda_i}{8C_c} - \frac{\lambda_i^3}{8C_c^3}\right)}$ for $\lambda_i \le C_c$ $F_a = \frac{12\pi^2 E}{23\lambda_i^2}$ for $\lambda_i > C_c$ $\lvert\sigma_i\rvert \le 58$ ksi $(400$ MPa$); i = 1,...,120$
Displacement constraint in the directions of X, Y and Z in all unsupported nodes
$\lvert\Delta_i\rvert \le 0.1969$ in $(0.500126$ cm$)$

4.4 Example 4: A 120-bar dome truss

A 120-bar dome structure is considered as the fourth design example. Geometry and member grouping structures are shown in Fig. 10. This structure has nine degrees of statical indeterminacy. Redundant forces are considered as the reactions at nodes 38, 42 and 46. The necessary data for

Table 13 Comparison of optimal designs for the 120-bar spatial truss

Area (in^2)	CBO	ECBO	VPS	CSS, Kaveh and Ahmadi (2013)
A1	2.2464	2.246417	2.246417	3.0129
A2	15.5525	16.21654	15.77258	14.7596
A3	5.6267	5.310306	5.393888	5.1118
A4	2.4648	2.454844	2.466867	3.1304
A5	9.0497	8.946653	8.946029	8.543
A6	3.5581	3.480555	3.720805	3.2026
A7	1.9181	1.978158	1.958689	2.4917
Weight (lb)	31891	31900	31888	33241.99

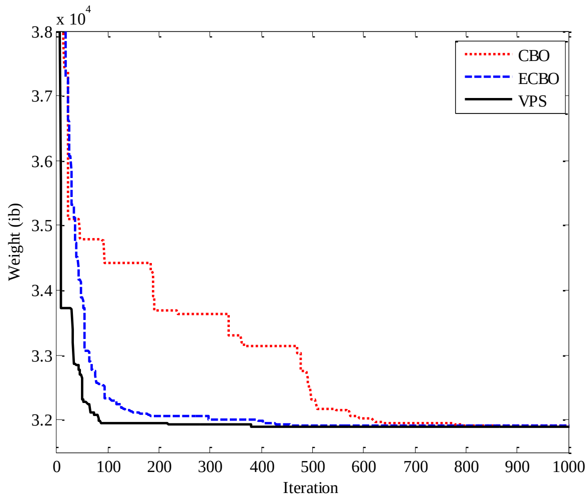


Fig. 11 Convergence curves obtained for the 120-bar spatial truss

the design and the constraints are provided in Table 12. Optimal design comparison for the 120-bar dome truss is attained in Table 13. It can be seen that the lightest design (31887.78 lb) are found by the VPS. Fig. 11 compares the convergence curves of the best results obtained by CBO, ECBO and VPS methods. The loading condition is considered as follow:

1. Vertical load at node 1 equal to −13.49 kips (−60 kN).
2. Vertical loads at nodes 2 through 14 equal to −6.744 kips (−30 kN).
3. Vertical loads at the rest of the nodes equal to −2.248 kips (−10 kN).

5. Conclusions

In this article, an efficient technique is presented for analysis, design and optimization of structures by CBO, ECBO and VPS algorithms to avoid finding the inverse of the large matrices, especially in large-scale structures. These metaheuristic algorithms and force method are applied simultaneously for the analysis and design of various kinds of large-scale structures. The results obtained for structural examples show the accuracy of these methods in handling the simultaneous analysis, design and optimization of this type of structure. The CBO, ECBO and VPS algorithms have a good performance in comparison to other optimization methods. The results of these examples illustrate the capability of the CBO, ECBO and VPS algorithms and force method when simultaneously utilized for analysis, design and optimization of structures.

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