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Free vibration analysis of chiral double-walled carbon nanotube embedded in an elastic medium using non-local elasticity theory and Euler Bernoulli beam model

Ahmed Dihaj[1], Mohamed Zidour*[3,4], Mustapha Meradjah[1], Kaddour Rakrak[2,3], Houari Heireche[2] and Awda Chemi[2]

[1]*Laboratoire des Matériaux et Hydrologie, Université de Sidi Bel Abbés, BP 89 Cité Ben M'hidi, 22000 Sidi Bel Abbés, Algerie*
[2]*Laboratoire de Modélisation et Simulation Multi-échelle, Université de Sidi Bel Abbés, Algerie*
[3]*Université Ibn Khaldoun, BP 78 Zaaroura, 14000 Tiaret, Algerie*
[4]*Laboratoire de Géomatique et Développement Durable, Université Ibn Khaldoun de Tiaret, Algérie*

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Abstract. The transverse free vibration of chiral double-walled carbon nanotube (DWCNTs) embedded in elastic medium is modeled by the non-local elasticity theory and Euler Bernoulli beam model. The governing equations are derived and the solutions of frequency are obtained. According to this study, the vibrational mode number, the small-scale coefficient, the Winkler parameter and chirality of double-walled carbon nanotube on the frequency ratio (χN) of the (DWCNTs) are studied and discussed. The new features of the vibration behavior of (DWCNTs) embedded in an elastic medium and the present solutions can be used for the static and dynamic analyses of double-walled carbon nanotubes.

Keywords: vibration; nanotube; zigzag; armchair; chirality; non-local

1. Introduction

In recent years, the multi-walled carbon nanotube (MWCNT) and single-walled carbon nanotube (SWCNT) are found by Iijima (1991) and Iijima *et al*. (1993). Carbon nanotubes are cylindrical macromolecules composed of carbon atoms a very great attention from various branches of science. CNTs are well known for their excellent rigidity, higher than that of steel and any other metal. There have been extensive researches on these nanomaterials which indicate that carbon nanotubes (CNTs) have high mechanical properties and good physical properties leading to many potential applications (Baghdadi *et al.* 2015) so they can be used for nanoelectronics, nanodevices and nanocomposites.

Other studies have showed that they have good properties (Bao *et al.* 2004). Bounouara *et al*. (2016) have studied a nonlocal zeroth-order shear deformation theory for free vibration of functionally graded nanoscale plates resting on elastic foundation. Gupta *et al*. (2012a) have studied the Dynamic analysis of fixed-free single-walled carbon nanotube-based bio-sensors because of various viruses.

To predict the responses of nanostructures under different loading conditions by the experiments methods are quite difficult because the CNTs are extremely small. Therefore, there are two major kinds for simulating of (CNTs) which are: molecular dynamics (MD) simulation and continuum mechanics.

The continuum mechanics approach has been successfully used to investigate the mechanical behavior of single-walled carbon nanotubes (SWCNTs) and (MWCNTs). In this context, Gupta *et al.* (2012b) Analysis the vibration of Carbon Nanotube based Mass Sensor using different Boundary Conditions. Several researchers have studied free vibration (Darılmaz *et al.* 2015, Bounouara *et al.* 2015, Belkorissat *et al.* 2015), the nonlinear vibration (Hasan Rahimi Pour *et al*. 2015, Hajnayeb and Khadem 2015), the buckling (Chemi *et al*. 2015, Adda Bedia *et al.* 2015, Benguediab *et al.* 2014, Larbi Chaht *et al.* 2015), the Thermal buckling (Bouazza *et al.* 2016), the static (Talha et al 2010), wave propagation (Moradi-Dastjerdi 2016), nanobeam (Ahouel *et al.* 2016, Al-Basyouni *et al.* 2016, Bouafia *et al.* 2017), nanocomposite (Esawi *et al*. 2007, Hu *et al*. 2015, Lei *et al*. 2013b, Thostensonet al 2003) and thermo-mechanical analysis of (CNTs) (Lei *et al.* 2013). More recently, utilize a higher order Shear and Normal Deformation Theory for vibration response of gradient plates (Gupta *et al.* 2017), (Gupta and Talha 2016). Becheri *et al*. (2016) have studied the Buckling of symmetrically laminated plates using nth-order shear deformation theory with curvature effects. Many studies have been developed to predict the mechanical behavior of functionally graded beam and plates. Bourada *et al.* (2015) used a new simple shear and normal deformations theory for functionally graded beams. Houari *et al.* (2016) utilized a new simple three-unknown sinusoidal shear deformation theory for functionally graded plates. Several researchers studied A new quasi-3D hyperbolic shear deformation

*Corresponding author, Ph.D.
E-mail: zidour.mohamed@yahoo.fr

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theory (Hebali *et al.* 2014, Tounsi *et al.* 2016), higher-order shear deformation theories (Ait Yahia *et al.* 2015, Belabed *et al.* 2014, Bousahla *et al.* 2014, Bessaim *et al.* 2013, Meradjah *et al.* 2015, Mahi *et al.* 2015, Ait Atmane *et al.* 2015, Boukhari *et al.* 2015, Bennoun *et al.* 2016). Fekrar *et al.* (2014), Bellifa *et al.* (2016), analysis the functionally graded plates using the concept the neutral surface position.

Molecular dynamics (MD) simulation is an atomistic method to analyse of different nanostructures (Griebel *et al*. 2004, Han *et al*. 2007). But the problem of this method is time consuming, complex, expensive in computational cost and limited to maximum system sizes of the atoms to be of the order of femto-seconds (10-15 s) (Ranjbartoreh *et al.* 2007). In this study the Young's modulus of (SWCNTs) using (MD) simulation obtained by Bao Wen Xing *et al*. (2004) is used in the formulations, when the Young's moduli of (SWCNTs) are in the range of 929.87±11.5 GPa. These results are in good agreement.

The vast majority of structural theories are derived using the nonlocal elasticity theory, based on the hypothesis that the main point of nonlocal elasticity theory is the stress tensor at a reference point x depends not only on strain tensor of the same point but also on all other points of the domain. Many studies have been developed to analysis of the vibration of CNTs, based on this nonclassical theory Eringen (1972), Lu *et al.* (2006) and Murmu and Adhikari (2010) have analyzed the longitudinal vibration of double nanorod systems using the non-local elasticity. The nonlocal elasticity theory is applied in various types of nanostructures (nano FGM structures, nanotube) such as the buckling (Pradhan et al 2009, Simsek *et al.* 2011, Chikh *et al.* 2017), the static (Foroutan et al 2012, Draiche et al 2016), free vibration (Hadji et al 2014, Jahangiri et al 2015, Attia *et al.* 2015). Thermomechanical bending (Bouderba *et al.* 2013, Tounsi *et al.* 2013, Hamidi *et al.* 2015). Thermal stability of functionally graded plates and sandwich plates using a simple shear deformation theory have used by Bouderba *et al.* (2016) and Bousahla *et al.* (2016). Bending analysis of FGM plates under hygro-thermo-mechanical loading using four variable refined plates theory have been used by Zidi *et al.* (2014) and Beldjelili *et al.* (2016).

In this paper, the effects of chirality of double-walled carbon nanotubes (DWCNTs) embedded in an elastic medium on the wave propagation will be investigated by using the non-local Euler Bernoulli elastic beam model.

The influences of the vibrational mode number, elastic medium and aspect ratio on the frequency of the (DWCNTs) are studied and discussed. It is expected that the present analysis will be useful to materials scientists and engineers for the design of CNTs and their composites.

2. Single and double-walled carbon nanotube (SWCNT/ DWCNT)

A single-walled carbon nanotube (SWCNT) is theoretically assumed to be made by rolling a graphene sheet (Fig. 1) and the double-walled carbon nanotubes are distinguished from traditional elastic beam by their hollow two-layer structures and associated intertube van der Waals forces. (Fig. 2).

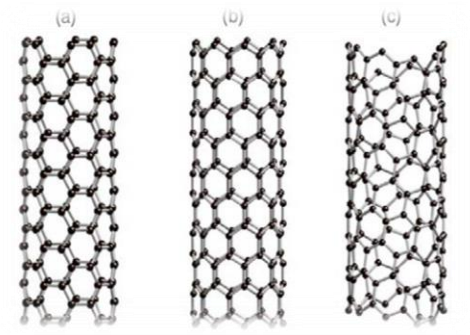

Fig. 1 single-walled carbon nanotube, (a) armchair, (b) zigzag, (c) chiral

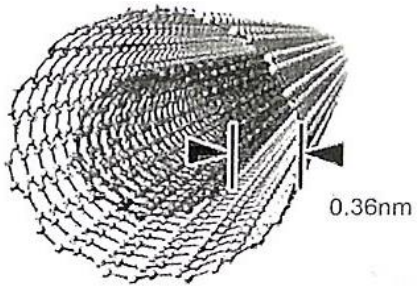

Fig. 2 double-walled carbon nanotube

The radius of the inner and the outer tube of carbon nanotubes can be expressed in terms of integers (n, m): (Tokio 1995)

$$R_1 = a\sqrt{3(n^2 + m^2 + nm)}\,/\,2\pi, \tag{1}$$

$$R_2 = R_1 + h \tag{2}$$

Where the integer pair (n, m) are the indices of translation (a) is the length of the carbon-carbon bond which is (1.42 $A°$) and h is the distance between the inner and the outer tube.

According to different values of integers (n, m), carbon nanotube can be classified into zigzag ((n *or* m)=0), armchair (n =m) and chiral (n≠m) (Fig. 1).

3. non-local Euler Bernoulli beam model of double-walled carbon nanotube

Recently, the Euler Bernoulli beam model has been widely and successfully used to study the vibration analysis of functionally graded plates using a simple shear deformation theory, such as the sandwich plates (Hadji *et al*. 2016) and static analysis of the FGM plate with porosities (Benferhat *et al*. 2016, Akavci 2014). The equations of motion for transversely vibrating Euler beam can be expressed as

$$\frac{\partial T}{\partial x} + p(x) + f(x) = \rho A \frac{\partial^2 w}{\partial t^2} \tag{3}$$

where f(x) is the interaction pressure per unite axial length between the nanotube and the surrounding elastic medium,

$p(x)$ is the spread transverse force along axis x, w is the transverse deflection, ρ is the density, A is the area of the cross section of the nanotube, and T is the resultant shear force on the cross section, which

The one-dimensional nonlocal constitutive relation for the nanotube can be approximated to (Eringen 1983)

$$\sigma_x - e0a^2 \frac{\partial^2 \sigma_x}{\partial x^2} = E_{SWNT}\left(-z\frac{\partial^2 w}{\partial x^2}\right) \tag{4}$$

Where ESWNT is the Young's modulus of single-walled nanotubes, w is the transverse displacement. Thus, the scale coefficient (e0a) in the modelling will lead to small-scale effect on the response of structures at nano size. In addition, e0 is a constant appropriate to each material, and a is an internal characteristic length of the material (e.g., length of C-C bond, lattice spacing, granular distance).

Starting point on Eringen nonlocal elasticity model (Eringen 1983) the stress at a reference point is considered to be functional of the strain field at every point in the body. In the limit when the effects of strains at points other than x are neglected, one obtains classical or local theory of elasticity.

Tu and Ou-Yang **(**2002) indicated that the relation between Young's modulus of multi-walled carbon nanotubes (MWCNTs) and the layer number N' can be expressed as

$$E_{MWNT} = \frac{N'}{N'-1+t/h}\frac{t}{h}E_{SWNT} \tag{5}$$

Where E_{MWNT}, E_{SWNT}, t, N' and h are respectively Young's modulus of multi-walled nanotubes, effective wall thickness of single-walled nanotubes, number of layers and layer distance. If $N'1=1$ and $E_{MWNT}=E_{SWNT}$, which corresponds to the case of single –walled carbon nanotubes and results are in good agreement with the existing experimental ones

According to Eq. (4) and The resultant bending moment M and shear force: $M = \int_A z\sigma_x dA$, $T = \frac{dM}{dx}$

Thus, can be expressed as

$$M - (e_O a)^2 \frac{\partial^2 M}{\partial x^2} = -EI\frac{\partial^2 w}{\partial x^2} \tag{6}$$

where ($I = \int_A z^2 dA$) is the moment of inertia.

By substituting Eq. (3) into Eq. (6), the bending moment M and the shear force T for the non-local model can be expressed as

$$M = -EI\frac{\partial^2 w}{\partial x^2} + (e_O a)^2\left[\rho A\frac{\partial^2 w}{\partial t^2} - f(x) - p(x)\right] \tag{7}$$

$$T = -EI\frac{\partial^3 w}{\partial x^3} + (e_O a)^2\left[\rho A\frac{\partial^3 w}{\partial x \partial t^2} - \frac{\partial f(x)}{\partial x} - \frac{\partial p(x)}{\partial x}\right] \tag{8}$$

Substituting Eq. (8) into Eq. (3) the following differential equation of a non-local Euler Bernoulli beam theory.

$$EI\frac{\partial^4 w}{\partial x^4} + \left(1 - e_O a^2\frac{\partial^2}{\partial x^2}\right)\left(\rho A\frac{\partial^2 w}{\partial t^2} - f(x) - p(x)\right) = 0 \tag{9}$$

The governing equation of the inner tube can also be obtained in the similar way as that of the outer tube. Note that the terms corresponding to elastic medium will not enter the equation of the inner tube since only the outer tube interacts with the elastic medium. Hence, the governing equations for free vibration of an embedded DWCNT can be written as.

$$\begin{cases} EI_1\frac{\partial^4 w_1}{\partial x^4} + \left(1 - e_O a^2\frac{\partial^2}{\partial x^2}\right)\left(\rho A_1\frac{\partial^2 w_1}{\partial t^2}_1 - p_{12}\right) = 0 \\ EI_2\frac{\partial^4 w_2}{\partial x^4} + \left(1 - e_O a^2\frac{\partial^2}{\partial x^2}\right)\left(\rho A_2\frac{\partial^2 w_2}{\partial t^2} - f(x) - p_{12}\right) = 0 \end{cases} \tag{10}$$

Where subscripts 1 and 2 denote the quantities associated with the inner and outer nanotubes, respectively. For the DWNTs, the van der Waals interaction forces between the two tubes can be expressed as

$$p_{12} = -p_{21} = c(w_2 - w_1) \tag{11}$$

where c is the intertube interaction coefficient per unit length between two tubes, which can be estimated as (Sudak 2003)

$$c = \frac{320(d_{in})erg/cm^2}{0.16a^2} \tag{12}$$

where d_{in} is the radius of the inner tube.

The pressure per unit axial length, acting on the outermost tube due to the surrounding elastic medium, can be described by a Winkler-type $f=-k_w w_2$.

Let us consider a double-walled carbon nanotube of length L in which the two ends are simply supported, so vibrational modes of the (DWCNT) are of the form (Batra 2007)

$$w_1 = \overline{W}_1 e^{i\omega t}\sin\lambda x,\ \ w_2 = \overline{W}_2 e^{i\omega t}\sin\lambda x \text{ et}$$
$$\lambda = \frac{k\pi}{L},\ (k = 1,2,.....) \tag{13}$$

where $\overline{W}_1$ and $\overline{W}_2$ are the amplitudes of deflections of the inner and outer tubes.

Substituting Eqs. (11)-(13) into Eq. (10), one can easily obtain the homogeneous system

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}\begin{Bmatrix} \overline{W}_1 \\ \overline{W}_2 \end{Bmatrix} = 0 \tag{16}$$

Where K_{11} , K_{12} , K_{21} and K_{22} in Eq. (16) are defined as

$$\begin{cases} K_{11} = EI_1\lambda^4 + \left(1 + (e_0 a)^2\lambda^2\right)\left(-\rho A_1\omega^2 + c\right) \\ K_{12} = K_{21} = c\left(1 + (e_0 a)^2\lambda^2\right) \\ K_{22} = EI_2\lambda^4 + \left(1 + (e_0 a)^2\lambda^2\right)\left(-\rho A_2\omega^2 + c + k_w\right) \end{cases}$$

Table 1 lists the values of Young's modulus of (SWNT) and (DWNT) for different chirality's

(n,m)	Young's modulus (SWNT) (GPa) Bao Wen Xing *et al.* (2004)	Young's modulus (DWNT) (GPa) Tu and Ou-Yang (2002)
Zigzag		
(14,0) (23,0)	939.032	810.268
(17,0) (26,0)	938.553	809.855
(21,0) (30,0)	936.936	808.460
(24,0) (33,0)	934.201	806.100
(28,0) (37,0)	932.626	804.741
(31,0) (40,0)	932.598	804.717
(35,0) (44,0)	933.061	805.116
Armchair		
(8,8) (12,12)	934.960	806.755
(10,10)(15,15)	935.470	807.195
(12,12)(17,17)	935.462	807.188
(14,14)(19,19)	935.454	807.181
(16,16)(21,21)	939.515	810.685
(18,18)(23,23)	934.727	806.554
(20,20)(25,25)	935.048	806.831
Chiral		
(12,6) (18,10)	927.671	800.465
(14,6) (20,10)	921.616	795.241
(16,8) 22,12)	928.013	800.760
(18,9) (24,13)	927.113	799.984
(20,12)(26,16)	904.353	780.345
(24,11)(30,15)	910.605	785.739
(30,8) (36,12)	908.792	784.175

Solving Eq. (16) the lower and higher natural frequency, of the DWCNT in which the effects of different parameters are shown.

$$\omega^2 = \frac{1}{2}\left(\alpha \pm \sqrt{\alpha^2 - 4\beta}\right) \tag{17}$$

Where α and β in equations yields are defined as

$$\alpha = \frac{c(A_1 + A_2)}{\rho A_1 A_2} + \frac{k_w}{\rho A_2} + \frac{E\lambda^4 (A_1 I_2 + A_2 I_1)}{\rho A_1 A_2 \left(1 + (e_0 a)^2 \lambda^2\right)}$$

$$\beta = c\lambda^4 \frac{(EI_1 + EI_2)}{\rho^2 A_1 A_2 \left(1 + (e_0 a)^2 \lambda^2\right)} + \lambda^8 \frac{E^2 I_1 I_2}{\rho^2 A_1 A_2 \left(1 + (e_0 a)^2 \lambda^2\right)^2} + \left(\frac{\lambda^4 EI_1}{\rho^2 A_1 A_2 \left(1 + (e_0 a)^2 \lambda^2\right)} + \frac{c}{\rho^2 A_1 A_2}\right) k_w$$

4. Results and discussion

The effect of both chirality and winkler modulus parameter on vibration properties of zigzag double-walled nanotubes are discussed here. The parameters used in calculations for the zigzag DWCNTs are given as follows:

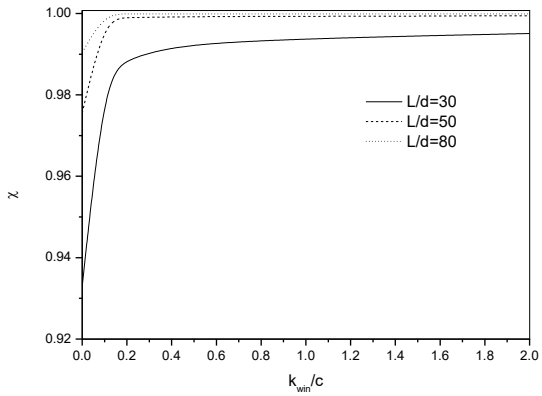

Fig. 3 Effect of the Winkler parameter on the ratio χ for various aspect ratio L/d of double armchair carbon nanotubes (N = 2 and e0a= 2 nm)

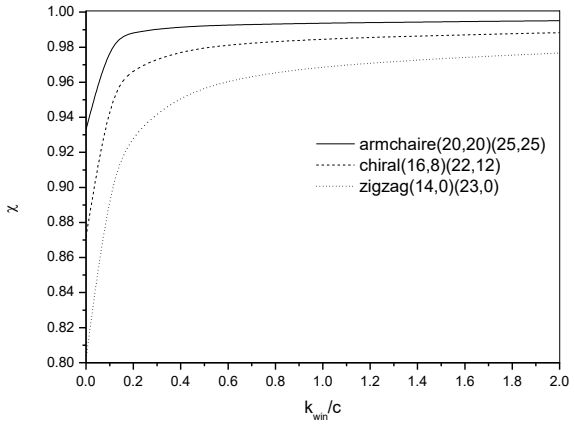

Fig. 4 Effect of the Winkler parameter on the ratio χ for various chiralities of double carbon nanotubes (N= 2, L/d=30 and e0a= 2 nm)

the mass density ρ=2.3 *g/cm³*, the effective thickness of (CNTs) taken to be 0.258 *nm*, poisson ratio υ=0.19 and layer distance *h*=0.34 *nm*.

The Young's modulus of (SWCNTs) and (DWCNTs) employed in this study (Table 1), are calculated by Bao Wen Xing *et al.* (2004) and Tu and Ou-Yang (2002) respectively. The decreasing of Young's modulus (DWCNTs) for some chirality nanotube is attributed to the weak van der Waals forces between the outer and inner tube.

The vibration properties of SWCNTs with different chirality are compared. To investigate the effect of scale parameter on vibrations of SWCNTs, the following ratio is introduced

$$\chi = \omega_{NE} / \omega_{LE} \tag{18}$$

Where ω_{LE} and ω_{NE} are the frequencies based on the local and nonlocal Euler Bernoulli beam models, respectively.

The (Fig. 3) illustrate the dependence of the frequency ratio on the aspect ratio L/d of double armchair carbon nanotubes. The value of mode number N=2 and small-scale coefficient e0a=2 nm. The frequency ratio (χ) serves as an index to assess quantitatively the scale effect on CNT vibration solution. This means that the application of the

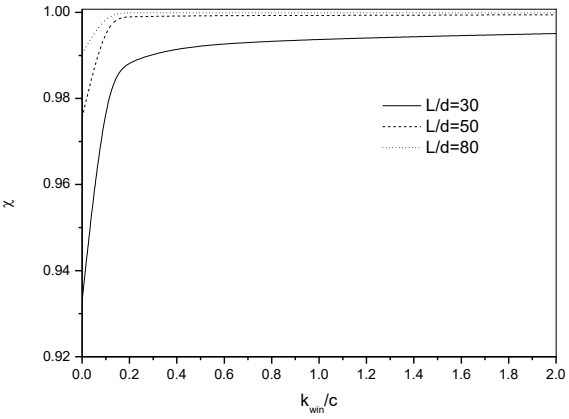

Fig. 5 Effect of elastic medium and mode number on vibration ratio of double Armchair (20,20)(25,25) carbon nanotube where (e0a=2 nm & L/d=30)

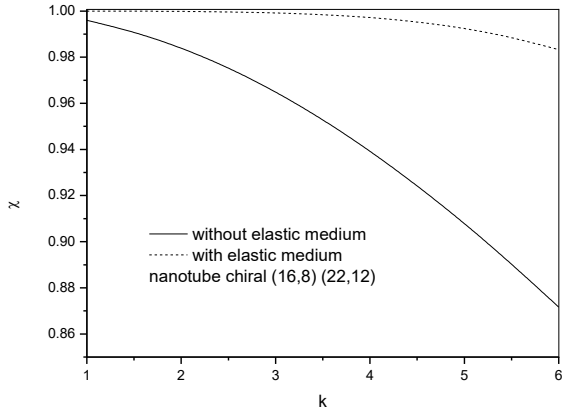

Fig. 6 Effect of elastic medium and mode number on vibration ratio of double chiral (16,8)(22,12) carbon nanotube where (e0a=2 nm & L/d=30)

local Euler Bernoulli beam model for CNT analysis would lead to an over prediction of the frequency if the scale effect between the individual carbon atoms in CNTs is neglected.

It is observed from this figure that the frequency ratio approach to the unity and the effect of the small-scale coefficient increase for increasing of the aspect ratio this effect is clearly for lower values of Winkler parameter. The reason for this phenomenon is that a carbon nanotube with larger long has a larger curvature, which results in a more significant distortion of (C-C) bonds.

It is clearly seen from (Fig. 4) that the ranges of frequency ratio for these chirality of double-walled carbon nanotubes (DWCNTs) are quite different, the range is the smallest for armchair (20,20)(25,25), but the range is the largest for zigzag (14,0)(23,0). The reason for this difference perhaps is attributed to the increasing or decreasing of carbon nanotube diameter. In additional, when the elastic medium parameter increases the effect of the small-scale coefficient decrease. The effect of mode number (N) on the frequency ratios (χ) for various chirality of double-carbon nanotube with and without elastic medium is demonstrated in (Figs. 5-7) with aspect ratio (L/d=30) and small-scale coefficient ($e0a$= 2 nm). In these figures, it is observed that as the mode number increase, the scale

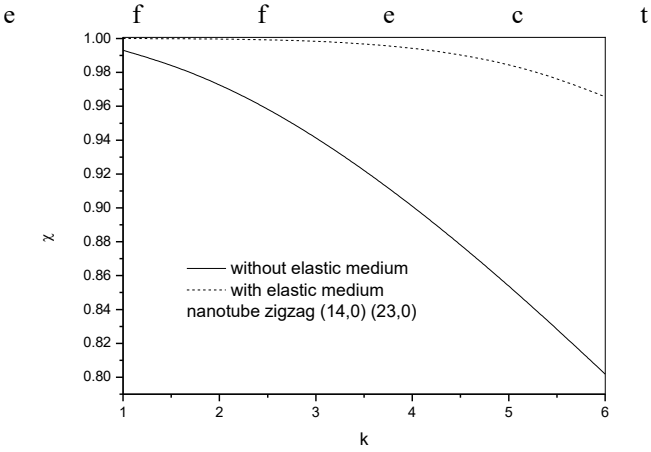

Fig. 7 Effect of elastic medium and mode number on vibration ratio of double Zigzag (14,0)(23,0) carbon nanotube where (e0a=2 nm & L/d=30)

Table 2 lists the values of the lower frequency ratio for different Zigzag chirality's and aspect ratios (L/d) of carbon nanotube with and without elastic medium

Zigzag	without elastic medium		with elastic medium	
	L/d =10	L/d =15	L/d =10	L/d =15
(14,0) (23,0)	0.94169	0.97308	0.99848	0.99974
(17,0) (26,0)	0.95365	0.97882	0.99888	0.99980
(21,0) (30,0)	0.96470	0.98401	0.99921	0.99986
(24,0) (33,0)	0.97062	0.98676	0.99936	0.99988
(28,0) (37,0)	0.97647	0.98944	0.99951	0.99991
(31,0) (40,0)	0.97979	0.99096	0.99959	0.99992
(35,0) (44,0)	0.98323	0.99252	0.99966	0.99994

Table 3 lists the values of the lower frequency ratio for different armchair chirality's and aspect ratios (L/d) of carbon nanotube with and without elastic medium

Armchair	without elastic medium		with elastic medium	
	L/d =10	L/d =15	L/d =10	L/d =15
(8,8) (12,12)	0.94101	0.97275	0.99846	0.99973
(10,10) (15,15)	0.95472	0.97932	0.99892	0.99981
(12,12) (17,17)	0.96421	0.98378	0.99919	0.99985
(14,14) (19,19)	0.97104	0.98695	0.99937	0.99989
(16,16) (21,21)	0.97610	0.98928	0.99950	0.99991
(18,18) (23,23)	0.97996	0.99104	0.99959	0.99992
(20,20) (25,25)	0.98296	0.99240	0.99966	0.99993

on the frequency ratios (χ) increase. The scale effect is becoming obvious for the higher vibration mode number. This significance in higher modes is attributed to the influence of small wavelength. For smaller wavelengths, the interactions between atoms are increasing and these loads to an increase in the nonlocal effects.

The scale effect increase as one transit from the armchair (20,20)(25,25) in (Fig. 5) to the chiral (16,8)(22,12) in (Fig. 6) and then, zigzag (14,0)(23,0) in (Fig. 7), when the diameter of nanotube is decreasing. The

Table 4 lists the values of the lower frequency ratio for different Chiral chirality's and aspect ratios (L/d) of carbon nanotube with and without elastic medium

Chiral	without elastic medium		with elastic medium	
	L/d =10	L/d =15	L/d =10	L/d =15
(12,6) (18,10)	0.94964	0.97690	0.99877	0.99978
(14,6) (20,10)	0.95618	0.98001	0.99898	0.99982
(16,8) (22,12)	0.96508	0.98419	0.99922	0.99986
(18,9) (24,13)	0.97030	0.98661	0.99936	0.99988
(20,12)(26,16)	0.97648	0.98945	0.99952	0.99991
(24,11)(30,15)	0.97980	0.99096	0.99960	0.99992
(30,8) (36,12)	0.98301	0.99242	0.99967	0.99994

reason for this difference perhaps is attributed to the increasing or decreasing of carbon nanotube diameter. In addition, the influence of the small-scale increase without elastic medium. This is explained by the influence of the small-scale effect as the Winkler parameter exists. Although the small-scale effect makes the carbone nanotube more flexible, as a CNT is assumed as atoms linked by springs, the elastic medium grips a nanotube and forces it to be stiffer. Therefore, it is clear that the small-scale effect is found to be more significant in the absence of an elastic medium.

The variation of vibration ratio of double-walled carbon nanotubes (DWCNTs) armchair, chiral and zigzag chirality with and without elastic medium with different length-to-diameter ratios are listed in (Tables 2-4). Their results show the dependence of the different chirality's of carbon nanotube, Aspect Ratio and, effect of elastic medium on the vibration ratio. It is observed that as the long of nanotube increase, the scale effect on the frequency ratios (χ) decrease. The scale effect is becoming obvious for the nanotube without elastic medium.

5. Conclusions

This article studies the vibration of double-carbon nanotube (DWCNTs) embedded in elastic based on the Euler–Bernoulli beam theory and Eringen's nonlocal elasticity theory. The influence of the stiffiness of the surrounding elastic medium on the frequency ratio of the DWCNT is shown. The results showed the dependence of the vibration-characteristics on the chirality of DWCNTs and the nonlocal parameter. It is observed that as the mode number increase, small-scale effects increases. The reason of these increases is attributed to the influence of small wavelength when the interactions between atoms are increasing. Besides, the increasing or decreasing of long or diameter of (DWCNTs) affects the frequency ratio. This affection is most pronounced for short nanotube. The cause for this phenomenon is that a carbon nanotube with higher long has a larger curvature, which results in a more significant distortion of (C–C) bonds. Although the small-scale effect makes the carbone nanotube more flexible, the elastic medium grips a nanotube and forces it to be stiffer. Therefore, it is clear that the small-scale effect is found to be more significant in the absence of an elastic medium. The investigation presented may be helpful in the application of DWCNTs, such as ultrahigh-frequency resonators, electron emission devices, high-frequency oscillators and mechanical sensors.

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