Structural Engineering and Mechanics, *Vol. 65, No. 3 (2018) 213-222*
DOI: https://doi.org/10.12989/sem.2018.65.3.213

Using fourth order element for free vibration parametric analysis of thick plates resting on elastic foundation

Y. I. Özdemir*

Department of Civil Engineering, Karadeniz Technical University, 61080 Trabzon, Turkey

(Received May 8, 2017, Revised November 8, 2017, Accepted November 13, 2017)

Abstract. The purpose of this paper is to study free vibration analysis of thick plates resting on Winkler foundation using Mindlin's theory with shear locking free fourth order finite element, to determine the effects of the thickness/span ratio, the aspect ratio, subgrade reaction modulus and the boundary conditions on the frequency paramerets of thick plates subjected to free vibration. In the analysis, finite element method is used for spatial integration. Finite element formulation of the equations of the thick plate theory is derived by using higher order displacement shape functions. A computer program using finite element method is coded in C++ to analyze the plates free, clamped or simply supported along all four edges. In the analysis, 17-noded finite element is used. Graphs are presented that should help engineers in the design of thick plates subjected to earthquake excitations. It is concluded that 17-noded finite element can be effectively used in the free vibration analysis of thick plates. It is also concluded that, in general, the changes in the thickness/span ratio are more effective on the maximum responses considered in this study than the changes in the aspect ratio.

Keywords: free vibration parametric analysis; thick plate; Mindlin's theory; fourth order finite element; Winkler foundation

1. Introduction

Plates are structural elements which are commonly used in the building industry. A plate is considered to be a thin plate if the ratio of the plate thickness to the smaller span length is less than 1/20; it is considered to be a thick plate if this ratio is larger than 1/20 (Ugural 1981).

The dynamic behavior of thin plates has been investigated by many researchers (Leissa 1973, 1977, 1981, 1981, 1987, Leissa 1977, Providakis and Beskos 1989, Warburton 1954, Caldersmith 1984, Grice and Pinnington 2002, Sakata and Hosokawa 1988, Lok and Cheng 2001, Si *et al.* 2005, Ayvaz and Durmuş 1995). There are also many references on the behavior of the thick plates subjected to different loads. The studies made on the behavior of the thick plates are based on the Reissner-Mindlin plate theory (Reissner 1945, 1947, 1950, Mindlin 1951). This theory requires only C^0 continuity for the finite elements in the analysis of thin and thick plates. Therefore, it appears as an alternative to the thin plate theory which also requires C^1 continuity. This requirement in the thin plate theory is solved easily if Mindlin theory is used in the analysis of thin plates. Despite the simple formulation of this theory, discretization of the plate by means of the finite element comes out to be an important parameter. In many cases, numerical solution can have lack of convergence, which is known as "shear-locking". Shear locking can be avoided by increasing the mesh size, i.e., using finer mesh, but if the thickness/span ratio is "too small", convergence may not be achieved even if the finer mesh is used for the low order displacement shape functions.

In order to avoid shear locking problem, the different methods and techniques, such as reduced and selective reduced integration, the substitute shear strain method, etc., are used by several researchers (Hinton and Huang 1986, Zienkiewich *et al.* 1971, Bergan and Wang 1984, Ozkul and Ture 2004, Hughes *et al.* 1977). The same problem can also be prevented by using higher order displacement shape function (Özdemir *et al.* 2007). Wanji and Cheung (2000) proposed a new quadrilateral thin/thick plate element based on the Mindlin-Reissner theory. Soh *et al.* (2001) improved a new element ARS-Q12 which is a simple quadrilateral 12 DOF plate bending element based on Reissner-Mindlin theory for analysis of thick and thin plates. Brezzi and Marini (2003) developped a locking free nonconforming element for the Reissner-Mindlin plate using discontinuous Galarkin techniques. Belounar and Guenfound (2005) improved a new rectangular finite element based on the strain approach and the Reissner-Mindlin theory is presented for the analysis of plates in bending either thick or thin. Vibration analysis made by Raju and Hinton (1980), they presented natural frequencies and modes of rhombic Mindlin plates. Si *et al.* (2005) studied vibration analysis of rectangular plates with one or more guided edges via bicubic B-spline method, Cen *et al.* (2006) developed a new high performance quadrilateral element for analysis of thick and thin plates. This distinguishing character of the new element is that all formulations are expressed in the quadrilateral area co-ordinate system. Shen *et al.* (2001) studied free and forced vibration of Reissner-Mindlin plates with free edges resting on elastic foundations. Woo *et al.*

*Corresponding author, Associate Professor
E-mail: yaprakozdemir@hotmail.com

http://www.techno-press.com/journals/sem&subpage=7
ISSN: 1225-4568 (Print), 1598-6217 (Online)

(2003) found accurate natural frequencies and mode shapes of skew plates with and without cutouts by p-version finite element method using integrals of Legendre polynomial for p=1-14. Qian *et al.* (2003) studied free and forced vibrations of thick rectangular plates using higher-order shear and normal deformable plate theory and meshless Petrov-Galarkin method. Özdemir and Ayvaz (2009) studied shear locking free earthquake analysis of thick and thin plates using Mindlin's theory. GuangPeng *et al.* (2012) studied free vibration analysis of plates on Winkler elastic foundation by boundary element method. Fallah *et al.* (2013) analyzed free vibration of moderately thick rectangular FG plates on elastic foundation with various combinations of simply supported and clamped boundary conditions. Governing equations of motion were obtained based on the Mindlin plate theory. Jahromi *et al.* (2013) analyzed free vibration analysis of Mindlin plates partially resting on Pasternak foundation. The governing equations which consist of a system of partial differential equations are obtained based on the first-order shear deformation theory. Özgan and Daloğlu (2013) studied free vibration analysis of thick plates on elastic foundations using modified Vlasov model with higher order finite elements, also same autors (2015) studied the effects of various parameters such as the aspect ratio, subgrade reaction modulus and thickness/span ratio on the frequency parameters of thick plates resting on Winkler elastic foundations. Authors used 4 and 8-noded finite elements this study. However, no references have been found in the technical literature for the free vibration analysis of thick plates resting on Winkler foundation by using fourth order 17-noded finite element.

The purpose of this paper is to study free vibration analysis of thick plates resting on Winkler foundation using Mindlin's theory with shear locking free fourth order finite element, to determine the effects of the thickness/span ratio, the aspect ratio, subgrade reaction modulus and the boundary conditions on the frequency paramerets of thick plates subjected to free vibration. A computer program using finite element method is coded in C++ to analyze the plates free, clamped or simply supported along all four edges. In the program, the finite element method is used for spatial integration. Finite element formulation of the equations of the thick plate theory is derived by using higher order displacement shape functions. In the analysis, 17-noded finite element is used to construct the stiffness and mass matrices since shear locking problem does not occur if this element is used in the finite element modelling of the thick and thin plates (Özdemir *et al.* 2007). No matter what the mesh size is unless it is less than 4x4. This is a new element, details of its formulation are presented in (Özdemir *et al.* 2007) and this is the first time this element is used in the free vibration analysis of thick plates. If this element is used in an analysis, it is not necessary to use finer mesh.

2. Mathematical model

The governing equation for a flexural plate (Fig. 1) subjected to free vibration without damping can be given as

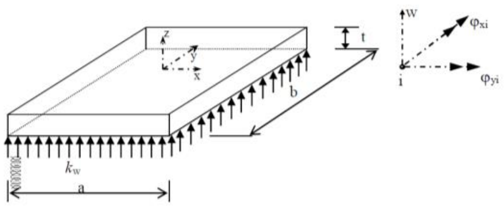

Fig. 1 The sample plate used in this study

$$[M]\{\ddot{w}\}+[K]\{w\}=0 \tag{1}$$

where (K) and (M) are the stiffness matrix and the mass matrix of the plate, respectively, w and $\ddot{w}$ are the lateral displacement and the second derivative of the lateral displacement of the plate with respect to time, respectively.

The total strain energy of plate-soil-structure system (see Fig. 1) can be written as

$$\Pi = \Pi_p + \Pi_s + V \tag{2}$$

where Π_p is the strain energy in the plate,

$$\Pi_p = \frac{1}{2}\int_A \left(-\frac{\partial\varphi_x}{\partial x} \quad \frac{\partial\varphi_y}{\partial y} \quad -\frac{\partial\varphi_x}{\partial y}+\frac{\partial\varphi_y}{\partial x}\right)^T E_\kappa \left(-\frac{\partial\varphi_x}{\partial x} \quad \frac{\partial\varphi_y}{\partial y} \quad -\frac{\partial\varphi_x}{\partial y}+\frac{\partial\varphi_y}{\partial x}\right) d_A + \frac{k}{2}\int_A \left(-\varphi_x+\frac{\partial w}{\partial x} \quad \varphi_y+\frac{\partial w}{\partial y}\right)^T E_\gamma \left(-\varphi_x+\frac{\partial w}{\partial x} \quad \varphi_y+\frac{\partial w}{\partial y}\right) d_A \tag{3}$$

where Π_s is the strain energy stored in the soil,

$$\Pi_s = \frac{1}{2}\int_0^H \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sigma_{ij}\varepsilon_{ij} \tag{4}$$

and V is the potential energy of the external loading

$$V = -\int_A \bar{q} w d_A \tag{5}$$

In this equation, E_κ and E_γ are the elasticity matrix and these matrices are given below at Eq. (17), $\bar{q}$ shows applied distributed load.

2.1 Evaluation of the stiffness matrix

The total strain energy of the plate-soil system according to Eq. (2) is

$$U_e = \frac{1}{2}\int_A \left(-\frac{\partial\varphi_x}{\partial x} \quad \frac{\partial\varphi_y}{\partial y} \quad -\frac{\partial\varphi_x}{\partial y}+\frac{\partial\varphi_y}{\partial x}\right)^T E_\kappa \left(-\frac{\partial\varphi_x}{\partial x} \quad \frac{\partial\varphi_y}{\partial y} \quad -\frac{\partial\varphi_x}{\partial y}+\frac{\partial\varphi_y}{\partial x}\right) d_A + \frac{k}{2}\int_A \left(-\varphi_x+\frac{\partial w}{\partial x} \quad \varphi_y+\frac{\partial w}{\partial y}\right)^T E_\gamma \left(-\varphi_x+\frac{\partial w}{\partial x} \quad \varphi_y+\frac{\partial w}{\partial y}\right) d_A + \frac{1}{2}\int_A (w_{x,y})^T K (w_{x,y}) d_A \tag{6}$$

At this equation the first and second part gives the conventional element stiffness matrix of the plate, (k_p^e), differentiation of the third integral with respect to the nodal parameters yields a matrix, (k_w^e), which accounts for the axial strain effect in the soil. Thus, the total energy of the plate-soil system can be written as

$$U_e = \frac{1}{2}\{w_e\}^T\left(\left[k_p^e\right]+\left[k_w^e\right]\right)\{w_e\}d_A \tag{7}$$

where

$$\{w_e\} = \begin{bmatrix} w_1 & \varphi_{y1} & \varphi_{x1} & \ldots & w_n & \varphi_{yn} & \varphi_{xn} \end{bmatrix}^T \tag{8}$$

Assuming that in the plate of Fig. 1 u and v are proportional to z and that w is the independent of z (Mindlin 1951), one can write the plate displacement at an arbitrary x, y, z in terms of the two slopes and a displacement as follows

$$u_i=\{w, v, u\}=\{w_0(x,y,t),\ z\varphi_y\,(x,y,t),\ -z\varphi_x\,(x,y,t)\} \tag{9}$$

where w_0 is average displacement of the plate, and φ_x and φ_y are the bending slopes in the x and y directions, respectively.

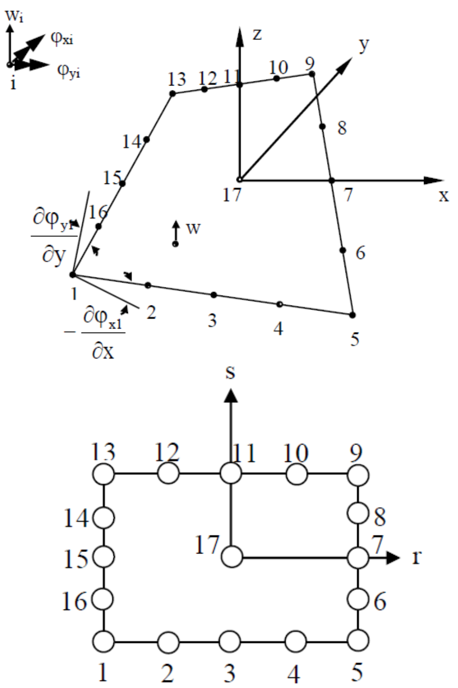

Fig. 2 17-noded quadrilateral finite element used in this study (Özdemir *et al*. 2007)

The nodal displacements for 17-noded quadrilateral serendipity element (MT17) (Fig. 2) can be written as follows

$$w=\sum_1^{17} h_i w_i\ ,\ v=z\varphi_y= z\sum_1^{17} h_i \varphi_{yi}\ ,\ u=-z\varphi_x= -z\sum_1^{17} h_i \varphi_{xi} \qquad i=1,\ldots,17 \text{ for 17 - noded element}, \tag{10}$$

The displacement function chosen for this element is

$$w = c_1 + c_2 r + c_3 s + c_4 r^2 + c_5 rs + c_6 s^2 + c_7 r^2 s + c_8 rs^2 + c_9 r^3 + c_{10} r^3 s + c_{11} rs^3 + c_{12} s^3 + c_{13} r^2 s^2 + c_{14} r^4 + c_{15} r^4 s + c_{16} rs^4 + c_{17} s^4. \tag{11}$$

From this assumption, it is possible to derive the displacement shape function to be (Özdemir *et al*. 2007)

$$h = [h_1, \ldots, h_{17}]. \tag{12}$$

Then, the strain-displacement matrix (B) for this element can be written as follows Cook *et al.* (1989)

$$[B] = \begin{bmatrix} 0 & 0 & -\frac{\partial h_i}{\partial x} & \ldots \\ 0 & \frac{\partial h_i}{\partial y} & 0 & \ldots \\ 0 & \frac{\partial h_i}{\partial x} & -\frac{\partial h_i}{\partial y} & \ldots \\ \frac{\partial h_i}{\partial x} & 0 & -h_i & \ldots \\ \frac{\partial h_i}{\partial y} & h_i & 0 & \ldots \end{bmatrix}_{5\times 51} \tag{13}$$

$$i=1,\ldots,17 \text{ for 17 - noded element}$$

The stiffness matrix for MT17 element can be obtained by the following equation (Cook *et al.* 1989).

$$[K] = \int_A [B]^T[D][B]dA.\int_{-1}^{1}\int_{-1}^{1}[B]^T[D][B]|J|drds \tag{14}$$

which must be evaluated numerically (Hughes *et al.* 1977). As seen from Eq. (14), in order to obtain the stiffness matrix, the strain-displacement matrix, (B), and the flexural rigidity matrix, (D), of the element need to be constructed.

The flexural rigidity matrix, (D), can be obtained by the following equation.

$$[D] = \begin{bmatrix} E_k & 0 \\ 0 & E_\gamma \end{bmatrix}. \tag{15}$$

In this equation, (E_k) is of size 3×3 and (E_γ) is of size 2×2. (E_k), and (E_γ) can be written as follows (Bathe 1996, Weaver and Johston 1984)

$$(E_k) = \frac{t^3}{12}\begin{bmatrix} \frac{E}{(1-\nu^2)} & \frac{\nu E}{(1-\nu^2)} & 0 \\ \frac{\nu E}{(1-\nu^2)} & \frac{E}{(1-\nu^2)} & 0 \\ 0 & 0 & \frac{E}{2(1-\nu)} \end{bmatrix};\quad (E_\gamma) = kt\begin{bmatrix} \frac{E}{2.4(1+\nu)} & 0 \\ 0 & \frac{E}{2.4(1+\nu)} \end{bmatrix} \tag{16}$$

Table 1 The first five natural frequency parameters of plates for b/a=0.1 and t/a=0.05

$\lambda_i=\omega^2$	Ozgan and Daloğlu (2015) PBQ8(FI)	This Study MT17 (4 element)	SAP2000
1	3990.42	4002.41	4000.00
2	3990.42	4002.41	4000.00
3	4000.40	4021.55	4000.00
4	8676.00	8650.67	8619.60
5	13957.64	13789.50	13292.31
6	17252.34	16939.10	16380.24

where E, υ, and t are modulus of the elasticity, Poisson's ratio, and the thickness of the plate, respectively, k is a constant to account for the actual non-uniformity of the shearing stresses. By assembling the element stiffness matrices obtained, the system stiffness matrix is obtained.

2.2 Evaluation of the mass matrix

The formula for the consistent mass matrix of the plate may be written as

$$M = \int_{\Omega} H_i^T \mu H_i d\Omega \quad . \tag{17}$$

In this equation, μ is the mass density matrix of the form (Tedesco *et al.* 1999)

$$\mu = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}, \tag{18}$$

where $m_1=\rho_p t$, $m_2=m_3=\frac{1}{12}\left(\rho_p t^3\right)$, and ρ_p is the mass densities of the plate. and H_i can be written as follows,

$$H_i = \left[dh_i / dx \quad dh_i / dy \quad h_i\right] \qquad i=1\ldots17. \tag{19}$$

It should be noted that the rotation inertia terms are not taken into account. By assembling the element mass matrices obtained, the system mass matrix is obtained.

2.3 Evaluation of frequency of plate

The formulation of lateral displacement, w, can be given as motion is sinusoidal

$$w= W \sin \omega t \tag{20}$$

Here ω is the circular frequency. Substitution of Eq. (20) and its second derivation into Eq. (1) gives expression as

$$(K- \omega^2 M) \{W\}=0 \tag{21}$$

Eq. (21) is obtained to calculate the circular frequency, ω, of the plate. Then natural frequency can be calculated with the formulation below

Table 2 Effects of aspect ratio and thickness/span ratio on the first six frequency parameters of the thick free plates resting on elastic foundation (a)Subgrade reaction modulus k=500

k	b/a	t/a	$\lambda = \omega^2$					
			λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
500	1.0	0.05	456.73	456.73	469.98	5048.72	10235.73	13366.95
		0.10	235.42	235.42	283.12	17448.03	37556.42	49322.00
		0.20	171.76	171.76	175.32	58681.13	126694.77	164490.80
		0.30	149.09	149.09	179.38	109100.98	229933.60	295362.76
	1.5	0.05	458.49	464.03	470.14	2492.94	2660.49	10937.27
		0.10	241.45	259.74	289.49	7970.41	8878.92	39117.93
		0.20	170.28	173.06	174.51	27346.73	31896.09	127052.66
		0.30	153.01	163.56	183.08	52430.17	63261.49	225607.89
	2.0	0.05	459.37	466.66	470.22	1161	1588.82	5784.55
		0.10	244.46	271.13	292.64	3031.39	4557.16	20388.11
		0.20	169.53	171.92	174.46	10730.89	15546.29	68560.22
		0.30	154.97	170.85	184.92	22168.05	30127.28	126587.63
	3.0	0.05	460.25	468.61	470.30	603.44	951.50	1519.07
		0.10	247.47	281.97	295.67	825.36	2129.95	4437.42
		0.20	168.79	170.78	173.44	2333.89	6956.53	15815.48
		0.30	156.93	177.91	186.77	4859.98	13529.71	32166.71

(b)Subgrade reaction modulus k=5000

k	b/a	t/a	$\lambda = \omega^2$					
			λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
5000	1.0	0.05	4048.88	4050.71	4050.71	8653.61	13833.74	16962.44
		0.10	1782.51	1828.73	1828.73	19105.26	39152.09	50908.80
		0.20	988.20	988.20	1100.65	59430.60	127406.10	165227.73
		0.30	769.51	769.51	797.73	109738.11	230496.95	295940.16
	1.5	0.05	4046.02	4049.58	4050.65	6091.26	6251.14	14537.14
		0.10	1768.59	1807.81	1815.91	9588.73	10441.43	40735.90
		0.20	998.69	1045.86	1112.15	28136.42	32704.57	127808.03
		0.30	765.62	785.60	792.93	53058.38	63852.85	226223.01
	2.0	0.05	4044.57	4049.01	4049.33	4747.36	5183.66	9370.14
		0.10	1761.48	1794.16	1809.51	4571.14	6154.11	21987.92
		0.20	1003.94	1071.72	1117.86	11584.64	16354.55	69347.86
		0.30	763.67	789.76	790.53	22769.41	30748.11	127203.92
	3.0	0.05	4043.04	4047.16	4048.44	4184.88	4542.92	5100.33
		0.10	1753.95	1778.27	1803.10	2337.21	3705.59	5942.54
		0.20	1009.18	1095.29	1123.49	3230.59	7781.66	16694.84
		0.30	761.73	788.11	790.91	5466.83	14142.39	32756.09

$$f= \omega /2\pi. \tag{22}$$

3. Numerical examples

3.1 Data for numerical examples

In the light of the results given in references (Özdemir *et*

Table 3 Effects of aspect ratio and thickness/span ratio on the first six frequency parameters of the thick simply supported plates resting on elastic foundation (a)Subgrade reaction modulus k=500

k	b/a	t/a	$\lambda = \omega^2$					
			λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
500	1.0	0.05	9047.49	53796.43	53796.43	132323.07	208613.59	208895.04
		0.10	31594.42	186728.44	186728.44	431328.03	675755.92	678833.00
		0.20	100289.55	524820.99	524820.99	1078904.01	1617046.34	1635881.04
		0.30	177442.04	817902.27	817902.27	1562458.24	2233537.15	2283380.53
	1.5	0.05	5015.82	17094.00	43388.98	54027.36	71060.68	133204.57
		0.10	17329.00	60561.66	154092.71	188245.00	242603.20	436597.42
		0.20	57457.29	186916.54	449433.28	530921.10	658117.17	1095658.73
		0.30	106026.94	318024.32	715938.90	828749.90	1001294.89	1585462.92
	2.0	0.05	3912.42	9138.54	23276.24	39998.98	54142.96	54143.89
		0.10	13407.92	32257.55	82636.39	143304.16	189004.43	189006.59
		0.20	45541.02	103507.34	250665.43	424221.46	533149.24	533974.84
		0.30	85976.33	183477.06	417846.00	682308.08	830510.86	834182.73
	3.0	0.05	3220.63	5043.23	9169.12	17200.91	31325.02	37650.27
		0.10	10941.00	17534.14	32481.80	61320.90	111087.04	135748.49
		0.20	38015.92	58510.72	104603.63	190395.95	330316.70	406397.06
		0.30	73378.65	108073.96	185541.50	324384.26	539235.37	658701.91

(b)Subgrade reaction modulus k=5000

k	b/a	t/a	$\lambda = \omega^2$					
			λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
5000	1.0	0.05	12609.04	57354.76	57354.76	135874.88	212157.76	212439.24
		0.10	32964.79	188091.77	188091.77	432688.04	677116.41	680193.55
		0.20	101247.30	525741.77	525741.77	1079799.04	1617942.25	1636779.96
		0.30	177977.58	818437.52	818437.52	1562998.28	2234080.37	2283934.68
	1.5	0.05	8577.44	20655.19	46948.06	57585.70	74617.70	136756.40
		0.10	18701.35	61929.72	155457.14	189608.42	243965.46	437957.71
		0.20	58426.83	187861.85	450362.11	531842.74	659030.25	1096555.70
		0.30	106567.74	318558.03	716477.03	829286.80	1001831.09	1586005.38
	2.0	0.05	7474.01	12700.10	26837.03	43558.29	57701.30	57702.23
		0.10	14781.01	33627.96	84003.25	144669.00	190367.90	190370.05
		0.20	46515.20	104465.61	251604.71	425153.30	534071.10	534896.91
		0.30	86519.76	184013.55	418380.12	682847.59	831047.66	834720.46
	3.0	0.05	6782.19	8604.85	12730.68	20762.10	34885.23	41209.73
		0.10	12314.64	18906.50	33852.23	62689.00	112452.74	137113.62
		0.20	38993.59	59480.44	105562.09	191341.81	331250.24	407331.09
		0.30	73924.28	108615.10	186078.32	324918.95	539770.74	659242.52

Table 4 Effects of aspect ratio and thickness/span ratio on the first six frequency parameters of the thick clamped plates resting on elastic foundation (a)Subgrade reaction modulus k=500

k	b/a	t/a	$\lambda = \omega^2$					
			λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
500	1.0	0.05	29399.26	116834.05	116834.05	249807.83	357368.47	360069.10
		0.10	101617.17	373256.14	373256.14	740887.88	1030395.66	1047986.95
		0.20	278374.14	861871.29	861871.29	1565922.64	2039946.24	2095998.94
		0.30	419370.68	1157254.77	1157254.77	2023171.94	2554811.38	2633517.10
	1.5	0.05	16908.99	39205.42	95258.36	97182.57	137631.77	216897.69
		0.10	59525.60	134919.66	310099.31	319495.60	435811.88	672182.96
		0.20	172125.70	363920.36	730210.43	780426.62	997378.19	1482599.24
		0.30	270116.88	542564.98	987430.79	1090396.18	1341214.91	1958332.72
	2.0	0.05	14117.41	23213.23	45070.22	88572.09	89337.85	109659.86
		0.10	49805.40	81344.93	154982.37	292222.57	295023.93	353890.91
		0.20	145485.47	230671.31	416980.07	691060.91	740329.64	826755.76
		0.30	229437.01	357622.38	622340.64	934274.34	1054405.91	1120423.97
	3.0	0.05	12644.54	15556.30	21711.76	32937.34	51635.82	80718.74
		0.10	44641.49	54835.14	76281.37	114829.11	177546.73	272162.77
		0.20	131110.32	159211.61	217922.26	319973.27	477559.95	666155.64
		0.30	206907.81	250437.15	340325.33	491310.53	714689.63	899579.80

(b)Subgrade reaction modulus k=5000

k	b/a	t/a	$\lambda = \omega^2$					
			λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
5000	1.0	0.05	32959.36	120390.09	120390.09	253356.69	360920.13	363621.48
		0.10	102989.69	374624.18	374624.18	742255.07	1031761.35	1049354.96
		0.20	279348.85	862818.66	862818.66	1566852.97	2040867.69	2096931.61
		0.30	419933.76	1157814.27	1157814.27	2023741.71	2555370.41	2634089.82
	1.5	0.05	20469.69	42765.14	98815.54	100738.89	141186.45	220447.55
		0.10	60898.68	136289.31	311467.67	320862.26	437178.85	673548.46
		0.20	173105.70	364881.24	731160.80	781368.75	998317.44	1483525.59
		0.30	270677.14	543121.22	987986.19	1090953.82	1341775.40	1958894.07
	2.0	0.05	17678.19	26773.78	48629.62	92128.68	92895.35	113216.20
		0.10	51179.10	82716.25	156350.84	293591.18	296389.98	355258.47
		0.20	146468.75	231640.84	417933.94	692013.00	741268.89	827700.22
		0.30	229997.53	358178.09	622894.67	934828.66	1054961.53	1120980.66
	3.0	0.05	16205.33	19117.11	25272.42	36497.46	55194.78	84275.67
		0.10	46015.71	56208.14	77652.62	116198.33	178913.95	273528.27
		0.20	132096.63	160190.39	218890.84	320930.69	478506.42	667109.26
		0.30	207469.48	250994.78	340879.27	491862.47	715241.53	900133.60

al. 2007, Özdemir 2012), the aspect ratios, b/a, of the plate are taken to be 1, 1.5, and 2.0. The thickness/span ratios, t/a, are taken as 0.01, 0.05, 0.1, 0.2, and 0.3 for each aspect ratio. The shorter span length of the plate is kept constant to be 10 m. The mass density, Poisson’s ratio, and the modulus of elasticity of the plate are taken to be 2.5 kN s^2/m^2, 0.2, and 2.7×10^7 kN/m^2. Shear factor k is taken to be 5/6. The subgrade reaction modulus of the Winkler-type foundation is taken to be 500 and 5000 kN/m^3.

For the sake of accuracy in the results, rather than starting with a set of a finite element mesh size, the mesh size required to obtain the desired accuracy were determined before presenting any results. This analysis was performed separately for the mesh size. It was concluded that the results have acceptable error when equally spaced 4×4 mesh size for 17-noded elements are used for a 10 m×10 m plate. Length of the elements in the x and y directions are kept constant for different aspect ratios as in the case of square plate.

In order to illustrate that the mesh density used in this paper is enough to obtain correct results, the first six frequency parameters of the thick plate with b/a=1 and t/a=0.05 is presented in Table 1 by comparing with the result obtained SAP2000 program and the results Özgan and Daloğlu (2015). In this study Özgan and Daloğlu used 4-noded and 8-noded quadrilateral finite element with 10×10 and 5×5 mesh size. It should be noted that the results presented for MT17 element are obtained by using equally spaced 2×2 mesh size. As seen from Table 1, the results obtained by using 17-noded quadrilateral finite element have excellent agreement with the results obtained by Özgan and Daloğlu (2015) and SAP200 even if 2×2 mesh size is used for MT17 element.

3.2 Results

The first six frequency parameters of thick plate resting on Winkler foundation with free edges are compared with the same thick plate modeled by Ozgan and Daloğlu (2010) and Sap2000 program and it is presented in Table1. The subgrade reaction modulus of the Winkler-type foundation for this example is taken to be 5000 kN/m^3. This thick plate is modeled with MT17 element 2×2 mesh size for b/a=1.0, t/a=0.05 ratios.

As seen from Table 1, the values of the frequency parameters of these analyses are so close even if this study mesh size is so poor. Then writers enlarged parameters of aspect ratio, b/a, thickness/span ratio, t/a, for help the researchers.

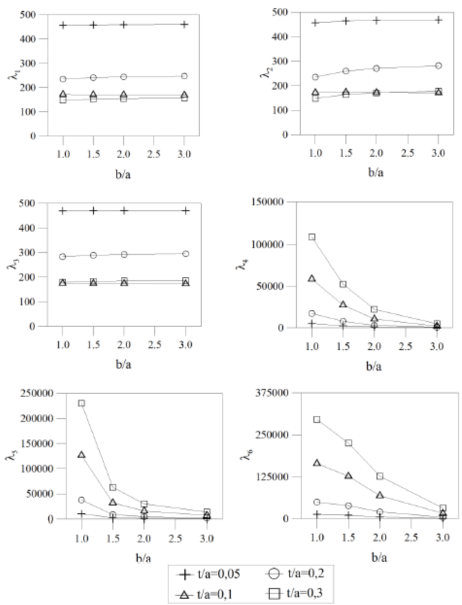

Fig. 3 Effects of aspect ratio and thickness/span ratio on the first six frequency parameters of the thick free plates with subgrade reaction modulus k=500

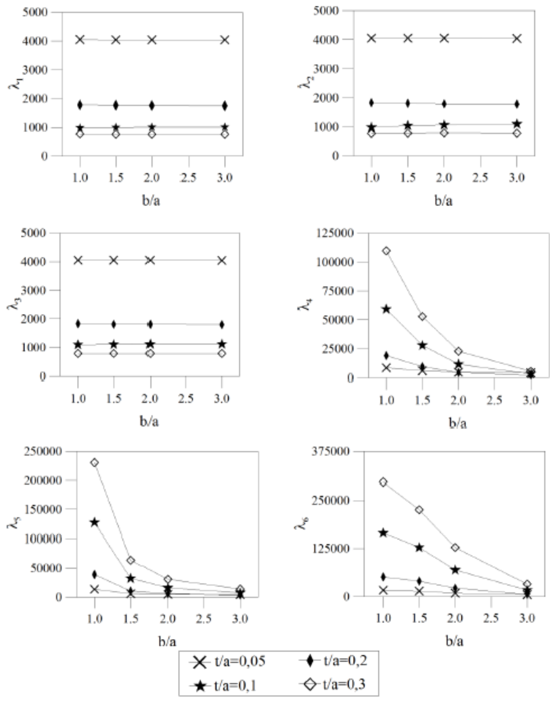

Fig. 4 Effects of aspect ratio and thickness/span ratio on the first six frequency parameters of the thick free plates with subgrade reaction modulus k=5000

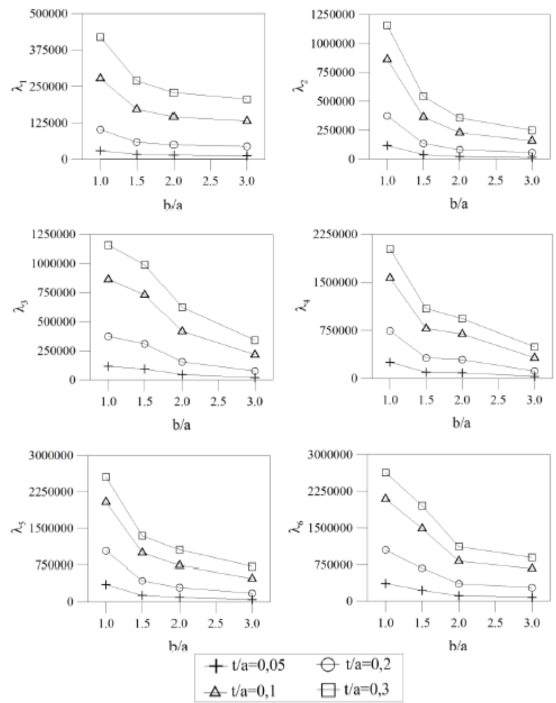

Fig. 5 Effects of aspect ratio and thickness/span ratio on the first six frequency parameters of the thick simply suppored plates with subgrade reaction modulus k=500

The first six frequency parameters of thick plates resting on Winkler foundation considered for different aspect ratio, b/a, thickness/smaller span ratio, t/a, are presented in Table 2 for the with free edges, in Table 3 for the thick simply

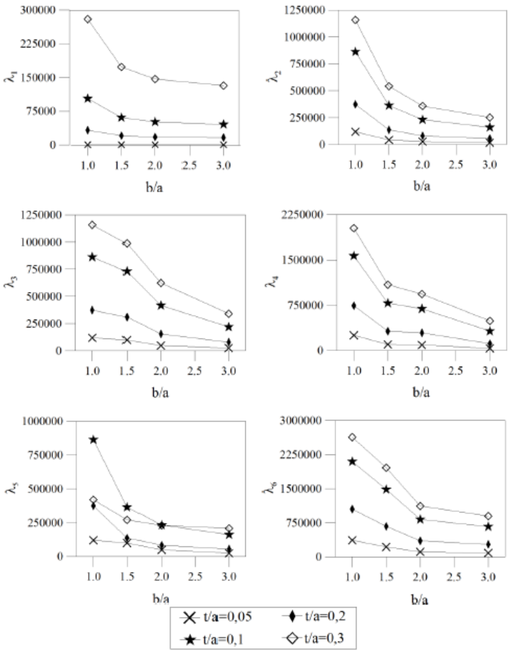

Fig. 6 Effects of aspect ratio and thickness/span ratio on the first six frequency parameters of the thick simply suppored plates with subgrade reaction modulus k=5000

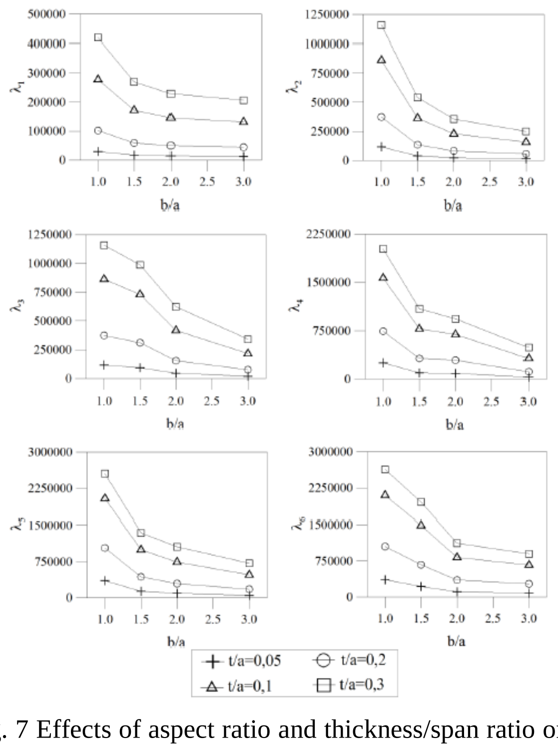

Fig. 7 Effects of aspect ratio and thickness/span ratio on the first six frequency parameters of the thick clamped plates with subgrade reaction modulus k=500

supported plates and in Table 4 for thick clamped plates. In order to see the effects of the changes in these parameters better on the first six frequency parameters, they are also presented in Figs. 3-4 for the thick free plates, in Figs. 5-6 for the thick simply supported plates and in Figs. 7-8 for the thick clamped plates.

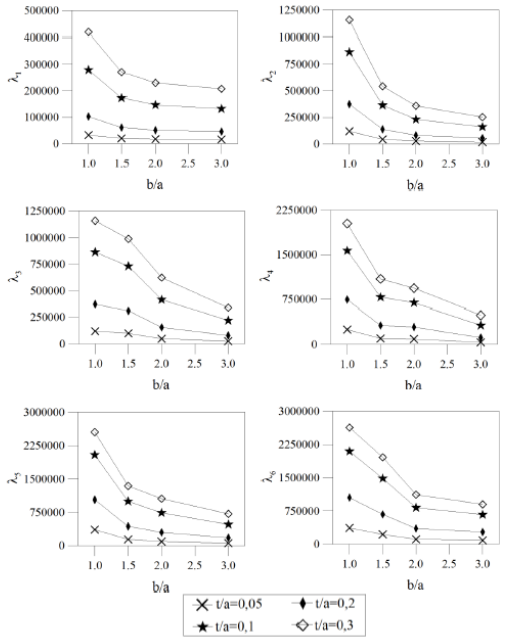

Fig. 8 Effects of aspect ratio and thickness/span ratio on the first six frequency parameters of the thick clamped plates with subgrade reaction modulus k=5000

As seen from Tables 2, and 3, and Figs. 3, and 4, the values of the first three frequency parameters for a constant value of *t/a* increase as the aspect ratio, *b*/*a*, increases up to the 3rd frequency parameters, but after the 3rd frequency parametrer, the values of the frequency parameters for a constant value of *t/a* decrease as the aspect ratio, *b*/*a*, increases.

As also seen from Tables 2, and 3, and Figs. 3, and 4, the values of the first three frequency parameters for a constant value of *b/a* decrease as the thickness/span ratio, *b*/*a*, increases up to the 3rd frequency parameters, but after the 3rd frequency parameters, the values of the frequency parameters for a constant value of *b/a* increase as the thickness/span ratio, t/*a*, increases.

The icnrease in the frequency parameters with increasing value of b/a for a constant t/a ratio gets less for larger values of b/a up to the 3rd frequency parameters. After the 3rd frequency parameters, the decrase in the frequency parameters with increasing value of b/a for a constant t/a ratio gets also less for larger values of b/a.

The changes in the frequency parameters with increasing value of b/a for a constant t/a ratio is larger for the smaller values of the b/a ratios. Also, the changes in the frequency parameters with increasing value of b/a for a constant t/a ratio is is less than that in the frequency parameters with increasing increasing t/a ratios for a constant valueof b/a.

These observations indicate that the effects of the change in the t/a ratio on the frequency parameter of the plate are generally larger than those of the change in the b/a ratios considered in this study.

As also seen from Tables 2, 3 and 4, and Figs. 3, and 4, the curves for a constant value of b/a ratio are fairly getting closer to each other as the value of t/a increases up to the 3rd

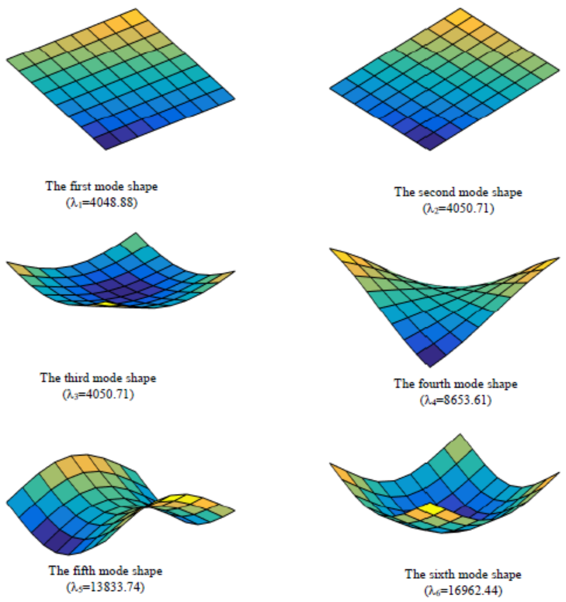

Fig. 9 The first six mode shapes of the thick free plates for b/a=1.0 and t/a=0.05 with subgrade reaction modulus k=5000

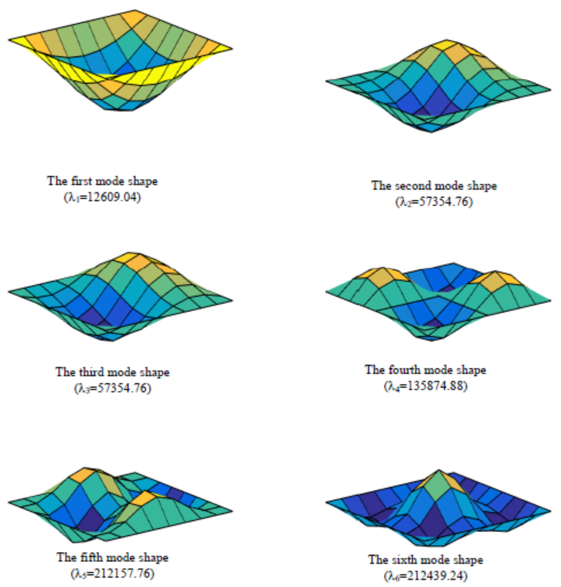

Fig. 10 The first six mode shapes of the thick simply supported plates for b/a=1.0 and t/a=0.05 with subgrade reaction modulus k=5000

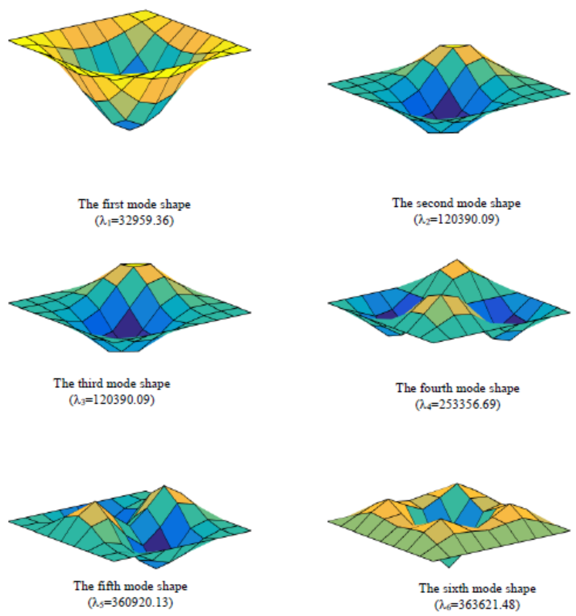

Fig. 11 The first six mode shapes of the thick clamped plates for b/a=1.0 and t/a=0.05 with subgrade reaction modulus k=5000

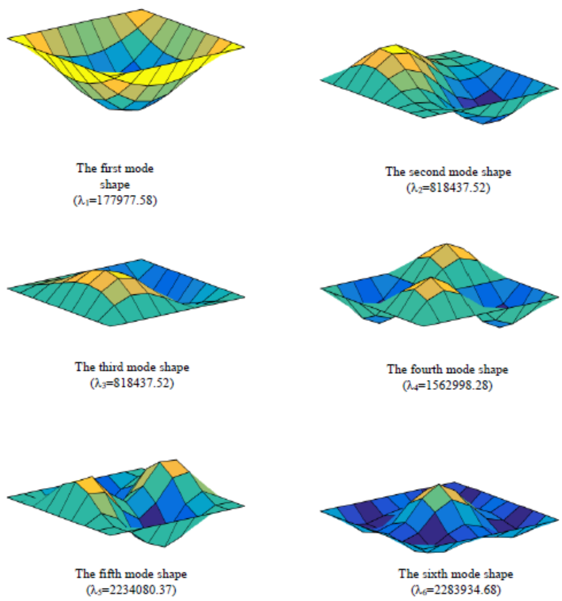

Fig. 12 The first six mode shapes of the thick simply supported plates for b/a=1.0 and t/a=0.3 with subgrade reaction modulus k=5000

frequency parameters. This shows that the curves of the frequency parameters will almost coincide with each other when the value of the ratio of t/a increases more. After the 3rd frequency parameters, the curves for a constant value of t/a ratio are fairly getting closer to each other as the value of b/a increases. This also shows that the curves of the frequency parameters will almost coincide with each other when the value of the ratio of b/a increases more.

In other words, up to the 3rd ferquency parameters, the increase in the t/a ratio will not affect the frequency parameters after a determined value of t/a. After the 3rd ferquency parameters, the increase in the b/a ratio will not affect the frequency parameters after a determined value of b/a.

As seen from Tables 2, 3, 4 and Figs. 3, 4, 5, 6, 7 and 8, the values of the frequency parameters for a constant value of t/a decrease as the aspect ratio, b/a, increases. This behavior is understandable in that a thick plate with a larger aspect ratio becomes more flexible and has smaller frequency parameters. The decreases in the frequency parameters with increasing value of b/a ratio gets less for a constant value of t/a.

As seen from Tables 2, 3, 4 and Figs. 3, 4, 5, 6, 7 and 8, the values of the frequency parameters for a constant value of b/a increase as the thickness/span ratio, b/a, increases.

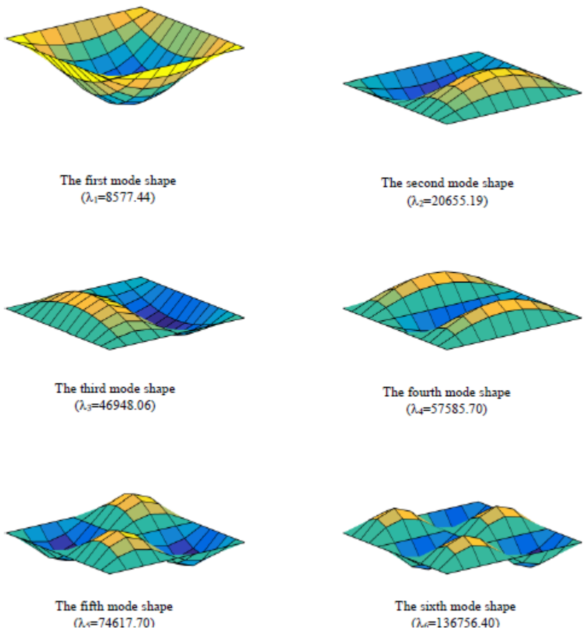

Fig. 13 The first six mode shapes of the thick simply supported plates for b/a=1.5 and t/a=0.05 with subgrade reaction modulus k=5000

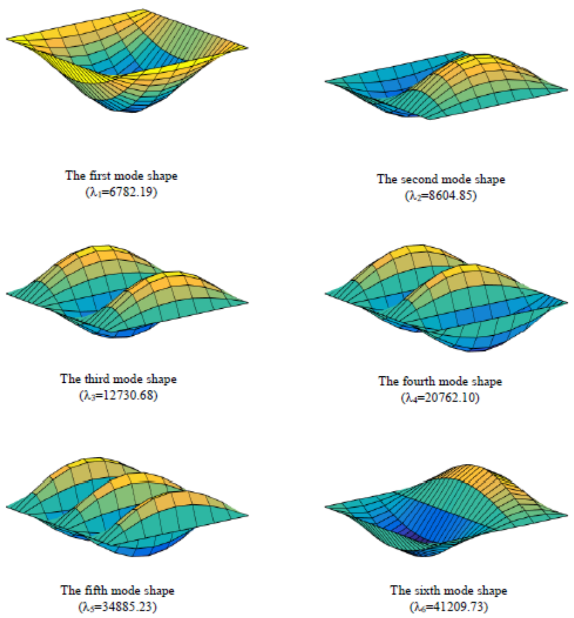

Fig. 14 The first six mode shapes of the thick simply supported plates for b/a=3.0 and t/a=0.05 with subgrade reaction modulus k=5000

This behavior is also understandable in that a thick plate with a larger thickness/span ratio becomes more rigid and has larger frequency parameters. The increases in the frequency parameters with increasing value of t/a ratio gets larger for a constant value of b/a.

It should be noted that the increase in the frequency parameters with increasing *t/a* ratios for a constant value of *b/a* ratio gets larger for larger values of the frequency parameters.

These observations indicate that the effects of the change in the *t/a* ratio on the frequency parameter of the thick plates simply supported or clamped along all four edges are always larger than those of the change in the aspect ratio.

As also seen from Figs. 3, 4, 5, 6, 7 and 8, the curves for a constant value of the aspect ratio, *b/a* are fairly getting closer to each other as the value of *t/a* decreases. This shows that the curves of the frequency parameters will almost coincide with each other when the value of the thickness/span ratio, *t/a*, decreases more. In other words, the decrease in the thickness/span ratio will not affect the frequency parameters after a determined value of *t/a*.

In this study, the mode shapes of the thick plates are also obtained for all parameters considered. Since presentation of all of these mode shapes would take up excessive space, only the mode shapes corresponding to the six lowest frequency parameters of the thick plate free, simply supported, clamped along all four edges for *b/a* = 1, 1.5, and 3 and *t/a* = 0.05, 0.3 are presented. These mode shapes are given in Figs. 9, 10, 11, 12, 13 and 14, respectively. In order to make the visibility better, the mode shapes are plotted with exaggerated amplitudes.

As seen from these figures, the number of half wave increases as the mode number increases. It should be noted that appearances of the mode shapes not given here for the thick plates clamped along all four edges are similar to those of the mode shapes presented here.

4. Conclusions

The purpose of this paper was to study parametric free vibration analysis of thick plates using higher order finite elements with Mindlin's theory and to determine the effects of the thickness/span ratio, the aspect ratio and the boundary conditions on the linear responses of thick plates subjected to vibration. As a result, free vibration analyze of the thick plates were done by using p version serendipity element, and the coded program on the purpose is effectively used. In addition, the following conclusions can also be drawn from the results obtained in this study.

The frequency parameters increase with increasing b/a ratio for a constant value of t/a up to the 3rd ferquency parameters, but after that the frequency parameters decrases with increasing b/a ratio for a constant value of t/a.

The frequency parameters decrease with increasing t/a ratio for a constant value of b/a up to the 3rd ferquency parameters, but after that the frequency parameters increases with increasing t/a ratio for a constant value of b/a.

The effects of the change in the t/a ratio on the frequency parameter of the thick plate are generally larger than those of the change in the b/a ratios considered in this study.

References

Ayvaz, Y. and Durmuş, A. (1995), "Earhquake analysis of simply supported reinforced concrete slabs", *J. Sound Vibr*., **187**(3), 531-539.

Bathe, K.J. (1996), *Finite Element Procedures*, Prentice Hall, Upper Saddle River, New Jersey, U.S.A.

Belounar, L. and Guenfound, M. (2005), "A new rectangular finite element based on the strain approach for plate bending", *Thin-Wall*. *Struct*., **43**, 47-63.

Bergan, P.G. and Wang, X. (1984), "Quadrilateral plate bending elements with shear deformations", *Comput*. *Struct*., **19**(1-2) 25-34.

Brezzi, F. and Marini, L.D. (2003), "A nonconforming element for the Reissner-Mindlin plate", *Comput*. *Struct*., **81**, 515-522.

Caldersmith, G.W. (1984), "Vibrations of orthotropic rectangular plates", *ACUSTICA*, **56**, 144-152.

Cen, S., Long, Y.Q., Yao, Z.H. and Chiew, S.P. (2006), "Application of the quadrilateral area co-ordinate method: Anew element for Mindlin-Reissner plate", *J*. *Numer*. *Meth*. *Eng*., **66**, 1-45.

Cook, R.D., Malkus, D.S. and Michael, E.P. (1989), *Concepts and Applications of Finite Element Analysis*, John Wiley & Sons, Inc., Canada.

Fallah, A., Aghdam, M.M. and Kargarnovin, M.H. (2013), "Free vibration analysis of moderately thick functionally graded plates on elastic foundation using the extended Kantorovich method", *Arch*. *Appl*. *Mech*., **83**(2), 177-191.

Grice, R.M. and Pinnington, R.J. (2002), "Analysis of the flexural vibration of a thin-plate box using a combination of finite element analysis and analytical impedances", *J*. *Sound Vibr*., **249**(3), 499-527.

Gunagpeng, Z., Tianxia, Z. and Yaohui, S. (2012), "Free vibration analysis of plates on Winkler elastic foundation by boundary element method", *Opt*. *Electr*. *Mater*. *Appl*. *II*, **529**, 246-251.

Hinton, E. and Huang, H.C. (1986), "A family of quadrilateral Mindlin plate element with substitute shear strain fields", *Comput*. *Struct*., **23**(3), 409-431.

Hughes, T.J.R., Taylor, R.L. and Kalcjai, W. (1977), "Simple and efficient element for plate bending", *J*. *Numer*. *Meth*. *Eng*., **11**, 1529-1543.

Jahromim, H.N., Aghdam, M.M. and Fallah, A. (2013), "Free vibration analysis of Mindlin plates partially resting on Pasternak foundation", *J*. *Mech*. *Sci*., **75**, 1-7.

Leissa, A.W. (1973), "The free vibration of rectangular plates", *J*. *Sound Vibr*., **31**(3), 257-294.

Leissa, A.W. (1981), "Plate vibration research, 1976-1980: complicating effects", *Shock Vibr*. *Dig*., **13**(10) 19-36.

Leissa, A.W (1987), "Plate vibration research, 1981-1985-part II: Complicating effects", *Shock Vibr*. *Dig*., **19**(3), 10-24.

Leissa, A.W. (1977), "Recent research in plate vibrations, 1973-1976: Complicating effects", *Shock Vibr*. *Dig*., **9**(11), 21-35.

Leissa, A.W. (1977), "Recent research in plate vibrations", 1973-1976: classical theory, *Shock Vibr*. *Dig*., **9**(10), 13-24.

Leissa, A.W. (1981), "Plate vibration research, 1976-1980: Classical theory", *Shock Vibr*. *Dig*., **13**(9), 11-22.

Leissa, A.W. (1987), "Plate vibration research, 1981-1985-part I: Classical theory", *Shock Vibr*. *Dig*., **19**(2), 11-18.

Lok, T.S. and Cheng, Q.H. (2001), "Free and forced vibration of simply supported, orthotropic sandwich panel", *Comput*. *Struct*., **79**(3), 301-312.

Mindlin, R.D. (1951), "Influence of rotatory inertia and shear on flexural motions of isotropic, elastic plates", *J*. *Appl*. *Mech*., **18**(1), 31-38.

Özdemir, Y.I. (2012), "Development of a higher order finite element on a Winkler foundation", *Fin*. *Elem*. *Anal*. *Des*., **48**, 1400-1408.

Özdemir, Y.I. and Ayvaz, Y. (2009), "Shear locking-free earthquake analysis of thick and thin plates using Mindlin's theory", *Struct*. *Eng*. *Mech*., **33**(3), 373-385.

Özdemir, Y.I., Bekiroğlu, S. and Ayvaz, Y. (2007), "Shear locking-free analysis of thick plates using Mindlin's theory", *Struct*. *Eng*. *Mech*., **27**(3), 311-331.

Özgan, K. and Daloglu, A.T. (2012), "Free vibration analysis of thick plates on elastic foundations using modified Vlasov model with higher order finite elements", *J*. *Eng*. *Mater*. *Sci*., **19**, 279-291.

Özgan, K. and Daloglu, A.T. (2015), "Free vibration analysis of thick plates resting on Winkler elastic foundation", *Chall*. *J*. *Struct*. *Mech*., **1**(2), 78-83.

Özkul, T.A. and Ture, U. (2004), "The transition from thin plates to moderately thick plates by using finite element analysis and the shear locking problem", *Thin-Wall*. *Struct*., **42**, 1405-1430.

Providakis, C.P. and Beskos, D.E. (1989), "Free and forced vibrations of plates by boundary and interior elements", *J*. *Numer*. *Meth*. *Eng*., **28**, 1977-1994.

Providakis, C.P. and Beskos, D.E. (1989), "Free and forced vibrations of plates by boundary elements", *Comput*. *Meth*. *Appl*. *Mech*. *Eng*., **74**, 231-250.

Qian, R.C., Batra, L.M. and Chen. (2003), "Free and forced vibration of thick rectangular plates using higher-order shear and normal deformable plate theory and meshless Petrov-Galerkin (MLPG) method", *Comput*. *Model*. *Eng*. *Sci*., **4**(5), 519-534.

Raju, K.K. and Hinton, E. (1980), "Natural frequencies and modes of rhombic Mindlin plates", *Earhq*. *Eng*. *Struct*. *Dyn*., **8**, 55-62.

Reissner, E. (1945), "The effect of transverse shear deformation on the bending of elastic plates", *J*. *Appl*. *Mech*., **12**, A69-A77.

Reissner, E. (1947), "On bending of elastic plates", *Quarter*. *Appl*. *Math*., **5**(1), 55-68.

Reissner, E. (1950), "On a variational theorem in elasticity", *J*. *Math*. *Phys*., **29**, 90-95.

Sakata, T. and Hosokawa, K. (1988), "Vibrations of clamped orthotropic rectangular plates", *J*. *Sound Vibr*., **125**(3), 429-439.

Shen, H.S., Yang, J. and Zhang, L. (2001), "Free and forced vibration of Reissner-Mindlin plates with free edges resting on elastic foundation", *J*. *Sound Vibr*., **244**(2), 299-320.

Si, W.J., Lam, K.Y. and Gang, S.W. (2005), "Vibration analysis of rectangular plates with one or more guided edges via bicubic B-spline method", *Shock Vibr*., **12**(5).

Soh, A.K., Cen, S., Long, Y. and Long, Z. (2001), "A new twelve DOF quadrilateral element for analysis of thick and thin plates", *Eur*. *J*. *Mech*. *A/Sol*., **20**, 299-326.

Tedesco, J.W., McDougal, W.G. and Ross, C.A. (1999), *Structural Dynamics*, Addison Wesley Longman Inc., California, U.S.A.

Ugural, A.C. (1981), *Stresses in Plates and Shells*, McGraw-Hill, New York, U.S.A.

Wanji, C. and Cheung, Y.K. (2000), "Refined quadrilateral element based on Mindlin/Reissner plate theory", *J*. *Numer*. *Meth*. *Eng*., **47**, 605-627.

Warburton, G.B. (1954), "The vibration of rectangular plates", *Proceedings of the İnstitude of Mechanical Engineers*, **168**, 371-384.

Weaver, W. and Johnston, P.R. (1984), *Finite Elements for Structural Analysis*, Prentice Hall, Inc., Englewood Cliffs, New Jersey, U.S.A.

Woo, K.S., Hong, C.H., Basu, P.K. and Seo, C.G. (2003), "Free vibration of skew Mindlin plates by p-version of F.E.M.", *J*. *Sound Vibr*., **268**, 637-656.

Zienkiewich, O.C., Taylor, R.L. and Too, J.M. (1971), "Reduced integration technique in general analysis of plates and shells", *J*. *Numer*. *Meth*. *Eng*., **3**, 275-290.

CC