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Flexural fatigue modeling of short fibers/epoxy composites

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Abstract. In the present research, an available flexural stiffness degradation model was modified and a new comprehensive model called "*X-NFSD*" was developed. The *X-NFSD* model is capable of predicting the flexural stiffness degradation of composite specimen at different states of stresses and at room temperature. The model was verified by means of different experimental data for chopped strand mat/epoxy composites under displacement controlled bending loading condition at different displacements and states of stresses. The obtained results provided by the present model are impressively in very good agreement with the experimental data and the mean value of error of 5.4% was achieved.

Keywords: flexural fatigue; displacement controlled loading; chopped strand mat/epoxy composites; degradation; composites

1. Introduction

There are two main methods for testing the fatigue behavior of materials; namely, load and displacement control conditions. In the load control method, the applied load in the test instrument will be set and continuously recorded by using a load cell and displacement will be changed based on the material degradation phenomenon. While, in the displacement control method the specimen is subjected to a known displacement and variable loadings. Amongst the various types of the composites, chopped strand mat (CSM) composites have many applications in various industries due to their intrinsic properties. In many applications, the CSM polymeric composites are under fatigue loading conditions. The fatigue life of CSM polymeric composites have been investigated by some researchers (Mandell 1990, Caprino and Amore 1998, Shokrieh *et al*. 2014a, Mortazavian and Fatemi 2015). The modeling approaches for the prediction of the fatigue life for composite materials can be categorized in three groups: residual strength, residual stiffness and empirical methodologies. For instance, Caprino and Amore (1998) investigated the flexural fatigue behavior of random continuous-fiber reinforced thermoplastic composites and the data obtained were used to assess the prediction of the fatigue response of a composite material. In residual stiffness models, the stiffness of the specimen is recorded during the test and the residual stiffness of the specimen is used as an indicator for the failure. Sidoroff and Subagio (1987) proposed a damage growth rate model and applied it to results of three-point bending tests on glass/epoxy unidirectional composites. Paepegem and Degrieck (2001a) developed an experimental setup for bending fatigue loading and they adopted a residual stiffness model which described the fatigue damage behavior of the composite material (2002). Baere *et al*. (2009) performed a comparison for different setups for fatigue testing of thin composites laminates. Also, Paepegem *et al*. (2001b) used a finite element approach based on a model represented by Sidoroff and Subagio for prediction of composites fatigue life. Paepegem and Degrieck (2001c) developed a model for fatigue damage behavior of fiber-reinforced polymer composites. Naderi and Khonsari (2011) analyzed the fatigue failure in composite laminates using a thermodynamic approach. Koricho *et al*. (2014) investigated the fatigue behavior of twill *E*-glass/epoxy composite under displacement controlled bending fatigue tests with stress ratio R of 0.1. Catangiu *et al*. (2011) introduced an exponential flexural degradation model for composite plates of orthogonal woven glass fibers and epoxy resin at different cut-out angels. Epaarachchi and Clausen (2003) studied the fatigue behavior of glass fiber-reinforced plastic composites for various stress ratios and frequencies. Tarar *et al*. (2010) developed an energy based finite element method for fatigue life prediction under uniaxial and bending loading conditions. In the present work, a new exponential normalized flexural stiffness degradation method was utilized for CSM/epoxy composites to predict the fatigue life. The model was developed based on the model presented by Catangiu *et al*. (2011) for a specific material, such as isotropic composites. Moreover, the capability of the model for predicting of the fatigue behavior of polymer CSM/epoxy composites was investigated. A series of displacement controlled flexural fatigue tests at room temperature and different load levels were performed and the material constants of the model

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were obtained.

2. Analytical model

Catangiu *et al*. (2011) introduced an exponential flexural degradation model for composite plates of orthogonal woven glass fibers and epoxy resin at different cut-out angels. Based on their assumptions, the evolution of the damage parameter can be divided in two separate regions at a specific critical number of fatigue cycles. They stated that the critical number of fatigue cycle is the number of cycles in which the damage evolution factor (D) is equal to 0.2. The damage evolution factor (D) is defined by the following expression

$$D = 1 - \frac{E}{E_s} \tag{1}$$

where E is the current stiffness and E_s is the initial static stiffness of the material.

Moreover, the relation between the stiffness degradation and fatigue number of cycles was utilized by them and the exponential stiffness degradation expression was defined as

$$E(n) = A.e^{-m.n} \tag{2}$$

where A and m are material constants that can be determined by testing specimens at various stress levels. In addition, $E(n)$ is the stiffness of composites after n cycles.

2.1 Exponential normalized flexural stiffness degradation model (X-NFSD) for nanoparticle/fibrous polymeric composites

In the present study, an exponential normalized flexural stiffness degradation model called *X-NFSD* is developed. The model predicts the residual stiffness of the specimen by the following expression

$$E(n) = aE^0 e^{-m.n} \tag{3}$$

where a is the material constant and E^0 is the static flexural stiffness of composite materials.

The normalized flexural stiffness can be expressed as follows

$$\overline{E}(n) = \frac{E(n)}{E^0} = ae^{-m.n} \tag{4}$$

where $\overline{E}(n)$ is the normalized flexural stiffness of composites after n cycles under flexural fatigue loading conditions.

The formerly existing model, introduced by Catangiu *et al*. (2011) was developed for predicting the flexural stiffness degradation behavior of unidirectional composites at one state of stress. The model was able to model the effect of cut-out angles in unidirectional composites under displacement controlled flexural fatigue testing conditions. On the other hand, the present model is able to predict the flexural stiffness degradation of isotropic composite specimens at different states of applied stress. This is due to the normalized material constant, which is used in the model. Moreover, this model was applied to predict the flexural behavior of the chopped strand mat/epoxy composites under displacement controlled bending loading conditions at different displacements, namely different stress levels.

Table 1 Specifications of *E*-glass CSM fiber

Parameter	Value
Fiber diameter (μm)	10-20
Nominal weight (g/m^2)	450
Density (kg/m^3)	2500
Poisson's ratio	0.27

Table 2 Properties of ML 526 epoxy resin

Physical Properties		Mechanical Properties	
Viscosity at 25℃ (Centipoise)	Glass temperature (℃)	Flexural modulus (GPa)	Flexural strength (MPa)
1190	72	3.00±0.02	110±1.95

3. Experimental study

3.1 Material

ML-526 epoxy resin is based on Bisphenol-*A*. The curing agent is HA-11 (Polyamine). The resin and hardener were supplied by Mokarrar Engineering Materials Company, Iran. The flexural stiffness and strength of epoxy resin were measured 3 GPa and 110 MPa, respectively (Shokrieh *et al*. 2014b). The *E*-glass fabric in form of chopped-strand mat (CSM) was supplied by Taishan Fiberglass Inc., China. The random distributed fibers have an average diameter of approximately 13 micron; around 5 mm length and an area density of 450 g/m^2. The specifications of both the *E*-glass CSM and the resin are presented in Tables 1-2.

3.2 Specimen preparation

For the fabrication of the specimens, the hardener was added to the epoxy resin at a ratio of 15:100 and stirred gently by using a mechanical stirrer (Heidolph RZR2102) for 5 min at 100 rpm. The stirring speed was low in order to avoid undesirable bubble formation. For the manufacturing of the CSM/epoxy composite specimens using the hand layup process, six layers of E-glass CSM were cut into a sheet. Then, layers were stacked with ML-526 epoxy resin and impregnated at room temperature. A roller was used in order to release the trapped air and voids. Later, samples were kept under 500 N static loading to get trapped bubbles out. The fabricated sheet was also pre-cured for 48 hr. For the post-curing process, the fabricated sheet was placed into an oven for 2 hr at 80℃ and further 1 hr at 110℃. The test specimens are shown in Fig. 1, were cut in accordance with the B593-96 ASTM Standard (2003) by the water jet cutting process. The flexural stiffness and strength of CSM/Epoxy composites were obtained 9.81 GPa and 241 MPa, respectively.

Fig. 1 CSM/epoxy composite specimens based on B593-96 ASTM Standard (2003)

Fig. 2 The experimental setup for the displacement-controlled flexural fatigue loading (BFM-110)

3.3 Test setup

The Santam universal testing machine STM-150 was utilized to perform flexural tests in accordance with the ASTM D790 (2010). The cross-head speed for bending tests was 16 mm/min. In order to perform the flexural fatigue tests, BFM-110 test apparatus shown in Fig. 2 was used. The CSM/Epoxy composite specimens were mounted into a fixed cantilever, constant deflection type fatigue testing machine. The specimen is held at one end, acting as a cantilever beam and cycled until a complete failure is achieved. Generally, the shaft of the motor has a rotational speed of 0-1450 rpm. The power is transmitted by a *V*-belt to the second shaft, which provides a fatigue testing frequency between 2-20 Hz and gives the possibility to investigate the influence of the frequency in this range of values. The power transmission through a *V*-belt ensures that the motor and the measurement system are electrically isolated. The second shaft bears a crank-linkage mechanism.

This mechanism imposes an alternating displacement on the hinge that connects the linkage with the lower clamp of the CSM/Epoxy composite specimen. At the upper end of the machine, the specimen is clamped. Hence the sample is loaded as a cantilever beam. The schematic picture of the clamping procedure is shown in Fig. 3.

The amplitude of the imposed displacement is a

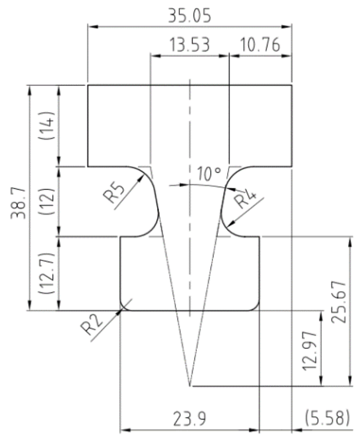

Fig. 3 Drawing of the flexural fatigue specimens (dimensions in mm)

controllable parameter and the adjustable crank allows choosing between single-sided and fully reversed bending, i.e., the deflection can vary from zero to a maximum deflection in one direction, or in two opposite directions, respectively.

The maximum deflection is measured by a displacement dial gauge at the back of the lower clamp. The number of fatigue cycles should be counted directly for each test specimen by PES-R18PO3MD reflector speed sensor supplied by IBEST electric, LTD., China. The counting signal was transferred to the counter fabricated by RASAM Madar Electronic Company, Iran. In this setup there are two parallel test stands with counting system implemented separately. To stop the counter of speed sensors, at the bottom of each specimen a thin wire as an electrical contact are used and after failure, the damaged specimen drops down and disconnects the wire and leads to stopping the counting system. Therefore, after failure of both specimens, control system acts and turns off the main current of the machine completely. It is worth mentioning that CSM/epoxy composite specimen bears stiffness degradation under the displacement controlled fatigue loading condition, In order to calculate the residual stiffness of the specimen, a 50 kg force load-cell is mounted on the linkage (Fig. 2). The DBBP-50 load-cell was supplied by Boshing Company, China. After a specific number of cycles, the maximum force induced on the specimen is recorded and the residual stiffness of the specimen is calculated.

3.3.1 Calculation of the bending stress and residual stiffness

In this research, high cycle fatigue properties of CSM/Epoxy composites are measured using a modified cantilever beam bending test. A typical fatigue life test specimen in the current research, presented in Fig. 3, is designed based on the B593-96 ASTM Standard (2003) and the method performed by Ramkumar and Gnanamoorthy (2010) and Rajeesh *et al*. (2010). The gauge length of the specimen is designed based on the stress concentration concept. The wide end of the specimen is clamped to a bed plate, while the narrow end is cyclically deflected (as displayed in Fig. 4).

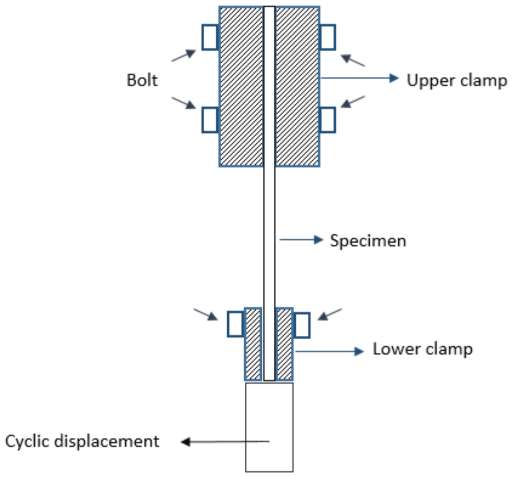

Fig. 4 Schematic of specimen clamping procedure (Shokrieh *et al.* 2014c)

To obtain reliable results, the surface area available for crack initiation must not be too small. If the cross section of a beam is symmetrical with respect to its neutral plane, then the maximum tension or compression stress at a given cross section for small displacements within the elastic deformation behavior is calculated according to the following equation

$$\sigma_{\max} = \frac{M \times H}{2 \times I} \tag{5}$$

where $\sigma_{\max}$ is the maximum stress, M is the local bending moment, H is the thickness of the beam, I is the second moment of area of the cross section.

According to Fig. 5, the specimen is a wedge-shaped beam and the cross section is not uniform and defined by means of a parameter called "local B". Therefore, the magnitude of the second moment of area of the cross section depends on the position along the x-axis.

$$B(x) = \frac{B_0}{L_0} \times (L_0 - x) \tag{6}$$

$$I(x) = \frac{B_0 (L_0 - x) H^3}{12 L_0} \tag{7}$$

where, L_0 is its overall length and B_0 is the width at the base of the wedge-shaped beam. The maximum tension or compression stress at a given cross section for small displacements within the elastic deformation behavior is calculated according to the following equation (Berchem and Hocking 2006)

$$\sigma_{\max} = \frac{-z_0.E.H}{L_0^{\ 2}} \tag{8}$$

where, z_0 is the displacement at point x=L_0. The curve of the neutral plane in a bent beam is described by the differential equation of the deflection curve. The well-known relationship between the deflection z and the applied moment M in the classical beam theory is expressed as the following equation

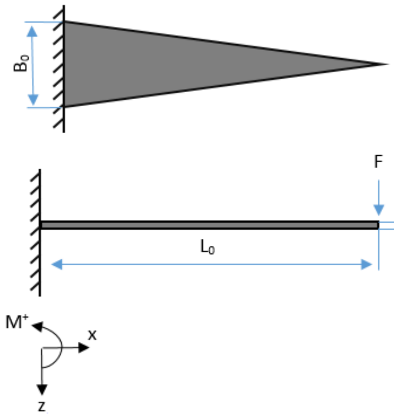

Fig. 5 Schematic view of a beam with coordinates (Shokrieh *et al.* 2014c)

Table 3 Properties of CSM/epoxy composites

Mechanical Properties		
Young's modulus (GPa)	Ultimate tensile strength (MPa)	*E*-glass CSM weight fraction (%)
9.8±0.2	248±15.1	50.5

$$\frac{d^2 z}{dx^2} = -\frac{M}{EI} \tag{9}$$

By integrating Eq. (9) and applying the boundary conditions, relationship between the applied force and the residual modulus of elasticity is as the following equation in terms of the number of fatigue cycles

$$E(n) = \frac{6F(n)L_0^{\ 3}}{z_0 B_0 H^3} \tag{10}$$

where $E(n)$ is the residual stiffness and $F(n)$ is the applied load on the linkage of test machine and can be measured by the load-cell.

4. Results and discussions

4.1 Static and bending strength

The flexural tests were carried out in accordance with the ASTM D790 standard (2010) and cross-head speed was set at 16 mm/min. The dimensions of standard flexural specimens are 80×10×3.6 mm. The measured flexural stiffness and strength for the CSM/epoxy composites were obtained 9.81 GPa and 248 MPa, respectively. The characterization of the CSM/epoxy composite specimen is shown in Table 3.

4.2 Cyclic bending fatigue stiffness degradation

The performance of CSM/epoxy composites under flexural fatigue loading condition were examined based on the ASTM B593-96 (2003) and the results presented by Ramkumar and Gnanamoorthy (2010).

The tests were performed at the state of stress equal to

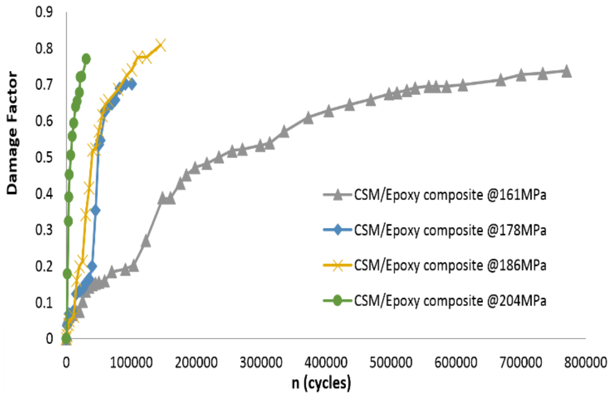

Fig. 6 Damage evolution factor versus number of fatigue cycles at different states of stress

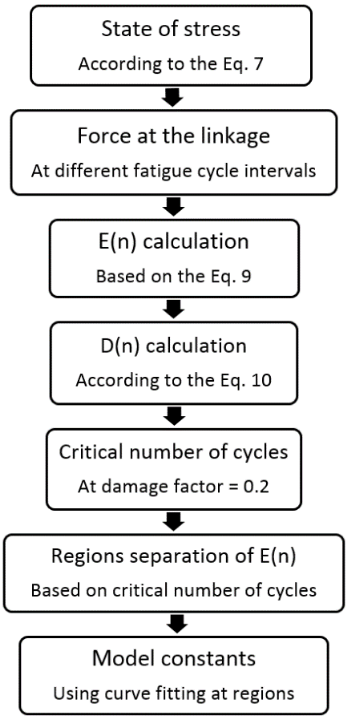

Fig. 7 The flowchart of the model

161, 178, 186 and 204 MPa and the frequency was equal to 5 Hz. During the test, at different number of cycle intervals, the maximum applied force on the linkage was measured and recorded using a load-cell. Therefore, the residual stiffness can be found using Eq. (10).

According to the performed research by Catangiu *et al*. (2011) the damage evolution factor versus number of fatigue cycles is shown in Fig. 6 and classified in two specified stages of degradation. In the first stage, due to dramatic decreasing of residual stiffness, damage factor is increased substantially. But, in the second stage, the damage factor is increased and changed slightly. Based on the results provided by Catangiu *et al*. (2011), at the transition between the two stages, the damage factor was assumed to be equal to 0.2 as a criterion. In the current study, this assumption was also applied in the *X-NFSD* model.

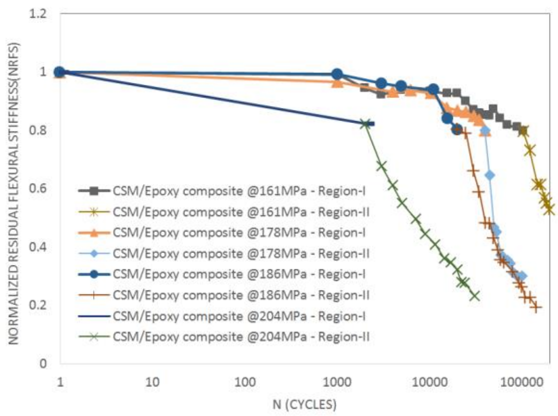

Fig. 8 Normalized flexural stiffness degradation behavior versus number of cycles at different states of stress

Table 4 The material constants of the model in two regions and critical number of cycles for different sates of stress

State of Stress (MPa)	Region 1			Region 2			Critical number of cycles.
	a	*A*	*m*	*a*	*A*	*m*	
161	0.950051	8.5103	-2.00E-06	0.7482	7.339842	-2.00E-06	104000
178	0.967472	8.6747	-5.00E-06	0.8341	8.182521	-1.00E-05	39783
186	1.0010398	9.0031	-1.00E-05	1.076	10.55556	-1.00E-05	20200
204	1.0001019	8.9672	-1.00E-04	0.7093	6.958233	-4.00E-05	2000

Table 5 Material constants of the *X-NFSD* mode, first region

Material	Material Constants of the *X-NFSD* Model	
	$\bar{a}_{comp}$	$\bar{m}_{comp}$
CSM/Epoxy composites	0.9409	-2E-06
Normalized stiffness equation	$\bar{E}_{comp}(n) = 0.9409\,.e^{-2E-6.n}$	

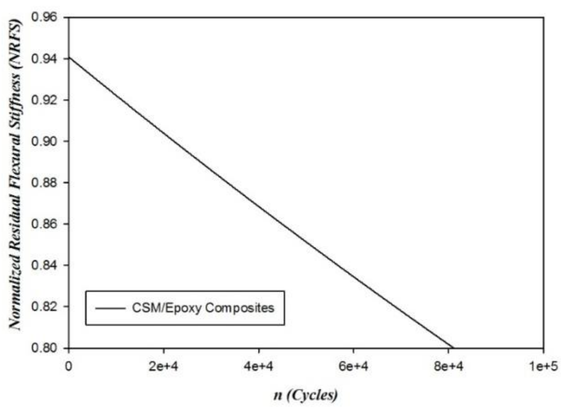

Fig. 9 Normalized residual flexural stiffness (NRFS) degradation behavior versus number of fatigue cycles

By means of this criterion, the stiffness degradation curves are divided into two regions. Based on the flowchart of model (see Fig. 7) for both regions, the normalized residual stiffness curves of two regions were depicted in Fig. 8 and material constants of the model in two regions

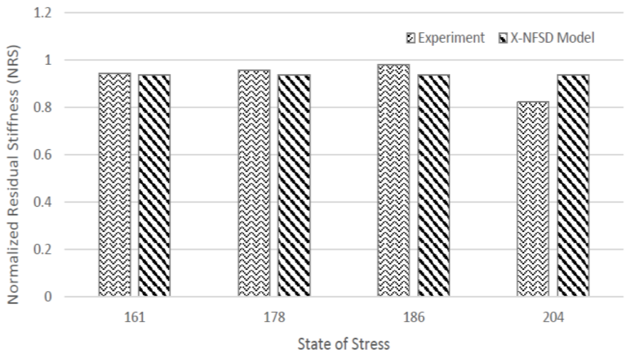

Fig. 10 Normalized residual flexural stiffness (NRFS) versus experimental data at different states of applied stress predicted by *X-NFSD* model

for different states of stress were summarized in Table 4.

The mean value of the parameter "*a*" for the first and second regions were obtained equal to 0.9409 and 0.8861, respectively. Moreover, the mean value of the parameter "*m*" for first and second regions were obtained about -2E-06 and -7.33E-06, respectively. Furthermore, Table 5 shows material constants of the *X-NFSD* model through the first region. Consequently, the normalized residual flexural stiffness (NRFS) degradation behavior versus number of fatigue cycles is represented in Fig. 9.

Fig. 10 shows that the results of model are in a good agreement with those of the experimental observations and the mean value of the error is 4.6%.

5. Conclusions

In the present study, a new normalized flexural stiffness degradation model, namely *X-NFSD* is developed. This model is capable of predicting the flexural stiffness degradation of composite specimen at different state of stresses. The formerly existing model was capable of predicting the flexural stiffness degradation behavior at one state of stress and was used to predict the flexural stiffness degradation of the unidirectional composites. While the newly developed *X-NFSD* model predicts the flexural stiffness degradation of composite specimen at different states of applied stress. Then, by means of different experimental data obtained from BFM-110 test setup for chopped strand mat/epoxy composites under displacement controlled bending loading conditions at different displacements, the developed model was verified. The damage factor versus number of cycles was divided in two regions and all parameters for the *X-NFSD* model were calculated. A comparison of the results obtained by the present model and experimental results show a very good agreement and a mean value of the error about 4.6% was observed.

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