**Structural Engineering and Mechanics**, *Vol. 63, No. 6 (2017) 789-797*
DOI: https://doi.org/10.12989/sem.2017.63.6.789


# On-line integration of structural identification/damage detection and structural reliability evaluation of stochastic building structures

Ying Lei[*1], Longfei Wang[1a], Lanxin Lu[1b] and Dandan Xia[2c]

[1]*School of Architecture and Civil Engineering, Xiamen University, Xiamen, 361005, China*
[2]*School of Civil & Architecture Engineering, Xiamen University of Technology, Xiamen, 361024, China*

*(Received April 5, 2017, Revised May 23, 2017, Accepted June 16, 2017)*

**Abstract.** Recently, some integrated structural identification/damage detection and reliability evaluation of structures with uncertainties have been proposed. However, these techniques are applicable for off-line synthesis of structural identification and reliability evaluation. In this paper, based on the recursive formulation of the extended Kalman filter, an on-line integration of structural identification/damage detection and reliability evaluation of stochastic building structures is investigated. Structural limit state is expanded by the Taylor series in terms of uncertain variables to obtain the probability density function (PDF). Both structural component reliability with only one limit state function and system reliability with multi-limit state functions are studied. Then, it is extended to adopt the recent extended Kalman filter with unknown input (EKF-UI) proposed by the authors for on-line integration of structural identification/damage detection and structural reliability evaluation of stochastic building structures subject to unknown excitations. Numerical examples are used to demonstrate the proposed method. The evaluated results of structural component reliability and structural system reliability are compared with those by the Monte Carlo simulation to validate the performances of the proposed method.

**Keywords:** structural identification; damage detection; uncertainties; probability; reliability evaluation; on-line; Integration; extended Kalman filter; partial measurements

## 1. Introduction

It is well known that structural system identification /damage detection and structural reliability evaluation are two important issues to ensure the serviceability and safety of structures (Yao and Natke 1994, Doeblin *et al*. 1998, Wong and Yao 2001, Li and Chen 2006, Ou and Li 2010, Fan and Qiao 2011, Li and Chen 2013). However, in most of previous investigations, structural system identification/damage detection and reliability evaluation were investigated separately. When uncertainties which are inevitably involved in civil structures are taken into account, the identified structural parameters are random parameters (Li and Law 2008). Under some circumstance, it is prohibitive to evaluate structural reliability by the current reliability analysis methods due to the lack of knowledge of the uncertainty propagation. Therefore, the methods of assessing structural reliability in conjunction with the structural identification of stochastic building structures are indeed desirable (Li and Law 2010, Zhang *et al*. 2011).

Research efforts devoted to the methodologies that accept the monitoring data as input and produce as output the reliability of the concerned building structure are very few in comparison with the vast literature addressing on structural identification/damage detection and reliability evaluation, respectively. Zhang *et al*. (2011) proposed an integrated system identification and reliability evaluation of stochastic building structures by combining a statistical moment-based damage detection method (SMBDD) (Zhang *et al*. 2008, Xu *et al*. 2009) with the probability density evolution equation (PDE) based structural reliability evaluation method (Li and Chen 2004). Li and Law (2010) investigate the possibility of updating the reliability of a bridge structure based on measured information and shown that the measured information of a structural system could be integrated with the reliability analysis to yield a safety estimate on the component or system. Their presented frameworks are innovative, but the presented integrations are off-line approaches as long time series of measured discrete time history of structural responses are needed for the evaluation of statistical moment in the SMBDD (Zhang *et al*. 2011) and in the sensitivity analysis of structural responses with respect to the coefficients of structural parameters and unknown interaction forces (Li and Law 2010).

In practical SHM, it is impossible to deploy so many sensors to measure all responses of structural systems. Thus, it is highly desirable to explore efficient algorithms which can detect structural damage utilizing partially measured responses of structures (Sun and Betti 2014). The extended Kalman filter (EKF) approach has been shown to be useful tool for this purpose and it has been widely used for structural identification and damage detection (Hoshiya and Saito 1984, Yang *et al*. 2006, Lei *et al*. 2012a, 2012b,

∗Corresponding author, Professor
E-mail: ylei@xmu.edu.cn
[a]Graduate Student
[b]Undergraduate Student
[c]Graduate Student

**Copyright © 2017 Techno-Press, Ltd.**

ISSN: 1225-4568 (Print), 1598-6217 (Online)

Lei *et al*. 2015). Especially the unique recursive formulation of the extended Kalman filter makes it suitable for on-line structural identification and damage detection. Some approaches have been proposed for the on-line integrations of structural damage detection and optimal vibration control (Chen *et al*. 2008, He *et al*. 2014, Lei *et al*. 2014).

Moreover, it is often difficult or even impossible to measure all structural external excitations under actual operating conditions. In the last decade, some approaches have been proposed for simultaneous identification of structural systems and unknown external excitations. Among these approaches, the approaches based on the extended Kalman filter (EKF) with consideration of unknown input have received great attention (Yang *et al*. 2007, Lei *et al*. 2014). The authors recently extended the traditional extended Kalman filter (EKF) approach to extended Kalman filter with unknown inputs (EKF-UI) (Liu *et al*. 2016). Based on the procedures of the traditional EKF, analytical recursive solutions for the EKF-UI are derived and presented, so it is a straightforward, intuitive and easy to implement method. Moreover, data fusion of displacement and acceleration measurements is used to prevent in real time the low-frequency drifts in the identification results. Such an analytical recursive solution for data fusion based EKF-UI is not available in the previous literature. Due to the recursive estimation by EKF-UI, it is suitable for on-line integration of on-line integration of structural identification/damage detection and structural reliability evaluation of stochastic building structures subject to unknown excitations.

In this paper, based on the recursive formulation of the extended Kalman filter, an on-line integration of structural identification/damage detection and reliability evaluation of stochastic building structures is investigated. Structural limit state is expanded by the Taylor series in terms of uncertain variables to obtain the probability density function (PDF). Both structural component reliability with only one limit state function and system reliability with multi-limit state functions are studied. Then, it is extended to adopt the recent extended Kalman filter with unknown input (EKF-UI) proposed by the authors for on-line integration of structural identification/damage detection and structural reliability evaluation of stochastic building structures subject to unknown excitations. Numerical examples are used to demonstrate the proposed method and the evaluated results are compared with those by the Monte Carlo simulation to validate the proposed method.

## 2. The proposed on-line integration for stochastic building structures under known excitation

Due to the inevitable uncertainties, structural dynamic response is a random process with the equation of motion expressed as

$$\boldsymbol{M}\ddot{\boldsymbol{x}}(t)+\boldsymbol{C}(\boldsymbol{\Theta})\dot{\boldsymbol{x}}(t)+\boldsymbol{K}(\boldsymbol{\Theta})\boldsymbol{x}(t)=\boldsymbol{\eta}\boldsymbol{f}(t) \tag{1}$$

where $\ddot{\boldsymbol{x}}(t)$, $\dot{\boldsymbol{x}}(t)$ and $\boldsymbol{x}(t)$ are $n$-dimensional vectors of structural acceleration, velocity and displacement, respectively; $\boldsymbol{M}$, $\boldsymbol{C}(\Theta)$, $\boldsymbol{K}(\Theta)$ are $n\times n$ structural mass, damping and stiffness matrices, respectively; $\boldsymbol{\Theta}$ is a $m$-dimensional random parameter vector reflecting the uncertainties in the structural identification procedure, with the known probability density function $p_{\boldsymbol{\Theta}}(\theta)$. In this paper, it is assumed that structural mass matrix is known, so only structural damping and stiffness matrices are functions of random parameter vector $\boldsymbol{\Theta}$ in the stochastic building model. $\boldsymbol{f}(t)$ is a deterministic or random external input vector, and $\boldsymbol{\eta}$ is the corresponding influence matrix associated with the external input $\boldsymbol{f}(t)$.

For simplicity, it is assumed that random variables in the random parameter vector $\boldsymbol{\Theta}$ are independent to each other. Then, structural limit state denoted by $\boldsymbol{Z}(\boldsymbol{\Theta}, t)$ can be expanded as the first-order Taylor series in terms of uncertain variable vector as (Li and Law 2008, Li and Law 2010)

$$\begin{aligned}\boldsymbol{Z}(\boldsymbol{\Theta},t_{k+1}) &= \boldsymbol{Z}(\boldsymbol{\Theta}_0,t_{k+1})\\ &+\sum_{i=1}^{m}\frac{\partial \boldsymbol{Z}(\boldsymbol{\Theta},t_{k+1})}{\partial \boldsymbol{\Theta}_i}\bigg|_{\boldsymbol{\Theta}_i=\boldsymbol{\Theta}_{i0}}(\Theta_i-\Theta_{i0})\end{aligned} \tag{2}$$

where $\boldsymbol{\Theta}_0$ is the vector of mean value $\boldsymbol{\Theta}$.

In Eq. (2), $\boldsymbol{Z}(\boldsymbol{\Theta}_0, t_{k+1})$ can be estimated recursively based on the extended Kalman filter approach using partial measurement of structural responses.

### *2.1 On-line integration with the EKF*

When the random variable vector $\boldsymbol{\Theta}$ takes its mean value $\boldsymbol{\Theta}_0$, structural dynamic responses become deterministic. By introducing an extended structural state vector $\boldsymbol{X}_e$ as

$$\boldsymbol{X}_e=\left[\boldsymbol{x}^T(t),\dot{\boldsymbol{x}}^T(t),\boldsymbol{\theta}^T\right]^T \tag{3}$$

Where $\boldsymbol{\theta}$ is the unknown structural parametric vector. Eq.(1) can be converted into the following state equation as

$$\begin{aligned}\dot{\boldsymbol{X}}_e &= \left\{\begin{matrix}\dot{\boldsymbol{x}}(t)\\ -\boldsymbol{M}^{-1}[\boldsymbol{C}(\boldsymbol{\Theta}_0,\boldsymbol{\theta})\dot{\boldsymbol{x}}(t)+\boldsymbol{K}(\boldsymbol{\Theta}_0,\boldsymbol{\theta})\boldsymbol{x}(t)-\boldsymbol{\eta}\boldsymbol{f}(t)]\\ \boldsymbol{0}\end{matrix}\right\}\\ &=\boldsymbol{g}(\boldsymbol{X}_e,\boldsymbol{\Theta}_0,\boldsymbol{f})\end{aligned} \tag{4}$$

in which $\boldsymbol{g}$ denotes a nonlinear function.

In practice, only some structural responses can be measured. Therefore, the discrete equation for the observation vector (measured output) can be expressed as

$$\boldsymbol{y}_{k+1}=\boldsymbol{h}(\boldsymbol{X}_{e,k+1},\boldsymbol{\Theta}_0,\boldsymbol{f}_{k+1})+\boldsymbol{v}_{k+1} \tag{5}$$

where $\boldsymbol{y}_{k+1}$ is the measured structural response vector at time $t=(k+1)\Delta t$ with $\Delta t$ being the sampling time step. $\boldsymbol{v}_{k+1}$ is the measurement noise vector of a Gaussian white noise vector with zero mean and a covariance matrix $\boldsymbol{E}\left[\boldsymbol{v}_{k+1}\,\boldsymbol{v}_{k+1}^T\right]=\boldsymbol{R}_{k+1}$.

Let $\hat{\boldsymbol{X}}_{e,k}$ and $\tilde{\boldsymbol{X}}_{e,k+1}$ be the estimates of $\boldsymbol{X}_{e,k}$ and $\boldsymbol{X}_{e,k+1}$ given the observations ($y_1, y_2, \ldots, y_k$), respectively, Eqs. (4)-(5) can be linearized at $\hat{\boldsymbol{X}}_{e,k}$ and $\tilde{\boldsymbol{X}}_{e,k+1}$ by the Taylor series expansion to the first order as

$$\begin{aligned}&\boldsymbol{g}(\boldsymbol{X}_e,\boldsymbol{\Theta}_0,\boldsymbol{f})=\boldsymbol{g}(\hat{\boldsymbol{X}}_{e,k},\boldsymbol{\Theta}_0,\boldsymbol{f})+\boldsymbol{G}_k(\boldsymbol{X}_e-\hat{\boldsymbol{X}}_{e,k});\\ &\boldsymbol{G}_k=\frac{\partial \boldsymbol{g}(\boldsymbol{X}_e,\boldsymbol{\Theta}_0,\boldsymbol{f})}{\partial \boldsymbol{X}_e}\bigg|_{\boldsymbol{X}_e=\hat{\boldsymbol{X}}_{e,k}}\end{aligned} \tag{6}$$

$$\begin{aligned}&\boldsymbol{h}(\boldsymbol{X}_{e,k+1},\boldsymbol{\Theta}_0,\boldsymbol{f}_{k+1})\\&=\boldsymbol{h}(\tilde{\boldsymbol{X}}_{e,k+1},\boldsymbol{\Theta}_0,\boldsymbol{f}_{k+1})+\boldsymbol{H}_{k+1}(\boldsymbol{X}_{e,k+1}-\tilde{\boldsymbol{X}}_{e,k+1});\\&\boldsymbol{H}_{k+1}=\left.\frac{\partial\boldsymbol{h}(\boldsymbol{X}_{e,k+1},\boldsymbol{\Theta}_0,\boldsymbol{f}_{k+1})}{\partial\boldsymbol{X}_{e,k+1}}\right|_{\boldsymbol{X}_{e,k+1}=\tilde{\boldsymbol{X}}_{e,k+1}}\end{aligned}\tag{7}$$

The EKF mainly consists of the two recursive procedures. The first one is the time update (prediction) procedure, in which

$$\tilde{\boldsymbol{X}}_{e,k+1}(\boldsymbol{\Theta}_0)=\hat{\boldsymbol{X}}_{e,k}(\boldsymbol{\Theta}_0)+\int_{k\Delta t}^{(k+1)\Delta t}\boldsymbol{g}(\hat{\boldsymbol{X}}_{e,k},\boldsymbol{\Theta}_0,\boldsymbol{f})dt\tag{8}$$

and the prediction error of $\tilde{\boldsymbol{X}}_{e,k+1}$ is $\tilde{\boldsymbol{e}}_{k+1}=\boldsymbol{X}_{e,k+1}-\tilde{\boldsymbol{X}}_{e,k+1}\left(\boldsymbol{\Theta}_0\right)$ with the prediction error covariance matrix $\tilde{\boldsymbol{P}}_{k+1}=\boldsymbol{E}\left[\tilde{\boldsymbol{e}}_{k+1}\ \tilde{\boldsymbol{e}}_{k+1}^{T}\right]$. It can be derived that

$$\tilde{\boldsymbol{P}}_{k+1}=\boldsymbol{\Phi}_{k+1}\hat{\boldsymbol{P}}_k\boldsymbol{\Phi}_{k+1}^T+\boldsymbol{Q}_k\tag{9}$$

where $\boldsymbol{\Phi}_{k+1}\approx\boldsymbol{I}_{2n+l}+\Delta t\boldsymbol{G}_k$, $\hat{\boldsymbol{P}}_k=\boldsymbol{E}\left[\hat{\boldsymbol{e}}_k\ \hat{\boldsymbol{e}}_k^T\right]$, in which, $\boldsymbol{I}_{2n+l}$ is a unit matrix of dimension $2n+l$.

The second process of EKF is the measurement update (correction) procedure, in which

$$\begin{aligned}\hat{\boldsymbol{X}}_{e,k+1}\left(\boldsymbol{\Theta}_0\right)&=\tilde{\boldsymbol{X}}_{e,k+1}\left(\boldsymbol{\Theta}_0\right)\\&\quad+\boldsymbol{K}_{k+1}\left[\boldsymbol{y}_{k+1}-\boldsymbol{h}(\tilde{\boldsymbol{X}}_{e,k+1},\boldsymbol{\Theta}_0,\boldsymbol{f}_{k+1})\right]\end{aligned}\tag{10}$$

where $\hat{\boldsymbol{X}}_{e,k+1}\left(\boldsymbol{\Theta}_0\right)$ is the optimal estimate of $\boldsymbol{X}_{e,k+1}$ given the observations ($\boldsymbol{y}_1$, $\boldsymbol{y}_{2,\ldots,}$ $\boldsymbol{y}_{k+1}$) and $\boldsymbol{K}_{k+1}$ is the Kalman gain matrix, which can be derived as

$$\boldsymbol{K}_{k+1}=\tilde{\boldsymbol{P}}_{k+1}\boldsymbol{H}_{k+1}^{T}\left(\boldsymbol{H}_{k+1}\tilde{\boldsymbol{P}}_{k+1}\boldsymbol{H}_{k+1}^{T}+\boldsymbol{R}_{k+1}\right)^{-1}\tag{11}$$

and

$$\hat{\boldsymbol{P}}_{k+1}=(\boldsymbol{I}_{2n+l}-\boldsymbol{K}_{k+1}\boldsymbol{H}_{k+1})\tilde{\boldsymbol{P}}_{k+1}(\boldsymbol{I}_{2n+l}-\boldsymbol{K}_{k+1}\boldsymbol{H}_{k+1})^T+\boldsymbol{K}_{k+1}\boldsymbol{R}_{k+1}\boldsymbol{K}_{k+1}^T\tag{12}$$

Based on the above recursive EKF approach, $\hat{\boldsymbol{X}}_{e,k+1}\left(\boldsymbol{\Theta}_0\right)$ can be estimated, i.e., structural model at time $t=(k+1)\Delta t$ is identified based on the measured data ($y_1$, $y_2,\ldots,$ $y_{k+1}$). The updated structural model can be used for on-line estimation of $\boldsymbol{Z}(\boldsymbol{\Theta}_0, t_{k+1})$ based on the defined structural limit state.

### *2.2 Structural reliability evaluation based on the recursively updated model*

Based on the above recursive results of $\hat{\boldsymbol{X}}_{e,k+1}\left(\boldsymbol{\Theta}_0\right)$ by the on-line integration of EKF, $\boldsymbol{Z}(\boldsymbol{\Theta}_0, t_{k+1})$ in Eq. (2) can be estimated based on the defined structural limit state.

Also, based on structural updated model at time $t=(k+1)\Delta t$ by the above recursive EKF approach, $\left.\frac{\partial\boldsymbol{Z}(\boldsymbol{\Theta},t_{k+1})}{\partial\Theta_i}\right|_{\Theta_i=\Theta_{i0}}$ can be evaluated in a recursive procedure.

The detailed evaluation of $\left.\frac{\partial\boldsymbol{Z}(\boldsymbol{\Theta},t_{k+1})}{\partial\Theta_i}\right|_{\Theta_i=\Theta_{i0}}$ depends on the type of $\boldsymbol{\Theta}$.

If structural first-modal damping ratio $\zeta_1$ is treated as an uncertain parameter, the following equation can be obtained from Eq. (1) by taking the derivative with respect to $\zeta_1$

$$\begin{aligned}&\boldsymbol{M}\frac{\partial\ddot{\boldsymbol{x}}}{\partial\xi_1}(t)+(a\boldsymbol{M}+b\boldsymbol{K})\frac{\partial\dot{\boldsymbol{x}}}{\partial\xi_1}(t)+\boldsymbol{K}\frac{\partial\boldsymbol{x}}{\partial\xi_1}(t)\\&=-\boldsymbol{M}\dot{\boldsymbol{x}}\frac{\partial a}{\partial\xi_1}-\boldsymbol{K}\dot{\boldsymbol{x}}\frac{\partial b}{\partial\xi_1}\end{aligned}\tag{13}$$

where $a$ and $b$ are two coefficients of Rayleigh damping defined by

$$\begin{Bmatrix}a\\b\end{Bmatrix}=\frac{2\omega_1\omega_2}{\omega_2^2-\omega_1^2}\begin{bmatrix}\omega_2&-\omega_1\\-\frac{1}{\omega_2}&\frac{1}{\omega_1}\end{bmatrix}\begin{Bmatrix}\zeta_1\\\zeta_2\end{Bmatrix}\tag{14}$$

So, $\partial a/\partial\zeta_1$ and $\partial b/\partial\zeta_1$ being estimated by

$$\frac{\partial a}{\partial\xi_1}=\frac{2\omega_1\omega_2^2}{\omega_2^2-\omega_1^2},\qquad\frac{\partial b}{\partial\xi_1}=\frac{2\omega_1}{\omega_1^2-\omega_2^2}\tag{15}$$

in which $\omega_1$ and $\omega_2$ are the first two order natural frequencies.

Finally, $\left.\frac{\partial\boldsymbol{Z}(\boldsymbol{\Theta},t_{k+1})}{\partial\Theta_i}\right|_{\Theta_i=\Theta_{i0}}$ can be evaluated. And based on Eq. (2), the probability density functions of structural limit state can be achieved when probability distribution of the uncertain parametric vector $\boldsymbol{\Theta}$ is known. This useful information can be used for on-line structural reliability evaluation.

#### *2.2.1 Structural component reliability evaluation based on the recursively updated model*

In this study, the structural reliability is evaluated through either the building top displacement or one particular inter-story drift, structural limit state is the structural component reliability can be expressed as

$$R(t)=P\left\{\left|\frac{x_{top}(\boldsymbol{\Theta},t)}{h}\right|\le L_{\mathrm{top}}\quad,\quad t\in\left[0,t\right]\right\}\tag{16a}$$

or

$$R(t)=P\left\{\left|\frac{x_{\text{inter}-story}(\boldsymbol{\Theta},t)}{h_{\text{inter}-story}}\right|\le L_{\text{inter-story}},\quad t\in\left[0,t\right]\right\}\tag{16b}$$

where $P\{\cdot\}$ is the probability of the random event; $x_{top}$(t) is the displacement response of the building at the top, $h$ is the total height of the building; $x_{inter\text{-}story}(t)$ is the particular story drift and $h_{inter\text{-}story}$ is the height of the particular story investigated; $L_{top}$ and $L_{inter\text{-}story}$ are the thresholds of the dimensionless top displacement and particular story drift, respectively.

#### *2.2.2 Structural system reliability evaluation based on the recursively updated model*

More general, for the serviceability of a multi-story building structure, not only the first inter-story angle is required not to exceed a threshold, but also all the other inter-story angles are required not to exceed corresponding thresholds. In this case, Structural system reliability system reliability with multi-limit state functions are studied.

By denoting the dimensionless inter-story drifts from the bottom to the top floor by $\Delta_1(\mathbf{\Theta}, t)$, $\Delta_2(\mathbf{\Theta}, t)$, … , $\Delta_n(\mathbf{\Theta}, t)$, Structural system reliability is evaluated by

$$R(t) = P\left\{\bigcap_{i=1}^{n}\left\{\left|\frac{\Delta_i(\boldsymbol{\Theta},t)}{h_i}\right| < L_i, \quad t \in [0,t]\right\}\right\} \tag{17}$$

where $L_i$ denotes the threshold value of the dimensionless $i$-th inter-story drift.

## 3. The proposed on-line integration for stochastic building structures under unknown excitation

For the building structure under some unknown inputs, the equation of motion can be expressed by

$$\boldsymbol{M}\ddot{\boldsymbol{x}}(t) + \boldsymbol{C}(\boldsymbol{\Theta})\dot{\boldsymbol{x}}(t) + \boldsymbol{K}(\boldsymbol{\Theta})\boldsymbol{x}(t) = \boldsymbol{\eta}\boldsymbol{f}(t) + \boldsymbol{\eta}^{\mathrm{u}}\boldsymbol{f}^{\mathrm{u}}(t) \tag{18}$$

where $\boldsymbol{f}^u$ denotes a unmeasured $q$-dimensional external excitation vector and $\boldsymbol{\eta}^u$ is the corresponding influence matrices associated with the unknown $\boldsymbol{f}^u$.

The traditional EKF approach is only applicable when the information of external inputs to structures is available. In order to solve above problem, the authors (Liu *et al.* 2016) proposed an extended Kalman filter with unknown inputs (EKF-UI) based on the procedures of the traditional EKF.

### *3.1 A brief review of the recent data fusion EKF-UI*

When some external inputs to the n-DOF structure are unknown, the equation of motion of can be expressed as

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{F}(\boldsymbol{x},\dot{\boldsymbol{x}},\boldsymbol{\theta}) = \boldsymbol{\eta}\boldsymbol{f} + \boldsymbol{\eta}^u\boldsymbol{f}^u \tag{19}$$

where $\boldsymbol{M}$ is the mass matrix of the structure, $\boldsymbol{x}$, $\dot{\boldsymbol{x}}$ and $\ddot{\boldsymbol{x}}$ are the vectors of the displacement, velocity and acceleration responses, respectively. $\boldsymbol{\theta}$ is a $l$-dimensional unknown structural parametric vector, $\boldsymbol{F}(\boldsymbol{x}, \dot{\boldsymbol{x}}, \boldsymbol{\theta})$ is the a force vector which can be linear or nonlinear function of the displacements, velocities and the structural parameters, $\boldsymbol{f}$ and$\boldsymbol{\eta}$denote an known external excitation vector and the influence matrix associated with the excitation $\boldsymbol{f}$, respectively. $\boldsymbol{f}^u$ and $\boldsymbol{\eta}^u$ are a unmeasured $p$-dimensional external excitation vector and the influence matrix associated with the excitation $\boldsymbol{f}^u$.

By introducing a 2n+$l$ dimensional extended state vector $\boldsymbol{Z}=(\boldsymbol{x}^T, \dot{\boldsymbol{x}}^T, \boldsymbol{\theta}^T)$ and considering modeling error, Eq. (19) can be converted into the following state equation as

$$\dot{\boldsymbol{Z}} = \boldsymbol{g}(\boldsymbol{Z}, \boldsymbol{f}, \boldsymbol{f}^u) + \boldsymbol{w}(\mathrm{t}) \tag{20}$$

in which, $\boldsymbol{g}(\cdot)$ is a nonlinear function and $\boldsymbol{w}(t)$ is the model noise (uncertainty) with zero mean and a covariance matrix $\boldsymbol{Q}(t)$.

The nonlinear discrete equation for an observation vector can be expressed as

$$\boldsymbol{y}_{k+1} = \boldsymbol{h}(\boldsymbol{Z}_{k+1}, \boldsymbol{f}_{k+1}, \boldsymbol{f}_{k+1}^u) + \boldsymbol{v}_{k+1} \tag{21}$$

in which $\boldsymbol{y}_{\mathbf{k}+1}$ is a $m$-dimensional measured acceleration response vector at time $t=(k+1)\Delta t$ with $\Delta t$ being the sampling time step and $\boldsymbol{v}_{k+1}$ is the measurement noise vector of a Gaussian white noise vector with zero mean and a covariance matrix $\boldsymbol{E}\left(\boldsymbol{v}_{k+1}\boldsymbol{v}_{k+1}^T\right) = \boldsymbol{R}_{k+1}$.

Let $\hat{\boldsymbol{Z}}_{k|k}$ and $\hat{\boldsymbol{f}}_{k|k}^u$ be the estimates of $\boldsymbol{Z}_k$ and $\boldsymbol{f}_k^u$ given the observations ($\boldsymbol{y}_1\boldsymbol{y}_{2,\ldots,}$ $\boldsymbol{y}_k$), respectively, Eq. (20) can be linearized at $\hat{\boldsymbol{Z}}_{k|k}$ and $\hat{\boldsymbol{f}}_{k|k}$ by Taylor series expansion to the first order as

$$\begin{aligned}\boldsymbol{g}(\boldsymbol{Z}, \boldsymbol{f}, \boldsymbol{f}^u) &\approx \boldsymbol{g}(\hat{\boldsymbol{Z}}_{k|k}, \boldsymbol{f}, \hat{\boldsymbol{f}}_{k|k}^u) \\ &\quad + \boldsymbol{G}_{k|k}(\boldsymbol{Z} - \hat{\boldsymbol{Z}}_{k|k}) + \boldsymbol{B}_{k|k}^{\mathrm{u}}(\boldsymbol{f}^u - \hat{\boldsymbol{f}}_{k|k}^u)\end{aligned} \tag{22}$$

where

$$\begin{aligned}\boldsymbol{G}_{k|k} &= \left.\frac{\partial \boldsymbol{g}(\boldsymbol{Z}, \boldsymbol{f}, \boldsymbol{f}^u)}{\partial \boldsymbol{Z}}\right|_{\boldsymbol{Z}=\hat{\boldsymbol{Z}}_{k|k};\, \boldsymbol{f}^u=\boldsymbol{f}_{k|k}^u}; \\ \boldsymbol{B}_{k|k}^u &= \left.\frac{\partial \boldsymbol{g}(\boldsymbol{Z}, \boldsymbol{f}, \boldsymbol{f}^u)}{\partial \boldsymbol{f}^u}\right|_{\boldsymbol{Z}=\hat{\boldsymbol{Z}}_{k|k};\, \boldsymbol{f}^u=\boldsymbol{f}_{k|k}^u}\end{aligned} \tag{23}$$

Analogous to the traditional EKF, the first time update (prediction) procedure is

$$\tilde{\boldsymbol{Z}}_{k+1|k} = \hat{\boldsymbol{Z}}_{k|k} + \int_{k\Delta t}^{(k+1)\Delta t} \boldsymbol{g}(\hat{\boldsymbol{Z}}_{t|k}, \boldsymbol{f}, \hat{\boldsymbol{f}}_{k|k}^u)dt \tag{24}$$

The prediction error of $\tilde{\boldsymbol{Z}}_{k+1|k}$ is $\tilde{\boldsymbol{e}}_{k+1|k}^{\boldsymbol{Z}} = \boldsymbol{Z}_{k+1} - \tilde{\boldsymbol{Z}}_{k+1|k} = \boldsymbol{A}_{k|k}^{\boldsymbol{Z}}\hat{\boldsymbol{e}}_{k|k}^{\boldsymbol{Z}} + \Delta \mathrm{t}\boldsymbol{B}_{k|k}^u\hat{\boldsymbol{e}}_{k|k}^{\boldsymbol{f}^u} + \boldsymbol{w}_k$, where $\boldsymbol{A}_{k|k}^{\boldsymbol{Z}} \approx \boldsymbol{I}_{2n+l} + \Delta t\boldsymbol{G}_{k|k}$ and $\hat{\boldsymbol{e}}_{k|k}^{\boldsymbol{f}^u}$ is the error of $\hat{\boldsymbol{f}}_{k|k}^u$, i.e., $\hat{\boldsymbol{e}}_{k|k}^{\boldsymbol{f}^u} = \boldsymbol{f}_k^u - \hat{\boldsymbol{f}}_{k|k}^u$

The observation equation can also be linearized at $\tilde{\boldsymbol{Z}}_{k+1|k}$ and $\hat{\boldsymbol{f}}_{k|k}^u$ by Taylor series expansion to the first order as

$$\begin{aligned}\boldsymbol{h}(\boldsymbol{Z}_{k+1}, \boldsymbol{f}_{k+1}, \boldsymbol{f}_{k+1}^u) &= \boldsymbol{h}(\tilde{\boldsymbol{Z}}_{k+1|k}, \boldsymbol{f}_{k+1}, \hat{\boldsymbol{f}}_{k|k}^u) \\ &\quad + \boldsymbol{H}_{k+1|k}(\boldsymbol{Z}_{k+1} - \tilde{\boldsymbol{Z}}_{k+1|k}) + \boldsymbol{D}_{k+1|k}^u(\boldsymbol{f}_{k+1}^u - \hat{\boldsymbol{f}}_{k|k}^u)\end{aligned} \tag{25}$$

where

$$\begin{aligned}\boldsymbol{H}_{k+1|k} &= \left.\frac{\partial \boldsymbol{h}(\boldsymbol{Z}, \boldsymbol{f}, \boldsymbol{f}^u)}{\partial \boldsymbol{Z}}\right|_{\boldsymbol{Z}=\tilde{\boldsymbol{Z}}_{k+1|k};\, \boldsymbol{f}^u=\boldsymbol{f}_{k|k}^u} \\ \boldsymbol{D}_{k+1|k}^u &= \left.\frac{\partial \boldsymbol{h}(\boldsymbol{Z}, \boldsymbol{f}, \boldsymbol{f}^u)}{\partial \boldsymbol{f}^u}\right|_{\boldsymbol{Z}=\tilde{\boldsymbol{Z}}_{k+1|k};\, \boldsymbol{f}^u=\boldsymbol{f}_{k|k}^u}\end{aligned} \tag{26}$$

Then, the second measurement update (correction) procedure is

$$\begin{aligned}&\hat{\boldsymbol{Z}}_{k+1|k+1} = \tilde{\boldsymbol{Z}}_{k+1|k} + \\ &\boldsymbol{K}_{k+1}\left[\boldsymbol{y}_{k+1} - \boldsymbol{h}(\tilde{\boldsymbol{Z}}_{k+1|k}, \boldsymbol{f}_{k+1}, \hat{\boldsymbol{f}}_{k|k}^u) - \boldsymbol{D}_{k+1|k}^u(\hat{\boldsymbol{f}}_{k+1|k+1}^u - \hat{\boldsymbol{f}}_{k|k}^u)\right]\end{aligned} \tag{27}$$

where $\hat{\boldsymbol{Z}}_{k+1|k+1}$ and $\hat{\boldsymbol{f}}^{u}_{k+1|k+1}$ are the estimate of $\boldsymbol{Z}_{k+1}$ and $\boldsymbol{f}^{u}_{k+1}$ given the observations ($\boldsymbol{y}_1,\boldsymbol{y}_{2,\ldots,}\boldsymbol{y}_{k+1}$), respectively, and $\boldsymbol{K}_{k+1}$ is the Kalman gain matrix, which is derived as

$$\boldsymbol{K}_{k+1} = \tilde{\boldsymbol{P}}^{\boldsymbol{Z}}_{k+1|k}\boldsymbol{H}^{\boldsymbol{T}}_{k+1|k}\left(\boldsymbol{H}_{k+1|k}\tilde{\boldsymbol{P}}^{\boldsymbol{Z}}_{k+1|k}\boldsymbol{H}^{\boldsymbol{T}}_{k+1|k} + \boldsymbol{R}_{k+1}\right)^{-1} \tag{28}$$

Under the condition that the number of acceleration response measurements is not large than the total number of unknown external excitations, i.e., $m>p$, $\hat{\boldsymbol{f}}^{u}_{k+1|k+1}$ can be estimated by the least-squares estimation as

$$\begin{aligned}\hat{\boldsymbol{f}}^{u}_{k+1|k+1} &= \boldsymbol{S}_{k+1}\boldsymbol{D}^{u\mathrm{T}}_{k+1|k}\boldsymbol{R}^{-1}_{k+1}\left(\boldsymbol{I}_m - \boldsymbol{H}_{k+1|k}\boldsymbol{K}_{k+1}\right)\\ &\left[\boldsymbol{y}_{k+1} - \boldsymbol{h}(\tilde{\boldsymbol{Z}}_{k+1|k}, \boldsymbol{f}_{k+1}, \hat{\boldsymbol{f}}^{u}_{k|k}) + \boldsymbol{D}^{u}_{k+1|k}\hat{\boldsymbol{f}}^{u}_{k|k}\right]\end{aligned} \tag{29}$$

where $\boldsymbol{S}_{k+1} = \left[\boldsymbol{D}^{uT}_{k+1|k}\boldsymbol{R}^{-1}_{k+1}\left(\boldsymbol{I}_m - \boldsymbol{H}_{k+1|k}\boldsymbol{K}_{k+1}\right)\boldsymbol{D}^{u}_{k+1|k}\right]^{-1}$.

The covariance matrix for error $\hat{\boldsymbol{e}}^{\boldsymbol{Z}}_{k+1|k+1}$ and $\hat{\boldsymbol{e}}^{\boldsymbol{f}}_{k+1|k+1}$ are derived as

$$\begin{aligned}\hat{\boldsymbol{P}}^{\boldsymbol{Z}}_{k+1|k+1} &= \left(\boldsymbol{I}_{2n+l} + \boldsymbol{K}_{k+1}\boldsymbol{D}^{u}_{k+1|k}\boldsymbol{S}_{k+1}\boldsymbol{D}^{uT}_{k+1}\boldsymbol{R}^{-1}_{k+1}\boldsymbol{H}^{T}_{k+1|k}\right)\\ &\left(\boldsymbol{I}_{2n+l} - \boldsymbol{K}_{k+1}\boldsymbol{H}_{k+1|k}\right)\tilde{\boldsymbol{P}}^{\boldsymbol{Z}}_{k+1|k}\end{aligned} \tag{30a}$$

$$\hat{\boldsymbol{P}}^{\boldsymbol{f}}_{k+1|k+1} = \boldsymbol{S}_{k+1} \tag{30b}$$

Also, the error covariance matrix $\hat{\boldsymbol{P}}^{\boldsymbol{Zf}}_{k+1|k+1}$ can be derived from Eq. (28) by

$$\begin{aligned}\hat{\boldsymbol{P}}^{\boldsymbol{Zf}}_{k+1|k+1} &= \left(\hat{\boldsymbol{P}}^{\boldsymbol{fZ}}_{k+1|k+1}\right)^{T} = E\left[\hat{\boldsymbol{e}}^{\boldsymbol{Z}}_{k+1|k+1}\left(\hat{\boldsymbol{e}}^{\boldsymbol{f}}_{k+1|k+1}\right)^{T}\right]\\ &= -\boldsymbol{K}_{k+1}\boldsymbol{D}^{u}_{k+1|k}\boldsymbol{S}_{k+1}\end{aligned} \tag{30c}$$

and $\tilde{\boldsymbol{P}}^{\boldsymbol{Z}}_{k+1|k}$ can be given from $\tilde{\boldsymbol{e}}^{\boldsymbol{Z}}_{k+1|k}$ by

$$\tilde{\boldsymbol{P}}^{\boldsymbol{Z}}_{k+1|k} = \left[\boldsymbol{\Phi}^{\boldsymbol{Z}}_{k|k} \quad \Delta t\boldsymbol{B}^{u}_{k|k}\right]\begin{bmatrix}\hat{\boldsymbol{P}}^{\boldsymbol{Z}}_{k|k} & \hat{\boldsymbol{P}}^{\boldsymbol{Zf}}_{k|k}\\ \hat{\boldsymbol{P}}^{\boldsymbol{fZ}}_{k|k} & \hat{\boldsymbol{P}}^{\boldsymbol{f}}_{k|k}\end{bmatrix}\begin{bmatrix}\boldsymbol{\Phi}^{\boldsymbol{Z}\,T}_{k|k}\\ \Delta t\boldsymbol{B}^{uT}_{k|k}\end{bmatrix} + \boldsymbol{Q}_k \tag{30d}$$

Moreover, previous EKF-UI approaches by only using sparse noisy acceleration measurements are inherently unstable which leads poor tracking and low-frequency drifts in the estimated unknown inputs and structural displacements. The authors (Liu *et al.* 2016) recently have developed a data fusion based EKF-UI to prevent the drifts in the estimated structural state vector and unknown external inputs by previous approaches.

### 3.2 On-line integration with the EKF-UI

Based on the above recursive EKF-UI approach, $\hat{\boldsymbol{X}}_{e,k+1}\left(\boldsymbol{\Theta}_0\right)$ can be estimated, i.e., structural model at time $t=(k+1)\Delta t$ is identified based on the measured data ($y_1$, $y_2$, …, $y_{k+1}$). The updated structural model can be used for on-line estimation of $\boldsymbol{Z}(\boldsymbol{\Theta}_0, t_{k+1})$ based on the identified unknown excitation and the defined structural limit state.

Finally, structural reliability evaluation based on the recursively updated model can be implemented according to the procedure in Sect. 2.2.

## 4. Numerical example validations of the proposed method

A six-story shear frame building under ground motion is chosen as an example to validate the proposed integration method. The mass of the building is $\boldsymbol{m}$={600 550 500 450 400 350}kg, the story stiffness are $\boldsymbol{k}$={1.4 1.3 1.1 0.9 0.7 0.5}×$10^6$ N/m, and the height of each floor are the same with $h$ =2.0 m. Rayleigh damping is adopted, and the first modal damping ratio $\zeta_1$ is considered as a random parameter with a lognormal distribution due to the uncertainty. The mean value of $\zeta_1$ is 3% and the standard deviation is $\sigma$=10%$\zeta_1$.

The ground acceleration is generated by the Kanai–Tajimi spectrum KT with the spectral density function in the form as

$$S_g(\omega) = \frac{1 + 4\xi_g^2(\dfrac{\omega}{\omega_g})^2}{\left[1 - (\dfrac{\omega}{\omega_g})^2\right]^2 + 4\xi_g^2(\dfrac{\omega}{\omega_g})^2}S_0 \tag{31}$$

in which $\omega_g$, $\zeta_g$ and $S_0$ are the characteristic parameters of the ground motion. These parameters are selected as $\omega_g$=15.0 rad/s, $\zeta_g$=0.6, $S_0$=4.64×$10^{-4}$ $m^2$/rads$^3$. The time duration of the simulated acceleration is 15s and the sampling frequency is 1000 Hz

The structural damage scenario of the building is assumed as a 10% story stiffness degradation at the1st story, and the measured accelerations of the 1st, 2nd, 4th and the 5th floors are polluted by 5% noise.

In this situation, the extended state vector $\boldsymbol{X}_e$ at time $t = k\Delta t$ can be written as $\boldsymbol{X}_{e,k}=[\boldsymbol{x},\ \dot{\boldsymbol{x}},\ \boldsymbol{k}]^T$ where $\boldsymbol{k}=[k_1,k_2,\ldots,k_6]^T$. The matrix $\boldsymbol{G}_k$ in Eq. (6) can be worked out as

$$\begin{aligned}\boldsymbol{G}_k &= \left.\frac{\partial \boldsymbol{g}(\boldsymbol{X}_e, \boldsymbol{f})}{\partial \boldsymbol{X}_e}\right|_{\boldsymbol{X}_e=\hat{\boldsymbol{X}}_{e,k}} = \left.\frac{\partial \dot{\boldsymbol{X}}_e}{\partial \boldsymbol{X}_e}\right|_{\boldsymbol{X}_e=\hat{\boldsymbol{X}}_{e,k}}\\ &= \begin{bmatrix}\boldsymbol{0}_{n\times n} & \boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times n}\\ \boldsymbol{G}_{1,k} & \boldsymbol{G}_{2,k} & \boldsymbol{G}_{3,k}\\ \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times n} & \boldsymbol{0}_{n\times n}\end{bmatrix}_{3n\times 3n}\end{aligned} \tag{32}$$

in which

$$\begin{aligned}\boldsymbol{G}_{1,k} &= -\boldsymbol{M}[\boldsymbol{K}]_k\\ &= \boldsymbol{M}\begin{bmatrix}k_{1,k}+k_{2,k} & -k_{2,k} & 0 & \cdots & 0\\ -k_{2,k} & k_{2,k}+k_{3,k} & -k_{3,k} & \cdots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & 0 & k_{n-1,k}+k_{n,k} & -k_{n,k}\\ 0 & 0 & 0 & -k_{n,k} & k_{n,k}\end{bmatrix}\end{aligned} \tag{33}$$

where $[\boldsymbol{K}]_k$ is the stiffness matrix consisting of the elements $k_{i,k}$ in matrix $\boldsymbol{X}_{e,k}$ at the time of $t=k\Delta t$.

$$\boldsymbol{G}_{2,k} = -\boldsymbol{MC} = a_k\boldsymbol{M} + b_k[\boldsymbol{K}]_k \tag{34}$$

where $a_k$ and $b_k$ denote the coefficient values identified at time $t_k$ based on Eq. (34).

$$\boldsymbol{G}_{3,k} = \frac{\partial\left\{-\boldsymbol{M}^{-1}[(a\boldsymbol{M}+b\boldsymbol{K})\dot{\boldsymbol{x}}+\boldsymbol{K}\boldsymbol{x}-\boldsymbol{\eta}\boldsymbol{f}]\right\}}{\partial \boldsymbol{k}}\Bigg|_{\boldsymbol{X}_e=\hat{\boldsymbol{X}}_{e,k}}$$
$$= -\boldsymbol{M}^{-1}\left(\boldsymbol{M}\dot{\boldsymbol{x}}\frac{\partial a}{\partial \boldsymbol{k}}+\boldsymbol{K}\dot{\boldsymbol{x}}\frac{\partial b}{\partial \boldsymbol{k}}+b\dot{\boldsymbol{x}}+\boldsymbol{x}\right)\Bigg|_{\boldsymbol{X}_e=\hat{\boldsymbol{X}}_{e,k}} \tag{35}$$

In Eq. (35), the value of $\frac{\partial a}{\partial \boldsymbol{k}}$ and $\frac{\partial b}{\partial \boldsymbol{k}}$ can be worked out as

$$\frac{\partial a}{\partial \boldsymbol{k}} = \frac{\partial a}{\partial \boldsymbol{\omega}}\frac{\partial \boldsymbol{\omega}}{\partial \boldsymbol{k}}\,;\ \frac{\partial b}{\partial \boldsymbol{K}_\theta} = \frac{\partial b}{\partial \boldsymbol{\omega}}\frac{\partial \boldsymbol{\omega}}{\partial \boldsymbol{k}} \tag{36}$$

where

$$\boldsymbol{\omega}=\begin{bmatrix}\omega_1\\ \omega_2\end{bmatrix} \tag{37a}$$

$$\frac{\partial a}{\partial \boldsymbol{\omega}} = \frac{1}{(\omega_2^2-\omega_1^2)^2}\begin{bmatrix}2\omega_2^4\zeta_1+2\omega_1^2\omega_2^2\zeta_1-4\omega_1\omega_2^3\zeta_2\\ 2\omega_1^4\zeta_2+2\omega_1^2\omega_2^2\zeta_2-4\omega_1^3\omega_2\zeta_1\end{bmatrix}^T \tag{37b}$$

$$\frac{\partial b}{\partial \boldsymbol{\omega}} = \frac{1}{(\omega_2^2-\omega_1^2)^2}\begin{bmatrix}-2\omega_1^2\zeta_1-2\omega_2^2\zeta_1+4\omega_1\omega_2\zeta_2\\ -2\omega_1^2\zeta_2-2\omega_2^2\zeta_2-4\omega_1\omega_2\zeta_1\end{bmatrix}^T \tag{37c}$$

Also, it is derived that

$$\frac{\partial \omega_i^2}{\partial \boldsymbol{k}} = \boldsymbol{\Phi}_i^T\frac{\partial \boldsymbol{K}}{\partial \boldsymbol{k}}\boldsymbol{\Phi}_i$$
$$=\left[\phi_{i,1}^2,(\phi_{i,1}-\phi_{i,2})^2,\cdots(\phi_{i,5}-\phi_{i,6})^2\right]\ (i=1,2) \tag{38}$$

where $\boldsymbol{\Phi}_i=[\phi_{i,1},\phi_{i,2},\phi_{i,3},\cdots,\phi_{i,6}]^T$. Thus,

$$\frac{\partial \boldsymbol{\omega}}{\partial \boldsymbol{k}} = \begin{bmatrix}\frac{\partial \omega_1}{\partial \boldsymbol{k}}\\ \frac{\partial \omega_2}{\partial \boldsymbol{k}}\end{bmatrix} = \begin{bmatrix}\frac{1}{2\omega_1}\left[\phi_{1,1}^2,(\phi_{1,1}-\phi_{1,2})^2,\cdots,(\phi_{1,5}-\phi_{1,6})^2\right]\\ \frac{1}{2\omega_2}\left[\phi_{2,1}^2,(\phi_{2,1}-\phi_{2,2})^2,\cdots,(\phi_{2,5}-\phi_{2,6})^2\right]\end{bmatrix} \tag{39}$$

### *4.1 Numerical integration results for stochastic building structures under known excitation*

Table 1 shows the results of the identification in the undamaged and damaged scenarios, where $k^u$, $k_{id}^u$ and $k_{id}^d$ are structural actual undamaged stiffness, identified mean value of undamaged stiffness and the identified mean value of damaged stiffness parameters of the *i*-th stroy, respectively. It can be seen that the error is acceptable, and the identified degree of damage, which is estimated by the differences between the mean values of the identified undamaged and damaged story stiffness parameters, is closed to the preset value.

Figs. 1-2 are the component reliability in terms of the threshold of top rotation angle of the undamaged structure and the damaged structure. As seen from these figures, the reliabilities increase when the threshold enlarges and the reliabilities decrease for the damaged structure. In these figures, the evaluated structural component reliability results are also compared with those by Monte Carlo simulation (MCS) and it is noted that evaluated structural component reliabilities are in good agreements with those by MCS. The reasons for the discrepancies between the evaluated reliabilities and those by MCS include the

Table 1 The results of the stiffness identification

| Story No. | $k^u$ (N/m) | $k_{id}^u$ (N/m) | Error | $k_{id}^d$ (N/m) | Damage degree |
|---|---|---|---|---|---|
| 1 | 1400000 | 1402206 | 0.16% | 1261966 | -9.86% |
| 2 | 1300000 | 1299441 | -0.04% | 1298667 | -0.10% |
| 3 | 1100000 | 1100631 | 0.06% | 1100959 | 0.09% |
| 4 | 900000 | 899829 | -0.02% | 900245 | 0.03% |
| 5 | 700000 | 700244 | 0.03% | 700187 | 0.03% |
| 6 | 500000 | 500165 | 0.03% | 500164 | 0.03% |

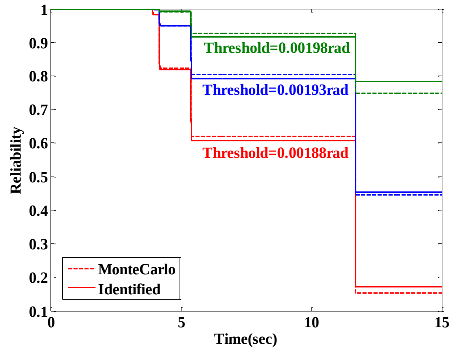


Fig. 1 Component reliability of the undamaged structure

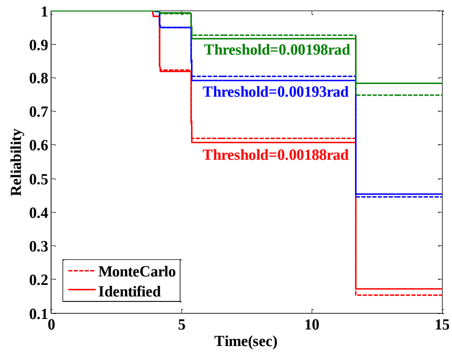


Fig. 2 Component reliability of the damaged structure

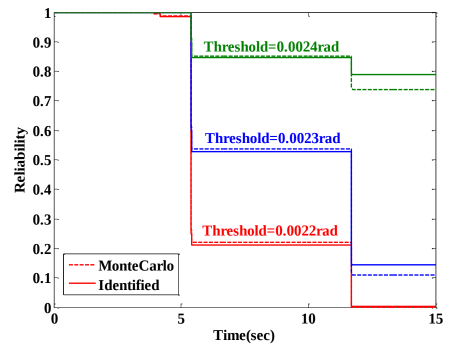


Fig. 3 The System Reliability of the undamaged structure

influences of partial measurements and measurement noise, structural reliability evaluation based on the first-order Taylor series in terms of uncertain variables, etc.

Figs. 3-4 show the comparisons of the evaluated structural system reliability by the proposed method with those by Monte Carlo simulation (MCS). The failure

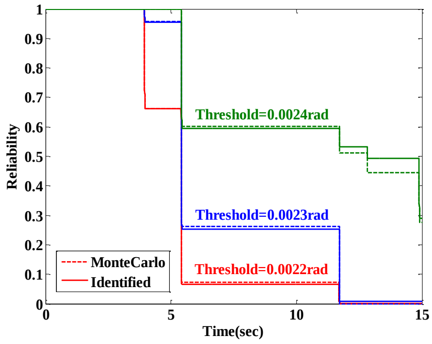
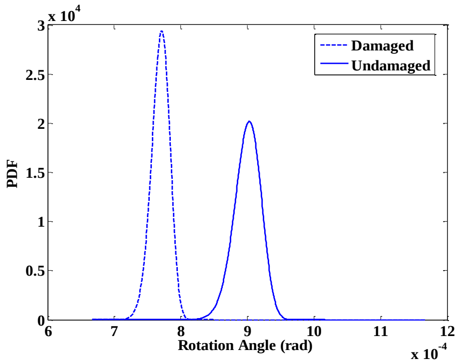
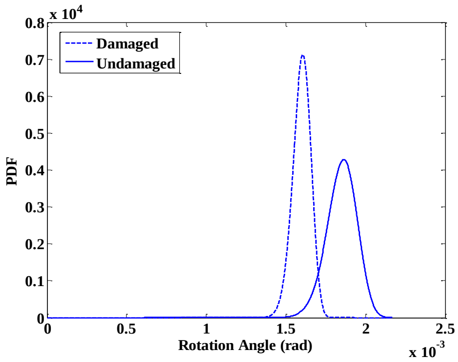


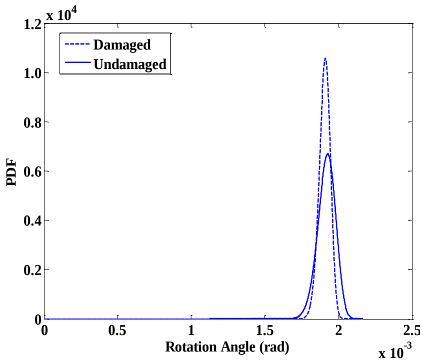

Fig. 4 Component reliability of the damaged structure




(a) PDFs at $t$=2.0 s




(b) PDFs at $t$=5.4 s




(c) PDFs at $t$=11.7 s

Fig. 5 Comparison of PDFs of top rotation angle of undamaged and damaged structure

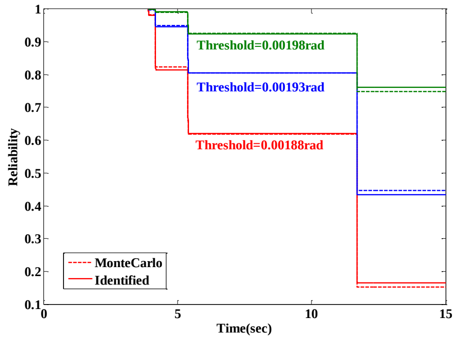



Fig. 6 Component reliability of the undamaged structure

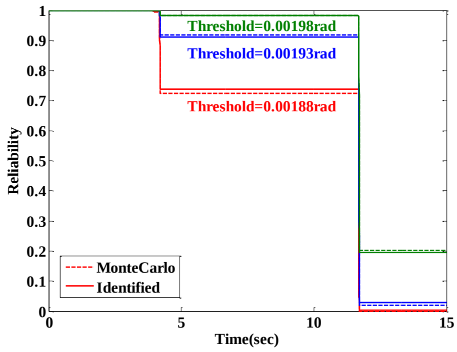



Fig. 7 Component reliability of the damaged structure

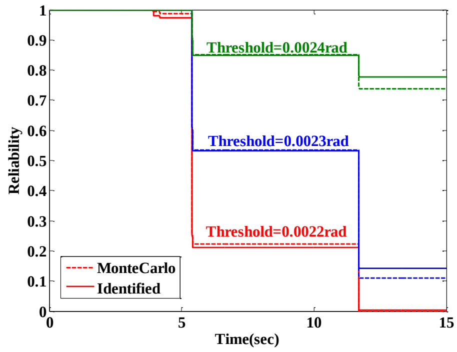



Fig. 8 The System reliability of the undamaged structure

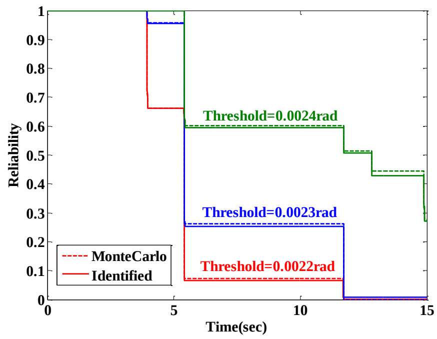



Fig. 9 Component reliability of the damaged structure

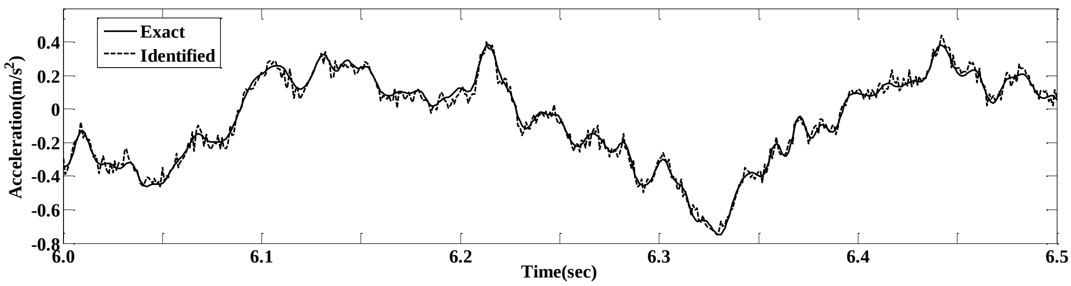



Fig. 10 Comparison of ground excitation in 6.0~6.5 seconds

probability of the whole structure equals to the maximum value of those of story drifts. Again, it is shown that that evaluated structural system reliabilities by the proposed method are also in good agreement with those by MCS.

The probability distribution functions (PDFs) of the extreme value of the top dimensionless displacement (rotational angle) of undamaged and damaged structure at $t$=2.0 s, 5.4 s, 11.7 s are calculated and presented in Fig. 5.

### *4.2 Numerical integration results for stochastic building structures under unknown excitation*

The same shear-building model and damaged scenario studied above is utilized. However, Figs. 6-7 show structural component reliabilities in terms of the threshold of top displacement of the undamaged structure and the damaged structure, respectively. Figs. 8-9 show the comparisons of the evaluated structural system reliability by the proposed method. In all the figures, the evaluated structural reliabilities are compared with those by Monte Carlo simulation (MCS). It is shown that that evaluated structural reliabilities by the proposed algorithm are in good are in good agreement with those by MCS.

Fig. 10 show the comparison of the identified unknown ground excitation with the actual excitation in the time segment of 6.0~6.5 sec., which indicates that the identification results of unknown input is acceptable.

## 5. Conclusions

The integration of structural reliability evaluation with the results from system identification (damage detection) is still in its infancy. To overcome the limitations of some previous techniques for integrated system identification and reliability evaluation of stochastic building structures, on-line integration of structural identification/damage detection and reliability evaluation of stochastic building structures is investigated in this paper. First, based on the recursive formulation of the extended Kalman filter, structural model with damage detection is recursively updated. Structural limit state is expanded by the Taylor series in terms of uncertain variables to obtain the probability density function (PDF). Both structural component reliability with only one limit state function and system reliability with multi-limit state functions are studied. Then, it is it is extended to adopt the recent extended Kalman filter with unknown input (EKF-UI) proposed by the authors for on-line integration of structural identification/damage detection and structural reliability evaluation of stochastic building structures subject to unknown excitations. Numerical examples demonstrate the proposed method. By comparing with the Monte Carlo simulation results for the evaluation of structural component reliability and structural system reliability, it is validated that the proposed method is suitable for on-line integration of structural identification/damage detection and reliability evaluation of stochastic building structures subject to known or unknown excitations.

In the proposed method, structural limit state is expended by the first-order Taylor series in terms of uncertain variables. Therefore, the deviations of uncertain variables cannot be significant in the proposed approach. It is necessary to adopt a more sophisticate reliability evaluation algorithm to overcome such limitation.

## Acknowledgments

This research described in this paper is financially supported by the Natural Science Foundation of China (NSFC) through the Grant No. 51378445 and Shanghai Municipal science and Technology Committee through the Scientific Research Project No.16DZ1200102.

## References

Chen, B., Xu, Y.L. and Zhao, X. (2010), “Integrated vibration control and health monitoring of building structures: a time-domain approach”, *Smart Struct. Syst.*, **6**, 811-833.

Doebling, S.W., Farrar, C.R. and Prime, M.B. (1998), “A summary review of vibration-based damage identification methods”, *Shock Vib. Dig*., **30**(2), 91-105.

Fan, W. and Qiao, P.Z. (2011), “Vibration-based damage identification methods: A review and comparative study”, *Struct. Hlth. Monit.*, **10**, 83-111.

Frangopol, D.M., Strauss, A. and Kim, S.Y. (2008). “Bridge reliability assessment based on monitoring”, *J. Bridge Eng.*, ASCE, **13**(3), 258-270.

He, J., Huang, Q. and Xu, Y.L. (2014), “Synthesis of vibration control and health monitoring of building structures under unknown excitation”, *Smart Mater. Struct.*, **23**,105025.

Hoshiya, M. and Saito, E. (1984), “Structural identification by extended Kalman filter”, *J. Eng. Mech.*, ASCE, **110**(12), 1757-1771.

Hosser, D., Klinzmann, C. and Schnetgoke, R.A. (2008), “Framework for reliability-based system assessment based on structural health monitoring", *Struct. Infrastr. Eng.*, **4**(4), 271-285.

Lei, Y., Chen, F. and Zhou, H. (2012a), "An algorithm based on two-step Kalman filter for intelligent structural damage detection", *Struct. Control Hlth. Monit.*, **22**(4), 694-706.

Lei, Y., Hua, W., Luo, S.J. and He, M.Y. (2015), "Detection and parametric identification of structural nonlinear restoring forces from partial measurements of structural responses", *Struct. Eng. Mech.*, **54**(2), 291-304.

Lei, Y., Jiang, Y.Q. and Xu, Z.Q. (2012b), "Structural damage detection with limited input and output measurement signals", *Mech. Syst. Signal Pr.*, **28**, 229-243.

Lei, Y., Liu, C. and Liu, L.J. (2014) "Identification of multistory shear buildings under unknown earthquake excitation using partial output measurements: numerical and experimental studies", *Struct. Control Hlth. Monit.*, **21**, 774-783.

Lei, Y., Zhou, H. and Liu, L.J. (2014), "An on-line integration technique for structural damage detection and active optimal vibration control", *Int. J. Struct. Stab. Dyn.*, **14**(5), 1440003.

Li, J. and Chen, J.B. (2004), "Probability density evolution method for dynamic response analysis of structures with uncertain parameters", *Comput. Mech.*, **34**, 400-409.

Li, X.Y. and Law, S.S. (2008), "Damage identification of structures including system uncertainties and measurement noise", *AIAA J.*, **46**(46), 263-276.

Li, Y. and Chen, Y. (2013), "A review on recent development of vibration-based structural robust damage detection", *Struct. Eng. Mech.*, **45**(2), 159-168.

Liu, L.J., Su, Y., Zhu, J.J. and Lei, Y. (2016) "Data fusion based EKF-UI for real-time simultaneous identification of structural systems and unknown external inputs", *Measur.*, **88**, 456-467

Ou, J.P. and Li, H. (2010), "Structural health monitoring in mainland China: Review and future trends", *Struct. Hlth. Monit.*, **9**, 219-231.

Sun, H. and Betti, R. (2014), "Simultaneous identification of structural parameter sand dynamic input with incomplete output-only measurements", *Struct. Control Hlth. Monit.*, **21**, 868-889.

Wong, F.S. and Yao, J.T.P. (2001), "Health monitoring and structural reliability as a value chain", *Comput. Aid. Civil Infrastr. Eng.*, **16**, 71-78.

Xu, Y.L., Zhang, J., Li, J.C. and Xia, Y. (2009), "Experimental investigation on statistical moment-based structural damage detection method", *Struct. Hlth. Monit.*, **8**(6), 555-571.

Yang, J.N., Lin, S.L., Huang, H.W. and Zhou, L. (2006), "An Adaptive Extended Kalman Filter for Structural Damage Identification", *Struct. Control Hlth. Monit.*, **13**(4), 849-867.

Yang, J.N., Pan, S. and Huang, H.W. (2007) "An adaptive extended Kalman filter for structural damage identification II: unknown inputs", *Struct. Control Hlth. Monit.*, **14**, 497-521.

Yao, J.T.P. and Natke, H.G. (1994) "Damage detection and reliability evaluation of existing structures", *Struct. Saf.*, **15**, 3-16.

Zhang, J., Xu, Y.L. and Li, J. (2011) "Integrated system identification and reliability evaluation of stochastic building structures", *Probab. Eng. Mech.*, **26**, 528-538.

Zhang, J., Xu, Y.L., Xia, Y. and Li, J. (2008) "A new statistical moment-based structural damage detection method", *Struct. Eng. Mech.*, **30**(4), 445-466.