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An original single variable shear deformation theory for buckling analysis of thick isotropic plates

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Abstract. This work proposes an original single variable shear deformation theory to study the buckling analysis of thick isotropic plates subjected to uniaxial and biaxial in-plane loads. This theory is built upon the classical plate theory (CPT) including the exponential function in terms of thickness coordinate to represent shear deformation effect and it involves only one governing differential equation. Efficacy of the present theory is confirmed through illustrative numerical examples. The obtained results are compared with those of other higher-order shear deformation plate theory results.

Keywords: complex single variable; buckling; plate

1. Introduction

The classical plate theory (CPT) was developed by Kirchhoff in 1850 and according to this theory, the straight line normal to the un-deformed mid-plane remains straight and normal to the deformed mid-plane and do not undergo thickness stretching. This model neglects the transverse shear deformation influence and thus cannot used to study thick plates where shear deformation influences are more significant. The first order shear deformation theory (FSDT) is proposed as improvement over the CPT and this by including the transverse shear deformation in the kinematic suppositions. It is based on the consideration that straight lines normal to un-deformed mid-plane remain straight but not necessarily normal to the deformed mid-plane. Mindlin *et al*. (1951) studied the free vibration of rectangular plate. Reissner (1945) was the first to propose a model which introduces the influence of shear. To overcome the problems of the FSDT (Sadoune *et al*. 2014, Meksi *et al*. 2015, Adda Bedia *et al*. 2015, Bellifa *et al*. 2016), a number of higher order shear deformation plate theories (HSDTs) are proposed. Recent reviews of such HSDTs are reported by Ghugal and Shimpi (2002), Wanji and Zhen (2008), Kreja (2001).

Levy (1877) has proposed a HSDT for thick plate for the first time by employing sinusoidal functions in the displacement field. Stein (1986) utilized theory via trigonometric functions for investigation of laminated beams and plates. Shimpi and Patel (2006) proposed a two variable refined plate theory for the free vibration of orthotropic plate. Reddy (1979, 1984) presented new mixed finite element models for nonlinear response of plates based on the CPT and FSDT. Global-local models are developed by Kapuria and Nath (2013) for bending and dynamic responses of laminated and sandwich plates. A number of HSDTs are also developed for analyzing beams and plates (Shi and Voyiadjis 2011, Bouderba *et al*. 2013, Bessaim *et al*. 2013, Tounsi *et al*. 2013, Zidi *et al*. 2014, Bousahla *et al*. 2014, Ait Amar Meziane *et al*. 2014, Fekrar *et al*. 2014, Belabed *et al*. 2014, Hebali *et al*. 2014, Ait Yahia *et al*. 2015, Mahi *et al*. 2015, Taibi *et al*. 2015, Kar and Panda 2015, Ait Atmane *et al*. 2015, Belkorissat *et al*. 2015, Hamidi *et al*. 2015, Attia *et al*. 2015, Bourada *et al*. 2015, Meradjah *et al*. 2015, Merdaci *et al*. 2016, Tounsi *et al*. 2016, Beldjelili *et al*. 2016, Akavci 2016, Boukhari *et al*. 2016, Bounouara *et al*. 2016, Bennoun *et al*. 2016, Houari *et al*. 2016, Draiche *et al*. 2016, Fahsi *et al*. 2017, Meksi *et al*. 2017, Bellifa *et al*. 2017, Besseghier *et al*. 2017). More recent works are already available on the buckling and post-buckling behaviour of structures with and without inclusion of geometrical distortion (Kar *et al*. 2017, Kar and Panda 2017, Chikh *et al*. 2017, Kar and Panda 2016, Bouderba *et al*. 2016, Bousahla *et al*. 2016, Kar *et al*. 2016, Bourada *et al*. 2016, Panda and Katariya 2015, Katariya and Panda 2014, Panda and Singh 2013a,b,c,d, Panda and Singh 2010a,b, Panda and Singh 2009).

This work presents a simple single variable shear deformation theory for buckling behavior of isotropic square plates under uniaxial and biaxial in-plane loads. The principal feature of this theory is that, in addition to including the shear deformation influence, the displacement field is modeled with only one unknown. The effectiveness of the developed theory is demonstrated through illustrative

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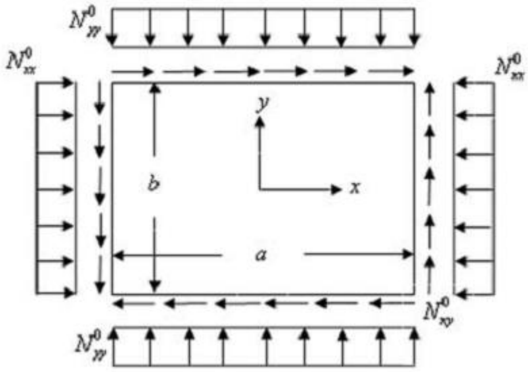


Fig. 1 Plate subjected to in-plane forces

examples.

2. Mathematical formulation

Consider a simply supported rectangular isotropic plate with the length a, width b, and thickness h. The plate is subjected to in-plane compressive forces ($N^0_{xx}, N^0_{yy}, N^0_{xy}$) as shown in Fig. 1.

The co-ordinate system (*x*,*y*,*z*) chosen and the coordinate parameters are such a that, the plate occupies a region given by Eq. (1)

$$0 \le x \le a\,,\quad 0 \le y \le b\,,\quad -h/2 \le z \le h/2 \tag{1}$$

2.1 Kinematics

The displacement field of the present single variable shear deformation theory is given as follows

$$\begin{aligned} u(x,y,z) &= -z\frac{\partial w_0}{\partial x} - \beta\, f(z)\frac{\partial^3 w_0}{\partial x^3} \\ v(x,y,z) &= -z\frac{\partial w_0}{\partial y} - \beta\, f(z)\frac{\partial^3 w_0}{\partial y^3} \\ w(x,y,z) &= w_0(x,y) \end{aligned} \tag{2}$$

Where *u*, *v* and *w* are the displacements in the *x*, *y* and *z*-directions respectively and β is a parameter of the proposed displacement model. *f*(*z*) is a shape function representing the distribution of the transverse shear strains and shear stresses through the thickness of the plate and is given as

$$f(z) = z\exp\left[-2\left(\frac{z}{h}\right)^2\right] \tag{3}$$

The nonzero strains associated with the displacement field in Eq. (2) are

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = z\begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} + \beta\, f(z)\begin{Bmatrix} \eta_x \\ \eta_y \\ \eta_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \beta\, g(z)\begin{Bmatrix} \gamma^0_{yz} \\ \gamma^0_{xz} \end{Bmatrix} \tag{4}$$

where

$$\begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_0}{\partial x^2} \\ -\dfrac{\partial^2 w_0}{\partial y^2} \\ -2\dfrac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix},\ \begin{Bmatrix} \eta_x \\ \eta_y \\ \eta_{xy} \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^4 w_0}{\partial x^2} \\ -\dfrac{\partial^4 w_0}{\partial y^2} \\ -\dfrac{\partial^2(\nabla^2 w_0)}{\partial x \partial y} \end{Bmatrix}, \tag{5}$$

$$\begin{Bmatrix} \gamma^0_{yz} \\ \gamma^0_{xz} \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^3 w_0}{\partial y^3} \\ -\dfrac{\partial^3 w_0}{\partial x^3} \end{Bmatrix}$$

And

$$g(z) = f'(z),\quad \nabla^2 w_0 = \frac{\partial^2 w_0}{\partial x^2} + \frac{\partial^2 w_0}{\partial y^2} \tag{6}$$

2.2 Constitutive relations

The constitutive relations of the isotropic plate can be written as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} \tag{7}$$

Where (σ_x, σ_y, τ_{yz}, τ_{xz}, τ_{xy}) and (ε_x, ε_y, γ_{yz}, γ_{xz}, γ_{xy}) are the stress and strain components, respectively. The stiffness coefficients, C_{ij}, can be defined as

$$C_{11} = C_{22} = \frac{E}{1-\nu^2}\,,\ C_{12} = \nu\, C_{11} \tag{8a}$$

$$C_{44} = C_{55} = C_{66} = G = \frac{E}{2(1+\nu)}, \tag{8b}$$

2.3 Governing equations

The governing equations can be derived using the principle of virtual work. The principle can be written in the following form

$$\begin{aligned} &\int_{-h/2}^{h/2}\int_{\Omega}\begin{pmatrix} \sigma_x\delta\ \varepsilon_x + \sigma_y\delta\ \varepsilon_y + \tau_{xy}\delta\ \gamma_{xy} \\ +\tau_{xz}\delta\ \gamma_{xz} + \tau_{yz}\delta\ \gamma_{yz} \end{pmatrix} d\Omega dz \\ &- \int_{\Omega}\left[N^0_{xx}\frac{\partial^2 w}{\partial x^2} + N^0_{yy}\frac{\partial^2 w}{\partial y^2} + 2N^0_{xy}\frac{\partial^2 w}{\partial x \partial y}\right]\delta w d\Omega = 0 \end{aligned} \tag{9}$$

Where Ω is the top surface

Substituting Eqs. (4), (2) and (7) into Eq. (9) and integrating through the thickness of the plate, Eq. (9) can be rewritten as

$$\int_{\Omega}\left[\begin{array}{l} M_x\delta\, k_x + M_y\delta\, k_y + M_{xy}\delta\, k_{xy} \\ +\beta(S_x\delta\,\eta_x + S_y\delta\,\eta_y + S_{xy}\delta\,\eta_{xy} \\ +Q_{yz}\delta\,\gamma^0_{yz} + Q_{xz}\delta\,\gamma^0_{xz})d\Omega \end{array}\right. \tag{10}$$

$$-\int_{\Omega}\left[N^0_{xx}\frac{\partial^2 w}{\partial x^2} + N^0_{yy}\frac{\partial^2 w}{\partial y^2} + 2N^0_{xy}\frac{\partial^2 w}{\partial x\partial y}\right]\delta\, w\, d\Omega = 0$$

In which the stress resultants M, S and Q are expressed by

$$\left(M_i, S_i\right) = \int_{-h/2}^{h/2}\left(z, \beta\, f\right)(\sigma_i)dz\,, \quad \left(i = x, y, xy\right)$$

$$\text{and} \quad Q_i = \int_{-h/2}^{h/2}(\tau_i)\,\beta\, g(z)dz\,, \quad \left(i = xz, yz\right) \tag{11}$$

Substituting Eqs. (4) and (7) into Eq. (10) and integrating through the thickness of the plate, the governing differential equations in-terms of stress resultants are as follows

$$\begin{array}{l} \dfrac{\partial^2 M_x}{\partial x^2} + 2\dfrac{\partial^2 M_{xy}}{\partial x\partial y} + \dfrac{\partial^2 M_y}{\partial y^2} + \beta\dfrac{\partial^4 S_x}{\partial x^4} + \beta\dfrac{\partial^4 S_{xy}}{\partial x^3\partial y} \\ +\beta\dfrac{\partial^4 S_{xy}}{\partial y^3\partial x} + \beta\dfrac{\partial^4 S_y}{\partial y^4} - \beta\dfrac{\partial^3 Q_{xz}}{\partial x^3} - \beta\dfrac{\partial^3 Q_{yz}}{\partial y^3} \\ +N^0_{xx}\dfrac{\partial^2 w}{\partial x^2} + N^0_{yy}\dfrac{\partial^2 w}{\partial y^2} + 2N^0_{xy}\dfrac{\partial^2 w}{\partial x\partial y} = 0 \end{array} \tag{12}$$

2.4 Governing equations in terms of displacements

From Eqs. (4), (7) and (11), the stress resultants can be expressed as below

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \\ S_x \\ S_y \\ S_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 & \beta D^s_{11} & \beta D^s_{12} & 0 \\ D_{12} & D_{22} & 0 & \beta D^s_{12} & \beta D^s_{22} & 0 \\ 0 & 0 & D_{66} & 0 & 0 & \beta D^s_{66} \\ \beta D^s_{11} & \beta D^s_{12} & 0 & \beta^2 H^s_{11} & \beta^2 H^s_{12} & 0 \\ \beta D^s_{12} & \beta D^s_{22} & 0 & \beta^2 H^s_{12} & \beta^2 H^s_{22} & 0 \\ 0 & 0 & \beta D^s_{66} & 0 & 0 & \beta^2 H^s_{66} \end{bmatrix} \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \\ \eta_x \\ \eta_y \\ \eta_{xy} \end{Bmatrix} \tag{13a}$$

$$\begin{Bmatrix} Q_{xz} \\ Q_{yz} \end{Bmatrix} = \beta^2 \begin{bmatrix} A^s_{55} & 0 \\ 0 & A^s_{44} \end{bmatrix} \begin{Bmatrix} \gamma^0_{xz} \\ \gamma^0_{yz} \end{Bmatrix} \tag{13 b}$$

Where

$$\begin{Bmatrix} D_{11} & D^s_{11} & H^s_{11} \\ D_{12} & D^s_{12} & H^s_{12} \\ D_{66} & D^s_{66} & H^s_{66} \end{Bmatrix} = \int_{-h/2}^{h/2} C_{11}\left(z^2, z\, f(z), f^2(z)\right)\begin{Bmatrix} 1 \\ \nu \\ \dfrac{1-\nu}{2} \end{Bmatrix} dz \tag{14a}$$

$$\left(D_{22}, D^s_{22}, H^s_{22}\right) = \left(D_{11}, D^s_{11}, H^s_{11}\right) \tag{14b}$$

$$A^s_{44} = A^s_{55} = \int_{-h/2}^{h/2} C_{44}\left[g(z)\right]^2 dz, \tag{14c}$$

The governing differential equations in-terms of unknown displacement variable used in the displacement field (w_0) obtained are as follows

$$\begin{array}{l} -D_{11}\dfrac{\partial^4 w_0}{\partial x^4} - 2\left(D_{12} + 2D_{66}\right)\dfrac{\partial^4 w_0}{\partial x^2\partial y^2} - D_{22}\dfrac{\partial^4 w_0}{\partial y^4} \\ +\beta\left(\begin{array}{l} -2D^s_{11}\dfrac{\partial^6 w_0}{\partial x^6} - 2\left(D^s_{12} + 2D^s_{66}\right)\dfrac{\partial^6 w_0}{\partial x^2\partial y^4} \\ -2\left(D^s_{12} + 2D^s_{66}\right)\dfrac{\partial^6 w_0}{\partial x^4\partial y^2} - 2D^s_{22}\dfrac{\partial^6 w_0}{\partial y^6} \end{array}\right) \\ -\beta^2\left(\begin{array}{l} H^s_{11}\dfrac{\partial^8 w_0}{\partial x^8} + 2\left(H^s_{12} + H^s_{66}\right)\dfrac{\partial^8 w_0}{\partial x^4\partial y^4} + \\ H^s_{66}\dfrac{\partial^8 w_0}{\partial x^6\partial y^2} + H^s_{66}\dfrac{\partial^8 w_0}{\partial x^2\partial y^6} + H^s_{22}\dfrac{\partial^8 w_0}{\partial y^8} \\ -A^s_{44}\dfrac{\partial^6 w_0}{\partial x^6} - A^s_{55}\dfrac{\partial^6 w_0}{\partial y^6} \end{array}\right) \\ +N^0_{xx}\dfrac{\partial^2 w}{\partial x^2} + N^0_{yy}\dfrac{\partial^2 w}{\partial y^2} + 2N^0_{xy}\dfrac{\partial^2 w}{\partial x\partial y} = 0 \end{array} \tag{15}$$

3. Analytical solutions

In this work, the studied simply supported rectangular plate is subjected to in-plane forces in two directions ($N^0_x = -N_0, N^0_y = kN^0_x$ and $N^0_{xy} = 0$). Based on Navier solution procedure, the displacements are assumed as follows

$$w(x, y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{mn}\sin\left(\lambda\, x\right)\sin\left(\mu\, y\right) \tag{16}$$

where $\lambda = m\pi/a$, $\mu = n\pi/b$, W_{mn} is the unknown maximum displacement coefficient, and ω is the angular frequency.

Substitute Eq. (16) in the governing differential Eq. (15) resulting the following equation form

$$S - N_0\left(\lambda^2 + k\mu^2\right)W_{mn} = 0 \tag{17}$$

With

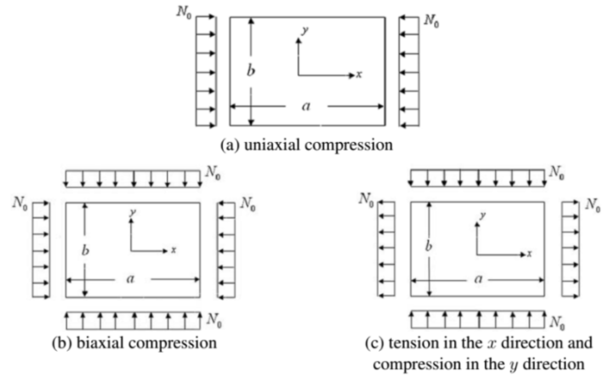


Fig. 2 The loading conditions of square plate for (a) uniaxial compression, (b) biaxial compression and (c) tension in the *x* direction and compression in the *y* direction

$$
\begin{aligned}
S &= D_{11}\lambda^4 + 2\left(D_{12}+2D_{66}\right)\lambda^2\mu^2 + D_{22}\mu^4 \\
&-\beta\begin{bmatrix} -2D_{11}^s\lambda^6 + 2\left(D_{12}^s+2D_{66}^s\right)\left(\lambda^2\mu^4+\lambda^4\mu^2\right) \\ -2D_{22}^s\mu^6 \end{bmatrix} \\
&+\beta^2\begin{bmatrix} H_{11}^s\lambda^8 + 2\left(H_{12}^s+2H_{66}^s\right)\lambda^4\mu^4 + H_{66}^s\lambda^6\mu^2 \\ +H_{66}^s\lambda^2\mu^6 + H_{22}^s\mu^8 - A_{44}^s\lambda^6 - A_{55}^s\mu^6 \end{bmatrix}
\end{aligned} \tag{18}
$$

4. Numerical results and discussion

In this section, a simply supported square plate under the loading conditions, as presented in Fig. 2, is examined to demonstrate the accuracy of the proposed theory in investigating the stability behavior of the isotropic plate.

The expression of shape parameter 'β' is evaluated in the post-processing phase and is found to be as follows

$$\beta = \frac{A_1 - A_2}{1+e^{(\theta - x_0)/d_x}} + A_2 \tag{19}$$

Where $\theta = a/h$ and the values of the other coefficients are computed in the post-processing phase and are found to be A_1=0.67515 A_2=0.58471, x_0=2.4099, d_x=1.60645.

For validation purpose, the computed results are compared with those obtained by the third shear deformation theory (TSDT) of Reddy (1984), the exponential shear deformation theory (ESDT) of Sayyada and Ghugal (2012) and CPT and FSDT of Mindlin (1951). Following material characteristics of isotropic plates are employed

$$E = 210 \text{ GPa and } \nu = 0.3 \tag{20a}$$

$$E = 77 \text{ GPa and } \nu = 0.33 \tag{20b}$$

For convenience, the following non-dimensional buckling load is utilized

$$\overline{N}_{cr} = \frac{N_0 a^2}{Eh^3} \tag{21}$$

Table 1 Comparison of non-dimensional critical stability load ($\overline{N}_{cr}$) of square plates under to uniaxial compression (k=0, E=210 GPa and ν=0.3)

Mode for the plate (m,n)	Theory	*a/h*				
		5	10	20	50	100
(1, 1)	Present	2.9552	3.4234	3.5652	3.6071	3.6132
	ESDT(*)	2.9603	3.4242	3.5654	3.6072	3.6132
	TSDT	2.9512	3.4224	3.5649	3.6068	3.6130
	FSDT	2.9498	3.4222	3.5649	3.6071	3.6130
	CPT	3.6152	3.6152	3.6152	3.6152	3.6152

(*) Taken from Sayyada and Ghugal (2012)

Table 2 Comparison of non-dimensional critical stability load ($\overline{N}_{cr}$) of square plates under to biaxial compression (k=1, E=210 GPa and ν=0.3)

Mode for the plate (m,n)	Theory	*a/h*				
		5	10	20	50	100
(1, 1)	Present	1.4776	1.7117	1.7826	1.8035	1.8066
	ESDT(*)	1.4802	1.7121	1.7827	1.8038	1.8065
	TSDT	1.4756	1.7112	1.7825	1.8034	1.8065
	FSDT	1.4749	1.7111	1.7825	1.8035	1.8065
	CPT	1.8076	1.8076	1.8076	1.8076	1.8076

(*) Taken from Sayyada and Ghugal (2012)

Table 3 Comparison of non-dimensional critical stability load ($\overline{N}_{cr}$) of square plates under tension in the x direction and compression in the y direction (k=1, E=210 GPa and ν=0.3)

Mode for the plate (m,n)	Theory	*a/h*				
		5	10	20	50	100
(1, 2)	Present	4.8444	6.6039	7.2798	7.4896	7.5211
	ESDT(*)	4.8798	6.6133	7.2777	7.4898	7.5212
	TSDT	4.8274	6.6024	7.2754	7.4893	7.5201
	FSDT	4.8158	6.6010	7.2753	7.4895	7.5211
	CPT	7.5317	7.5317	7.5317	7.5317	7.5317

(*) Taken from Sayyada and Ghugal (2012)

Tables 1-3 presents the comparison of critical stability load for the steel plates whereas Tables 4-6 present the comparison of critical stability load for the aluminum plates under in-plane loads. In case of plate under to uniaxial compression (Fig. 2(a)) and biaxial compression (Fig. 2(b)), buckling force is critical when mode for the plate is (1, 1) whereas in case of plate under tension in x direction and compression in y direction (Fig. 2(c)), buckling force is critical when mode for the plate is (1, 2).

The examination of Tables 1-6 show that the critical stability load predicted by proposed single variable shear deformation theory (SVSDT) and Reddy's theory (TSDT) is in excellent agreement with each other even though the plate is very thick due to inclusion of effect of transverse shear deformation. It is noted that, the present theory involves only one unknown variable against the three unknown variables in case of TSDT, ESDT and FSDT.

Table 4 Comparison of non-dimensional critical stability load ($\overline{N}_{cr}$) of square plates under to uniaxial compression (k=0, E=70 GPa and v=0.33)

Mode for the plate (m,n)	Theory	a/h				
		5	10	20	50	100
(1, 1)	Present	3.0154	3.4956	3.6408	3.6836	3.6898
	ESDT(*)	2.9991	3.4886	3.6388	3.6833	3.6898
	TSDT	2.9893	3.4866	3.6383	3.6833	3.6896
	FSDT	2.9877	3.4865	3.6383	3.6832	3.6900
	CPT	3.6919	3.6919	3.6919	3.6919	3.6919

(*) Taken from Sayyada and Ghugal (2012)

Table 5 Comparison of non-dimensional critical stability load ($\overline{N}_{cr}$) of square plates under to biaxial compression (k=1, E=70 GPa and v=0.33)

Mode for the plate (m,n)	Theory	a/h				
		5	10	20	50	100
(1, 1)	Present	1.5077	1.7478	1.8204	1.8418	1.8449
	ESDT(*)	1.4995	1.7443	1.8194	1.8416	1.8449
	TSDT	1.4947	1.7433	1.8192	1.8416	1.8448
	FSDT	1.4939	1.7433	1.8192	1.8415	1.8450
	CPT	1.8459	1.8459	1.8459	1.8459	1.8459

(*) Taken from Sayyada and Ghugal (2012)

Table 6 Comparison of non-dimensional critical stability load ($\overline{N}_{cr}$) of square plates under tension in the x direction and compression in the y direction (k=1, E=70 GPa and v=0.33)

Mode for the plate (m,n)	Theory	a/h				
		5	10	20	50	100
(1, 2)	Present	4.9142	6.7392	7.4332	7.6483	7.6806
	ESDT(*)	4.9083	6.7172	7.4208	7.6468	7.6803
	TSDT	4.8523	6.7055	7.4184	7.6465	7.6804
	FSDT	4.8398	6.7040	7.4183	7.6465	7.6810
	CPT	7.6915	7.6915	7.6915	7.6915	7.6915

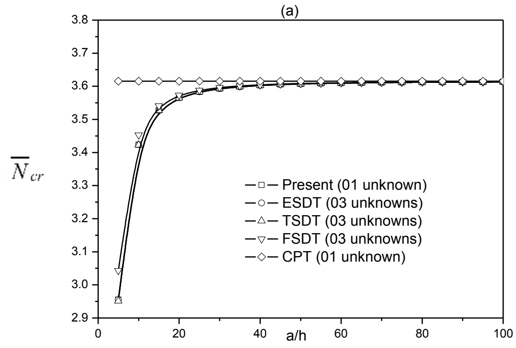
(*) Taken from Sayyada and Ghugal (2012)

Furthermore, it can be also noted that, the FSDT requires the use of a shear correction factor. In contrast, present theory does not require a shear correction factor. From the results that CPT overestimates the values of critical stability load due to neglect of transverse shear deformation. Also, we found that in case of CPT, critical stability load is independent of side-to-thickness ratio (a/h).

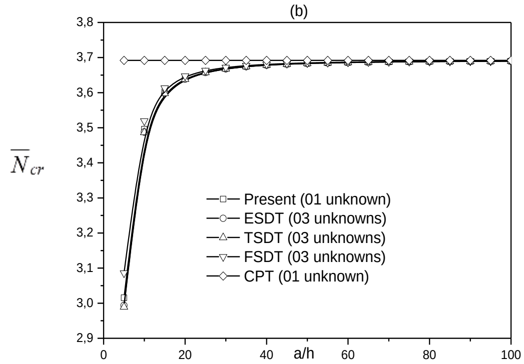
Figs. 3-5 demonstrate that, for the higher value of side to-thickness ratio (a/h), the results computed by the present SVSDT, ESDT, TSDT, FSDT and CPT are more or less same.

5. Conclusions

In this article, a simple single variable shear deformation theory for buckling behavior of isotropic thick plates is

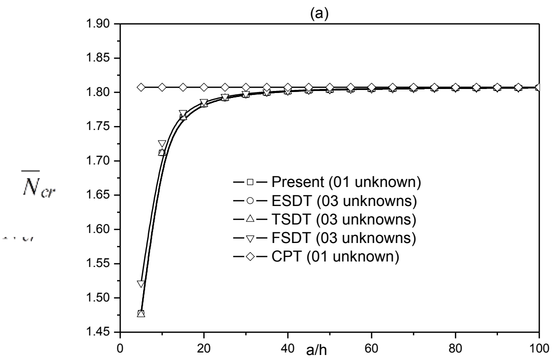


(a) (E=210 GPa and v=0.3)

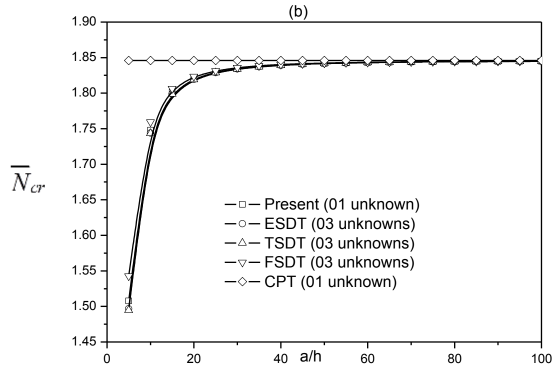


(b) (E=70 GPa and v=0.33)

Fig. 3 The influence of side-to-thickness ratios on the critical buckling load of square plate subjected to uniaxial compression

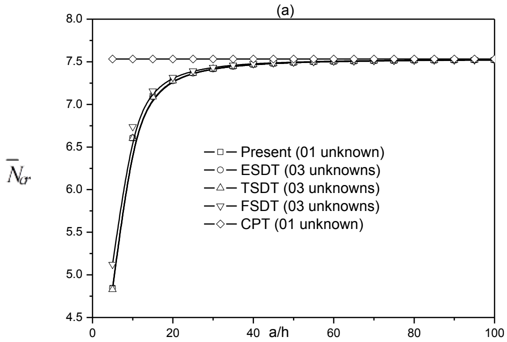


(a) (E=210 GPa and v=0.3)

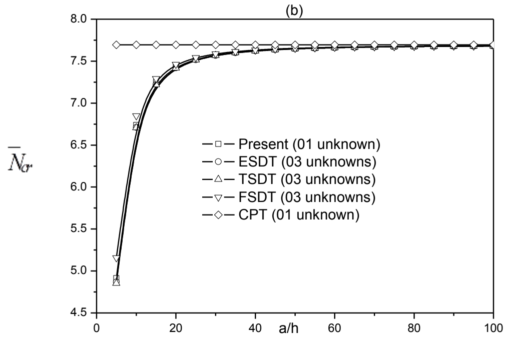


(b) (E=70 GPa and v=0.33)

Fig. 4 The influence of side-to-thickness ratios on the critical buckling load of square plate subjected to biaxial compression



(a) (E=210 GPa and v=0.3)



(b) (E=70 GPa and v=0.33)

Fig 5 The influence of side-to-thickness ratios on the critical buckling load of square plate subjected to tension in the x direction and compression in the y direction

presented. Some of the important aspects of the plate theory presented herein can be summarized as follows:

- The governing differential equation of the theory involves only one unknown variable.
- The displacement field of the present plate theory gives rise to a realistic parabolic variation of transverse shear stress across the thickness. Furthermore, present theory does not require a shear correction factor.
- Efficacy of the proposed theory is demonstrated through illustrative examples for buckling of thick isotropic plates. The obtained numerical results are compared with those of other first-order and higher-order shear deformation plate theory results. The results obtained are found to be accurate.
- It can be concluded that the proposed theory with only one unknown variable can accurately predict the critical buckling loads of the isotropic plates.

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