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Elasto-plastic thermal stress analysis of functionally graded hyperbolic discs

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Abstract. The objective of this analytical study is to calculate the elasto-plastic stresses of Functionally Graded (FG) hyperbolic disc subjected to uniform temperature. The material properties (elastic modulus, thermal expansion coefficient and yield strength) and the geometry (thickness) of the disc are assumed to vary radially with a power law function, but Poisson's ratio does not vary. FG disc material is assumed to be non-work hardening. Radial and tangential stresses are obtained for various thickness profile, temperature and material properties. The results indicate that thickness profile and volume fractions of constituent materials play very important role on the thermal stresses of the FG hyperbolic discs. It is seen that thermal stresses in a disc with variable thickness are lower than those with constant thickness at the same temperature. As a result of this, variations in the thickness profile increase the operation temperature. Moreover, thickness variation in the discs provides a significant weight reduction. A disc with lower rigidity at the inner surface according to the outer surface should be selected to obtain almost homogenous stress distribution and to increase resistance to temperature. So, discs, which have more rigid region at the outer surface, are more useful in terms of resistance to temperature.

Keywords: analytical method; elasto-plastic; functionally graded; residual stress/strain; yield criterion

1. Introduction

Functionally Graded Materials (FGMs) are obtained by changing gradually the volume fractions of constituents from one surface to the other. This variation in the structure brings significant advantages compared to laminated composites and isotropic structures. FGMs are used as interfacial zone to improve the bonding strength of layered composites and to reduce the residual and thermal stresses in bonded dissimilar materials in machine and engine components (Pindera 1997, Erdoğan 1995). Moreover, the material properties of the structure can be adjusted according to demand. Therefore, FGMs have been the subject of intense research and attracted considerable attention in recent years. In addition, the mechanical and mathematical modeling of FGMs are a very important research area.

The discs, such as structural parts of fly-wheels, high-speed gears and turbine rotors have a wide range of applications in engineering (Bayat *et al*. 2008). With increasing demand to achieve high strength to weight ratios, optimizing the geometrical and physical properties of the disc configuration becomes more significant. For example, an optimal design is required to assess a suitable radial thickness profile of a gas turbine disc in turbo jet engines. A thickness variation in the gas turbine discs provides a significant weight reduction while keeping all other performance characteristics the same.

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You *et al*. (2007) investigated the effect of varying material properties, different temperature changes and radius ratios on stresses and deformations in FG rotating circular discs. Zenkour (2005) proposed a new material properties and density profile in exponential form containing four geometric parameters for rotating annular discs. In that study, he obtained an exact elasticity solution for FG discs with plane stress assumption and exponential material properties variation. Çallıoğlu (2011) and Çallıoğlu *et al*. (2011) analyzed the stresses and deformation in a FG disc under mechanical and thermal loads. It is found from the results that the grading indexes play an important role in determining the mechanical responses of FG disc and in optimal design of these structures. Nie and Batra (2010) analyzed axisymmetric deformations of a rotating disc varying thickness, mass density, thermal expansion coefficient and shear modulus. Kordkheili and Naghdabadi (2007) presented a semi-analytical thermoelasticity solution for hollow and solid rotating axisymmetric discs made of FGMs. They also presented the analytical solutions of stress, strain and displacement components along the radius and compared with those of a finite element analysis in the literature. Finally, they noted that the property gradation correlates with thermomechanical responses of FG rotating discs.

Mohammadi and Dryden (2008) examined the role of nonhomogeneous stiffness on the thermoelastic stress field in FG curved beam and ring. They found that the flexural stress in the ring is high in comparison with the beam. Mohammadi (2015) studied the effects of varying elastic modulus, Poisson's ratio and thermal expansion coefficient on the thermoplastic field in the graded axisymmetric and one-dimensional problems. He reported that the effect of varying Poisson's ratio on the thermal stresses was

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considerable for the solid ring whereas it was negligible for the curved beam. Chiba (2009) derived analytically the second-order statistics in an axisymmetrically heated FG annular disc with spatially random heat transfer coefficients on the upper and lower surfaces using integral transform method and a perturbation method. Finally, the method has been verified and its applicability has been examined through comparisons with the results obtained by a direct Monte Carlo simulation. Damircheli and Azadi (2011) analyzed thermal and mechanical stresses in a FG rotating disc with variable thickness by using finite element method (FEM). To model the disc by FEM, one-dimensional two-degree elements with three nodes are used. The effect of varying thicknesses and dependency of material properties on temperature distribution were also investigated. Khanna *et al*. (2015) studied steady-state creep in a rotating Al-SiCp disc having different thickness profiles. They concluded that the radial and tangential stresses increased by increasing SiCp gradient. Liew *et al*. (2003) presented an analysis of the thermomechanical behavior of hollow circular cylinders made of FGM. They provided a new solution method for thermal stresses in homogeneous cylinders as a special case. Kar and Kanoria (2007) investigated the distribution of stresses due to step input of temperature on the boundaries of a homogeneous transversely isotropic circular disc by applying Laplace transform technique in the context of generalized theories of thermo-elasticity. They computed numerically the stresses of the disc. Garg *et al*. (2015) investigated the steady state creep behaviors of a FG rotating disc under varying thermal gradient. Their results indicated that with increase in the temperature, the radial stress increases over the entire disc but the tangential and effective stresses increase near the inner radius and decrease toward the outer radius. Wang *et al*. (2013) investigated semi-analytically the effects of thickness variations on the annular discs with fully constrained at the outer boundary under thermal loading. They assumed that the disc material properties depend on the temperature, but the Poisson's ratio is independent. They found for the disc with small holes that the index values have strong effects on the normalized temperature difference.

Alexandrova and Vila Real (2007) analyzed plastic analytical stresses of a rotating annular disc with its contours being free from the radial pressure and with specifically variable thickness in terms of the Mises-yield criterion and its associated flow rule. They found that the existence of the thickness gradient influences the size of plastic zone, limit angular velocities and stress distributions significantly. Çallıoğlu *et al*. (2015) analyzed the elasto-plastic stresses of rotating FG discs. They showed that the analytical results were compatible with numerical results.

As can be understood from the above literature survey, most of the studies mentioned above are dealt with the elastic stress analysis of FG discs which subjected to mechanical and/or thermal loads. They have been done in order to see the effects of FGMs on the isotropic discs. Nevertheless, the studies on the elasto-plastic stress analysis of FG discs are found few in the open literature. A small part of the above studies are related to elasto-plastic stress analysis of homogeneous isotropic, composite or FG discs. In this analytical study, the elasto-plastic thermal stresses are analyzed in a FG hyperbolic disc. The material properties, such as elastic modulus, thermal expansion coefficient and yield strength of FG disc, vary radially according to a power law function, and gradient parameter is taken in the range of -1.0 to 1.0. The geometric parameter for the thickness profile is also chosen between -0.9 and 0.0 (Alexandrova and Vila Real, 2007). In the elasto-plastic solution, the yielding behavior of the disc material is supposed as a non-work hardening case using von Mises' yield condition.

2. Elastic solution

Assuming that the stresses do not vary along the thickness of the disc, the governing differential equation of equilibrium, which is used for a thin disc of constant thickness, can also be extended to disc of variable thickness (Timoshenko and Goodier 1970)

$$\frac{d}{dr}\left(h(r)\, r\sigma_r\right) - h(r)\,\sigma_\theta = 0 \tag{1}$$

where σ_r and σ_θ are radial and tangential stresses, h is thickness of the disc and it is assumed to vary along the radial direction, r is the radial distance ($a \le r \le b$), a and b are inner and outer radii of the disc, respectively, as illustrated in Fig. 1.

Due to the symmetry, the strain-displacement relations are given by

$$\varepsilon_r = \frac{du}{dr}, \qquad \varepsilon_\theta = \frac{u}{r} \tag{2}$$

where u is the displacement component in the radial direction. The strain compatibility equation is

$$\varepsilon_r = \frac{d}{dr}\left(r\,\varepsilon_\theta\right) \tag{3}$$

The strain-stress relations, which includes thermal effects, can be given by

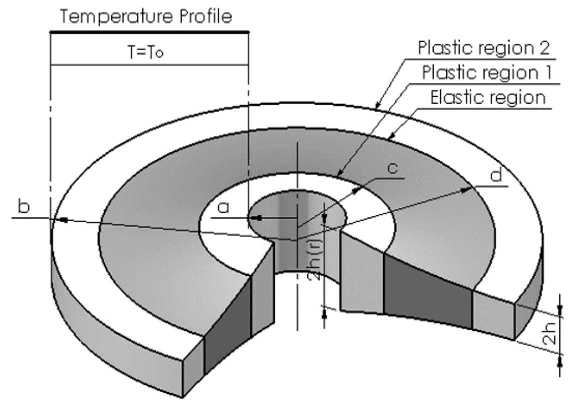


Fig. 1 Schematic of annular FG disc of variable thickness subjected to uniform temperature

$$\varepsilon_r = \frac{1}{E(r)}\left(\sigma_r - \nu\,\sigma_\theta\right) + \alpha\left(r\right)T_o \tag{4a}$$

$$\varepsilon_\theta = \frac{1}{E(r)}\left(\sigma_\theta - \nu\,\sigma_r\right) + \alpha\left(r\right)T_o \tag{4b}$$

where $E(r)$, $\alpha(r)$ and T_0 are radially varying elastic modulus, thermal expansion coefficient and applied uniform temperature, respectively. Poisson's ratio, ν is assumed as a constant because variation of Poisson's ratio has much less practical significance than those in the others.

Eq. (1) is satisfied by the stress function F defined as

$$\sigma_r = \frac{F}{h(r)\,r}, \qquad \sigma_\theta = \frac{1}{h(r)}\frac{dF}{dr} \tag{5}$$

Substituting Eqs. (4) and (5) into compatibility Eq. (3) gives

$$r^2F'' + r\,F'\left(1 - r\frac{E'(r)}{E(r)} - r\frac{h'(r)}{h(r)}\right) - F\left(1 - \nu\,r\frac{E'(r)}{E(r)} - \nu\,r\frac{h'(r)}{h(r)}\right) = -E(r)\,h(r)\,r^2\,\alpha'\left(r\right)T_o \tag{6}$$

where upper apostrophes indicate derivative according to r.

Now suppose that

$$E(r) = E\left(\frac{r}{b}\right)^{n_1} \tag{7a}$$

$$\alpha(r) = \alpha\left(\frac{r}{b}\right)^{n_2} \tag{7b}$$

$$h(r) = h\left(\frac{r}{b}\right)^{n_3} \tag{7c}$$

where n_1, n_2 and n_3 are gradient indexes. Substituting Eq. (7) into Eq. (6), the differential equation reduces to

$$r^2F'' + r\,F'\left(1 - n_1 - n_3\right) + F\left(1 - \nu\,n_1 - \nu\,n_3\right) = -\frac{E\,h\,\alpha\,n_2\,T_o}{b^{n_1+n_2+n_3}}\,r^{n_1+n_2+n_3+1} \tag{8}$$

As gradation indexes are almost equal to zero, Eq. (8) reduces to equation of the homogenous, isotropic disc (Timoshenko and Goodier 1970)

$$r^2F'' + r\,F' + F = -E\,h\,\alpha\,n_2\,rT_o \tag{9}$$

As Eq. (8) is solved, the stress function F can be written as

$$F = C_1\,r^{\frac{n_1+n_3+m}{2}} + C_2\,r^{\frac{n_1+n_3-m}{2}} + A\,r^{n_1+n_2+n_3+1} \tag{10}$$

where C_1 and C_2 are the integration constants and the positive root of the equation, m, is

$$m = \left[(n_1+n_3)^2 - 4(\nu\,n_1 + \nu\,n_3 - 1)\right]^{1/2} \tag{11}$$

and the term A is

$$A = -\frac{E\,h\,\alpha\,n_2\,T_o}{b^{n_1+n_2+n_3}\left[n_1n_2 + n_2n_3 + n_2^2 + n_1 + 2n_2 + n_3 + \nu n_1 + \nu n_3\right]} \tag{12}$$

The stress components can be obtained from the stress function as

$$\sigma_r = \frac{1}{h(r)}\left[C_1\,r^{\frac{n_1+n_3+m-2}{2}} + C_2\,r^{\frac{n_1+n_3-m-2}{2}} + A\,r^{n_1+n_2+n_3}\right] \tag{13}$$

$$\sigma_\theta = \frac{1}{h(r)}\left[\left(\frac{n_1+n_3+m}{2}\right)C_1\,r^{\frac{n_1+n_3+m-2}{2}} + \left(\frac{n_1+n_3-m}{2}\right)C_2\,r^{\frac{n_1+n_3-m-2}{2}} + \left(n_1+n_2+n_3+1\right)A\,r^{n_1+n_2+n_3}\right] \tag{14}$$

The integration constants C_1 and C_2 can be obtained from the boundary conditions. Since the disc is assumed to be connected to the shaft by means of splines, where small axial movement is permitted, σ_r is equal to zero at the inner and outer surfaces of the disc (Timoshenko and Goodier 1970)

$$\sigma_r = 0 \;\; at \;\; r = a \;\; and \;\; r = b \tag{15}$$

By using these conditions, C_1 and C_2 are determined as

$$C_1 = \frac{A\left(b^{\frac{n_1+2n_2+n_3+m+2}{2}} - a^{\frac{n_1+2n_2+n_3+m+2}{2}}\right)}{a^m - b^m} \tag{16a}$$

$$C_2 = -\frac{A\left(a^m\,b^{\frac{n_1+2n_2+n_3+m+2}{2}} - b^m\,a^{\frac{n_1+2n_2+n_3+m+2}{2}}\right)}{a^m - b^m} \tag{16b}$$

3. Plastic solution

In the solution, von Mises hypothesis is used as a yield criterion due to the same yield strengths (σ_0) in tension and compression for ductile materials. The equivalent stress can be written as

$$\bar{\sigma} = \sqrt{\sigma_\theta^2 - \sigma_r\sigma_\theta + \sigma_r^2} \tag{17}$$

The material is assumed to be non-work hardening. In this case, the yield strength is written as

$$\sigma_Y = \sigma_0(r) = \sigma_0\left(\frac{r}{b}\right)^{n_4} \tag{18}$$

where σ_Y and n_4 are the yield strength and its gradient index, respectively.

As the tangential stress is isolated from Eq. (1), the following equation can be obtained

$$\sigma_\theta = \sigma_r\left(n_3+1\right) + r\frac{d\sigma_r}{dr} \tag{19}$$

Substituting σ_θ into Eq. (17), yield criterion produces the following equation,

$$r^2\left(\frac{d\sigma_r}{dr}\right)^2 + r\sigma_r(1+2n_3)\left(\frac{d\sigma_r}{dr}\right) + \sigma_r^2(n_3^2+n_3+1) - \sigma_Y^2 = 0 \tag{20}$$

and thus $d\sigma_r$ is obtained from above second degree equation.

$$d\sigma_r = \frac{-\sigma_r(1+2n_3) \pm \sqrt{-3\sigma_r^2 + 4\sigma_Y^2}}{2r}dr \tag{21}$$

By the increase in the temperature, the yielding starts at the inner surface because the stress components exceed the yield strength at the inner surface (Owen and Hinton 1986). If the temperature is continuing to increase, the yielding, which starts at the inner surface, expands more along the radial direction. When the temperature further increases, the second yielding region takes place from outer surface to radius *d*. As seen in Fig. 1, the radii *c* and *d* represent the boundary of the elastic and plastic regions.

The tangential and radial stresses are obtained from Eq. (19) and the following equation using the numerical integration, in the plastic region

$$\left(\sigma_r\right)_{i+1} = \left(\sigma_r\right)_i + d\sigma_r \tag{22}$$

As the plastic region reaches to radius *c*, the following boundary conditions can be written as

$$\sigma_r = p \;\; at \;\; r = c \tag{23}$$

and

$$\sigma_r = 0 \;\; at \;\; r = b \tag{24}$$

where *p* is the radial stress at radius *c*.

For the elastic region after radius *c*, the radial and tangential stresses in Eqs. (13) and (14) are used, however the integration constants C_1 and C_2 given below are used instead of C_1 and C_2 in Eq. (16)

$$C_1 = \frac{A\left(c^{\frac{n_1+2n_2+n_3+m+2}{2}} - b^{\frac{n_1+2n_2+n_3+m+2}{2}}\right) - p\,h(r)\,c^{\frac{-n_1-n_3+m+2}{2}}}{b^m - c^m} \tag{25a}$$

$$C_2 = \frac{A\left(c^m b^{\frac{n_1+2n_2+n_3+m+2}{2}} - b^m c^{\frac{n_1+2n_2+n_3+m+2}{2}}\right) + p\,h(r)\,b^m c^{\frac{-n_1-n_3+m+2}{2}}}{b^m - c^m} \tag{25b}$$

In the second yielding region, the equations obtained for first yielding region are also valid.

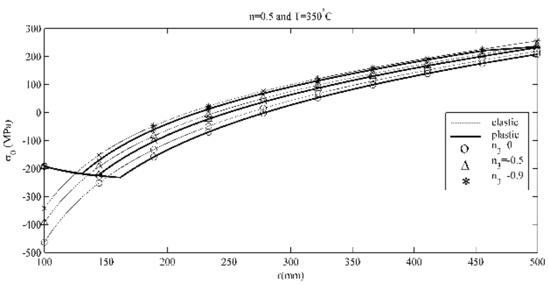
4. Results and discussion

In the analytical study, the elasto-plastic thermal stresses are analyzed in an annular disc made of Functionally Graded Materials (FGMs). The inner and outer radii of the disc are *a*=100 mm and *b*=500 mm, and thickness of the outer surface is *h*=10 mm, as shown in Fig. 1. The FG disc material is assumed to be composed of various contents of two different metal powder particles by using powder metallurgy. The material properties (elastic modulus, thermal expansion coefficient and yield strength) and thickness of FG disc vary along the radial direction by a power law function. In order to apply the mathematical modelling, the material of the disc is selected as steel at the outer surface and its elastic modulus is *E*=207000 MPa, thermal expansion coefficient is α=12.5×10^{-6} 1/℃ and yielding stress is σ_0=235 MPa. The gradient parameters n_1, n_2, n_4 are assumed to *n*, *-n* and *n/4* for elastic modulus, thermal expansion coefficient and yield strength, respectively. In the elasto-plastic solution, the yielding behavior of the disc material is supposed as non-work hardening case using von Mises' yield condition.

The gradient parameter n for material properties is chosen from -1.0 to 1.0. For thickness profile, the geometric parameter n_3 is also used as 0.0, -0.5 and -0.9 (Alexandrova and Vila Real 2007). Elastic and elasto-plastic stresses are obtained by using analytical solutions for uniform temperatures. The temperatures are chosen between *T*=100℃ and *T*=400℃.

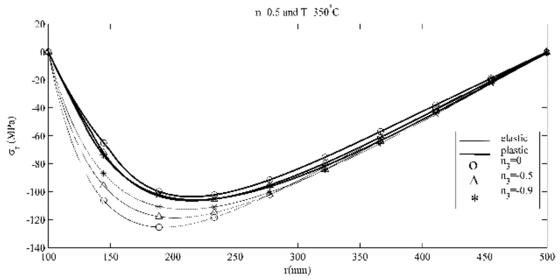
4.1 The effect of thickness

Fig. 2 shows distributions of the elastic and elasto-plastic stresses of the discs for the various power law indexes (*n*=0.5 and n_3=0.0, -0.5 and -0.9) at 350℃. The thickness profile in Eq. (7c) is commonly used in many industrial parts, particularly the value of n_3=-0.5 is used in the gas turbine rotors (Reddy and Srinath 1974). In this figure, it is given the effect of the thickness gradient (n_3) on the stresses of a FG disc with *n*=0.5. The dashed and continuous lines represent values obtained from elastic and plastic solutions, respectively. It is seen from Fig. 2(a) that all discs with *n*=0.5 at *T*=350℃ are entered the first plastic region at the inner surface, and even the plastic region of the disc with n_3=0 expands more from inner surface towards to the outer surface of disc. Nevertheless, although the plastic region of the disc with n_3=-0.9 expands less at the inner surface, it is newly entered the second plastic region at the outer surface.



(a) tangential stresses

Fig. 2 The effect of the thickness profile on the elastic and elasto-plastic tangential and radial stresses for n=0.50 and T=350℃



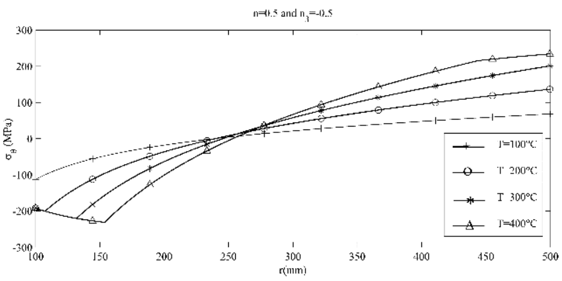
(b) radial stresses

Fig. 2 Continued

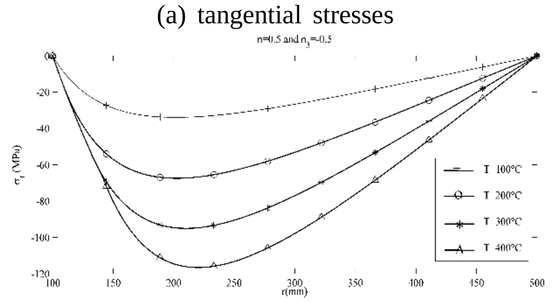
The yielding stress of the disc with all n_3-values is the same and it is 235 MPa at the inner surface. Then, the elasto-plastic tangential stresses decrease until elasto-plastic boundary (radius *c*). After radius *c*, the stresses increase gradually. It indicates that the discs whose thickness varies hyperbolically are more efficient than the disc with constant thickness in terms of resistance to temperature. This conclusion is also supported by the lower tangential stress distribution in Fig. 2(a). But, it should be paid attention to start of the second plastic region. As can be seen from Fig. 2(b), the maximum elastic stress values of the radial stresses are obtained for the disc of constant thickness (n_3=0.0), however, they decrease gradually with varying n_3-values. But, the values of the elasto-plastic radial stresses are almost the same.

4.2 The effect of temperature

Fig. 3 shows distributions of the elastic and elasto-plastic stresses of the discs under different temperatures (*T*=100℃, *T*=200℃, *T*=300℃ and *T*=400℃) for *n*=0.5 and n_3=-0.5. In this figure, it is given the effect of the temperatures on the stresses of a FG disc with *n*=0.5 and n_3=-0.5. It is seen from Fig. 3(a) that as the disc exposed to 100℃ has not entered the plastic region at the inner surface, yet; the plastic region of others expands gradually from inner surface towards outer surface. As different from the others, the elasto-plastic region of the disc under 400℃ expands from both inner and outer surfaces towards the middle region.



(a) tangential stresses



(b) radial stresses

Fig. 3 The effect of the temperature on the elastic and elasto-plastic tangential and radial stresses for n=0.50 and n_3=-0.50

The elastic tangential stress of the disc exposed to 100℃ doesn't reach the yielding stress at the inner surface, whereas the elasto-plastic tangential stresses of the other discs are the same at the inner surface and equal to yielding stress. While the elastic tangential stress of the disc under 100℃ is increasing gradually, the elasto-plastic tangential stresses of the other discs decrease firstly until elasto-plastic boundary (radius *c*) then the stresses increase gradually.

It is seen from Fig. 3(b) that the radial stresses increase gradually with increasing temperature and the maximum radial stress is obtained for the disc exposed to 400℃.

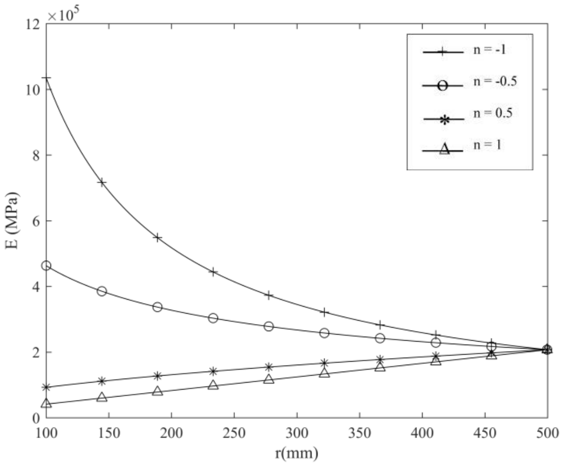
4.3 The effect of material

In Fig. 4, the variations of elastic modulus, thermal expansion coefficient and yield stress along the radius are shown. Elastic modulus decreases at the inner surface for *n*=-0.5 and -1.0 and it increases for *n*=0.5 and 1.0 according to material properties at the outer surface. Unlike, thermal expansion coefficient increases at the inner surface for *n*=-0.5 and -1.0, and it decreases for *n*=0.5 and 1.0. The characteristic of yield stress curve is similar to the one of the elastic modulus, as can be seen in Fig. 4(c).

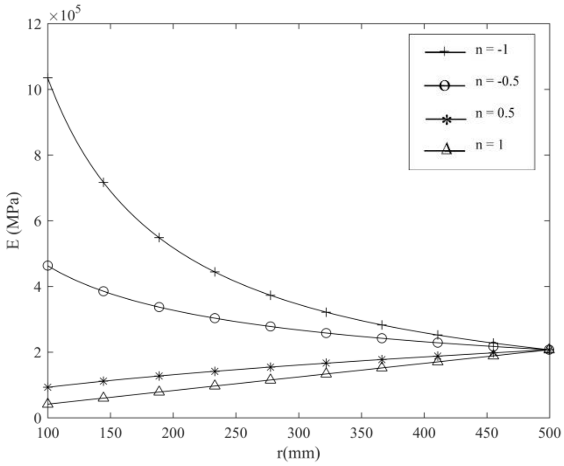
Fig. 5 shows distributions of the elastic and elasto-plastic stresses of the discs with different power law indexes (*n*=-1.0, -0.5, 0.5 and 1) for n_3=-0.5 and *T*=175℃. If Fig. 5 is examined step by step: The stresses that are occurred in the disc with *n*=-0.5 are completely elastic stresses. When the plastic region occurs just shortly before at the inner surface of the disc with *n*=0.5, it does not occur at the outer surface. The plastic region of the disc with *n*=-1.0 occurs at the inner surface and it expands some, it does not also occur at the outer surface. The plastic regions in the disc with *n*=1.0 occur at the both inner and outer surfaces, and they more expand into the inner region of the disc.

As examined in Fig. 4(c), the yield stresses at the inner surface of the discs with different n-values can be sorted from small to large as *n*=1, 0.5, -0.5 and -1. However, it is seen in Fig. 5(a) that the disc with *n*=1 yields first and then one with n=-1 yields unexpectedly at the inner surface. The first yield of the disc with n=1 is normal. Because, it has minimum yield stress. The reason why the disc with n=-1 yields secondly is that it has minimum thermal expansion coefficient.

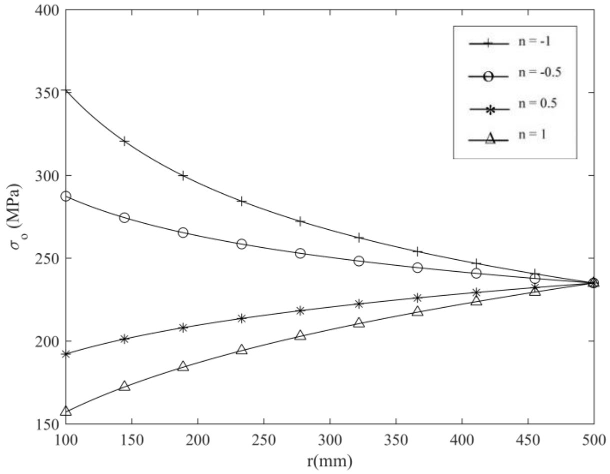
It indicates that discs, which have more rigid region at the outer surface, are more useful in terms of resistance to temperature. This conclusion is also supported by Fig. 5(a). Because, it's seen that the maximum tangential stresses occurred on the disc with *n*=-1. But, it should be paid attention to start of the second plastic region. Because, the plastic regions of the discs with positive *n*-values expand more than those of the discs with negative *n*-values. It is



(a) elastic modulus

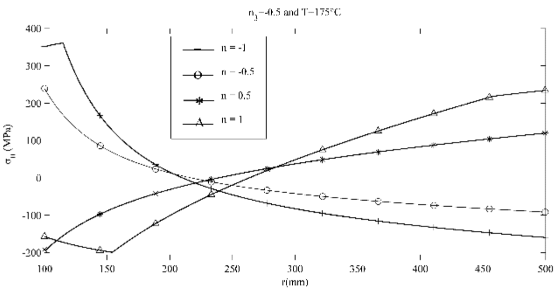


(b) thermal expansion coefficient

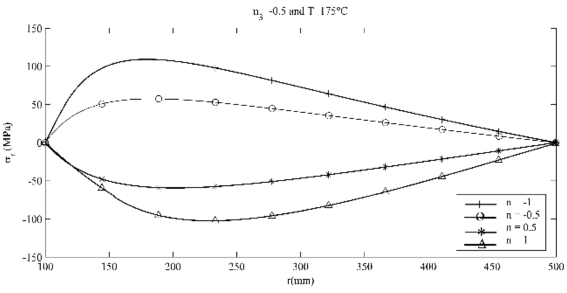


(c) yield stress

Fig. 4 The variations of elastic modulus, thermal expansion coefficient and yield stress along the radius



(a) tangential stresses



(b) radial stresses

Fig. 5 The effect of the material properties on the elastic and elasto-plastic tangential and radial stresses for n_3=-0.50 and T=175°C

seen that the entire radial stresses increase with increasing absolute *n*-values.

5. Conclusions

The following conclusions are derived from the elasto-plastic thermal stress analysis of a FG disc with variable thickness profile:

• Plastic yielding in the disc happens firstly at the inner surface where the greatest value of σ_θ is.

• Yielding region moves to the outer surface of the disc with gradually increase in the temperature. After then, as the temperature further increase the second plastic yielding can take place at the outer surfaces.

• The stresses in a disc of variable thickness are lower than those in constant thickness at the same temperature.

• The discs whose thickness varies hyperbolically are more useful than the disc with constant thickness in terms of resistance to temperature.

• The size of the elasto-plastic boundary (radius *c*) decreases when n_3-value decreases.

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