Structural Engineering and Mechanics, *Vol. 62, No. 1 (2017) 107-111*
DOI: https://doi.org/10.12989/sem.2017.62.1.107

Buckling load optimization of laminated composite stepped columns

Umut Topal*

Department of Civil Engineering, Karadeniz Technical University, Faculty of Technology, 61830, Trabzon, Turkey

(Received October 27, 2016, Revised January 26, 2017, Accepted February 10, 2017)

Abstract. This paper deals with critical buckling load optimization of symmetric angle-ply laminated stepped flat columns under axial compression load. The design objective is the maximization of the critical buckling load and the design variable is the fiber orientations in the layers of the laminates. The classical laminate plate theory is used for the finite element solution of the laminated stepped flat columns. The modified feasible direction (MFD) method is used for the optimization routine. For this purpose, a program based on FORTRAN is exploited. Finally, the optimization results are presented for width ratios (b/B), ratios of fillet radius (r_1/r_2), aspect ratios (L/B) and boundary conditions. The results are presented in graphical and tabular forms and the results are compared.

Keywords: laminated stepped columns; critical buckling load; modified feasible direction method; optimization

1. Introduction

In many applications, some or all parts of a high-performance structure are made of composite materials. The basis of the superior structural performance of composite materials, compared to the conventional materials, lies in their low density, high strength, high stiffness, and heterogeneous properties. Optimization is the main concept in the design of composite structures due to the adaptability of composite materials to a specific design situation. It should be also declared that the optimal design of composite laminates, on the other hand, has become a challenge for designers. Because of numerous design variables, the complex behavior, and multiple failure modes, these structures essentially require sophisticated analytical techniques There have been extensive numerical and analytical studies on the determination of critical buckling load of laminated composites in the literatures. Walker and Hamilton (2005) maximized critical bucking load of symmetrically laminated composite plates with manufacturing uncertainty in the ply angle using golden section method. Fares *et al*. (2004) presented a multiobjective optimization problem to determine the optimal layer thickness and optimal closed loop control function for symmetric cross-ply laminated composite plates subjected to thermomechanical loadings. Sciuva *et al.* (2003) performed optimization of laminated and sandwich plates with respect to buckling load and thickness using genetic and simulated annealing algorithms. Adali *et al.* (2003) studied optimal designs of symmetrically laminated composites to maximize the biaxial buckling. Sebaey *et al.* (2013) presented buckling load optimization of laminated plates using ant colony optimization algorithm. Correia *et al.* (2003) investigated optimal design of laminated composite plates with integrated piezoelectric actuators. Walker (2002) presented optimal designs of symmetrically laminated composite plates with different stiffener arrangements using golden section method. Walker (2001) presented for the optimal multiobjective design of symmetrically laminated composite plates using golden section method. Mateus *et al.* (1997) presented a general formulation for maximization of the buckling load. Walker *et al.* (1996) presented optimal buckling designs of symmetrically laminated composite plates under in-plane uniaxial loads using golden section method. Joshi and Biggers (1996) studied thickness optimization for laminated composite plates using the method of feasible directions. Fukunaga *et al.* (1995) presented an optimization approach for symmetrically laminated plates to maximize buckling loads under combined loading using lamination parameters. Hu and Lin (1995) maximized the buckling resistance of symmetrically laminated plates subjected to uniaxial compression with respect to fiber orientations by using a sequential linear programming method. Adali *et al.* (1995) presented two design problems for hybrid symmetric laminated plates.. Setoodeh *et al.* (2009) presented a generalized reciprocal approximation for design of variable-stiffness laminated composite panels for maximum buckling load.

While considerable researches have gone into the buckling load optimization of the laminated composite plates, only limited investigations have been reported on the buckling of laminated composite stepped columns in the literature. Lellep and Sakkov (2008) studied the stability of elastic multistepped columns subjected to axial pressure load. Lellep and Kraav (2008) established minimum weight design of laminated stepped column subjected to axial compressive load using non-linear programming. Akbulut *et al.* (2010) investigated theoretical prediction of buckling loads for symmetric angle-ply and cross-ply laminated flat composite columns.

*Corresponding author, Associate Professor
E-mail: utopal@ktu.edu.tr

http://www.techno-press.org/?journal=sem&subpage=8 ISSN: 1225-4568 (Print), 1598-6217 (Online)

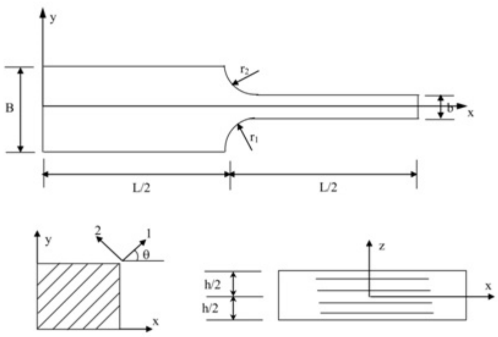

Fig. 1 Geometry of a laminated stepped column (Akbulut *et al.* 2010)

On the other hand, to the authors' knowledge, there is no any research on the buckling load optimization of laminated composite stepped columns connected by fillets in the literature. Therefore, in this research, buckling load optimization of laminated composite stepped columns connected by fillets is investigated to fill this gap. The design objective is the maximization of the critical buckling load and the design variable is the fiber orientations in the layers of the laminates. The classical laminate plate theory is used for the finite element solution of the laminated stepped flat columns. The modified feasible direction (MFD) method is used for the optimization routine. For this purpose, a program based on FORTRAN is exploited. Finally, the optimization results are presented for width ratios (b/B), ratios of fillet radius (r_1/r_2), aspect ratios (L/B) and boundary conditions. The results are presented in graphical and tabular forms and the results are compared.

2. Basic equations

In Fig. 1, a laminated stepped column is illustrated. The displacement field of the laminates based on the classical laminated plate theory is given by the following expressions

$$u = u_o(x,y) - z\frac{\partial w}{\partial x},\ v = v_o(x,y) - z\frac{\partial w}{\partial y},\ w = w(x,y) \quad (1)$$

where u, v and w represent the displacements in the x, y and z directions, respectively and also, u_0 and v_0 are the plate mid-plane displacements.

The membrane strains (ε_x, ε_y, γ_{xy}) and the bending curvatures (κ_x, κ_y, κ_{xy}) are defined as follows

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} u_{o,x} + \frac{1}{2}w_{o,x}^2 \\ v_{o,y} + \frac{1}{2}w_{o,y}^2 \\ u_{o,y} + v_{o,x} + w_{o,x}w_{o,y} \end{Bmatrix} \quad (2)$$

$$\{\kappa\} = \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} = \begin{Bmatrix} -w_{o,xx} \\ -w_{o,yy} \\ -2w_{o,xy} \end{Bmatrix} \quad (3)$$

The in-plane stress resultants N_{ij} and the moment resultants M_{ij} are defined by the constitutive relations that given by

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ & & A_{66} & B_{16} & B_{26} & B_{66} \\ & & & D_{11} & D_{12} & D_{16} \\ & sym & & & D_{22} & D_{26} \\ & & & & & D_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \quad (4)$$

where A_{ij}, B_{ij} and D_{ij} denote the extensional, coupling and bending stiffnesses, respectively. A_{ij}, B_{ij} and D_{ij} can be calculated as follows

$$\left(A_{ij}, B_{ij}, D_{ij}\right) = \int_{-h/2}^{h/2} \overline{Q_{ij}}\,(1, z, z^2)dz \quad (5)$$

Since presently only symmetric laminates are considered, the coupling stiffness terms B_{ij} vanish.

In this study, four-noded Lagrangian finite elements having three degrees of freedom per node is used for the buckling of the laminated stepped columns. Using the same shape functions associated with node i (i=1, 2,…, n), Φ_i, for interpolating the variables in each element, the displacement vectors can be written as

$$u = \sum_{i=1}^{n} \Phi_i(x,y)u_i \quad (6)$$

where u_i is the value of the displacement vector corresponding to the node i, and is given by

$$u_i = \left\{u_o^{(i)}, v_o^{(i)}, w^{(i)}\right\}^T \quad (7)$$

The generalized form of the buckling eigenvalue problem using the finite element discretization can be written as

$$\left([K] - \lambda[K_g]\right)\{u\} = 0 \quad (8)$$

where K and K_g are the stiffness matrix and geometric matrix, respectively. For a non-trivial solution, the eigenvalues (λ) which make the determinant to be equal to zero, correspond to the critical buckling loads. The subspace iteration technique is used for the buckling solution of the laminated stepped columns.

3. Modified feasible direction method

The MFD method is one of the most powerful methods for optimization problems. This method takes into account not only the gradients of objective function and constraints, but also the search direction in the former iteration. In this study, there is not any constraint. The iterative process of modified feasible direction method is given below step by step.

Step 1 q=0, $X^q = X^m$

Step 2 q=q+1, evaluate objective function $F(X^{q-1})$

Step 3 Calculate gradient of the objective function $\nabla F(X^{q-1})$

Step 4 Find the usable-feasible direction S^q

Step 5 Perform a one-dimensional search, $X^q = X^{q-1} + \alpha S^q$

Step 6 Check convergence. If satisfied, go to step 7, otherwise go to step 2

Step 7 $X^m = X^q$

The objective function $F(X_i)$ is accurately modelled as a quadratic polynomial approximation around the current iterate X_i as in Eq. (9).

$$\mathrm{F}(X_i) = a_o + \sum_{i=1}^{N_d} a_i X_i + \sum_{i=1}^{N_d} b_i X_i^2 \tag{9}$$

where N_d and X_i are number of design variables and *i*th design variable, respectively. a_i and b_i are the coefficients of polynomial function determined by a least squares regression. After the objective function is approximated, their gradients with respect to the design variables are calculated by finite differences methods. The solving process is iterated until convergence is achieved.

Convergence or termination checks are performed at the end of each optimization loop. The optimization process continues until either convergence or termination occurs. The process may be terminated before convergence in two cases:

- The number of design sets so far exceeds the maximum number of optimization loops.
- If the initial design is infeasible and the allowed number of consecutive infeasible designs has been exceeded.

The optimization problem is considered converged if all of the following conditions are satisfied:

- The current design is feasible,
- Changes in the objective function F:

a) The difference between the current value and the best design so far is less than the tolerance τ_F.

$$\left| F_{current} - F_{best} \right| \le \tau_F$$

b) The difference between the current value and the previous design is less than the tolerance,

$$\left| F_{current} - F_{current-1} \right| \le \tau_F$$

- Changes in the design variables X^i:

a) The difference between the current value of each design variable and the best design so far is less than the respective tolerance τ^i.

$$\left| X^i_{current} - X^i_{best} \right| \le \tau^i$$

b)The difference between the current value of each design variable and the previous design is less than the respective tolerance,

$$\left| X^i_{current} - X^i_{current} \right| \le \tau_F$$

The optimization process was solved to obtain a global maximum from different initial points to check if other solutions were possible. The convergence tolerance ratio was considered 0.01 for the objective function.

4. Optimization problem

In this study, the optimization problem is the maximization of the critical buckling load by designing the fibre orientations in the layers. The optimal design problem can be stated mathematically as follows

$$\begin{gathered} \text{Find: } \theta_1, \theta_2 \\ \text{Maximize: } (\lambda_{cr})_{max} = \max_{\theta} \lambda_{cr}(\theta) \\ \text{Subjected to: } 0^\circ \le \theta_k \le 90^\circ \end{gathered} \tag{10}$$

The critical buckling load λ_{cr} for a given fibre orientation is determined from the finite element solution of the eigenvalue problems given by Eq. (8). The optimization procedure involves the stages of evaluating the critical buckling load and improving the fiber orientation θ to maximise λ_{cr}. Thus, the computational solution consists of successive stages of analysis and optimization until a convergence is obtained and the optimal angle θ_{opt} is determined within a specified accuracy.

5. Numerical results and discussion

In this study, four layered angle-ply ($\theta_1/\theta_2/\theta_2/\theta_1$) laminated stepped columns subjected to axial pressure load is investigated for the optimization problem. Each of the lamina is assumed to be same thickness. The numerical results are given for a graphite/epoxy T300/5208 composite with the following the material properties:

E_1=181 GPa, E_2=10.3 GPa, G_{12}=7.17 GPa, v_{12}=0.28,

The nondimensional buckling load is calculated as

$$\overline{N}_{cr} = \lambda_{cr} B^2 / E_2 h^3 \tag{11}$$

In this study, effect of width ratios (b/B) on the optimum results is investigated for symmetric angle-ply laminated stepped columns for both ends clamped (B/h=100, r_1/r_2=1, L/B=2.5). In Fig. 2, effect of b/B ratios on the maximum critical buckling load is illustrated. As seen, the maximum critical buckling load decreases with increase in the width ratio except for unstepped laminated column (b/B=1). It can be attributed that, as width ratio increases distribution of the axial pressure load on the edge of the laminated stepped column increases. On the other hand, unstepped column gives the highest critical buckling load. In Table 1, effect of b/B ratio on the optimum fibre orientations is given.

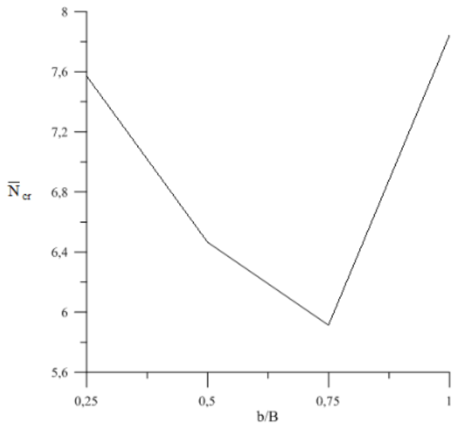

Fig. 2 Effect of b/B ratios on the maximum critical buckling load

Table 1 Optimum fibre orientations depending on b/B ratios

b/B	θ_{opt}
0.25	(0°/90°/90°/0°)
0.50	(0°/30°/30°/0°)
0.75	(0°/43°/43°/0°)
1.00	(0°/0°/0°/0°)

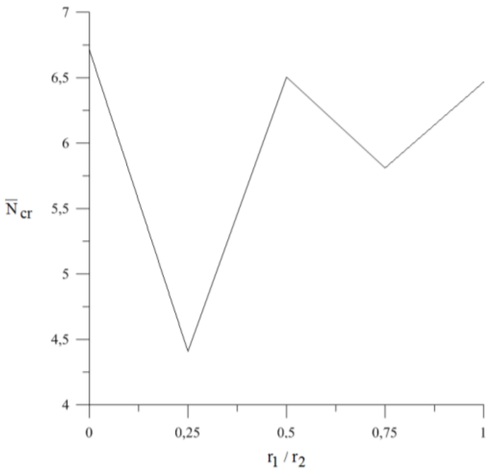

Fig. 3 Effect of r_1/r_2 ratios on the maximum critical buckling load

Table 2 Optimum fibre orientations depending on r_1/r_2 ratios

r_1/r_2	θ_{opt}
0.25	(0°/25°/25°/0°)
0.50	(0°/90°/90°/0°)
0.75	(0°/30°/30°/0°)
1.00	(0°/30°/30°/0°)

In Fig. 3, effect of r_1/r_2 ratio on the maximum critical buckling load is given. As seen from Fig. 4, the curve of the maximum critical buckling load fluctuates with increase in the r_1/r_2 ratio. The effect of r_1/r_2 ratio on the maximum critical buckling load diminishes as r_1/r_2 ratio increases. On the other hand, the maximum critical buckling load occurs for r_1/r_2=0. In Table 2, effect of r_1/r_2 ratio on the optimum fibre orientations is shown.

In this study, effect of aspect ratio (L/B) on the optimum design is investigated for symmetric angle-ply laminated stepped columns for both ends clamped (B/h=100, b/B=0.5, r_1/r_2=1). In Fig. 4, effect of L/B ratio on the maximum critical buckling load is illustrated. As observed in Fig. 3, the maximum critical buckling load decreases as L/B ratio increases because of the decreasing of the rigidity of the laminated stepped column. In Table 3, effect of L/B ratio on the optimum fibre orientations is shown.

In this study, effect of different boundary conditions on the optimum results is investigated for symmetric angle-ply laminated stepped columns (B/h=100, r_1/r_2=1, L/B=2.5, b/B=0.5). Boundary conditions are given as below:

1. Clamped-Clamped (C-C)
   At x=0, $u=v=w=\theta_y=\theta_z=0$
   At $x=L$, $v=w=\theta_y=\theta_z=0$
2. Clamped-Simply supported (C-S)
   At x=0, $u=v=w=\theta_y=\theta_z=0$
   At $x=L$, $v=w=\theta_z=0$
3. Simply supported-Simply supported (S-S)
   At x=0, $u=v=w=\theta_z=0$
   At $x=L$, $v=w=\theta_z=0$
4. Clamped-Free (C-F)
   At x=0, $u=v=w=\theta_y=\theta_z=0$

As seen from Table 4, as expected the maximum and minimum critical buckling loads occur for (C-C) and (C-F) boundary conditions, respectively.

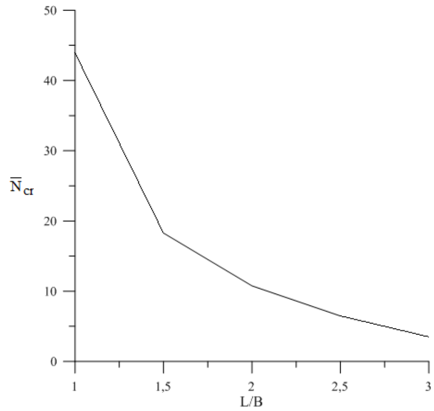

Fig. 4 Effect of L/B ratios on the maximum critical buckling load

Table 3 Optimum fibre orientations depending on L/B ratios

L/B	θ_{opt}
1.00	(4°/0°/0°/4°)
1.50	(0°/0°/0°/0°)
2.00	(0°/0°/0°/0°)
2.50	(0°/30°/30°/0°)
3.00	(0°/6°/6°/0°)

Table 4 Optimum results depending on the boundary conditions

Boundary conditions	θ_{opt}	$\overline{N}_{cr}$
(C-C)	(0°/30°/30°/0°)	6.465
(C-S)	(0°/0°/0°/0°)	3.459
(S-S)	(0°/0°/0°/0°)	1.587
(C-F)	(0°/0°/0°/0°)	0.580

6. Conclusions

This paper deals with critical buckling load optimization of laminated stepped flat columns under axial compression load. The design objective is the maximization of the critical buckling load and the design variable is the fibre orientation. It is observed that width ratios, fillet radius, aspect ratios and boundary conditions have important effects on the buckling behavior of laminated steeped column. As seen from the results, the maximum buckling load decreases as width ratio increases except for unstepped laminated column. Unstepped column gives the highest critical buckling load. The fillet radius has a substantial effect on the critical buckling loads of stepped columns. The curve of the maximum critical buckling load fluctuates with inrease in the r_1/r_2 ratio. The maximum critical buckling load decreases as L/B ratio increases because of the decreasing of the rigidity of the laminated stepped column. This problem can be investigated for other optimization methods and the results can be compared.

References

Adali, S., Lene, F., Duvaut, G. and Chiaruttini, V. (2003), "Optimization of laminated composites subject to uncertain buckling loads", *Compos. Struct*., **62**, 261-269.

Adali, S., Richter, A., Verijenko, V.E. and Summers, E.B. (1995), "Optimal design of hybrid laminates with discrete ply angles for maximum buckling load and minimum cost", *Compos. Struct.*, **32**, 409-415.

Akbulut, H., Gundogdu, O. and Sengul, M. (2010), "Buckling behaviors of laminated composite stepped flat columns", *Finite Elem. Anal. Des.*, **46**, 1061-1067.

Correia, V.M.F., Soares, C.M.M. and Soares, C.A.M. (2003), "Buckling optimization of composite laminated adaptive structures", *Compos. Struct.*, **62**, 315-321.

Fares, M.E., Youssif, Y.G. and Hafiz, M.A. (2004), "Structural and control optimization for maximum thermal buckling and minimum dynamic response of composite laminated plates", *Int. J. Solid. Struct.*, **41**, 1005-1019.

Fukunaga, H., Sekine, H., Sato, M. and Iino, A. (1995), "Buckling design of symmetrically laminated plates using lamination parameters", *Comput. Struct*., **57**, 643-649.

Hu, H.T. and Lin, B.H. (1995), "Buckling optimization of symmetrically laminated plates with various geometries and end conditions", *Compos. Sci. Tech.*, **55**, 277-285.

Joshi, M.G. and Biggers, S.B. (1996), "Thickness optimization for maximum buckling loads in composite laminated plates", *Compos. Part B: Eng.*, **27**, 105-114.

Lellep, J. and Kraav, T. (2008), "Optimization of stepped columns under compression", *Proceedings of the 20th EURO Mini Conference on Continuous Optimization and Knowledge-Based Technologies (EurOPT-2008)*, Neringa, Lithuania.

Lellep, J. and Sakkov, E. (2008), "Buckling of stepped composite columns", *Mech. Compos. Mater.*, **42**, 63-72.

Mateus, H.C., Soares, C.M.M. and Soares, C.A.M. (1997), "Buckling sensitivity analysis and optimal design of thin laminated structures", *Compos. Struct.*, **64**, 461-472.

Sciuva, D.M., Gherlone, M. and Lomario, D. (2003), "Multiconstrained optimization of laminated and sandwich plates using evolutionary algorithms and higher-order plate theories", *Compos. Struct.*, **59**, 149-154.

Sebaey, T.A., Lopes, C.S., Blanco, C. and Costa, C. (2013), "Ant colony optimization for dispersed laminated composite panels under biaxial loading", *Compos. Struct.*, **94**, 31-36.

Setoodeh, S., Abdalla, M.M., Ijsselmuiden, S.T. and Gürdal, Z. (2009), "Design of variable-stiffness composite panels for maximum buckling load", *Compos. Struct.*, **87**, 109-117.

Walker, M. (2001), "Multiobjective design of laminated plates for maximum stability using the finite element method", *Compos. Struct.*, **54**, 389-393.

Walker, M. (2002), "The effect of stiffeners on the optimal ply orientation and buckling load of rectangular laminated plates", *Compos. Struct.*, **80**, 2229-2239.

Walker, M. and Hamilton, R. (2005), "A methodology for optimally designing fibre-reinforced laminated structures with design variable tolerances for maximum buckling strength", *Thin Wall. Struct.*, **43**, 161-174.

Walker, M., Adali, S. and Verijenko, V.E. (1996), "Optimal design of symmetric angle-ply laminates subject to nonuniform buckling loads and inplane restraints", *Thin Wall. Struct.*, **26**, 45-60.

CC