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Application of OMA on the bench-scale earthquake simulator using micro tremor data

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Abstract. In this study was investigated of possibility using the recorded micro tremor data on ground level as ambient vibration input excitation data for investigation and application Operational Modal Analysis (OMA) on the bench-scale earthquake simulator (The Quanser Shake Table) for model steel structures. As known OMA methods (such as EFDD, SSI and so on) are supposed to deal with the ambient responses. For this purpose, analytical and experimental modal analysis of a model steel structure for dynamic characteristics was evaluated. 3D Finite element model of the building was evaluated for the model steel structure based on the design drawing. Ambient excitation was provided by shake table from the recorded micro tremor ambient vibration data on ground level. Enhanced Frequency Domain Decomposition is used for the output only modal identification. From this study, best correlation is found between mode shapes. Natural frequencies and analytical frequencies in average (only) 2.8% are differences.

Keywords: experimental modal analysis; modal parameter; EFDD; shake table

1. Introduction

There are many varieties of the structural and architectural structures in the world. Common features of these structures, it can be managed to survive under static and dynamic loads. Structures under dynamic loads and vibrations occurred impact consists of vibrations that do not require or require intervention on the structure brings many damage occurs. In this case, the vibration should be known and may occur in nature and will be focused on the effects generated by these vibrations. In recent years, several earthquakes have occurred in the world and are given as a result of heavy losses. If the resulting perceived to pose several problems for the countries of heavy losses, the structure of the receipt of the knowledge of the current situation and how important it is understood that the necessary measures. In this case, experimental determination of the behavior they showed against vibrations from the structures and obtained the theoretical and the creation of finite element model to represent the actual structure by comparing the experimental value are emerging requirements. As known forced (shaker, impact, pull back or quick release tests) and ambient vibration techniques are available for vibration testing of large structures. Force vibration methods more complex and are generally more expensive than ambient vibration tests. Ambient vibration testing (also called Operational Modal Analysis) is the most economical non-destructive testing method to acquire vibration data from large civil engineering structures for Output-Only Model Identification. General characteristics of structural response (appropriate frequency, displacement, velocity, acceleration rungs), suggested measuring quantity (such as velocity or acceleration) depends on the type of vibrations (Traffic, Acoustic, Machinery inside, Earthquakes, Wind…) are given in Vibration of Buildings (1990).

This structures Response characteristics gives a general idea of the preferred quantity and its rungs to be measured. A few studies the analysis of ambient vibration measurements of buildings from 1982 until 1996 are discussed in Ventura and Schuster (1996). Last ten years Output-Only Model Identification studies of buildings are given in appropriate references structural vibration solutions. For the modal updating of the structure it is necessary to estimate sensitivity of reaction of examined system to change of parameters of a building. Kasimzade (2006) System identification is the process of developing or improving a mathematical representation of a physical system using experimental data investigated in HO and Kalman (1966), Kalman (1960), Ibrahim and Miculcik (1977), Ibrahim (1977), Bendat (1998), Ljung (1999), Juang (1994), Van Overschee and De Moor (1996), and system identification applications in civil engineering structures are presented in works Trifunac (1972), Turker (2014), Altunisik *et al.* (2010), Brincker *et al.* (2000), Roeck (2003), Peeters (2000), Cunha *et al.* (2005), Wenzel and Pichler (2005), Kasimzade and Tuhta (2007a, b, 2009). Extracting system physical parameters from identified state space representation was investigated in references. Alvin and Park (1994), Balmes (1997), Juang *et al.* (1988), Juang and Pappa (1985), Lus *et al.* (2003), Phan *et al.* (2003), Sestieri and Ibrahim (1994), Tseng *et al.* (1994). The solution of a matrix algebraic Riccati equation and

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orthogonality projection more intensively and inevitably used in system identification was deeply investigated in works of Aliev (1998). In engineering structures there are three types of identification: modal parameter identification; structural-modal parameter identification; control-model identification methods are used. In the frequency domain the identification is based on the singular value decomposition of the spectral density matrix and it is denoted Frequency Domain Decomposition (FDD) and its further development Enhanced Frequency Domain Decomposition (EFDD). In the time domain there are three different implementations of the Stochastic Subspace Identification (SSI) technique: Unweighted Principal Component (UPC); Principal component (PC); Canonical Variety Analysis (CVA) is used for the modal updating of the structure Friswell and Mottershead (1995), Marwala (2010). It is necessary to estimate sensitivity of reaction of examined system to change of random or fuzzy parameters of a structure. Investigated measurement noise perturbation influences to the identified system modal and physical parameters. Estimated measurement noise border, for which identified system parameters are acceptable for validation of finite element model of examine system. System identification is realized by observer Kalman filter (Juang *et al.* 1993) and Subspace Overschee and De Moor (1996), algorithms. In special case observer gain may be coincide with the Kalman gain. Stochastic state-space model of the structure are simulated by Monte-Carlo method.

The Quanser Shake Table is a bench-scale earthquake simulator ideal for teaching structural dynamics, control topics related to earthquake, aerospace and mechanical engineering and widely used in applications. In this study was investigated of possibility using the recorded micro tremor data on ground level as ambient vibration input excitation data for investigation and application Operational Modal Analysis (OMA) on the bench-scale earthquake simulator (The Quanser Shake Table) for model steel structures.

For this purpose, analytical and experimental modal analysis of a model steel structure for dynamic characteristics was evaluated. 3D Finite element model of the building was evaluated for the model steel structure based on the design drawing. Ambient excitation was provided by shake table from the recorded micro tremor ambient vibration data on ground level. Enhanced Frequency Domain Decomposition is used for the output only modal identification.

2. Modal parameter extractions

The (FDD) ambient modal identification is an extension of the Basic Frequency Domain (BFD) technique or called the Peak-Picking technique. This method uses the fact that modes can be estimated from the spectral densities calculated, in the case of a white noise input, and a lightly damped structure. It is a non parametric technique that determines the modal parameters directly from signal processing. The FDD technique estimates the modes using a Singular Value Decomposition (SVD) of each of the measurement data sets. This decomposition corresponds to a Single Degree of Freedom (SDOF) identification of the measured system for each singular value
(Brincker *et al.* 2000).

The Enhanced Frequency Domain Decomposition technique is an extension to Frequency Domain Decomposition (FDD) technique. This t technique is a simple technique that is extremely basic to use. In this technique, modes are easily picked locating the peaks in Singular Value Decomposition (SVD) plots calculated from the spectral density spectra of the responses. FDD technique is based on using a single frequency line from the Fast Fourier Transform analysis (FFT), the accuracy of the estimated natural frequency based on the FFT resolution and no modal damping is calculated. On the other hand, EFDD technique gives an advanced estimation of both the natural frequencies, the mode shapes and includes the damping ratios (Jacobsen *et al.* 2006). In EFDD technique, the single degree of freedom (SDOF) Power Spectral Density (PSD) function, identified about a peak of resonance, is taken back to the time domain using the Inverse Discrete Fourier Transform (IDFT). The natural frequency is acquired by defining the number of zero crossing as a function of time, and the damping by the logarithmic decrement of the correspondent single degree of freedom (SDOF) normalized auto correlation function Peeters (2000).

In this study modal parameter identification was implemented by the Enhanced Frequency Domain Decomposition. The relationship between the input and responses in the EFDD technique can be written as, In this method, unknown input is represented with $x(t)$ and measured output is represented with $y(t)$

$$\left[G_{yy}(j\omega)\right]=[H(j\omega)]^*[G_{xx}(j\omega)][H(j\omega)]^T \tag{1}$$

Where $G_{xx}(j\omega)$ is the $r\,x\,r$ Power Spectral Density (PSD) matrix of the input. $G_{yy}(j\omega)$ is the $m\,x\,m$ Power Spectral Density (PSD) matrix of the output, $H(j\omega)$ is the $m\,x\,r$ Frequency Response Function (FRF) matrix, and * and superscript T denote complex conjugate and transpose, respectively. The FRF can be reduced to a pole/residue form as follows

$$[H(\omega)]=\frac{[Y(\omega)]}{[X(\omega)]}=\sum_{k=1}^{m}\frac{[R_k]}{j\omega\text{-}\lambda_k}+\frac{[R_k]^*}{j\omega\text{-}\lambda_k^*} \tag{2}$$

Where n is the number of modes λ_k is the pole and, R_k is the residue. Then Eq. (1) becomes as

$$G_{yy}(j\omega)=\sum_{k=1}^{n}\sum_{s=1}^{n}\left[\frac{[R_k]}{j\omega\text{-}\lambda_k}+\frac{[R_k]^*}{j\omega\text{-}\lambda_k^*}\right] G_{xx}(j\omega)\left[\frac{[R_s]}{j\omega\text{-}\lambda_s}+\frac{[R_s]^*}{j\omega\text{-}\lambda_s^*}\right]^{\bar{H}} \tag{3}$$

Where s the singular values, superscript is H denotes complex conjugate and transpose. Multiplying the two partial fraction factors and making use of the Heaviside partial fraction theorem, after some mathematical manipulations, the output PSD can be reduced to a pole/residue form as fallows

$$\left[G_{yy}(j\omega)\right]=\sum_{k=1}^{n}\frac{[A_k]}{j\omega-\lambda_k}+\frac{[A_k]^*}{j\omega-\lambda_k^*}+\frac{[B_k]}{-j\omega-\lambda_k}+\frac{[B_k]^*}{-j\omega-\lambda_k^*} \tag{4}$$

Where A_k is the k th residue matrix of the output PSD. In the EFDD identification, the first step is to estimate the PSD matrix. The estimation of the output PSD known at discrete frequencies is then decomposed by taking the SVD (singular value decomposition) of the matrix

$$G_{yy}(j\omega_i)=U_iS_iU_i^{\bar{H}} \tag{5}$$

Where the matrix $U_i=[u_{i1},u_{i2},\ldots,u_{im}]$ is a unitary matrix holding the singular vectors u_{ij} and s_{ij} is a diagonal matrix holding the scalar singular values. The first singular vector u_{ij} is an estimation of the mode shape. PSD function is identified around the peak by comparing the mode shape estimation u_{ij} with the singular vectors for the frequency lines around the peak. From the piece of the SDOF density function obtained around the peak of the PSD, the natural frequency and the damping can be obtained.

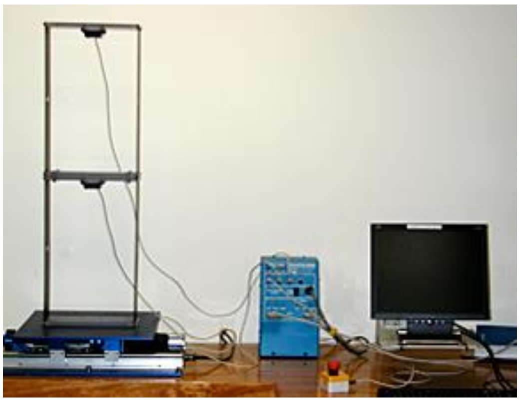
3. Description of model steel structure

The Quanser shake table II is a uniaxial bench-scale shake table. This unit can be controlled by appropriate software was illustrated in Figs. 1(a), (b), (c). It is effective for a wide variety of experiments for civil engineering structures and models. Shake table specifications are as follows Quanser (2008):

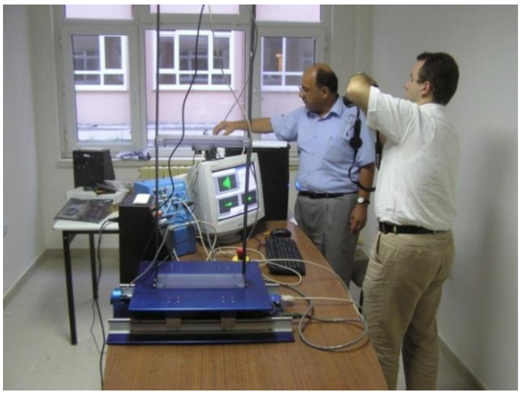
Table 1 Shake table specifications

Dimensions (H×L×W)	(61×46×13) cm
Total mass	27.2 kg
Payload area (L×W)	(46×46) cm
Maximum payload at 2.5 g	7.5 kg
Maximum travel	±7.6 cm
Operational bandwidth	10 Hz
Maximum velocity	66.5 cm/s
Maximum acceleration	2.5 g
Lead screw pitch	1.27 cm/rev
Servomotor power	400 W
Amplifier maximum continuous current	12.5 A
Motor maximum torque	7.82 N.m
Lead screw encoder resolution	8192 counts/rev
Effective stage position resolution	1.55 µm/count
Accelerometer range	±49 m/s²
Accelerometer sensitivity	1.0 g/V

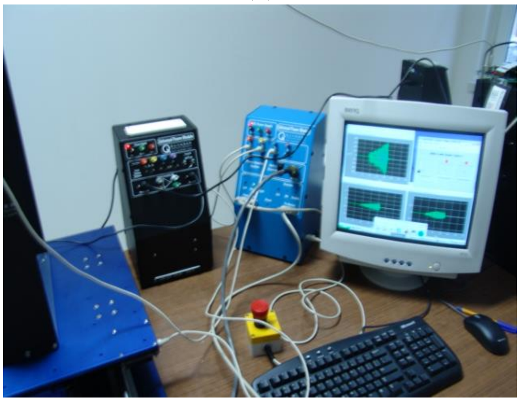
Model steel structure is 1.03 m height. Thickness of elements is 0.001588 m. The structure dimensions are shown in Fig. 2.



(a)



(b)



(c)

Fig. 1(a), (b), (c) Illustration of model steel structure and shake table

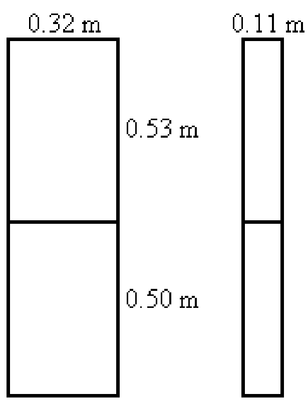


Fig. 2 View of model steel structure and shake table

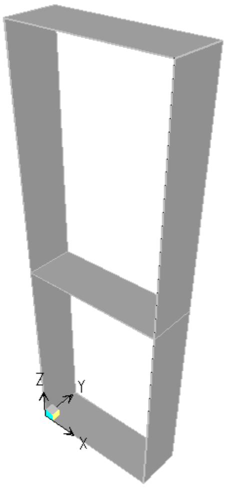


Fig. 3 Finite element model of model steel structure

Table 2 Analytical modal analysis result at the first at the Finite Element (FE) model

Mode number	1	2	3	4	5
Frequency (Hz)	2.075	5.890	7.025	7.994	9.246

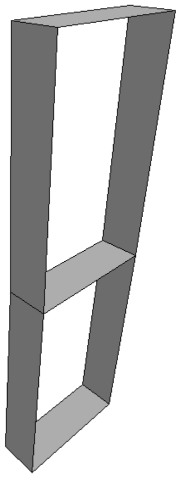
4. Analytical modal analysis of model steel structure

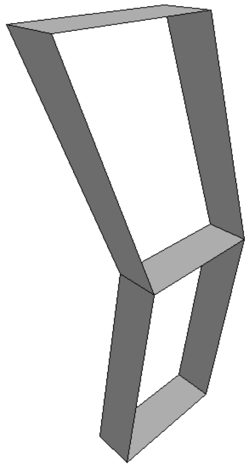
A finite element model was generated in SAP2000 (1997). Beams and columns were modeled as 3D beam-column elements (in Fig. 3 shown by the black color). Structure modeled as an absolutely rigidity floor (rigid diaphragm). The selected structure is modeled as a space frame structure with 3D elements. Beams and columns were modeled as 3D beam-column elements which have degrees of freedom. At the base of the structure in the model, the ends of every element were fixed against translation and rotation for the 6 degree of freedom (DOF) then creating finite element model of the structure in SAP2000. The following assumptions were taken into account. Model steel structure is modeled using an equivalent thickness and shell elements with isotropic property. All supports are modeled as fully fixed. The members of steel frame are modeled as rigidly connected together at the intersection points. In modeling of beams and columns the modulus of elasticity E=2.000E11 N/m^2, Poisson ratio μ=0.3, mass per unit volume ρ=78500 N/m^3

Natural frequencies and vibration modes are concerned a significant impact on the dynamic performance of buildings is an important dynamic properties. A total of five natural frequencies of the structure are attained which range between 2 and 9 Hz. The first five vibration mode of the structure is shown in Fig. 4. Analytical modal analysis results at the finite element model are shown in Table 2.

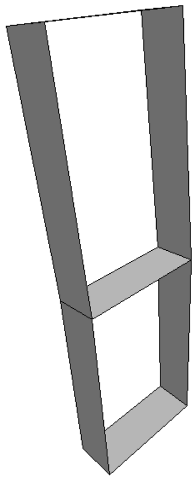
5. Experimental modal analysis of model steel structure

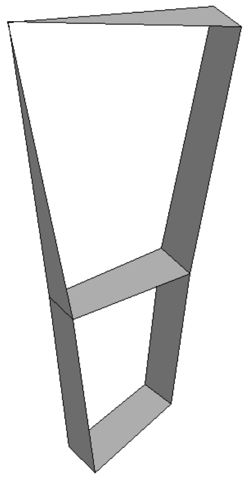
Ambient excitation was provided by the recorded micro tremor data on ground level. Three accelerometers (with



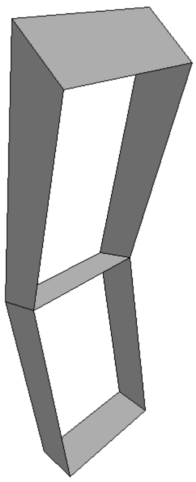


1st Mode Shape (f=2.075 Hz) 2nd Mode Shape (f=5.890 Hz)





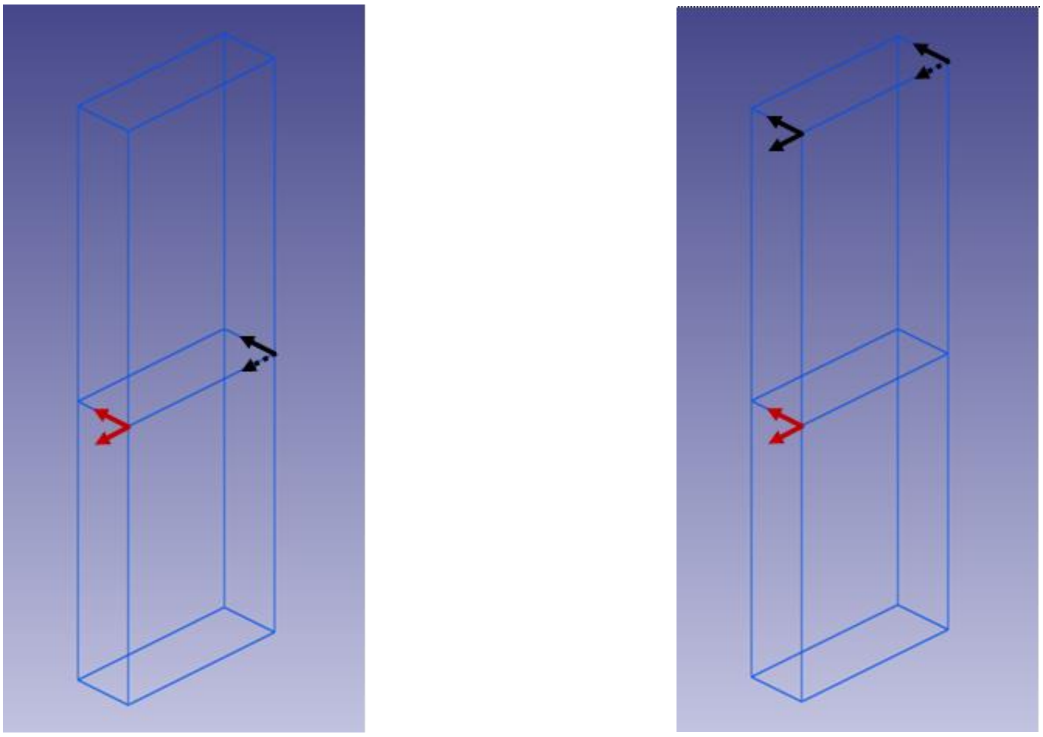
3rd Mode Shape (f=7.025 Hz) 4th Mode Shape (f=7.994 Hz)



5th Mode Shape (f=9.246 Hz)

Fig. 4 Analytically identified mode shapes of model steel structure

both x and y directional measures) were used for the ambient vibration measurements, one of which were allocated as reference sensor always located in the first floor (they are shown by the red line in Fig. 5(a), (b)). Two



(a) First setup (b) Second setup

Fig. 5 Accelerometers location of experimental model in the 3D view

Fig. 6 (a) Ambient vibrations recorded by the accelerometer

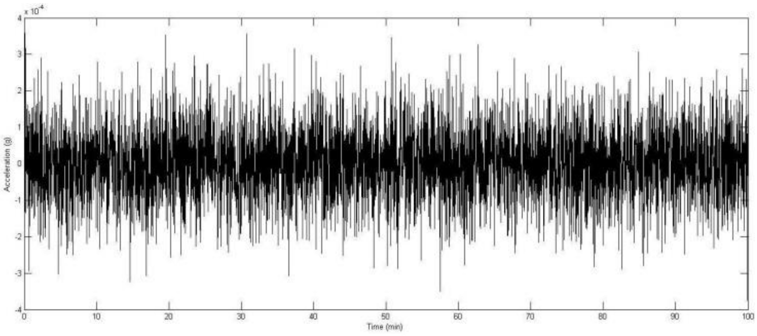


Fig. 6 (b) Ambient excitation data from the recorded micro tremor data on ground level used in the shake table

accelerometers were used as roving sensors (they are shown by the black line in Fig. 5(a), (b)). The response was measured in two data sets (Fig. 5(a), (b)). For two data sets were used 3 and 5 degree of freedom records respectively (Fig. 5(a), (b)). Every data set (Fig. 5(a), (b)) was measured 100 min. The selected measurement points and directions are shown in Fig. 5(a), (b).

The data acquisition computer was dedicated to acquiring the ambient vibration records. In between measurements, the data files from the previous setup were transferred to the data analysis computer using a software package. This arrangement allowed data to be collected on the computer while the second, and faster, computer could be used to process the data in site. This approach maintained a good quality control that allowed preliminary analyses of the collected data. If the data showed unexpected signal drifts or unwanted noise or for some unknown reasons, was corrupted, the data set was discarded and the measurements were repeated.

Before the measurements could begin, the cable used to connect the sensors to the data acquisition, equipment had to be laid out. Following each measurement, the roving sensors were systematically located from floor to floor until the test was completed. The equipment used for the measurement includes three quanser accelerometers (with both *x* and *y* directional measures) and geosig uni-axial accelerometer, matlab data acquisition toolbox (wincon). For modal parameter estimation from the ambient vibration data, the operational modal analysis (OMA) software ARTeMIS Extractor (1999) is used.

The simple peak-picking method (PPM) finds the eigenfrequencies as the peaks of nonparametric spectrum estimates. This frequency selection procedure becomes a subjective task in case of noisy test data, weakly excited

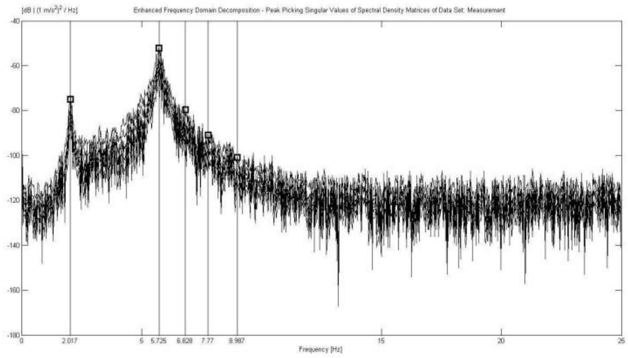


Fig. 7 Singular values of spectral density matrices

Table 3 Experimental modal analysis result at the model steel structure

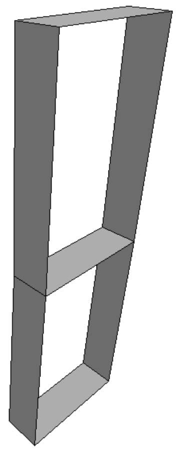
Mode number	1	2	3	4	5
Frequency (Hz)	2.017	5.725	6.828	7.770	8.987
Modal damping ratio (ξ)	0.672	1.822	1.035	0.551	0.670

Table 4 Comparison of analytical and experimental modal analysis results

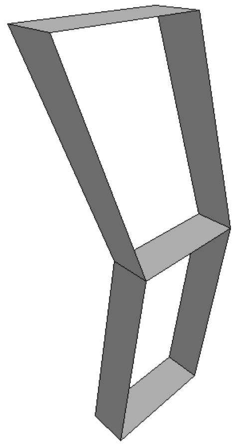
Mode number	1	2	3	4	5
Analytical frequency (Hz)	2.075	5.890	7.025	7.994	9.246
Experimental frequency (Hz)	2.017	5.725	6.828	7.770	8.987
Difference (%)	2.795	2.801	2.804	2.802	2.801

modes and relatively close eigenfrequencies. Also for damping ratio estimation the related half-power bandwidth method is not reliable at all. Frequency domain algorithms have been the most popular, mainly due to their convenience and operating speed.

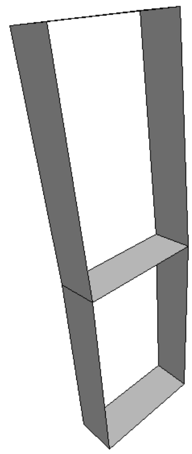
Singular values of spectral density matrices, attained from vibration data using PP (Peak Picking) technique are shown in Fig. 7. Natural frequencies acquired from the all measurement setup are given in Table 3. The first five mode shapes extracted from experimental modal analyses are given in Fig. 8. When all measurements are examined, it can be seen that there are best accordance is found between experimental mode shapes. When the analytically and experimentally identified modal parameters are checked with each other, it can be seen that there is a best agreement between the mode shapes in experimental and analytical modal analyses (Table 4).



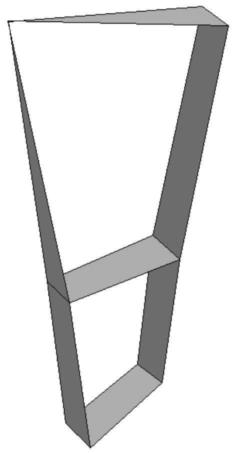
1st Mode Shape (f=2.017 Hz, ξ=0.672)



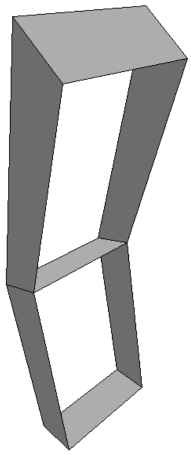
2nd Mode Shape (f=5.725 Hz, ξ=1.822)



3rd Mode Shape (f=6.828 Hz, ξ=1.035)



4th Mode Shape (f=7.770 Hz, ξ=0.551)



5th Mode Shape (f=8.987 Hz, ξ=0.670)

Fig. 8 Experimentally identified mode shapes of model steel structure

6. Conclusions

In this paper, analytical and experimental modal analysis of model steel structure was presented. Comparing the result of study, the following observation can be made:

From the finite element model of model steel structure a total of 5 natural frequencies were attained analytically, which range between 2 and 9 Hz. 3D finite element model of model steel structure is constructed with SAP2000 software and dynamic characteristics are determined analytically. The ambient vibration tests are conducted under provided by shake table from ambient vibration data on ground level. Modal parameter identification was implemented by the Enhanced Frequency Domain Decomposition (EFDD) technique. Comparing the result of analytically and experimentally modal analysis, the following observations can be made:

From the finite element model of the model steel structure, the first five mode shapes are attained analytically that range between 2 and 10 Hz.

• From the ambient vibration test, the first five natural frequencies are attained experimentally, which range between 2 and 9 Hz.

• When comparing the analytical and experimental results, it is clearly seen that there is best agreement between mode shapes.

• Analytical and experimental modal frequencies differences between 2.795%-2.804%.

• Presented investigation results are shown and confirm of possibility using the recorded micro tremor data on ground level as ambient vibration input excitation data for investigation and application Operational Modal Analysis (OMA) on the bench-scale earthquake simulator (The Quanser Shake Table) for model steel structures and shed light on the development of related research.

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