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Rotational effect on thermoelastic Stoneley, Love and Rayleigh waves in fibre-reinforced anisotropic general viscoelastic media of higher order

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Abstract. In this paper, we investigated the propagation of thermoelastic surface waves in fibre-reinforced anisotropic general viscoelastic media of higher order of nth order including time rate of strain under the influence of rotation. The general surface wave speed is derived to study the effectsof rotation andthermal onsurface waves. Particular cases for Stoneley, Love and Rayleighwaves are discussed.The results obtained in this investigation are more general in the sense that some earlier published results are obtained from our result as special cases. Our results for viscoelastic of order zero are well agreed to fibre-reinforced materials. Comparison was made with the results obtained in the presence and absence of rotation and parameters for fibre-reinforced of the material medium. It is also observed that, surface waves cannot propagate in a fast rotating medium.Numerical results for particular materials are given and illustrated graphically. The results indicate that the effect of rotation on fibre-reinforced anisotropic general viscoelastic media are very pronounced.

Keywords: fibre-reinforced; viscoelastic; surface waves; rotation; anisotropic; thermoelastic

1. Introduction

These problems are based on the more realistic elastic model since thermoelastic waves are propagating on the surface of earth, moon and other planets which are rotating about an axis. Schoenberg and Censor (1973) were the first to study the propagation of plane harmonic waves in a rotating elastic medium where it is shown that the elastic medium becomes dispersive and anisotropic due to rotation. Later on, many researchers introduced rotation in different theories of thermoelasticity. Agarwal (1979) studied thermo-elastic plane wave propagation in an infinite non-rotating medium. The normal mode analysis was used to obtain the exact expression for the temperature distribution, the thermal stresses and the displacement components. The purpose of the present work is to show the thermal and rotational effects on the surface waves.Surface waves have been well recognized in the study of earthquake, seismology, geophysics and geodynamics. A good amount of literaturefor surface waves is available (in Refs. Bullen 1965, Ewing and Jardetzky 1957, Rayleigh 1885, Stoneley 1924). Acharya andSingupta (1978), Pal and Sengupta (1987) and Sengupta and Nath (2001) and his research collaborators have studied surface waves. These waves usually have greater amplitudes ascompared with body waves and travel more slowly than body waves. There are many types of surface waves but we only discussed Stoneley, Love and Rayleigh waves. Earthquakeradiate seismic energy as both body and surface waves. These are also used for detecting cracks and other defects in materials. The idea of continuous self-reinforcement at every point of an elastic solid was introduced by Belfield *et al*. (1983). The superiority of fibre-reinforced composite materials over other structural materials attracted many authors to study different types of problems in this field. Fibre-reinforced composite structures are used due to their low weight and high strength. Two important components, namely concrete and steel of a reinforced medium are bound together as a single unit so that there can be no relative displacement between them i.e. they act together as a single anisotropic unit. The artificial structures on the surface of the earth are excited during an earthquake, which give rise to violent vibrations in some cases (Refs. Acharya 2009, Samaland and Chattaraj 3011). Engineers and architects are in search of such reinforced elastic materials for the structures that resist the oscillatory vibration. The propagation of waves depends upon the ground vibration and the physical properties of the material structure. Surface wave propagation in fiber reinforced media was discussed by various authors (Sing 2006, Kakar *et al*. 2013). Abd-Alla *et al*. (2012) investigated the transient coupled thermoelasticity of an annular fin. Reflection of quasi-P and quasi-SV waves at the free and rigid boundaries of a fibre-reinforced medium was also discussed by Chattopadhyay *et al*. (2012). Abd-Alla and Mahmoud (2011) investigated the magneto-thermoelastic problem in rotating non-homogeneous orthotropic hollow cylinder under the hyperbolic, heat conduction model.The extensive literature on the topic is now available and we can only mention a

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Singh 2004, Abd-Alla 2013, Singh 2007, Abd-Alla 2011, Abo-Dahab *et al*. 2016, Alla *et al*. 2015, Kumar *et al*. 2016, Said and Othman 2016, Bakora and Tounsi 2015).The temperature-rate dependent theory of thermoelasticity, which takes into account two relaxation times, was developed by Green and Lindsay (1972). Kumar *et al*. (2016) investigated the thermomechanical interaction transversely isotropic magnetothermoelastic medium with vacuum and with and without energy dissipation with the combined effects of rotation. Marin (1996) studied the Lagrange identity method in thermoelasticity of bodies with microstructure. Marin (1995) presented the existence and uniqueness in thermoelasticity of micropolar bodies. Marin and Marinescu (1998) investigated the thermoelasticity of initially stressed bodies. Asymptotic equipartition of energies.

The aim of this paper is to investigate the propagation of thermoelastic surface wavesin a rotating fibre-reinforced viscoelastic anisotropic media of higher order. The general surface wave speed is derived to study the effect of rotation and thermal on surface waves. The wave velocity equations have been obtained for Stoneley waves, Rayleigh waves and Love waves, and are in well agreement with the corresponding classical result in the absence of viscosity, temperature, rotation as well as homogeneity of the material medium. The results obtained in this investigation are more general in the sense that some earlier published results are obtained from our result as special cases. For order zero our results are well agreedto fibre-reinforced materials. It is also observed that the corresponding classical results follow from this analysis, in viscoelastic media of order zero, by neglecting reinforced parameters, rotational and thermal effects. Numerical results are given and illustrated graphically. It is important to note that Love wave remains unaffected by thermal and rotational effects.

2. Formulation of the problem

The constitutive relation of an anisotropic and elastic solid is expressed by the generalized Hooke's law, which can be written as

$$\tau_{ij} = C_{ijkl}\varepsilon_{kl} \qquad i, j, k, l=1, 2, 3. \tag{1}$$

Let T_o be the reference temperature at which the system is in equilibrium and let it be subjected to a temperature change $T-T_0$ where $|T-T_0|<<T_0$. Thus the coupled thermoelastic equations for the material Kakar *et al*. (2013) may be written as

$$\tau_{ij} = C_{ijkl}\varepsilon_{kl} - \beta_{ij}\left(1+\nu_o\frac{\partial}{\partial t}\right)(T-T_o) \tag{2}$$

$$\frac{\partial}{\partial x_i}\left(\kappa_{ij}\left(n^*+t_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_j}\right)=\rho c_v\left(n_1\frac{\partial}{\partial t}+\tau_o\frac{\partial^2}{\partial t^2}\right)T+T_o\beta_{ij}\left(n_1\frac{\partial}{\partial t}+n_o\tau_o\frac{\partial^2}{\partial t^2}\right)\varepsilon_{ij} \tag{3}$$

The thermal constant ν_0 and τ_0 appearing in the above equations satisfy the inequalities $\nu_0\geq\tau_0\geq0$. It is evident that if $\tau_0>0$, consequently $\nu_0>0$, the Eq. (3) Predicts a finite speed of propagation of thermal signals and that if, the Eqs. (2) and (3) reduce to the equations of the coupled thermoelasticity (C-T theory). The assumption $\tau_0=0$ and $\nu_0>0$ is also a valid one; in this case the equation of motion continues to be affected by the temperature rate, while Eq. (3) predicts an infinite speed for the propagation of heat.

In Eq. (3) We have made use of the condition $|T-T_0|<<T_0$ to replace T by T_0 in the last term of Eq. (3). The k_{ij} is the conductivity tensor, $c\nu$ is the specific heat at constant deformation, β_{ij} are the thermal moduli, τ_{ij} are the Cartesian components of the stress and ε_{kl} is the strain tensor which is related to the displacement vector, u_i, C_{ijkl} are the components of a fourth-order tensor called the elasticities of the medium. The Einstein convention for repeated indices is used.

If a body is rotating about an axis with a constant angular velocity Ω then the equation of motioncan be written as follows.

$$\tau_{ij,j} = \rho\left\{\ddot{u}_i+\Omega_j u_j\Omega_i-\Omega^2 u_i+2\varepsilon_{ijk}\Omega_j\dot{u}_k\right\} \tag{4}$$

where ε_{ijk} is the Levi-Civita tensor, by using (2), the equation of motionin a thermoelastic medium becomes

$$C_{ijkl}u_{k,jl} = \rho\left\{\ddot{u}_i+\Omega_j u_j\Omega_i-\Omega^2 u_i+2\varepsilon_{ijk}\Omega_j\dot{u}_k\right\}$$
$$+\beta_{ij}(1+\nu_o\frac{\partial}{\partial t})T_{,j} \tag{5}$$

In isotropic medium $\kappa_{ij}=\kappa\delta_{ij}$ and $\beta_{ij}=\beta\delta_{ij}$, β is the coefficient of linear thermal expansion and $\kappa$is the thermal conductivity of the medium.

We consider a homogeneous thermally conducting anisotropic two fibre-reinforced medium. Let M_1 and M_2 be two fibre-reinforced thermoelastic semi-infinite solid media. They are perfectly welded in contact to prevent any relative motion or sliding before and after the disturbances and that the continuity of displacement, stress etc. hold good across the common boundary surface. Further the mechanical properties of M_1 are different from those of M_2. These media extend to an infinite great distance from the origin and are separated by a plane horizontal boundary and M_2 is to be taken above M_1.

Let $Ox_1x_2x_3$ be a set of orthogonal Cartesian coordinates and let O be the any point of the plane boundary and Ox_3 points vertically downward to the medium M_1. We consider the possibility of a type of wave travelling in the direction Ox_1, in such a manner that the disturbance is largely confined to the neighborhood of the boundary and at any instant, all particles in any line parallel to x_2-*axis* have equal displacements. These two assumptions conclude that the wave is a surface wave and all partial derivatives with respect to x_2 are a zero. It is assumed that the waves travel in the positive direction of x_1-axis and at any instant, all particles have equal displacements in any direction parallel to Ox_3. In view of those assumptions, the propagation of waves will be independent of x_3. The general equation for a fibre-reinforced linear elastic anisotropic media (Kakar *et al*. 2013) and $\bar{a}=(a_1, a_2, a_3)$ where, $a_1^2+a_2^2+a_3^2=1$.

If $\bar{a}$ has components that are (1,0,0), so that the preferred direction is the x-axis

$$C_{ijkl} = D_\lambda \varepsilon_{kk}\delta_{ij} + 2D_{\mu T}\varepsilon_{ij} + D_\alpha(a_k a_m \varepsilon_{km}\delta_{ij} + \varepsilon_{kk}a_i a_j) \\ +2(D_{\mu L} - D_{\mu T})(a_i a_k \varepsilon_{ki}) + +D_\beta(a_k a_m \varepsilon_{km} a_i a_j), \quad (5)$$

where the strain tensor is $\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$ and D_λ, $D_{\mu T}$ are elastic parameters. D_α, D_β and $(D_{\mu_L} - D_{\mu_T})$ are reinforced anisotropic viscoelastic parameters of higher order, s, defined as

$$D_\lambda = \lambda_k \frac{\partial^k}{\partial t^k}, \quad D_\mu = \mu_k \frac{\partial^k}{\partial t^k}, \\ D_\alpha = \alpha_k \frac{\partial^k}{\partial t^k}, \quad D_{\mu L} = \mu_{l_k} \frac{\partial^k}{\partial t^k}, \\ D_\beta = \beta_k \frac{\partial^k}{\partial t^k}, \quad D_{\mu T} = \mu_{T_k} \frac{\partial^k}{\partial t^k}. \\ k = 0,1,2...s. \quad (6)$$

An Einstein summation convention for repeated indices over "k" is used and comma followed by an index denotes the derivative with respect to coordinate.

u_i are the displacement vector components. The components of stress becomes as follows

$$\left.\begin{aligned} &\tau_{11} = (D_\lambda + 2D_\alpha + 4D_{\mu_L} - 2D_{\mu_T} + D_\beta)\varepsilon_{11} + (D_\lambda + D_\alpha)\varepsilon_{22} \\ &\quad +(D_\lambda + D_\alpha)\varepsilon_{33} ? \beta(1+\nu_o \frac{\partial}{\partial t})(T - T_o, \\ &\tau_{22} = (D_\lambda + D_\alpha)\varepsilon_{11} + (D_\lambda + 2D_{\mu_T})\varepsilon_{22} + D_\lambda\varepsilon_{33} - \beta(1+\nu_o \frac{\partial}{\partial t})(T - T_o), \\ &\tau_{33} = (D_\lambda + D_\alpha)\varepsilon_{11} + D_\lambda\varepsilon_{22} + (D_\lambda + 2D_{\mu_T})\varepsilon_{33} ? \beta(1+\nu_o \frac{\partial}{\partial t})(T - T_o, \\ &\tau_{13} = 2D_{\mu_L}\varepsilon_{13}, \\ &\tau_{12} = 2D_{\mu_L}\varepsilon_{12}, \\ &\tau_{23} = 2D_{\mu_T}\varepsilon_{23}. \end{aligned}\right\} \quad (7)$$

It is assumed that thebody is rotated about thez-axis with an angular frequency, Ω i.e., $\mathbf{\Omega}=\Omega(0,0,1)$. The Eq. (5) of motion take the following form

$$(D_\lambda + 2D_\alpha + 4D_{\mu_L} - 2D_{\mu_T} + D_\beta)u_{1,11} + (D_\alpha + D_\lambda + D_{\mu_L})u_{2,21} \\ +D_{\mu_L}u_{1,22} = \rho\{\ddot{u}_1 - \Omega^2 u_1 - 2\Omega\dot{u}_2\} + \beta(1+\nu_o \frac{\partial}{\partial t})T_{,1} \quad (8)$$

$$(D_\alpha + D_{\lambda_k} + D_{\mu_L})u_{1,12} + D_{\mu_L}u_{2,11} + (D_{\lambda_k} + 2D_{\mu_T})u_{2,22} \\ = \rho\{\ddot{u}_2 - \Omega^2 u_2 + 2\Omega\dot{u}_1\} + \beta(1+\nu_o \frac{\partial}{\partial t})T_{,2} \quad (9)$$

$$(D_{\mu_L} u_{3,11} + D_{\mu_T} u_{3,22}) = \rho\ddot{u}_3, \quad (10)$$

From Eq. (3), we have

$$\kappa\left(n^* + t_1 \frac{\partial}{\partial t}\right)T_{,ii} = \rho c_v \left(n_1 \frac{\partial}{\partial t} + \tau_o \frac{\partial^2}{\partial t^2}\right)T \\ + T_o\beta\left(n_1 \frac{\partial}{\partial t} + n_o\tau_o \frac{\partial^2}{\partial t^2}\right)u_{i,i} \quad (11)$$

Similarly, we can get similar relations in M_1 with ρ, α, β, κ, c_v, D_α, D_λ, D_{μ_L}, D_{μ_T} and D_β are replaced by ρ', β', κ', c'_v, $D_{\alpha'}$, $D_{\lambda'}$, $D_{\mu'_L}$, $D_{\mu'_T}$ and $D_{\beta'}$ i.e., all the parameters in medium M_1 are denoted by superscript "dash".

Thus the above set of Eqs. (8)-(11) becomes (For convenience dashes are omitted)

$$h_3 u_{1,11} + h_2 u_{2,21} + h_1 u_{1,22} = \rho(\ddot{u}_1 - \Omega^2 u_1 - 2\Omega\dot{u}_2) \\ +\beta(1+\nu_o \frac{\partial}{\partial t})T_{,1} \quad (12)$$

$$h_4 u_{2,22} + h_2 u_{1,12} + h_1 u_{2,11} = \rho(\ddot{u}_2 - \Omega^2 u_2 + 2\Omega\dot{u}_1) \\ +\beta(1+\nu_o \frac{\partial}{\partial t})T_{,2} \quad (13)$$

$$\kappa\left(n^* + t_1 \frac{\partial}{\partial t}\right)T_{,ii} = \rho c_v \left(n_1 \frac{\partial}{\partial t} + \tau_o \frac{\partial^2}{\partial t^2}\right)T \\ + \beta T_o\left(n_1 \frac{\partial}{\partial t} + n_o\tau_o \frac{\partial^2}{\partial t^2}\right)u_{i,i}\ ; i = 1,2 \quad (14)$$

where

$h_1 = D_{\mu_L}$, $h_2 = D_\alpha + D_\lambda + D_{\mu_L}$,

$h_3 = D_\lambda + 2D_\alpha + 4D_{\mu_L} - 2D_{\mu_T} + D_\beta$

and $h_4 = D_{\lambda_k} + 2D_{\mu_T}$

3. Solution of the problem

Now our main objective to solve Eqs. (12), (13) and (14). We seek the solution of (12), (13), (14) in the following forms

$$\{u_1(x_1,x_2,t), u_2(x_1,x_2,t), u_3(x_1,x_2,t)\} = \\ \{\hat{u}_1(x_2), \hat{u}_2(x_2), u_3(x_2)\}\exp(i\omega(x_1 - ct)) \quad (15)$$

and

$$\theta(x_1, x_2, t) = \hat{\theta}(x_2)\exp(i\omega(x_1 - ct)) \quad (16)$$

where ω is the angular frequency, c is the phase velocity and $\theta = T - T_0$.

Thus the coupled Eqs. (12)-(14) become

$$(\hbar_1 D^2 - \omega^2\hbar_3 + \omega^2\rho c^2 + \rho\Omega^2)\hat{u}_1 \\ +i\omega(\hbar_2 D - 2c\rho\Omega)\hat{u}_2 - i\omega\beta(1 - i\omega c\nu_o)\hat{\theta} = 0 \quad (17)$$

$$(\hbar_4 D^2 - \omega^2\hbar_1 + \omega^2\rho c^2 + \rho\Omega^2)\hat{u}_2 \\ +i\omega(\hbar_2 D + 2c\rho\Omega)\hat{u}_1 - \beta(1 - i\omega c\nu_o)D\hat{\theta} = 0 \quad (18)$$

and

$$\beta T_0(i\omega c n_1 - n_0\omega^2 c^2\tau_0)(D\hat{u}_1 - i\omega\hat{u}_2) + \\ \{(D^2 - \omega^2)(n^* - i\omega c t_1) + i\omega c n_1 + \omega^2 c^2\tau_0\}\hat{\theta} = 0 \quad (19)$$

where,

$$\hbar_1 = \mu_{Lk}(-i\omega c)^k,\ \hbar_2 = (\alpha_k + \lambda_k + \mu_{Lk})(-i\omega c)^k,$$
$$\hbar_3 = (\lambda_k + 2\alpha_k + 4\mu_{Lk} - 2\mu_{Tk} + \beta_k)(-i\omega c)^k$$
$$and\ \hbar_4 = (\lambda_k + 2\mu_{Tk})(-i\omega c)^k$$

Eq. (10) has the following solution

$$u_3 = Ee^{-\eta\omega x_2} e^{i\omega(x_1 - ct)}, \tag{20}$$

where , $\eta^2 = \dfrac{\rho c^2 - \mu_{Lk}(-i\omega c)^k}{\mu_{Tk}(-i\omega c)^k}$.

For positive real root η, it is necessary that $0<\rho c^2<\mu_{Lk}$.

The above set of Eqs. (17)-(19) can be written as

$$\left.\begin{array}{c}(\hbar_1 D^2 - A_1)\hat{u}_1 + i\omega(\hbar_2 D - 2c\rho\Omega)\hat{u}_2 - i\omega Q\hat{\theta} = 0 \\ (\hbar_4 D^2 - A_2)\hat{u}_2 + i\omega(\hbar_2 D + 2c\rho\Omega)\hat{u}_1 - Q\hat{\theta} = 0 \\ A_4(i\omega\hat{u}_1 + D\hat{u}_2) + (D^2 - A_3)\hat{\theta} = 0\end{array}\right\} \tag{21}$$

where

$$A_1 = \omega^2\hbar_3 - \omega^2\rho c^2 - \rho\Omega^2$$
$$A_2 = \omega^2\hbar_1 - \omega^2\rho c^2 - \rho\Omega^2$$
$$A_3 = \omega^2 - \frac{i\omega c n_1 + \omega^2 c^2\tau_o}{(n^* - i\omega c t_1)}$$
$$A_4 = \beta T_o \frac{\left(i\omega c n_1 - n_o\omega^2 c^2\tau_o\right)}{(n^* - i\omega c t_1)}$$
$$Q = \beta(1 - i\omega c v_o)$$

From above set of Eq.s (21), we have

$$\begin{vmatrix} (\hbar_1 D^2 - A_1) & i\omega(\hbar_2 D - 2c\rho\Omega) & -i\omega Q \\ i\omega(\hbar_2 D + 2c\rho\Omega) & (\hbar_4 D^2 - A_2) & -QD \\ i\omega A_4 & A_4 D & (D^2 - A_3) \end{vmatrix} = 0, \tag{22}$$

This implies

$$(D^6 - AD^4 + BD^2 - C) = 0, \tag{23}$$

where

$$A = \frac{1}{\hbar_1\hbar_4}\left(\hbar_4 A_1 + \hbar_1(A_2 + \hbar_4 A_3 - A_4 Q) - \omega^2\hbar_2^2\right)$$
$$B = \frac{1}{\hbar_1\hbar_4}\left\{\left(A_1 A_2 + \hbar_4 A_1 A_3 + \hbar_1 A_2 A_3 - \omega^2 A_3\right) + \right.$$
$$\left. + QA_4(A_1 + 2\omega^2\hbar_2 - \hbar_4\omega^2) - 4c^2\omega^2\rho^2\Omega^2)\right\}$$
$$C = \frac{1}{\hbar_1\hbar_4}\left(A_1 A_2 A_3 - \omega^2 A_2 A_4 Q - 4c^2\omega^2\rho^2\Omega^2\right)$$
$$Let\ \ D^2 = m$$

The auxiliary Eq. (23) becomes

$$m^3 - Am^2 + Bm - C = 0, \tag{24}$$

A, B and C must be positive for real positive roots (m). If there is no thermal effect, then the above equation is quadratic in m and it is easy to solve. But in the case of thermoelastic, it is cubic. A, B and C must positively impose a necessary and sufficient condition upon the frequency of rotation of the medium. Through which a surface wave cannot propagate in a fast rotating medium. If there is no thermal effect then

$$\Omega^2 < \omega^2\hbar_3 / \rho,$$

This means that in a fast rotating medium, surface wave cannot propagate.

Let m_1, m_2 and m_3 be three positive real roots, then the solution by normal mode method has the following form

$$\hat{u}_1 = \sum_{n=1}^{3} M_n\ e^{-m_n x_2}, \tag{25}$$

$$\hat{u}_2 = \sum_{n=1}^{3} M_{1n}\ e^{-m_n x_2}, \tag{26}$$

$$\hat{\theta} = \sum_{n=1}^{3} M_{2n}\ e^{-m_n x_2}, \tag{27}$$

Where, M_n, M_{1n} and M_{2n}, are some parameters depending on c and ω. By using Eqs. (25)-(27) into Eq. (21), we get the following relations

$$\left.\begin{array}{l} M_{1n} = H_{1n} M_n \\ M_{2n} = H_{2n} M_n \end{array}\right\} \tag{28}$$

Where

$$H_{1n} = \frac{i\omega\left(A_2 + (\hbar_2 - \hbar_4)m_n^2 + 2\rho c\Omega m_n\right)}{\hbar_1 m_n^3 + (\hbar_2\omega^2 - A_1)m_n - 2\rho c\omega^2\Omega},$$

$$H_{2n} = \frac{m_n^2 - A_3}{A_4(m_n H_{1n} - i\omega)}, \qquad n = 1, 2, 3.$$

Hence we obtain the expressions of the displacement components, temperature distribution function and stresses as follows

$$u_1 = \sum_{n=1}^{3} M_n\ e^{-m_n x_2}\ exp\left\{i\omega(x_1 - ct)\right\}, \tag{29}$$

$$u_2 = \sum_{n=1}^{3} H_{1n} M_n\ e^{-m_n x_2}\ exp\left\{i\omega(x_1 - ct)\right\}, \tag{30}$$

$$u_3 = Ee^{-\eta\omega x_2}\ exp\left\{i\omega(x_1 - ct)\right\}, \tag{31}$$

$$\theta = \sum_{n=1}^{3} H_{2n} M_n\ e^{-m_n x_2}\ exp\left\{i\omega(x_1 - ct)\right\}. \tag{32}$$

Also it is found that

$$\tau_{11} = \sum_{n=1}^{3}\{i\omega\hbar_3 - (\hbar_2 - \hbar_1)m_n H_{1n} - \beta(1 - i\omega c v_0)H_{2n}\}$$
$$M_n e^{-m_n x_2}\ \ exo\{i\omega(x_1 - ct)\} \tag{33}$$

$$\tau_{22} = \sum_{n=1}^{3}\left\{i\omega(\hbar_2 - \hbar_1) - \hbar_4 m_n H_{1n} - \beta(1 - i\omega c v_o)H_{2n}\right\}\times$$
$$M_n e^{-m_n x_2}\ exp\left\{i\omega(x_1 - ct)\right\}, \tag{34}$$

$$\tau_{12} = \sum_{n=1}^{3} D_{\mu_L}\left(-m_n + i\omega H_{1n}\right)\times$$

$$M_n e^{-m_n x_2} \exp\{i\omega(x_1 - ct)\}, \tag{35}$$

Similar expressions can be obtained for second medium and present them with dashes as follows

$$u_1' = \sum_{n=1}^{3} M_n' \, e^{-m_n' x_2} \exp\{i\omega(x_1 - ct)\}, \tag{36}$$

$$u_2' = \sum_{n=1}^{3} H_{1n}' M_n' \, e^{-m_n' x_2} \exp\{i\omega(x_1 - ct)\}, \tag{37}$$

$$u_3' = F e^{-\eta'\omega x_2} \exp\{i\omega(x_1 - ct)\}, \tag{38}$$

$$\theta' = \sum_{n=1}^{3} H_{2n}' M_n' \, e^{-m_n' x_2} \exp\{i\omega(x_1 - ct)\}, \tag{39}$$

Also it is found that

$$\tau_{11}' = \sum_{n=1}^{3} \begin{aligned}&\{i\omega\hbar_3' - (\hbar_2' - \hbar_1')m_n' H_{1n}' - \beta'(1 - i\omega c v_o')H_{2n}'\} \times \\ &M_n' e^{-m_n' x_2} \exp\{i\omega(x_1 - ct)\},\end{aligned} \tag{40}$$

$$\begin{aligned}\tau_{22}' = &\sum_{n=1}^{3} \{i\omega(\hbar_2' - \hbar_1') - \hbar_4' m_n' H_{1n}' - \beta'(1 - i\omega c v_o')H_{2n}'\} \times \\ &M_n' e^{-m_n' x_2} \exp\{i\omega(x_1 - ct)\},\end{aligned} \tag{41}$$

$$\begin{aligned}\tau_{12}' = &\sum_{n=1}^{3} \hbar_1' \left(-m_n' + i\omega H_{1n}'\right) \times \\ &M_n' e^{-m_n' x_2} \exp\{i\omega(x_1 - ct)\},\end{aligned} \tag{42}$$

In order to determine the secular equations, we have the following boundary conditions.

4. Boundary conditions

The stresses components, the displacement components, and temperature at the boundary surface between the media M_1 and M_2 must be continued at all times and positions.

Consider the following:

1) The displacement components between the mediums are continuous, i.e.
2) $u_1 = u_1'$, $u_2 = u_2'$ and $\theta = \theta'$ on $x_2 = 0$, for all x_2 and t.
3) Stress continuity exists, i.e., $\tau_{12} = \tau_{12}'$, $\tau_{22} = \tau_{22}'$ on $x_2 = 0$, for all x_1 and t.
4) Thermal boundary conditions (Singh and Singh 2004) gives

$$\left(\frac{\partial\theta}{\partial x_2} + h\theta\right)_{M_1} = \left(\frac{\partial\theta'}{\partial x_2} + h'\theta'\right)_{M_2},$$

on the plane $x_2 = 0$, $\forall x_1$ and t, where h is non negative thermal constant.

The boundary conditions imply the following equations

$$\left.\begin{aligned} M_1 + M_2 + M_3 &= M_1' + M_2' + M_3' \\ H_{11}M_1 + H_{12}M_2 + H_{13}M_3 &= H_{11}'M_1' + H_{12}'M_2' + H_{13}'M_3' \\ H_{21}M_1 + H_{22}M_2 + H_{23}M_3 &= H_{21}'M_3' + H_{22}'M_2' + H_{23}'M_3' \\ E &= F \end{aligned}\right\} \tag{43}$$

$$\left.\begin{aligned} &\sum_{n=1}^{3} \hbar_1(-m_n + i\omega H_{1n})M_n = \sum_{n=1}^{3} \hbar_1(-m_n' + i\omega H_{1n}')M_n', \\ &\sum_{n=1}^{3} \{i\omega(\hbar_2 - \hbar_1) - \hbar_4 m_n H_{1n} - \beta(1 - i\omega c v_0)H_{2n}\}M_n = \\ &\sum_{n=1}^{3} \{i\omega(\hbar_2' - \hbar_1') - \hbar_4' m_n' H_{1n}' - \beta'(1 - i\omega c v_0')H_{2n}'\}M_n', \\ &\hbar_5\eta_3 E = \hbar_5'\eta_3' F, \\ &(h - m_n)H_{2n}M_n = (h' - m_n')H_{2n}'M_n' \end{aligned}\right\} \tag{44}$$

From the above equations containing E and F, we have

$E = F = 0$.

This implies that there is no propagation in the transverse component of displacement.

Elimination of constants M_n and M'_n, (n=1,2,3) from the above set of relation, gives the following secular equation for thermoelastic surface wave in a rotating fibre reinforced viscoelastic material of order n.

$$\det(a_{pq}) = 0; \quad p = q = 1,2,3,4,5,6. \tag{45}$$

where

$$a_{11} = 1,\ a_{12} = 1,\ a_{13} = 1,\ a_{14} = -1,\ a_{15} = -1,\ a_{16} = -1,$$
$$a_{21} = H_{11},\ a_{22} = H_{12},\ a_{23} = H_{13},\ a_{24} = -H_{11}',$$
$$a_{25} = -H_{12}',\ a_{26} = -H_{13}', a_{31} = H_{21},\ a_{32} = H_{22},$$
$$a_{33} = H_{23},\ a_{34} = -H_{21}',\ a_{35} = -H_{22}',\ a_{36} = -H_{23}',$$
$$a_{41} = D_{\mu_L}\left(-m_1 + i\omega H_{11}\right),\ a_{42} = D_{\mu_L}\left(-m_2 + i\omega H_{12}\right),$$
$$a_{43} = D_{\mu_L}\left(-m_3 + i\omega H_{13}\right), a_{44} = -D_{\mu_L}'\left(-m_1' + i\omega H_{11}'\right),$$
$$a_{45} = -D_{\mu_L}'\left(-m_2' + i\omega H_{12}'\right),\ a_{46} = -D_{\mu_L}'\left(-m_3' + i\omega H_{13}'\right)$$
$$a_{51} = \{i\omega(\hbar_2 - \hbar_1) - \hbar_4 m_1 H_{11} - \beta(1 - i\omega c v_o)H_{21}\}$$
$$a_{52} = \{i\omega(\hbar_2 - \hbar_1) - \hbar_4 m_2 H_{12} - \beta(1 - i\omega c v_o)H_{22}\}$$
$$a_{53} = \{i\omega(\hbar_2 - \hbar_1) - \hbar_4 m_3 H_{13} - \beta(1 - i\omega c v_o)H_{23}\}$$
$$a_{54} = -\{i\omega(\hbar_2' - \hbar_1') - \hbar_4' m_1 H_{11}' - \beta'(1 - i\omega c v_o)H_{21}'\}$$
$$a_{55} = -\{i\omega(\hbar_2' - \hbar_1') - \hbar_4' m_2 H_{12}' - \beta'(1 - i\omega c v_o)H_{22}'\}$$
$$a_{56} = -\{i\omega(\hbar_2' - \hbar_1') - \hbar_4' m_3 H_{13}' - \beta'(1 - i\omega c v_o)H_{23}'\}$$
$$a_{61} = (h - m_1)H_{22}, \qquad a_{62} = (h - m_2)H_{22},$$
$$a_{63} = (h - m_3)H_{23},$$
$$a_{64} = -(h' - m_1')H_{21}', \qquad a_{65} = -(h' - m_2')H_{22}',$$
$$a_{66} = -(h' - m_3')H_{23}',$$

From Eq. (45), we get the velocity of surface waves in common boundary between two viscoelastic, fiber-reinforced solid media of Voigt type, where the viscosity is of general nth order involving time rate of change of strain.

5. Particular cases

5.1 Stoneley waves

It is the generalized form of Rayleigh waves in which we assume that the waves are propagated along the common boundary of two semi-infinite media M_1 and M_2. Therefore, Eq. (45) determines the wave velocity equation for Stoneley waves in the case of general viscoelastic, fibre-reinforced solid medal of nth order involving time rate of strain. Clearly from the Eq. (45), it follows that wave velocity of the Stoneley waves depends upon the parameters for fibre-reinforced of the material medium and the viscosity. Since the wave velocity Eq. (45) for Stoneley waves under the present circumstances does not contain ω explicitly, such types of waves are not dispersive like the classical one. In case of absence of parameters for fibre-reinforced and isotropic viscoelastic medium of 1st order involving time rate of change of strain is taken. Eq. (45) is the secular equation for Stonely waves in a fibre reinforced viscoelastic media of orders. For k=0, results are similar to Abd-Alla *et al*. (2013)and Lotfy (2012). If rotational, thermal and fiber-reinforced parameters are ignored, then for k=0, the results are same as Stoneley (1924).

Then Eq. (45) reduces to

$$\left|b_{ij}\right|=0, \qquad i,j=1,2,3,4,5,6; \tag{46}$$

where

$$b_{11}=1,\quad b_{12}=1,\ b_{13}=1,\ b_{14}=-1,\ b_{15}=-1,\ b_{16}=-1,$$
$$b_{21}=H_{11},\quad b_{22}=H_{12},\ b_{23}=H_{13},\quad b_{24}=-H'_{11},$$
$$b_{25}=-H'_{12},\quad b_{26}=-H'_{13}, b_{31}=H_{21},\ b_{32}=H_{22},$$
$$b_{33}=H_{33},\quad b_{34}=-H'_{21},\ b_{35}=-H'_{22},\quad b_{36}=-H'_{23},$$
$$b_{41}=D_{\mu L}(-m'_1+i\omega H_{11}),\quad b_{42}=D_{\mu L}(-m_2+i\omega H_{12}),$$
$$b_{43}=D_{\mu L}(-m_3+i\omega H_{13}), b_{44}=-D'_{\mu L}(-m'_1+i\omega H'_{11}),$$
$$b_{45}=-D'_{\mu L}(-m'_2+i\omega H'_{12}),\ b_{46}=-D'_{\mu L}(-m'_3+i\omega H'_{13}),$$
$$b_{51}=\left\{i\omega(h_2-h_1)-h_4 m_1 H_{11}-\beta(1-i\omega c v_0)H_{21}\right\},$$
$$b_{52}=\left\{i\omega(h_2-h_1)-h_4 m_2 H_{12}-\beta(1-iwcv_0)H_{22}\right\},$$
$$b_{53}=\left\{i\omega(h_2-h_1)-h_4 m_3 H_{13}-\beta(1-i\omega c v_0)H_{23}\right\},$$
$$b_{54}=-\left\{i\omega(h'_2-h'_1)-h'_4 m'_1 H'_{11}-\beta'(1-i\omega c v_0)H'_{21}\right\},$$
$$b_{55}=-\left\{i\omega(h'_2-h'_1)-h'_4 m'_2 H'_{12}-\beta'(1-i\omega c v_0)H'_{22}\right\},$$
$$b_{56}=-\left\{i\omega(h'_2-h'_1)-h'_4 m'_3 H'_{13}-\beta'(1-i\omega c v_0)H'_{33}\right\},$$
$$b_{61}=(h-m_1)H_{22},\quad b_{62}=(h-m_2)H_{22},\quad b_{63}=(h-m_3)H_{23},$$
$$b_{64}=-(h'-m'_1)H'_{21},\quad b_{65}=-(h'-m'_2)H'_{22},\quad b_{66}=-(h'-m'_3)H'_{23}$$

Eq. (46) gives the wave velocity equation of Stoneley waves in a viscoelastic medium of Voigt type where the viscosity is of Ist order involving time rate of change of strain which is completely in agreement with classical results given by Sengupta and Nath (2001). Further Eq. (46), of course, is in complete agreement with the corresponding classical result, when the effect of rotation, viscosity and parameters of fibre-reinforcement are ignored.

5.2 Love waves

To investigate the rotational effects on Love waves in a fibre reinforced viscoelastic media of higher order, we replace medium M_1 by an infinitely extended horizontal plate of finite thickness d and bounded by two horizontal plane surfaces x_2=0 and $x_2=d$. Medium M is semi infinite as in the general case.

The boundary conditions of Love wave are as follows

The displacement component u_3 and τ_{12} between the mediums is continuous, i.e.,

$u_3=u'_3$ and $\tau_3=\tau'_{23}$ on x_2=0

τ'_{23}=0 on $x_2=d$, for all x_1 and t,

where

$$u_3=Ee^{-\eta\omega x_2}e^{i\omega(x_1-ct)},$$
$$u'_3=E'e^{\eta'\omega x_2}e^{i\omega(x_1-ct)}+F'e^{-\eta'\omega x_2}e^{i\omega(x_1-ct)},$$

This implies

$$E-E'-F'=0,$$
$$\mu_{Tk}\left(-i\omega c\right)^k\eta_3 E+\mu'_{Tk}\left(-i\omega c\right)^k\eta'_3 E'-\mu'_{Tk}\left(-i\omega c\right)^k\eta'_3 F'=0,$$
$$e^{\omega\eta_3 d}E'-e^{-\omega\eta_3 d}F'=0.$$

For non trivial solution implies

$$\begin{vmatrix} 1 & -1 & -1 \\ \mu_{Tk}\left(-i\omega c\right)^k\eta_3 & \mu'_{Tk}\left(-i\omega c\right)^k\eta'_3 & -\mu'_{Tk}\left(-i\omega c\right)^k\eta'_3 \\ 0 & e^{\omega\eta'_3 d} & -e^{-\omega\eta'_3 d}\end{vmatrix}=0$$

This gives the wave velocity of Love waves propagating in a fiber-reinforced viscoelastic medium of orders. For k=0, the results are exactly same as in (Stoneley 1924). It is interesting to note that rotation and thermal does not affect the velocity of Love waves.

5.3 Rayleigh waves

Rayleigh wave is a special case of the above general surface wave. In this case we consider a model where the medium M_1 is replaced by vacuum. Since the boundary x_2=0 is adjacent to vacuum. It is free from surface traction. So the stress boundary condition in this case may be expressed as

τ_{12}=0, τ_{22}=0 on x_2=0, for all x_1 and t,

$$\frac{\partial\theta}{\partial x_2}+h\theta=0,\ \text{on the plane}\ x_2=0\ ,\forall x_1\ and\ t,$$

Thus the above set of equations reduces to

$$\sum_{n=1}^{3}\ (-m_n+i\omega H_{1n})M_n=0\ ,$$
$$\sum_{n=1}^{3}\left\{i\omega(\hbar_2-\hbar_1)-\hbar'_4 m_n H_{1n}-\beta(1-i\omega c v_o)H_{2n}\right\}M_n=0$$
$$\sum_{n=1}^{3}(h-m_n)H_{2n}M_n=0$$

An Einstein summation convention for repeated indices upon k is applied.

Eliminating the constants M_1, M_2 and M_3 we get the wave velocity equation for Rayleigh waves in the rotating thermoelastic fibre-reinforced viscoelastic media of order n as under

$$\det(b_{lm})=0;\quad l=m=1,2,3. \tag{47}$$

where

$$b_{11}=\left(-m_1+i\omega H_{11}\right),\ b_{12}=\left(-m_2+i\omega H_{12}\right),$$
$$b_{13}=\left(-m_3+i\omega H_{13}\right),$$
$$b_{21}=\left\{i\omega(\hbar_2-\hbar_1)-\hbar_4 m_1 H_{11}-\beta(1-i\omega c v_o)H_{21}\right\}$$

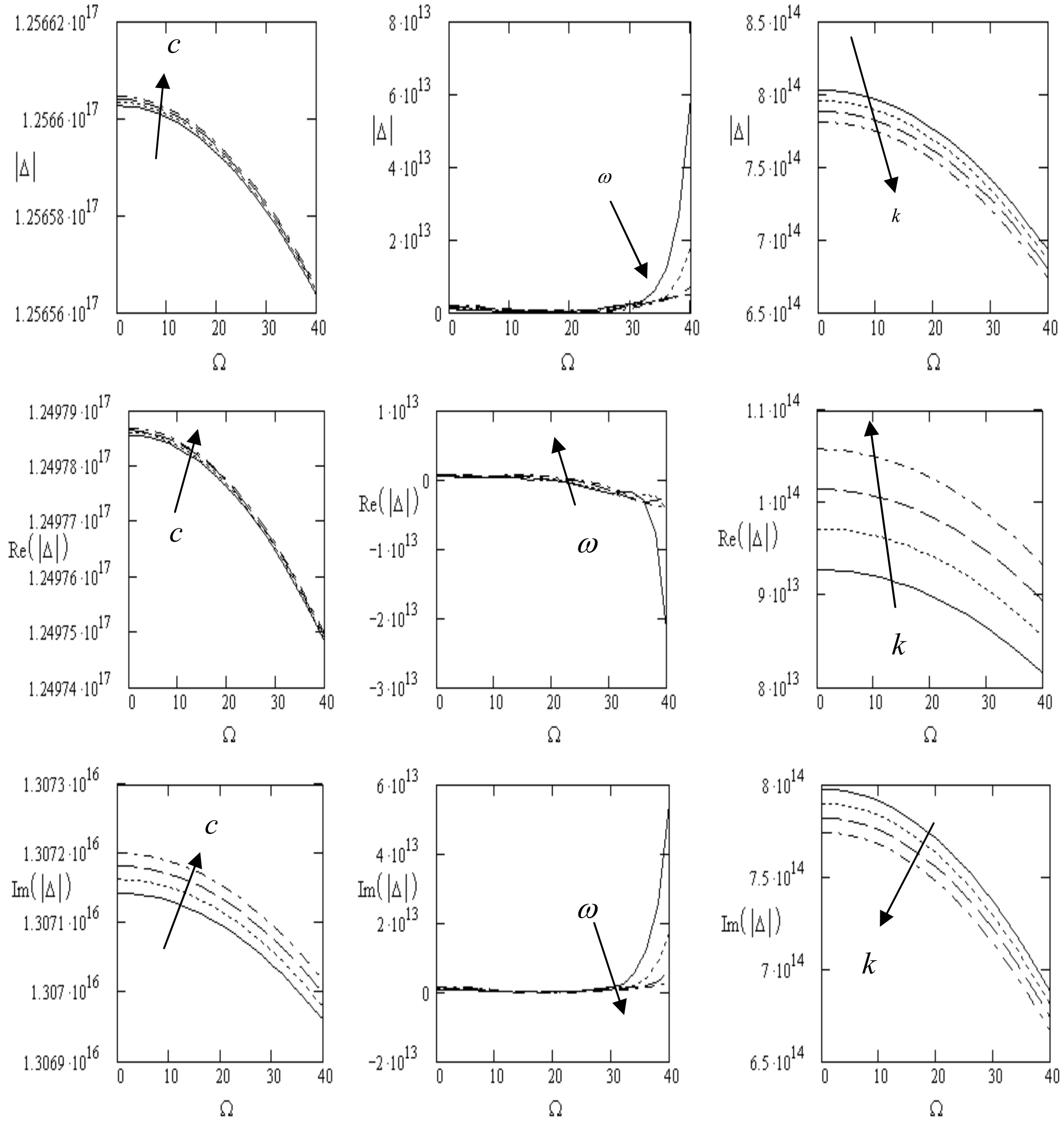

Fig. 1 Variation of the magnitude of the frequency equation |Δ| and attenuation coefficient Im(|Δ|) for Stoneley waves velocity Re(|Δ|) with respect to Ω with variation of *c*, *ω* and *k*

$$b_{23} = \{i\omega(\hbar_2 - \hbar_1) - \hbar_4 m_3 H_{13} - \beta(1 - i\omega c\nu_o)H_{23}\}$$

$$b_{31} = (h - m_1)H_{21}, \qquad b_{32} = (h - m_2)H_{22},$$

$$b_{33} = (h - m_3)H_{23},$$

The Eq. (5.1) is the magnitude of the frequency Eq. for Rayleigh wave for the medium M_1. For k=0, that is, our results are similar to Abd-Alla *et al.* (2013). For a non-rotating media, we have to put Ω=0, then for k=0 our results are same as that of Kakar *et al*. (2013). If one ignores the fiber-reinforced parameters, then the results are same as Rayleigh (1885).

6. Numerical results and discussion

The following values of elastic constants are considered Chattopadhyay *et al*. (2002), Singh (2006), for mediums M and M_1 respectively.

$$\rho = 2660\text{kG}/\text{m}^3, \quad \lambda = 5.65\text{x}10^{10}\,\text{Nm}^{-2},$$

$$\mu_{\text{T}} = 2.46\text{x}10^{9}\,\text{Nm}^{-2}, \quad \mu_{\text{L}} = 5.66\text{x}10^{9}\,\text{Nm}^{-2}$$

$$\alpha = -1.28\text{x}10^{9}\,\text{Nm}^{-2}, \quad \beta = 220.90\text{x}10^{9}\,\text{Nm}^{-2},$$

$$\rho = 7800\text{kG}/\text{m}^3, \quad \lambda = 5.65\text{x}10^{10}\,\text{Nm}^{-2},$$

$$\mu_{\text{T}} = 2.46\text{x}10^{10}\,\text{Nm}^{-2}, \quad \mu_{\text{L}} = 5.66\text{x}10^{10}\,\text{Nm}^{-2}$$

$$\alpha = -1.28\text{x}10^{10}\,\text{Nm}^{-2}, \quad \beta = 220.90\text{x}10^{10}\,\text{Nm}^{-2}$$

$$T_0 = 293K, \quad \tau_0 = 0.1, \quad \nu_0 = 0.2$$

Taking into consideration Green-Lindsay theory, the numerical technique outlined above was used to obtain secular equation, surface wave velocity and attenuation coefficients under the effects of rotation in two models. For the sake of brevity, some computational results are being presented here. The variations are shown in Figs. 1-3 respectively.

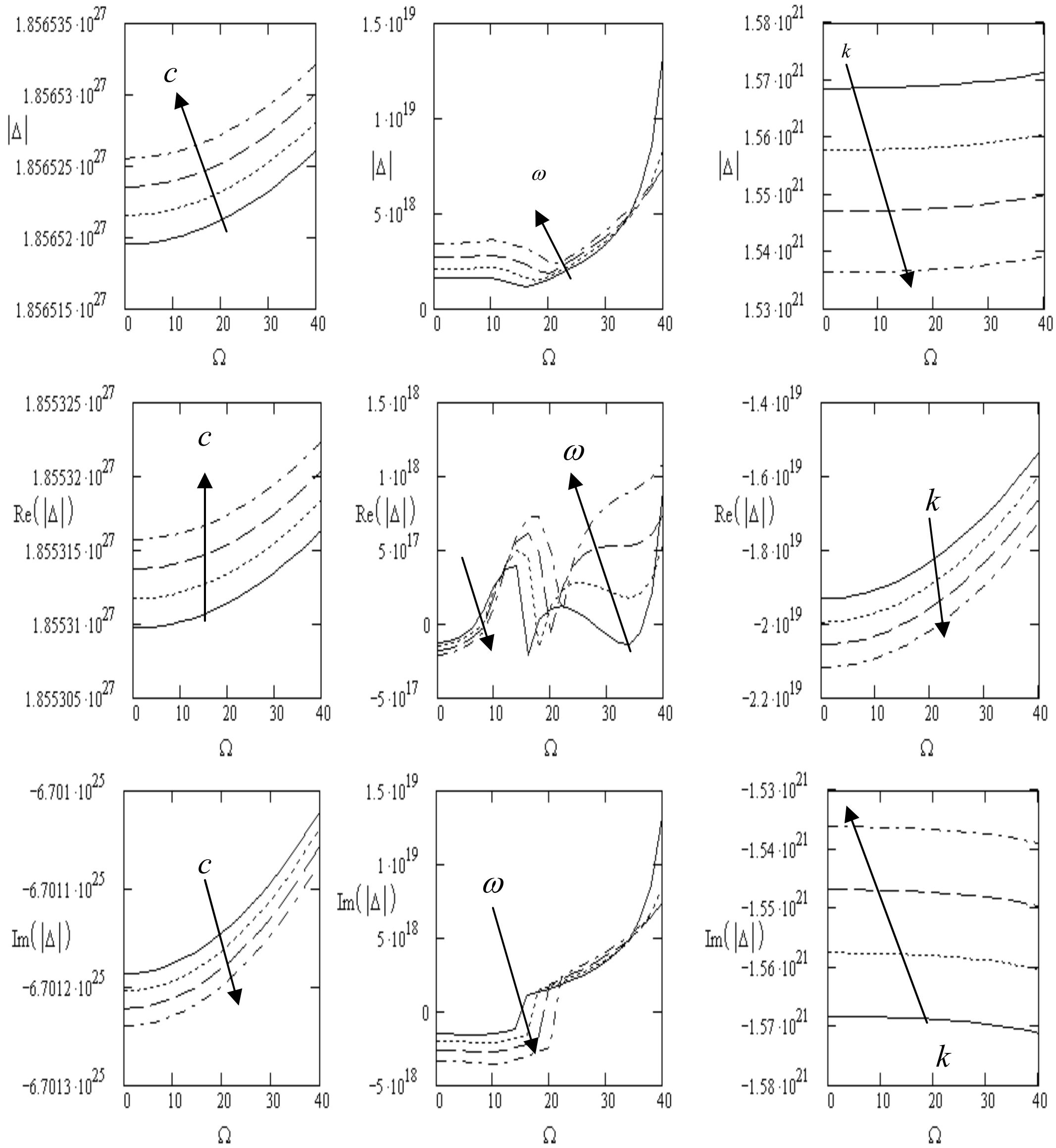

Fig. 2 Variation of the magnitude of the frequency equation |Δ|, velocity (Re(|Δ|)) and attenuation coefficient (Im(|Δ|)) for Love waves with respect to Ω with the variation of *c*, *ω* and *k*

Figs. (1(a)-1(i)) Show that the variation of the magnitude of the frequency equation |Δ|, Stoneley wave velocity Re(|Δ|) and attenuation coefficient Im(|Δ|) with respect to rotation Ω for different values of phase velocity *c*, frequency *ω* and higher order *k* of nth order including time rate of strain. The magnitude of the frequency equation decreases with increasing of rotation and frequency when effect of phase velocity and higher order, while it increases with increasing of rotation, as well secular equation increases with increasing of phase velocity and frequency, while it decreases with increasing of higher order, the Stoneley wave velocity decreases with increasing of rotation, while it increases with increasing of phase velocity, frequency and higher order, the attenuation coefficient decreases with increasing of rotation when effect of phase velocity and higher order, while it increases with increasing of rotation when effect of frequency, as well it increases with increasing of phase velocity, while it decreases with increasing of frequency and higher order.

Figs. 2(a)-2(i) show that the variation of the magnitude of the frequency equation |Δ|, Love wave velocity Re(|Δ|) and attenuation coefficient Im(|Δ|) with respect to rotation Ω for different values of phase velocity *c*, frequency *ω* and higher order *k* of nth order including time rate of strain. The the magnitude of the frequency equation increases with increasing of rotation, phase velocity and frequency, while it decreases with increasing of higher order, Love wave velocity increases with increasing of rotation, while it has oscillatory behavior with effect of frequency in the whole range of rotation, as well it increases with increasing of phase velocity and frequency, while it decreases with increasing of higher order, the attenuation coefficient increases with increasing of rotation when effect of phase velocity and frequency, while it increases with increasing of rotation when effect of wave number, as well it decreases with increasing of phase velocity and frequency, while it

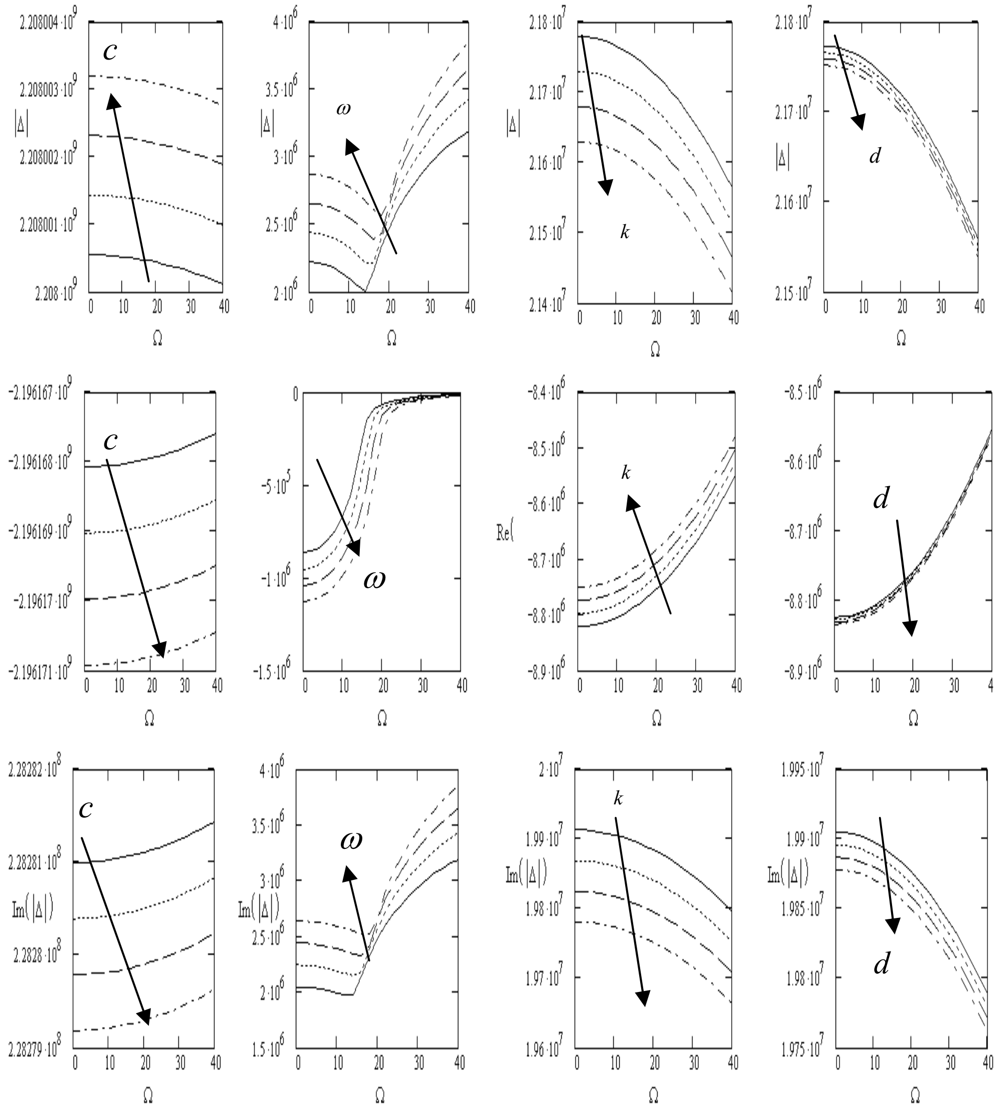

Fig. 3 Variation of the magnitude of the frequency equation $|\Delta|$, velocity (Re($|\Delta|$)) and attenuation coefficient (Im($|\Delta|$)) for Rayleigh waves with respect to Ω with variation of c, ω, k, and d

increases with increasing of higher order.

Figs. 3(a)-3(l) display the variation of the magnitude of the frequency equation $|\Delta|$, Rayleigh wave velocity Re($|\Delta|$) and attenuation coefficient Im($|\Delta|$) with respect to rotation Ω for different values of phase velocity c, frequencies ω, higher order k of nth order including time rate of strain and thickness d. The magnitude of the frequency equation decreases with increasing of rotation, while it has oscillatory behavior with effect of frequency in the whole range of rotation, as well it increases with increasing of phase velocity and frequency, while it decreases with increasing of higher order and thickness, the Rayleigh wave velocity increases with increasing of rotation, while it increases with increasing of phase velocity, frequency and thickness, while it increases with increasing of wave number, the attenuation coefficient increases with increasing of rotation, while it decreases with increasing of phase velocity, as well it decreases with increasing of rotation, higher order k and thickness, while it has oscillatory behavior with effect of frequency in the whole range of rotation.

7. Conclusions

The analysis of graphs permits us some concluding remarks

1. The surface waves in a homogeneous, anisotropic, fibre-reinforced viscoelastic solid media under the rotation and higher order k of nth order including time rate of strain are investigated.
2. Love waves do not depend on temperature; these are only affected by viscosity, rotation, frequency, higher order k of net order, including time rate of strain, phase velocity and thickness of the medium. In the absence of all fields, the dispersion equation is incomplete agreement with the corresponding classical result.
3. Rayleigh waves in a homogeneous, general thermo

viscoelastic solid medium of higher order, including time rate of change of strain we find that the wave velocity equation, proves that there is a dispersion of waves due to the presence of rotation, temperature, frequency, phase velocity and viscosity. The results are incomplete agreement with the corresponding classical results in the absence of all fields.

4. The wave velocity equation of Stoneley waves is very similar to the corresponding problem in the classical theory of elasticity. The dispersion of waves is due to the presence of rotation, phase velocity, frequency, temperature and viscosity of the solid. Also, wave velocity equation of this generalized type of surface waves is incomplete agreement with the corresponding classical result in the absence of all fields.

5. The results presented in this paper will be very helpful for researchers in geophysics, designers of new materials and the study of the phenomenon of rotation is also used to improve the conditions of oil extractions.

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