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On thermal stability of plates with functionally graded coefficient of thermal expansion

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Abstract. In this article, a four-variable refined plate theory is presented for buckling analysis of functionally graded plates subjected to uniform, linear and non-linear temperature rises across the thickness direction. The theory accounts for parabolic distribution of the transverse shear strains, and satisfies the zero traction boundary conditions on the surfaces of the plate without using shear correction factor. Young's modulus and Poisson ratio of the FGM plates are assumed to remain constant throughout the entire plate. However, the coefficient of thermal expansion of the FGM plate varies according to a power law form through the thickness coordinate. Equilibrium and stability equations are derived based on the present theory. The influences of many plate parameters on buckling temperature difference such ratio of thermal expansion, aspect ratio, side-to-thickness ratio and gradient index will be investigated.

Keywords: thermal properties; buckling; refined plate theory; functionally graded material; thermal expansion ratio; analytical modeling

1. Introduction

Functionally graded materials (FGMs) are a new class of composite structures that is of great interest for engineering design and manufacture. These kinds of materials possess desirable properties for specific applications, particularly for aircrafts, space vehicles, optical, biomechanical, electronic, chemical, mechanical, shipbuilding and other engineering structures under stress concentration, high thermal and residual stresses. In general, FGMs are both macroscopically and microscopically heterogeneous advanced composites which are made for example from a mixture of ceramics and metals with continuous composition gradation from pure ceramic on one surface to full metal on the other (Bouderba *et al.* 2013, Ould Larbi *et al.* 2013, Bousahla *et al.* 2014, Al-Basyouni *et al.* 2015, Belkorissat *et al.* 2015, Kar and Panda 2015,

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Pradhan and Chakraverty 2015, Bennai *et al.* 2015, Ebrahimi and Dashti 2015, Larbi Chaht *et al.* 2015, Bounouara *et al.* 2016, Ehyaei *et al.* 2016, Ahouel *et al.* 2016). This is achieved by gradually varying the volume fraction of the constituent materials.

Due to the importance and wide engineering applications of FGMs, the static, vibrational, thermomechanical and buckling analyses of FGM plate have been addressed by many investigators.

Javaheri and Eslami (2002) derived equilibrium equations of a rectangular FGP under thermal loads based on higher order theory. Na and Kim (2004) investigated the three dimensional thermomechanical buckling of an FGP composed of ceramic, FGM, and metal layers. The thermal buckling behaviors of FGM composite structures due to FGM thickness ratios, volume fraction distributions, and system geometric parameters were analyzed. Praveen and Reddy (1998) investigated the response of functionally graded ceramic-metal plate, using finite element method. Reddy (2000) presented solutions for rectangular functionally graded plates based on the third-order shear deformation plate theory. Najafizadeh and Eslami (2002) predicted the buckling analysis of clamped and simply supported circular FGM plate using classical plate theory (CPT). Latifi *et al*. (2013) studied the effect of various boundary conditions, using Fourier series expansion, on the buckling of thin rectangular functionally graded plates subjected to proportional biaxial compressive loadings based on classical plate theory. Yaghoobi and Yaghoobi (2013) proposed an analytical investigation on the buckling analysis of symmetric sandwich plates with FG face sheets resting on an elastic foundation based on the first-order shear deformation plate theory and subjected to mechanical, thermal and thermo-mechanical loads. Shen (2002) presented nonlinear bending analysis for a simply supported functionally graded rectangular plate subjected to a transverse uniform or sinusoidal load and in thermal environments based on Reddy's higher-order shear deformation plate theory. Matsunaga (2009) presented a higher order deformation theory for thermal buckling of FGPs. By using the method of power series expansion of displacement components, a set of fundamental equations of rectangular FGPs was derived. Akil (2014) presented a higher order theory for the buckling and post buckling behavior of sandwich beams having FG faces. Ait Amar Meziane *et al*. (2014) proposed an efficient and simple refined theory to investigate the buckling and free vibration responses of exponentially graded sandwich plates under various boundary conditions. Bouazza *et al*. (2016) presented an analytical solution to obtain the critical buckling temperature of cross-ply laminated plates with simply supported edge by using a refined hyperbolic shear deformation theory. Abdelhak *et al*. (2016) studied the buckling response of FG sandwich plates using a refined shear deformation theory. Abdelhak *et al*. (2015) presented a simple n-order four variable refined theory for buckling analysis of FG plates. Houari *et al*. (2013) developed a new higher-order shear and normal deformation theory for the thermo-elastic bending analysis of FGM sandwich plates. The same theory was used by Bessaim *et al*. (2013) for the static and free vibration analysis of FGM sandwich plates. Saidi *et al*. (2013) used the new hyperbolic shear deformation theory in which the stretching effect is included to investigate the thermo-mechanical bending response of FGM sandwich plates. Again, Kettaf *et al*. (2013) studied the thermal buckling behavior of FGM sandwich plates using the same model. Lanhe (2004) investigated the thermal buckling analysis of moderately thick functionally graded plates. Based on the Mindlin's plate theory, he obtained the critical buckling load for a simply supported rectangular plate subjected to two types of thermal loading, uniform temperature rise and gradient through the thickness. Shariat and Eslami (2005) studied the buckling analysis of thick functionally graded rectangular plates under different kinds of mechanical and thermal loads. They used third order shear deformation plate theory to obtain the closed form solution for the critical buckling loads of a simply supported rectangular plate. They reported that the plate under temperature variation across the plate thickness buckle at higher temperature in comparison with the uniform temperature rise and in the case of mechanical loads, the critical buckling mode varies with respect to the load ratio and/or the aspect ratio. Using a new four-variable refined plate theory, Bourada *et al*. (2012) investigated the thermal buckling response of sandwich FGM plates. Tounsi *et al*. (2013) presented a refined trigonometric shear deformable plate theory for thermoelastic bending of FGM sandwich plates. Bachir Bouiadjra *et al.* (2013) analysed the nonlinear thermal buckling behavior of FGM plates using an efficient sinusoidal shear deformation theory. Bachir Bouiadjra *et al*. (2012) studied the thermal buckling of FG plates based on the refined plate theory. Hadji *et al*. (2014) proposed a higher order shear deformation theory for static and free vibration of FG beam. Zidi *et al*. (2014) studied the bending response of FG plates under hygro-thermo-mechanical loading using a four variable refined plate theory. Hamidi *et al*. (2015) presented a sinusoidal plate theory with 5-unknowns and stretching effect for thermo-mechanical bending of FG sandwich plates. Belabed *et al*. (2014) presented an efficient and simple higher order shear and normal deformation theory for FG plates. Mahi *et al*. (2015) developed a new hyperbolic shear deformation theory for bending and free vibration analysis of isotropic, functionally graded, sandwich and laminated composite plates. Ait Atmane *et al*. (2015) presented a computational shear displacement model for vibrational analysis of FG beams with porosities. Ait Yahia *et al*. (2015) discussed the wave propagation in FG plates with porosities using various higher-order shear deformation plate theories. Also, Akbaş (2015) studied the wave propagation of a FG beam in thermal environments. Arefi (2015) given an elastic solution of a curved beam made of FGMs with different cross sections. Attia *et al*. (2015) examined the free vibration analysis of FG plates with temperature-dependent properties using various four variable refined plate theories. Bakora and Tounsi (2015) investigated the thermo-mechanical post-buckling behavior of thick FG plates resting on elastic foundations. Tebboune *et al*. (2015) investigated the thermal buckling analysis of FG plates resting on elastic foundation based on an efficient and simple trigonometric shear deformation theory. Bellifa *et al*. (2016) presented the bending and free vibration analysis of FG plates using a simple shear deformation theory and the concept the neutral surface position. Bennoun *et al*. (2016) proposed a novel five variable refined plate theory for vibration analysis of FG sandwich plates. Bourada *et al*. (2015) developed a novel simple shear and normal deformations theory for FG beams. Bouderba *et al*. (2016) investigated the thermal stability of FG sandwich plates using a simple shear deformation theory. Chikh *et al*. (2016) analyzed the thermo-mechanical post-buckling of symmetric S-FGM plates resting on Pasternak elastic foundations using hyperbolic shear deformation theory. El-Hassar *et al*. (2016) studied the thermal stability analysis of solar FG plates on elastic foundation using an efficient hyperbolic shear deformation theory. Tounsi *et al*. (2016) proposed for the first time a new 3-unknowns non-polynomial plate theory for buckling and vibration of FG sandwich plate.

The present article develops the thermal buckling of a new class of functionally graded rectangular plates. The FG plate is assumed to have constant Young's modulus and Poisson's ratio; however, the coefficients of thermal expansion of the FGM plates is assumed to vary continuously through the thickness, according to a simple power law distribution of the volume fraction of the constituents. Hence, the novelty of this work is to investigate for the first time the thermal buckling of plate with functionally graded coefficient of thermal expansion and keeping the other mechanical properties such as Young's modulus and Poisson's ratio constant. The theory presented is variationally consistent, does not require shear correction factor, and gives rise to transverse shear stress variation such that the transverse shear stresses vary parabolically across

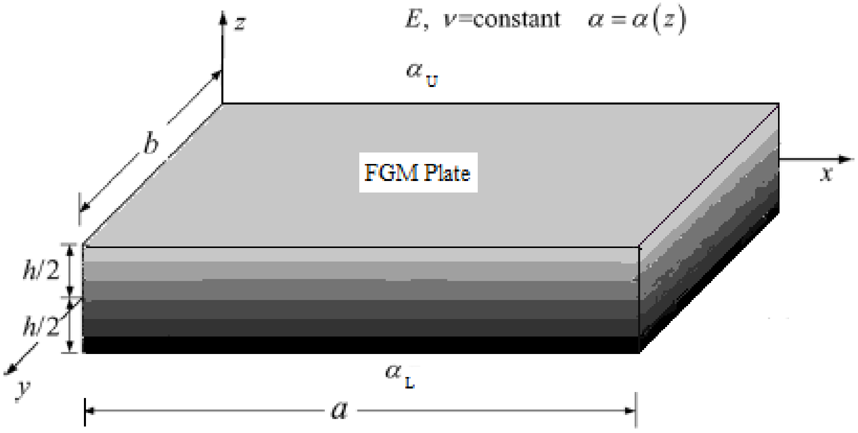


Fig. 1 Geometry of the FGM plate having constant Young's modulus and Poisson's ratio but functionally graded coefficient of thermal expansion

the thickness satisfying shear stress free surface conditions. Unlike any other theory, the number of unknown functions involved is only four, as against five in case of other shear deformation theories. The thermal loads are assumed as uniform, linear and non-linear temperature rises across the thickness direction. Illustrative examples are given so as to demonstrate the efficacies of the present model. The effects of various variables, such as thickness and aspect ratios, gradient index, loading on the critical buckling temperature difference are all discussed.

2. Problem formulation

Consider a rectangular plate made of FGMs of thickness *h*, length *a*, and width *b* made by mixing two distinct materials (metal and ceramic) is studied here. The coordinates *x*, *y* are along the in-plane directions and *z* is along the thickness direction (Fig. 1).

The properties of FGM vary continuously due to gradually changing the volume fraction of the constituent materials, usually in the thickness direction only. Power-law function is commonly used to describe these variations of materials properties. However, the material properties of the FGM plate are assumed as follows. The Young's modulus and Poisson's ratio are assumed to be constant and the coefficient of thermal expansion of the FGM plate is assumed to vary continuously through the thickness as.

$$\alpha(z) = \alpha_L + \alpha_{UL} V(z)^k \tag{1a}$$

$$\alpha_{UL} = \alpha_U - \alpha_L \tag{1b}$$

$$V(z) = \left(\frac{z}{h} + \frac{1}{2}\right)^k \tag{2}$$

α_L and α_U are the coefficient of thermal expansion at the bottom and the top of the FG plate, respectively. *k* is the material parameter.

Unlike the other shear deformation theory, just four unknowns functions are needed in the proposed hyperbolic shear deformation theory.

2.1 Basic assumptions

Assumptions of the present theory are as follows (Benachour *et al.* 2011, Nedri *et al.* 2014, Draiche *et al.* 2014, Nguyen *et al.* 2015, Sallai *et al.* 2015, Bouchafa *et al.* 2015, Hadji *et al.* 2015, Meradjah *et al.* 2015, Ould Youcef *et al.* 2015, Boukhari *et al.* 2016, Barati *et al.* 2016):

(i) The displacements are small in comparison with the plate thickness and, therefore, strains involved are infinitesimal.

(ii) The transverse displacement w includes two components of bending w_b, and shear w_s. These components are functions of coordinates x, y only.

$$w(x,y,z)=w_b(x,y)+w_s(x,y) \tag{3}$$

(iii) The transverse normal stress σ_z is negligible in comparison with in-plane stresses σ_x and σ_y.

(iv) The displacements u in x-direction and v in y-direction consist of extension, bending, and shear components.

$$u=u_0+u_b+u_s\,,\quad v=v_0+v_b+v_s \tag{4}$$

The bending components u_b and v_b are assumed to be similar to the displacements given by the classical plate theory. Therefore, the expression for u_b and v_b can be given as

$$u_b=-z\,\frac{\partial w_b}{\partial x}\,,\quad v_b=-z\,\frac{\partial w_b}{\partial y} \tag{5}$$

The shear components u_s and v_s give rise, in conjunction with w_s, to the parabolic variations of shear strains γ_{xz}, γ_{yz} and hence to shear stresses τ_{xz}, τ_{yz} through the thickness of the plate in such a way that shear stresses τ_{xz}, τ_{yz} are zero at the top and bottom faces of the plate. Consequently, the expression for u_s and v_s can be given as

$$u_s=-f(z)\frac{\partial w_s}{\partial x}\,,\quad v_s=-f(z)\frac{\partial w_s}{\partial y} \tag{6}$$

where $f(z)$ is given by (Hebali *et al.* 2014)

$$f(z)=\frac{(h/\pi)\sinh\left(\dfrac{\pi}{h}z\right)-z}{[\cosh(\pi/2)-1]} \tag{7}$$

2.2 Kinematics

Based on the assumptions made in the preceding section, the displacement field can be obtained using Eqs. (3)-(7) as

$$u(x,y,z)=u_0(x,y)-z\frac{\partial w_b}{\partial x}-f(z)\frac{\partial w_s}{\partial x} \tag{8a}$$

$$v(x,y,z)=v_0(x,y)-z\frac{\partial w_b}{\partial y}-f(z)\frac{\partial w_s}{\partial y} \tag{8b}$$

$$w(x,y,z)=w_b(x,y)+w_s(x,y) \tag{8c}$$

The non-linear von Karman strain-displacement equations are as follows

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}+z\begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix}+f(z)\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix},\quad \begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}=g(z)\begin{Bmatrix}\gamma_{yz}^s\\ \gamma_{xz}^s\end{Bmatrix} \tag{9}$$

where

$$\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}=\begin{Bmatrix}\dfrac{\partial u_0}{\partial x}+\dfrac{1}{2}\left(\dfrac{\partial w_b}{\partial x}+\dfrac{\partial w_s}{\partial x}\right)^2\\ \dfrac{\partial v_0}{\partial y}+\dfrac{1}{2}\left(\dfrac{\partial w_b}{\partial y}+\dfrac{\partial w_s}{\partial y}\right)^2\\ \dfrac{\partial u_0}{\partial y}+\dfrac{\partial v_0}{\partial x}+\left(\dfrac{\partial w_b}{\partial x}+\dfrac{\partial w_s}{\partial x}\right)\left(\dfrac{\partial w_b}{\partial y}+\dfrac{\partial w_s}{\partial y}\right)\end{Bmatrix},\quad \begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix}=\begin{Bmatrix}-\dfrac{\partial^2 w_b}{\partial x^2}\\ -\dfrac{\partial^2 w_b}{\partial y^2}\\ -2\dfrac{\partial^2 w_b}{\partial x\partial y}\end{Bmatrix},$$

$$\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix}=\begin{Bmatrix}-\dfrac{\partial^2 w_s}{\partial x^2}\\ -\dfrac{\partial^2 w_s}{\partial y^2}\\ -2\dfrac{\partial^2 w_s}{\partial x\partial y}\end{Bmatrix},\quad \begin{Bmatrix}\gamma_{yz}^s\\ \gamma_{xz}^s\end{Bmatrix}=\begin{Bmatrix}\dfrac{\partial w_s}{\partial y}\\ \dfrac{\partial w_s}{\partial x}\end{Bmatrix} \tag{10a}$$

and

$$g(z)=1-\frac{df(z)}{dz} \tag{10b}$$

2.3 Constitutive relations

The plate is subjected to a thermal load $T(x,y,z)$. The linear constitutive relations are

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}=\begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_x-\alpha T\\ \varepsilon_y-\alpha T\\ \gamma_{xy}\end{Bmatrix}\ \text{and}\ \begin{Bmatrix}\tau_{yz}\\ \tau_{zx}\end{Bmatrix}=\begin{bmatrix}Q_{44} & 0\\ 0 & Q_{55}\end{bmatrix}\begin{Bmatrix}\gamma_{yz}\\ \gamma_{zx}\end{Bmatrix} \tag{11}$$

where (σ_x, σ_y, τ_{xy}, τ_{yz}, τ_{yx}) and (ε_x, ε_y, γ_{xy}, γ_{yz}, γ_{yx}) are the stress and strain components, respectively. The stiffness coefficients, Q_{ij}, are expressed by

$$Q_{11} = Q_{22} = \frac{E}{1-\nu^2}, \tag{12a}$$

$$Q_{12} = \frac{\nu E}{1-\nu^2}, \tag{12b}$$

$$Q_{44} = Q_{55} = Q_{66} = \frac{E}{2(1+\nu)}, \tag{12c}$$

2.4 Stability equations

The total potential energy of the FG plate may be written as

$$U = \frac{1}{2}\iiint\left[\sigma_x(\varepsilon_x - \alpha T) + \sigma_y(\varepsilon_y - \alpha T) + \tau_{xy}\gamma_{xy} + \tau_{yz}\gamma_{yz} + \tau_{xz}\gamma_{xz}\right]dzdydx, \tag{13}$$

The principle of virtual work for the present problem may be expressed as follows

$$\begin{aligned}\iint\Big[&N_x\delta\varepsilon_x^0 + N_y\delta\varepsilon_y^0 + N_{xy}\delta\gamma_{xy}^0 + M_x^b\delta k_x^b + M_y^b\delta k_y^b + M_{xy}^b\delta k_{xy}^b + M_x^s\delta k_x^s \\ &+ M_y^s\delta k_y^s + M_{xy}^s\delta k_{xy}^s + S_{yz}^s\delta\gamma_{yz}^s + S_{xz}^s\delta\gamma_{xz}^s\Big]dxdy = 0\end{aligned} \tag{14}$$

where

$$\begin{Bmatrix} N_x, & N_y, & N_{xy} \\ M_x^b, & M_y^b, & M_{xy}^b \\ M_x^s, & M_y^s, & M_{xy}^s \end{Bmatrix} = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \tau_{xy}) \begin{Bmatrix} 1 \\ z \\ f(z) \end{Bmatrix} dz, \tag{15a}$$

$$\left(S_{xz}^s, S_{yz}^s\right) = \int_{-h/2}^{h/2} (\tau_{xz}, \tau_{yz})\, g(z)dz. \tag{15b}$$

Using Eq. (11) in Eq. (15), the stress resultants of the FG plate can be related to the total strains by

$$\begin{Bmatrix} N \\ M^b \\ M^s \end{Bmatrix} = \begin{bmatrix} A & 0 & B^s \\ 0 & D & D^s \\ B^s & D^s & H^s \end{bmatrix} \begin{Bmatrix} \varepsilon \\ k^b \\ k^s \end{Bmatrix} - \begin{Bmatrix} N^T \\ M^{bT} \\ M^{sT} \end{Bmatrix}, \quad S = A^s\gamma \tag{16}$$

where

$$N = \{N_x, N_y, N_{xy}\}^t, \quad M^b = \{M_x^b, M_y^b, M_{xy}^b\}^t, \quad M^s = \{M_x^s, M_y^s, M_{xy}^s\}^t \tag{17a}$$

$$N^T = \left\{N_x^T, N_y^T, 0\right\}^t, \quad M^{bT} = \left\{M_x^{bT}, M_y^{bT}, 0\right\}^t, \quad M^{sT} = \left\{M_x^{sT}, M_y^{sT}, 0\right\}^t \tag{17b}$$

$$\varepsilon = \left\{\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0\right\}^t, \quad k^b = \left\{k_x^b, k_y^b, k_{xy}^b\right\}^t, \quad k^s = \left\{k_x^s, k_y^s, k_{xy}^s\right\}^t \tag{17c}$$

$$A = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}, \quad D = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \tag{17d}$$

$$B^s = \begin{bmatrix} B_{11}^s & B_{12}^s & 0 \\ B_{12}^s & B_{22}^s & 0 \\ 0 & 0 & B_{66}^s \end{bmatrix}, \quad D^s = \begin{bmatrix} D_{11}^s & D_{12}^s & 0 \\ D_{12}^s & D_{22}^s & 0 \\ 0 & 0 & D_{66}^s \end{bmatrix}, \quad H^s = \begin{bmatrix} H_{11}^s & H_{12}^s & 0 \\ H_{12}^s & H_{22}^s & 0 \\ 0 & 0 & H_{66}^s \end{bmatrix} \tag{17e}$$

$$S = \left\{S_{yz}^s, S_{xz}^s\right\}^t, \quad \gamma = \left\{\gamma_{yz}, \gamma_{xz}\right\}^t, \quad A^s = \begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix} \tag{17f}$$

where A_{ij}, D_{ij}, etc., are the plate stiffness, defined by

$$\begin{Bmatrix} A_{11} & D_{11} & B_{11}^s & D_{11}^s & H_{11}^s \\ A_{12} & D_{12} & B_{12}^s & D_{12}^s & H_{12}^s \\ A_{66} & D_{66} & B_{66}^s & D_{66}^s & H_{66}^s \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{11}\left(1, z^2, f(z), z\, f(z), f^2(z)\right) \begin{Bmatrix} 1 \\ \nu \\ \dfrac{1-\nu}{2} \end{Bmatrix} dz \tag{18a}$$

and

$$\left(A_{22}, D_{22}, B_{22}^s, D_{22}^s, H_{22}^s\right) = \left(A_{11}, D_{11}, B_{11}^s, D_{11}^s, H_{11}^s\right) \tag{18b}$$

$$A_{44}^s = A_{55}^s = \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{E(z)}{2(1+\nu)} \left[g(z)\right]^2 dz \;, \tag{18c}$$

The stress and moment resultants, $N_x^T = N_y^T$, $M_x^{bT} = M_y^{bT}$, and $M_x^{sT} = M_y^{sT}$ due to thermal loading are defined by

$$\begin{Bmatrix} N_x^T \\ M_x^{bT} \\ M_x^{sT} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{E(z)}{1-\nu} \alpha(z) T \begin{Bmatrix} 1 \\ z \\ f(z) \end{Bmatrix} dz, \tag{19}$$

The stability equations of the plate may be derived by the adjacent equilibrium criterion. Assume that the equilibrium state of the FG plate under thermal loads is defined in terms of the displacement components (u_0^0, v_0^0, w_b^0, w_s^0). The displacement components of a neighboring stable state differ by (u_0^1, v_0^1, w_b^1, w_s^1) with respect to the equilibrium position. Thus, the total displacements of a neighboring state are

$$u_0 = u_0^0 + u_0^1,\quad v_0 = v_0^0 + v_0^1,\quad w_b = w_b^0 + w_b^1,\quad w_s = w_s^0 + w_s^1 \tag{20}$$

where the superscript 1 refers to the state of stability and the superscript 0 refers to the state of equilibrium conditions.

Substituting Eqs. (9) and (20) into Eq. (14) and integrating by parts and then equating the coefficients of δu_0^1, δv_0^1, δw_b^1 and δw_s^1 to zero, separately, the governing stability equations are obtained for the shear deformation plate theories as

$$\begin{aligned}
&\frac{\partial N_x^1}{\partial x} + \frac{\partial N_{xy}^1}{\partial y} = 0\\
&\frac{\partial N_{xy}^1}{\partial x} + \frac{\partial N_y^1}{\partial y} = 0\\
&\frac{\partial^2 M_x^{b1}}{\partial x^2} + 2\frac{\partial^2 M_{xy}^{b1}}{\partial x \partial y} + \frac{\partial^2 M_y^{b1}}{\partial y^2} + \overline{N} = 0\\
&\frac{\partial^2 M_x^{s1}}{\partial x^2} + 2\frac{\partial^2 M_{xy}^{s1}}{\partial x \partial y} + \frac{\partial^2 M_y^{s1}}{\partial y^2} + \frac{\partial S_{xz}^{s1}}{\partial x} + \frac{\partial S_{yz}^{s1}}{\partial y} + \overline{N} = 0
\end{aligned} \tag{21}$$

with

$$\overline{N} = \left[N_x^0 \frac{\partial^2 \left(w_b^1 + w_s^1\right)}{\partial x^2} + N_y^0 \frac{\partial^2 \left(w_b^1 + w_s^1\right)}{\partial y^2} \right] \tag{22}$$

where the terms N_x^0 and N_y^0 are the pre-buckling force resultants obtained as

$$N_x^0 = N_y^0 = -\int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{\alpha(z)E(z)T}{1-\nu}\,dz, \tag{23}$$

The stability equations in terms of the displacement components may be obtained by substituting Eq. (16) into Eq. (21). Resulting equations are four stability equations based on the present refined shear deformation theory for FG plates.

$$A_{11}\frac{\partial^2 u_0^1}{\partial x^2} + A_{66}\frac{\partial^2 u_0^1}{\partial y^2} + \left(A_{12} + A_{66}\right)\frac{\partial^2 v_0^1}{\partial x \partial y} - B_{11}^s\frac{\partial^3 w_s^1}{\partial x^3} - \left(B_{12}^s + 2B_{66}^s\right)\frac{\partial^3 w_s^1}{\partial x \partial y^2} = 0 \tag{24a}$$

$$\left(A_{12} + A_{66}\right)\frac{\partial^2 u_0^1}{\partial x \partial y} + A_{66}\frac{\partial^2 v_0^1}{\partial x^2} + A_{22}\frac{\partial^2 v_0^1}{\partial y^2} - B_{22}^s\frac{\partial^3 w_s^1}{\partial y^3} - \left(B_{12}^s + 2B_{66}^s\right)\frac{\partial^3 w_s^1}{\partial x^2 \partial y} = 0 \tag{24b}$$

$$-D_{11}\frac{\partial^4 w_b^1}{\partial x^4}-2(D_{12}+2D_{66})\frac{\partial^4 w_b^1}{\partial x^2\partial y^2}-D_{22}\frac{\partial^4 w_b^1}{\partial y^4}-D_{11}^s\frac{\partial^4 w_s^1}{\partial x^4}$$
$$-2(D_{12}^s+2D_{66}^s)\frac{\partial^4 w_s^1}{\partial x^2\partial y^2}-D_{22}^s\frac{\partial^4 w_s^1}{\partial y^4}+\overline{N}^1=0 \tag{24c}$$

$$B_{11}^s\frac{\partial^3 u_0^1}{\partial x^3}+(B_{12}^s+2B_{66}^s)\frac{\partial^3 u_0^1}{\partial x\partial y^2}+(B_{12}^s+2B_{66}^s)\frac{\partial^3 v_0^1}{\partial x^2\partial y}+B_{22}^s\frac{\partial^3 v_0^1}{\partial y^3}-D_{11}^s\frac{\partial^4 w_b^1}{\partial x^4}-2(D_{12}^s+2D_{66}^s)\frac{\partial^4 w_b^1}{\partial x^2\partial y^2}$$
$$-D_{22}^s\frac{\partial^4 w_b^1}{\partial y^4}-H_{11}^s\frac{\partial^4 w_s^1}{\partial x^4}-2(H_{12}^s+2H_{66}^s)\frac{\partial^4 w_s^1}{\partial x^2\partial y^2}-H_{22}^s\frac{\partial^4 w_s^1}{\partial y^4}+A_{55}^s\frac{\partial^2 w_s^1}{\partial x^2}+A_{44}^s\frac{\partial^2 w_s^1}{\partial y^2}+\overline{N}^1=0 \tag{24d}$$

2.5 Trigonometric solution to thermal buckling

Rectangular plates are generally classified in accordance with the type of support used. We are here concerned with the exact solution of Eq. (21) for a simply supported FGM plate. The following boundary conditions are imposed for the present refined shear deformation theory at the side edges

$$v_0^1=w_b^1=w_s^1=\frac{\partial w_s^1}{\partial y}=N_x^1=M_x^{b1}=M_x^{s1}=0 \text{ at } x=0,\ a, \tag{25a}$$

$$u_0^1=w_b^1=w_s^1=\frac{\partial w_s^1}{\partial x}=N_y^1=M_y^{b1}=M_y^{s1}=0 \text{ at } y=0,\ b. \tag{25b}$$

The following approximate solution is seen to satisfy both the differential equation and the boundary conditions

$$\begin{Bmatrix} u_0^1 \\ v_0^1 \\ w_b^1 \\ w_s^1 \end{Bmatrix}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\begin{Bmatrix} U_{mn}^1\cos(\lambda\, x)\sin(\mu\, y) \\ V_{mn}^1\sin(\lambda\, x)\cos(\mu\, y) \\ W_{bmn}^1\sin(\lambda\, x)\sin(\mu\, y) \\ W_{smn}^1\sin(\lambda\, x)\sin(\mu\, y) \end{Bmatrix} \tag{26}$$

where U_{mn}^1, V_{mn}^1, W_{bmn}^1, and W_{smn}^1 are arbitrary parameters to be determined and $\lambda=m\pi/a$ and $\mu=n\pi/b$. Substituting Eq. (26) into Eq. (24), one obtains

$$[K]\{\Delta\}=0, \tag{27}$$

where $\{\Delta\}$ denotes the column

$$\{\Delta\}=\{U_{mn}^1,V_{mn}^1,W_{bmn}^1,W_{smn}^1\}^t \tag{28}$$

and [*K*] is the symmetric matrix given by

$$[K]=\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{12} & a_{22} & a_{23} & a_{24} \\ a_{13} & a_{23} & a_{33} & a_{34} \\ a_{14} & a_{24} & a_{34} & a_{44} \end{bmatrix}, \tag{29}$$

in which

$$\begin{aligned}
a_{11} &= -\left(A_{11}\lambda^2 + A_{66}\mu^2\right) \\
a_{12} &= -\lambda\,\mu\left(A_{12} + A_{66}\right) \\
a_{13} &= 0 \\
a_{14} &= \lambda\,[B_{11}^s\lambda^2 + (B_{12}^s + 2B_{66}^s)\,\mu^2\,] \\
a_{22} &= -\left(A_{66}\lambda^2 + A_{22}\mu^2\right) \\
a_{23} &= 0 \\
a_{24} &= \mu\,[(B_{12}^s + 2B_{66}^s)\,\lambda^2 + B_{22}^s\mu^2\,] \\
a_{33} &= -\left(D_{11}\lambda^4 + 2(D_{12} + 2D_{66})\lambda^2\mu^2 + D_{22}\mu^4 + N_x^0\lambda^2 + N_y^0\mu^2\right) \\
a_{34} &= -\left(D_{11}^s\lambda^4 + 2(D_{12}^s + 2D_{66}^s)\lambda^2\,\mu^2 + D_{22}^s\,\mu^4 + N_x^0\lambda^2 + N_y^0\mu^2\right) \\
a_{44} &= -\left(H_{11}^s\lambda^4 + 2(H_{12}^s + 2H_{66}^s)\lambda^2\mu^2 + H_{22}^s\mu^4 + A_{55}^s\lambda^2 + A_{44}^s\mu^2 + N_x^0\lambda^2 + N_y^0\mu^2\right)
\end{aligned} \tag{30}$$

By applying the static condensation approach to eliminate the coefficients associated with the in-plane displacements, Eq. (27) can be rewritten as

$$\begin{bmatrix} [K^{11}] & [K^{12}] \\ [K^{12}]^T & [K^{22}] \end{bmatrix} \begin{Bmatrix} \Delta^1 \\ \Delta^2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{31}$$

where

$$[K^{11}]=\begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix},\ [K^{12}]=\begin{bmatrix} 0 & a_{14} \\ 0 & a_{24} \end{bmatrix},\ [K^{22}]=\begin{bmatrix} a_{33} & a_{34} \\ a_{34} & a_{44} \end{bmatrix} \tag{32a}$$

$$\Delta^1 = \begin{Bmatrix} U_{mn}^1 \\ V_{mn}^1 \end{Bmatrix},\ \Delta^2 = \begin{Bmatrix} W_{bmn}^1 \\ W_{smn}^1 \end{Bmatrix} \tag{32b}$$

Eq. (31) represents a pair of two matrix equations

$$[K^{11}]\Delta^1 + [K^{12}]\Delta^2 = 0 \tag{33a}$$

$$[K^{12}]^T\,\Delta^1 + [K^{22}]\Delta^2 = 0 \tag{33b}$$

Solving Eq. (33a) for Δ^1 and then substituting the result into Eq. (33b), the following equation is obtained

$$\left[\overline{K}^{22}\right]\Delta^2 = 0 \tag{34}$$

where

$$\left[\overline{K}^{22}\right] = \left[K^{22}\right] - \left[K^{12}\right]^T \left[K^{11}\right]^{-1} \left[K^{12}\right] = \begin{bmatrix} \overline{a}_{33} & \overline{a}_{34} \\ \overline{a}_{43} & b_{44} \end{bmatrix} \tag{35a}$$

and

$$\overline{a}_{33} = a_{33}, \quad \overline{a}_{34} = a_{34}$$

$$\overline{a}_{43} = a_{34}, \quad b_{44} = a_{44} - a_{14}\frac{b_1}{b_0} - a_{24}\frac{b_2}{b_0}$$

$$b_0 = a_{11}a_{22} - a_{12}^2, \quad b_1 = a_{14}a_{22} - a_{12}a_{24}, \quad b_2 = a_{11}a_{24} - a_{12}a_{14} \tag{35b}$$

For nontrivial solution, the determinant of the coefficient matrix in Eq. (34) must be zero. This gives the following expression for the thermal buckling load

$$N_x^0 = N_y^0 = \frac{1}{\lambda^2 + \mu^2}\frac{a_{33}b_{44} - a_{34}^2}{a_{33} + a_{44} - 2a_{34}} \tag{36}$$

2.5.1 Buckling of FG plates under uniform temperature rise

The plate initial temperature is assumed to be T_i. The temperature is uniformly raised to a final value T_f in which the plate buckles. The temperature change is $\Delta T = T_f - T_i$. Using this distribution of temperature, the critical buckling temperature change ΔT_{cr} becomes b5 using Eqs. (22) and (36)

$$\Delta T_{cr} = \frac{1}{\overline{\beta}_1\left(\lambda^2 + \mu^2\right)}\frac{a_{33}b_{44} - a_{34}^2}{a_{33} + a_{44} - 2a_{34}} \tag{37a}$$

where

$$\overline{\beta}_1 = -\int_{-h/2}^{h/2}\frac{\alpha(z\ )E(z)}{1-\nu}dz. \tag{37b}$$

2.5.2 Buckling of FG plates under linear temperature rise

For FG plates, the temperature change is not uniform. The temperature is assumed to be varied linearly through the thickness as follows

$$T(z) = \Delta T\left(\frac{z}{h} + \frac{1}{2}\right) + T_M, \tag{38}$$

where the buckling temperature difference $\Delta T = T_C - T_M$ and T_C and T_M are the temperature of the top surface which is ceramic-rich and the bottom surface which is metal-rich, respectively.

Similar to the previous loading case, the critical buckling temperature difference ΔT_{cr} can be determined as

$$\Delta T_{cr} = \frac{a_{33}b_{44} - a_{34}^2 + T_M \overline{\beta}_1\left(\lambda^2 + \mu^2\right)\left(a_{33} + a_{44} - 2a_{34}\right)}{\overline{\beta}_2(\lambda^2 + \mu^2)(a_{33} + a_{44} - 2a_{34})} \tag{39a}$$

where

$$\overline{\beta}_2 = -\int_{-h/2}^{h/2} \frac{\alpha(z)E(z)}{1-\nu}\left(\frac{z}{h} + \frac{1}{2}\right)dz \tag{39b}$$

2.5.3 Buckling of FG plates subjected to graded temperature change across the thickness

We assume that the temperature of the top surface is T_M and the temperature varies from T_M, according to the power law variation through-the-thickness, to the bottom surface temperature T_M in which the plate buckles. In this case, the temperature through-the-thickness is given by

$$T(z) = \Delta T\left(\frac{z}{h} + \frac{1}{2}\right)^{\gamma} + T_M\,, \tag{40}$$

where the buckling temperature difference $\Delta T = T_C - T_M$ and γ is the temperature exponent ($0<\gamma<\infty$). Note that the value of γ equal to unity represents a linear temperature change across the thickness. While the value of γ excluding unity represents a non-linear temperature change through-the-thickness.

Similar to the previous loading case, the critical buckling temperature change ΔT_{cr} becomes by using Eqs. (23) and (36)

$$\Delta T_{cr} = \frac{a_{33}b_{44} - a_{34}^2 + T_M \overline{\beta}_1\left(\lambda^2 + \mu^2\right)\left(a_{33} + a_{44} - 2a_{34}\right)}{\overline{\beta}_3(\lambda^2 + \mu^2)(a_{33} + a_{44} - 2a_{34})} \tag{41a}$$

where

$$\overline{\beta}_3 = -\int_{-h/2}^{h/2} \frac{\alpha(z\ )E(z)}{1-\nu}\left(\frac{z}{h} + \frac{1}{2}\right)^{\gamma} dz \tag{41b}$$

3. Results and discussion

3.1 Comparative studies

In order to prove the validity of the present hyperbolic shear deformation theory, results were obtained for isotropic plates (k=0) and compared with the existing ones in the literature.
The material properties used in the present study are:

Ceramic E_U=380 GPa , α_U=7.410^{-6}/C

Metal E_L=70 Gpa, α_L=2310^{-6}/C.

Table 1 Critical buckling temperature change T_{cr} of FGM plate under *uniform temperature* rise for different values of aspect ratio a / b. (k=0) (a=100 h)

Theory	a/b=1	2	3	4	5
SPT	17.0894	42.6876	85.2554	144.6500	220.6729
HPT	17.0894	42.6875	85.2551	144.6490	220.6706
FPT	17.0894	42.6875	85.2551	144.6489	220.6704
CPT	17.0991	42.7477	85.4955	145.3424	222.2883
present	17.0894	42.6876	85.2553	144.6496	220.6721

Table 2 Critical buckling temperature change T_{cr} of FGM plate under *linear temperature* rise for different values of aspect ratio a / b. (k=0) (a=100 h)

Theory	a/b=1	2	3	4	5
SPT	24.1789	75.3753	160.5109	279.3000	431.3459
HPT	24.1789	75.3751	160.5102	279.2980	431.3412
FPT	24.1789	75.3751	160.5102	279.2979	431.3409
CPT	24.1982	75.4955	160.9910	280.6848	434.5767
present	24.1789	75.3752	160.5107	279.2993	431.3442

The correlation between the present four variable plate theory and different higher-order (HPT and SPT) and first-order shear deformation theories (FPT) and classical plate theory (CPT) is illustrated in Tables 1 and 2. Theses tables give the effects of aspect ratio a/b on critical buckling temperature change T_{cr} of isotropic plate under uniform and linear temperature rise across thickness respectively.

From the results presented in Tables 1 and 2, it is observed that results have a good agreement. In considering the results presented in Tables 1 to 2, it should be noted that the quantity of unknown variables in the present formulation is four, whereas the number unknown function in HPT, SPT and FPT is five. It can be concluded that the present theory is not only accurate but also comparatively simple and quite elegant in predicting the thermal buckling response of FGM plates.

3.2 Parametric investigations

In this parametric study, the general approach outlined in the previous sections for thermal buckling of FGM plates has been illustrated through numerical examples. For this, we consider a FGM plate with the following properties:

The Poisson's ratio and Young's modulus are taken as ν=0,3 and E=205, 8 GPa, respectively. The coefficient of thermal expansion at the bottom (z=-$h/2$) of the FGM plate is $\alpha_L=10^{-5}$/°C and that at the top of the plate varies according to the ratio α_U / α_L, whereas the coefficient of thermal expansion of the FGM plate is assumed to vary continuously through the thickness (Yen-Ling C and Hao-Xuan C 2008).

Fig. 2 shows the variation of the critical buckling temperature difference t_{cr} of FG plate (k=2 and a=10h) versus the aspect ratio a/b under uniform, linear and non linear temperature rise across the thickness for different values of α_U / α_L ratio.

It is noted from these figures that the critical buckling temperature increases with the increase of the aspect ratio a/b and this report for the three thermal loading. In addition, the highest critical

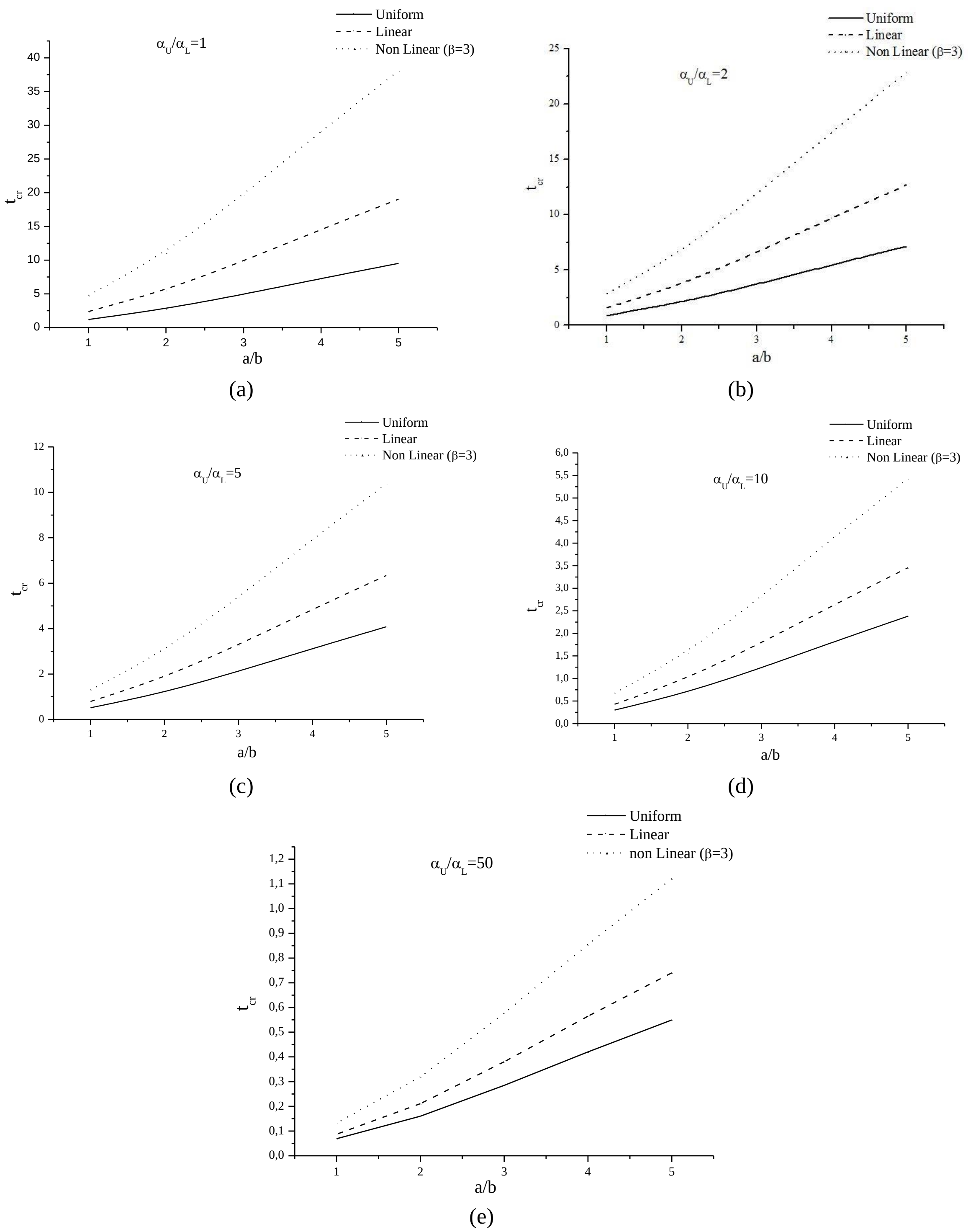


Fig. 2 Critical buckling temperature difference t_{cr} due to uniform, linear and non-linear temperature rise across the thickness versus the aspect ratio a/b for different values of α_U / α_L ratio (k=2) (a) α_U / α_L=1, (b) α_U / α_L=2, (c) α_U / α_L=5, (d) α_U / α_L=10, (e) α_U / α_L=50

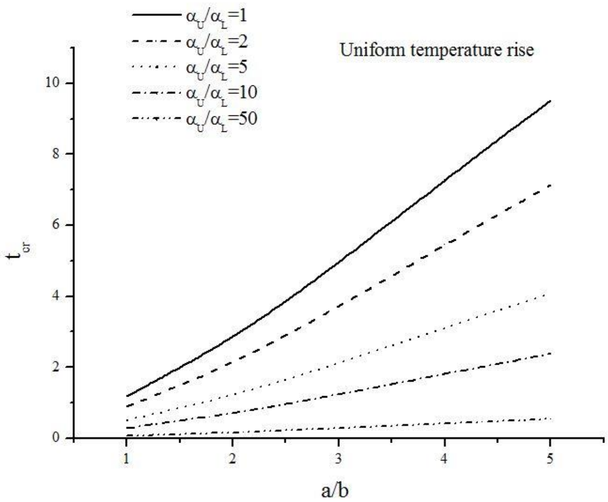
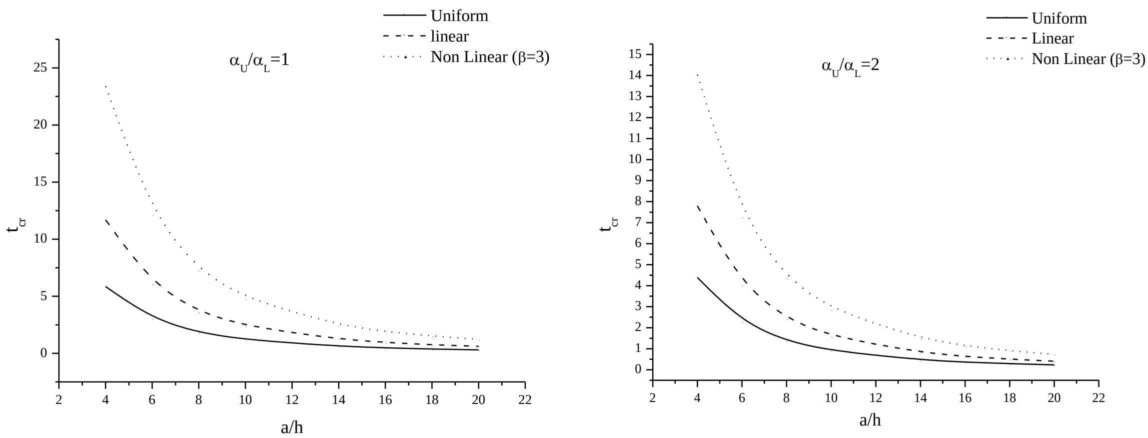


Fig. 3 Critical buckling temperature difference t_{cr} due to uniform temperature rise across the thickness versus the aspect ratio a/b for different values of α_U / α_L ratio (k=2)

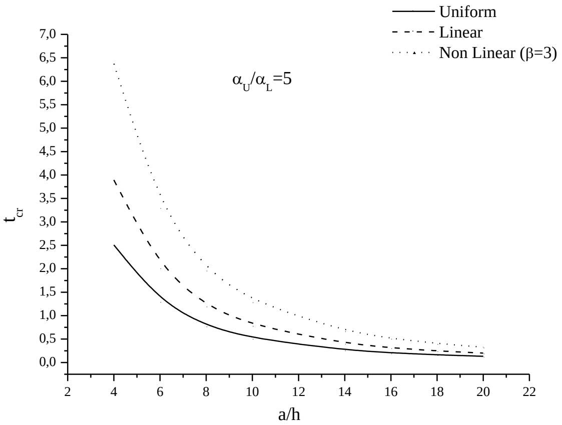
buckling temperature values are obtained for the case of a non-linear thermal loading.

Fig. 3 depict the critical buckling temperature difference t_{cr} due to uniform temperature rise across the thickness versus the aspect ratio a/b for different values of α_U / α_L ratio. It is seen from this figure that the critical buckling temperature decreases with the increase of the α_U / α_L ratio. In other words, the FG plates with coefficients of thermal expansion of the upper and lower faces close are such that provide the highest critical buckling temperature. In addition, when the ratio of thermal expansion α_U / α_L is close to 50, the critical buckling temperature difference becomes insensitive to the aspect ratio a/b.

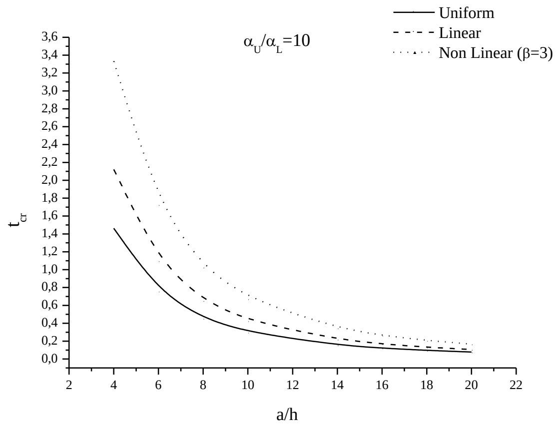


(a) (b)

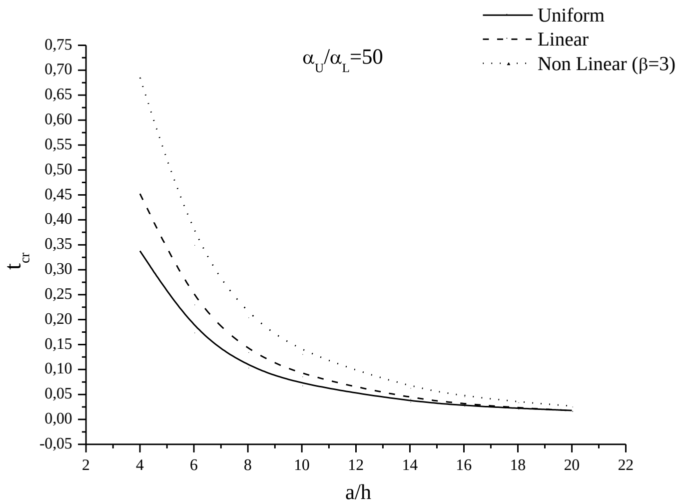
Fig. 4.Critical buckling temperature difference t_{cr} due to uniform, linear and non-linear temperature rise across the thickness versus the side-to thickness ratio a/h for different values of α_U / α_L ratio (k=2) (a) α_U / α_L=1, (b) α_U / α_L=2, (c) α_U / α_L=5, (d) α_U / α_L=10, (e) α_U / α_L=50



(c)



(d)



(e)

Fig. 4 Continued

The effect of the side-to-thickness ratio on the critical buckling temperature difference t_{cr} of FG square plate (k = 2) is shown in Figs. 4 and 5 for different values of α_U / α_L ratio. It can be seen that the increase of side to thickness ratio a / h leads to a decrease of the critical buckling temperature difference of the FG plate. In addition, these results reveal that the variation of the critical buckling temperature difference t_{cr} is very sensitive to the variation of the α_U / α_L ratio for values less or equal to 10. Moreover, when this ratio takes higher values, the critical buckling temperature difference becomes insensitive to the side to thickness ratio a / h.

Fig. 6 demonstrates the critical buckling temperature difference t_{cr} versus the material graded index k for linear temperature rise. From this figure we can see that for all cases of α_U / α_L ratio, the critical temperature difference demonstrates an increasing trend with increasing gradient index. It can be seen that the critical buckling temperature difference changes very slowly.

The critical buckling temperature difference t_{cr} versus the aspect ratio *a/b* of FG plates under various thermal loading types is exhibited in Fig. 7. It can be seen from these figures that,

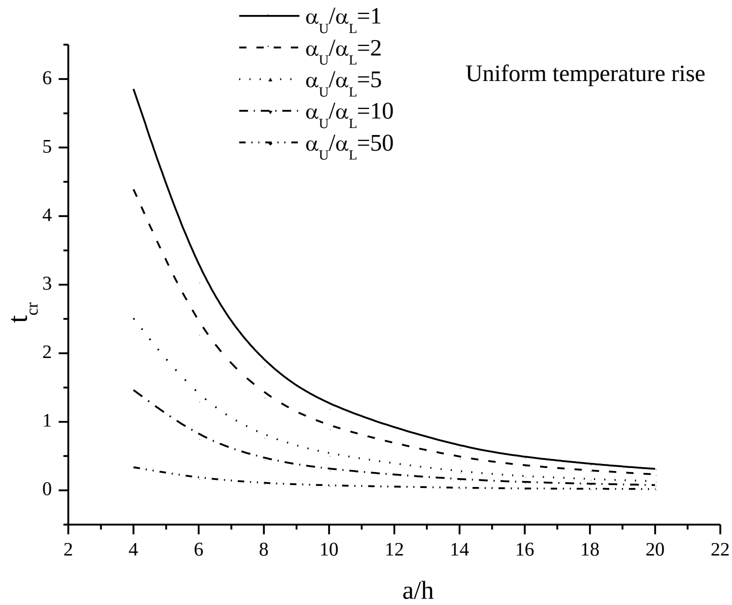


Fig. 5 Critical buckling temperature difference t_{cr} due to uniform temperature rise across the thickness versus the side-to thickness ratio *a/h* for different values of α_U / α_L ratio (*k*=2)

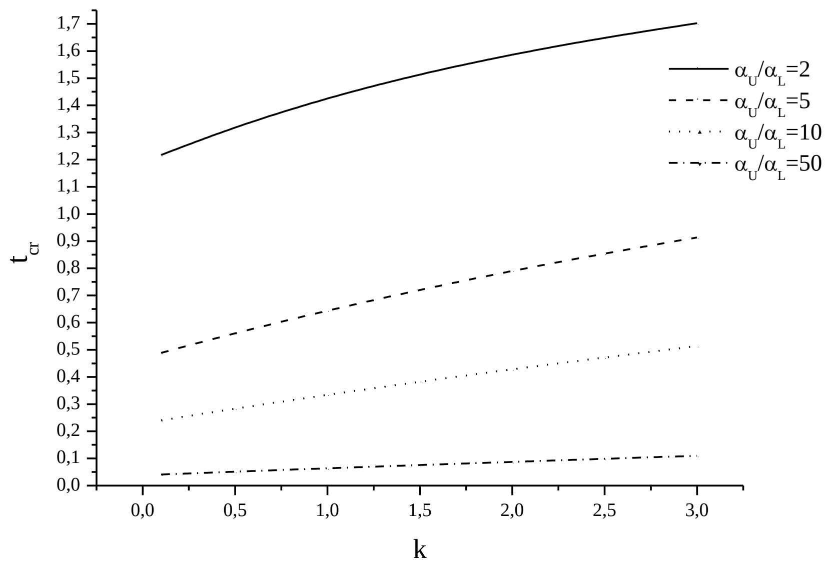
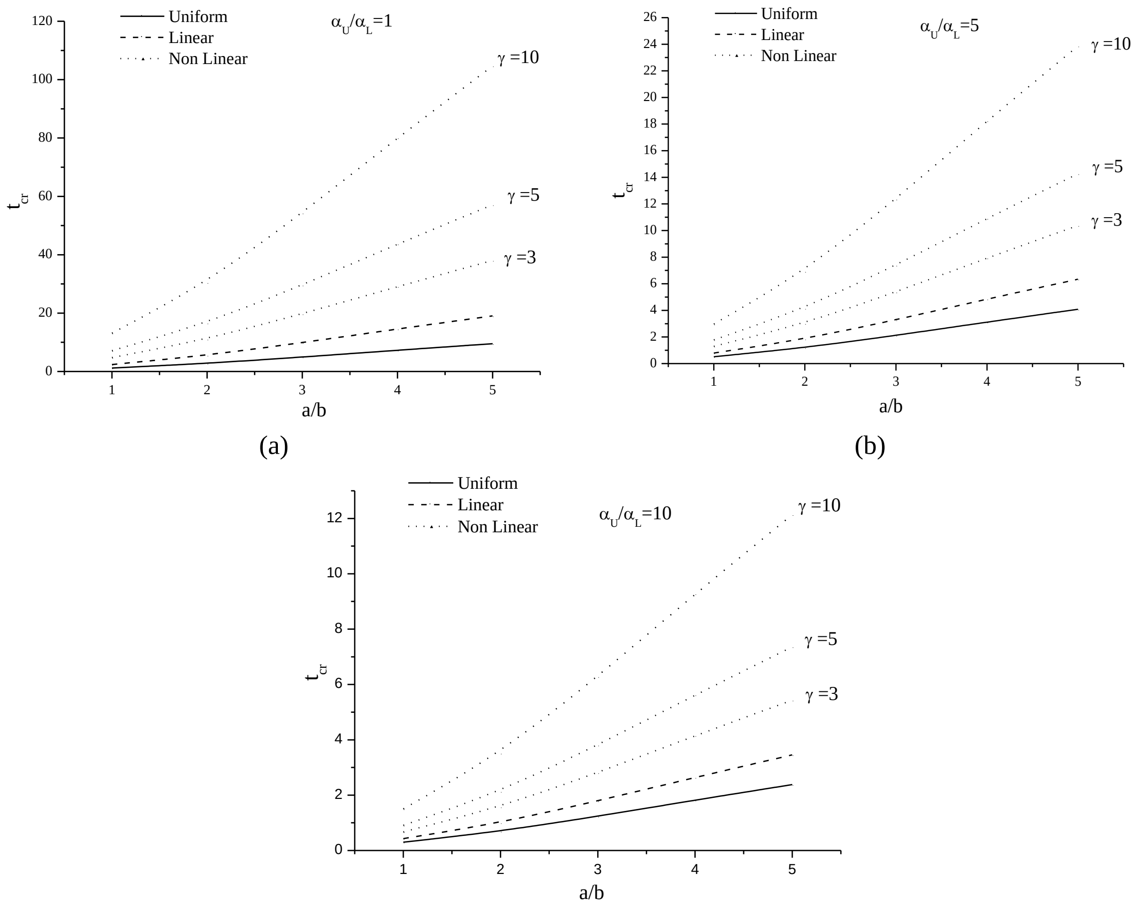


Fig. 6 Critical buckling temperature difference t_{cr} due to linear temperature rise across the thickness versus the power law index *k* (*a=10h*, *a=b*)

regardless of the loading type and α_U / α_L ratio, the critical buckling temperature difference t_{cr} increases as the aspect ratio *a/b* increases. It is also observed that the t_{cr} increases with the increase of the non-linearity parameter $\boldsymbol{\gamma}$.



(a) (b)

(c)

Fig. 7 Critical buckling temperature difference t_{cr} due to uniform, linear and non-linear temperature rise across the thickness versus the aspect ratio *a/b* and for different values of the non-linearity parameter γ. (k=2 and a/h=10) (a) α_U / α_L=1, (b) α_U / α_L=5, (c) α_U / α_L=10

4. Conclusions

In the present study, thermal buckling behavior of functionally graded plates subjected to uniform, linear and non-linear temperature rises across the thickness direction has been investigated. The FG plate is assumed to have constant Young's modulus and Poisson's ratio; but, the coefficients of thermal expansion of the FGM plates is assumed to vary continuously through the thickness, according to a simple power law distribution of the volume fraction of the constituents. A refined plate theory is successfully developed. The theory accounts for a quadratic variation of the transverse shear strains across the thickness, and satisfies the zero traction boundary conditions on the top and bottom surfaces of the plate without using shear correction factors.

The accuracy of the present theory is ascertained by comparing it with other higher-order shear deformation theories where an excellent agreement was observed in all cases. Furthermore, the influences of plate parameters such as coefficients of thermal expansion ratio, power law index, aspect ratio, the side to thickness ratio and thermal loading types on the critical buckling temperature difference of FG plate have been comprehensively investigated.

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