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Natural frequency characteristics of composite plates with random properties

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Abstract. Exercise of complete control on all aspects of any manufacturing / fabrication process is very difficult, leading to uncertainties in the material properties and geometric dimensions of structural components. This is especially true for laminated composites because of the large number of parameters associated with its fabrication. When the basic parameters like elastic modulus, density and Poisson's ratio are random, the derived response characteristics such as deflections, natural frequencies, buckling loads, stresses and strains are also random, being functions of the basic random system parameters. In this study the basic elastic properties of a composite lamina are assumed to be independent random variables. Perturbation formulation is used to model the random parameters assuming the dispersions small compared to the mean values. The system equations are analyzed to obtain the mean and the variance of the plate natural frequencies. Several application problems of free vibration analysis of composite plates, employing the proposed method are discussed. The analysis indicates that, at times it may be important to include the effect of randomness in material properties of composite laminates.

Key words: composite laminates; material property randomness; perturbation method; natural frequencies; standard deviation of response.

1. Introduction

Laminated composites have a large number of parameters associated with its fabrication. These are due to inherent limitations in exercising of complete control on all aspects of any manufacturing/fabrication process (Zweben *et al.* 1989). The slackness of control results in composite laminates exhibiting uncertainties in several factors like the fiber volume fraction, fiber orientation, fibre matrix interface parameters, etc. These uncertainties are reflected as random variations in material properties of composite laminates. In the present study parameters like elastic modulus and Poisson's ratio are considered random. Chen and Zhang (1990) adopted the perturbation approach to analyse stochastic structures. In earlier papers the authors (Salim *et al.* 1993, 1994) had used the above technique, extending the approach outlined by them and studied the effect of such uncertainties on the static and free vibration response of laminated fiber reinforced composite plates. This paper presents a discussion on several application problems to plate natural frequencies and some of the recent results of the investigations.

2. Analysis for laminated composite plates

The analysis presented here is based on the Classical Laminate Theory (CLT). The elastic

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properties of a composite lamina are selected as the basic variables and are assumed to be independent random variables. These random variables b_l^R-elastic moduli, Poisson's ratio, etc. can be represented as:

$$
\begin{aligned}
b_1^R &= E_{11}^R \\
b_2^R &= E_{22}^R \\
b_3^R &= \mu_{12}^R \\
b_4^R &= G_{12}^R \\
&\cdots\cdots\cdot
\end{aligned}
\tag{1}
$$

where the superscript R indicates a random variable. If Q_{ij}s are the reduced stiffness matrix terms, the simplified relationships between these and the basic lamina properties are given by (Vinson and Sierakowski 1986)

$$
\begin{aligned}
Q_{11} &= \frac{E_{11}}{(1-\mu_{12}\mu_{21})} \\
Q_{22} &= \frac{E_{22}}{(1-\mu_{12}\mu_{21})} \\
Q_{12} &= \frac{\mu_{21}E_{11}}{(1-\mu_{12}\mu_{21})} = \frac{\mu_{12}E_{22}}{(1-\mu_{12}\mu_{21})} \\
Q_{21} &= Q_{12} \\
Q_{44} &= G_{23} \\
Q_{55} &= G_{13} \\
Q_{66} &= G_{12}
\end{aligned}
\tag{2}
$$

This system of equations can be written as a series in the primary variables (Salim *et al.* 1994):

$$Q_{ij} = \sum_{n=0}^{\infty} f_{ij} \left\{ \prod_{l=1}^{5} (b_l)^{[e_{ijl}(n)]} \right\} \tag{3}$$

Here f_{ij} are constants and $e_{ijl}(n)$ are functions of n, the counter index for the infinite series approximation of Q_{ij}. The transformed reduced stiffness matrix terms $\overline{Q}_{ij}$ are related to the reduced stiffness matrix terms Q_{ij} through equation (Salim *et al.* 1994):

$$(\overline{Q}_{ij})_k = \sum_{q=1}^{7} (C_{ijq})_k \sum_{i=1}^{6} \sum_{j=1}^{6} a_{qij} Q_{ij} \tag{4}$$

where $(C_{ijq})_k$ are known functions of θ_k, the fiber orientation and a_{qij} are known constants. The elements of the extensional stiffness matrix A_{ij}, coupling stiffness matrix B_{ij} and bending stiffness matrix D_{ij} are given by (Vinson and Sierakowski 1986):

$$
\begin{aligned}
A_{ij} &= \sum_{k=1}^{N_{lay}} (\overline{Q}_{ij})_k (h_k - h_{k-1}) \\
B_{ij} &= \frac{1}{2} \sum_{k=1}^{N_{lay}} (\overline{Q}_{ij})_k (h_k^2 - h_{k-1}^2) \\
D_{ij} &= \frac{1}{3} \sum_{k=1}^{N_{lay}} (\overline{Q}_{ij})_k (h_k^3 - h_{k-1}^3)
\end{aligned}
$$

where h_k is the thickness of the k^{th} lamina and N_{lay} is the number of layers in the laminate. The solution X, of a structural analysis problem involving fiber reinforced composites will in general, be a function of the stiffness matrix terms of A_{ij}, B_{ij}, and D_{ij} as represented by:

$$X = F(A_{ij}, B_{ij}, D_{ij}) \tag{6}$$

So, the expectation of X will be a function of expectations of the stiffness matrix elements – $E[A_{ij}]$, $E[B_{ij}]$, and $E[D_{ij}]$, as the randomness in the stiffness elements depends on the basic material properties which are independent RVs. This may be expressed as:

$$E[X] = F(E[A_{ij}], E[B_{ij}], E[D_{ij}]) \tag{7}$$

The unknown expectations in Eq. (7) can be found with the help of the known quantities. Thus the expectation of the stiffness matrix elements – $E[A_{ij}]$, $E[B_{ij}]$, and $E[D_{ij}]$ are given by:

$$\begin{aligned} E[A_{ij}] &= \sum_{k=1}^{N_{lay}} E[(\bar{Q}_{ij})_k](h_k - h_{k-1}) \\ E[B_{ij}] &= \frac{1}{2}\sum_{k=1}^{N_{lay}} E[(\bar{Q}_{ij})_k](h_k^2 - h_{k-1}^2) \\ E[D_{ij}] &= \frac{1}{3}\sum_{k=1}^{N_{lay}} E[(\bar{Q}_{ij})_k](h_k^3 - h_{k-1}^3) \end{aligned} \tag{8}$$

The derivatives of the stiffness matrix terms with respect to b_l^R are required for obtaining the response statistics (Salim *et al.* 1993, 1994). These can be expressed using Eqs. (8):

$$\begin{aligned} \frac{\partial E[A_{ij}]}{\partial b_l^R} &= \sum_{k=1}^{N_{lay}} \frac{\partial E[(\bar{Q}_{ij})_k]}{\partial b_l^R}(h_k - h_{k-1}) \\ \frac{\partial E[B_{ij}]}{\partial b_l^R} &= \frac{1}{2}\sum_{k=1}^{N_{lay}} \frac{\partial E[(\bar{Q}_{ij})_k]}{\partial b_l^R}(h_k^2 - h_{k-1}^2) \\ \frac{\partial E[D_{ij}]}{\partial b_l^R} &= \frac{1}{3}\sum_{k=1}^{N_{lay}} \frac{\partial E[(\bar{Q}_{ij})_k]}{\partial b_l^R}(h_k^3 - h_{k-1}^3) \end{aligned} \tag{9}$$

Thus, with the help of the above equations, the characteristics of the random component as well as the deterministic component of the unknown terms can be found out. This method can be applied to a wide variety of problems for the static and dynamic analysis of composite structures.

The approach presented here is used to solve problems concerning free vibration of fiber reinforced composite plates, in the next section.

3. Free vibration of composite laminates

Some studies related to the free vibration of rectangular composite laminates with randomness in system parameters are attempted. The effect of randomness in system parameters on the response is introduced using the perturbation approach. The governing equations obtained using energy formulation are solved by the Rayleigh Ritz technique. The validation of the results is done by Monte Carlo simulation. The statistics of random natural frequencies in first few modes are obtained for some types of laminates.

3.1. Formulation procedure

As an example we consider the case of a specially orthotropic rectangular plate of size $a \times b$, simply supported along all the four edges.

In specially orthotropic laminates all the elements of $[B]$ matrix are identically zero. Other coupling terms like A_{16}, A_{26} and D_{16}, D_{26} are also absent. The only stiffness matrix terms present are D_{11}, D_{22}, D_{12} and D_{66}. Again here we are concerned only with the transverse oscillations of the plate. Owing to these simplifications several terms drop out from the general equations.

We try to attempt the solution using energy formulation. The simplified energy criterion for this problem assuming no lateral or in plane loads is:

$$\text{Total potential, } \Pi_p = U + T = \text{stationary value} \tag{10}$$

where U is the strain energy and T is the kinetic energy. Thus for the present case the total potential Π_p is:

$$\Pi_p = U + T = \frac{1}{2}\int_0^b\int_0^a \left[D_{11}\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + 2D_{12}\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} \right.$$
$$\left. + D_{22}\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + 4D_{66}\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \rho\omega^2 w^2 \right] dxdy = \text{stationary value} \tag{11}$$

Here ρ is the effective mass density of the composite laminate and ω is the natural frequency. The boundary conditions along the edges of the plate for the case when all the four edges of the plate are simply supported are defined by:

along $x = 0$ and $x = a$ for all y

$$w = 0$$
$$M_x = -D_{11}\frac{\partial^2 w}{\partial x^2} - D_{12}\frac{\partial^2 w}{\partial y^2} = 0 \tag{12}$$

along $y = 0$ and $y = b$ for all x

$$w = 0$$
$$M_y = -D_{12}\frac{\partial^2 w}{\partial x^2} - D_{22}\frac{\partial^2 w}{\partial y^2} = 0 \tag{13}$$

The Rayleigh-Ritz technique can be used to solve Eq. (11) for the boundary conditions specified by Eqs. (12) and (13). A series approximation for the transverse deflection $w(x, y)$, satisfying the boundary conditions can be written as:

$$w(x,y) = \sum_{m=1}^{M}\sum_{n=1}^{N} a_{mn} X_m(x) Y_n(y) \tag{14}$$

Here $X_m(x)$ and $Y_n(y)$ are eigen-functions that satisfy the set of boundary conditions. Substituting Eq. (14) in (11) and applying the condition of stationary value of total potential we get $M \times N$ homogeneous algebraic equations:

$$\sum_{i=1}^{M}\sum_{j=1}^{N}\left\{ D_{11}\int_0^a \frac{d^2 X_i}{dx^2}\frac{d^2 X_m}{dx^2}\,dx \int_0^b Y_j Y_n\, dy \right.$$

$$+D_{12}\left[\int_0^a X_m \frac{d^2X_i}{dx^2}dx\int_0^b Y_j \frac{d^2Y_n}{dy^2}dy+\int_0^a X_i \frac{d^2X_m}{dx^2}dx\int_0^b Y_n \frac{d^2Y_j}{dy^2}dy\right]$$

$$+D_{22}\int_0^a X_iX_m dx\int_0^b \frac{d^2Y_j}{dy^2}\frac{d^2Y_n}{dy^2}dy+4D_{66}\int_0^a \frac{dX_i}{dx}\frac{dX_m}{dx}dx\int_0^b \frac{dY_j}{dy}\frac{dY_n}{dy}dy$$

$$\left.-\rho\omega_{mn}^2\int_0^a X_iX_m dx\int_0^b Y_jY_n dy\right\} a_{ij}=0$$

$$m=1,2,\cdots,M \text{ and } n=1,2,\cdots,N \tag{15}$$

Substituting the admissible eigen functions X and Y and having carried out the required integration the above set of $M\times N$ equations can be written in a compact form using matrix notation:

$$[L-\omega^2B]\{A\}=0 \tag{16}$$

Where matrices L and B consist of system elements, A is the eigen vector giving mode shapes and ω is the natural frequency corresponding to the values of m and n used. If the inverse of B exists the above equation can be written as:

$$[B^{-1}L-\omega^2I]\{A\}=0 \tag{17}$$

If $R=B^{-1}L$ and considering the material parameter randomness at this point R, A and ω would be random in nature Eq. (17) an be reformatted as:

$$[R^R-\omega^{2R}I]\{A^R\}=0 \tag{18}$$

Any random variable (RV) may be expressed in two parts as its mean and a zero mean random part. For many practical engineering situations the random scatter in the system parameter is small compared to its mean value. Combining these two arguments we can write:

$$RV^R=RV^d+\varepsilon RV^r \tag{19}$$

Where RV^d and RV^r are mean and scaled up zero mean random component of the random variable RV^R and ε is a small perturbation parameter. Splitting up the random terms in the above equation gives:

$$[[R^d+\varepsilon R^r]-[\omega^{2d}+\varepsilon\omega^{2r}]I]\{A^d+\varepsilon A^r\}=0 \tag{20}$$

$$[R^d-\omega^{2d}I+\varepsilon R^r-\varepsilon\omega^{2r}I]\{A^d+\varepsilon A^r\}=0 \tag{21}$$

From the above terms corresponding to 0th and 1st powers of ε are collected together to get the following:

$$\varepsilon^0\longrightarrow[R^d-\omega^{2d}I]\{A^d\}=0 \tag{22}$$

$$\varepsilon^1\longrightarrow[R^r-\omega^{2r}I]\{A^d\}+[R^d-\omega^{2d}I]\{A^r\}=0 \tag{23}$$

Eigenvalue problems for evaluating natural frequencies can be formed from these two equations. Eq. (22) gives the deterministic part of the natural frequencies. Substituted for ω^d in Eq. (23) and considering it's second order expectation leads to the variance of the natural frequency. The assumption of independent input random variables results in substantial simplification of the variance relation.

4. Numerical results

Results have been computed for some typical problems to study the effect of randomness in material properties on the free vibration response of rectangular composite plates. Except when specified, all the input RVs are assumed to have the same standard deviation (SD) to Mean ratio. All the plate configurations under consideration are assumed to have the same total thickness h. For the problems studied, the basic material properties like the longitudinal modulus, transverse modulus and shear modulus are considered to be random variables. The randomness in material properties is handled using the perturbation technique discussed earlier. Most of the results computed are for Graphite/Epoxy plates. Towards the end some results for plates made of Glass/Epoxy composite are also presented. This is done to investigate the effect of modular ratio since the ratio of E_{11}/E_{22} is more for Graphite/Epoxy material, in comparison with Glass/Epoxy. The assumed mean values of the random variables used are given in Table 1.

Validation of the perturbation technique used for the present problem was done by comparing it with the results from a Monte Carlo simulation study as reported in an earlier paper (Salim *et al.* 1994). Fig. 1, shows results from the Monte Carlo simulation, compared with the present approximation for a [90°] square Graphite/Epoxy plate with all the four sides simply supported. The ratio $SD/Mean$ is assumed to be fixed at 0.10 for all the four primary random variables. Here the SD of fundamental and higher natural frequencies ω^2 are plotted against the change in SD of input RVs. As indicated by the plots, for the range of SD of input RVs considered both results are quite close. One may, therefore, conclude that the present approximation employing first order perturbation gives sufficiently accurate results for the free vibration problem under consideration.

The mean values of natural frequencies for a [90°] lamina simply supported along all the edges is given in Table 2, for various aspect ratios. Table 3 gives mean of natural frequencies for a square laminate, with various stacking sequences.

Fig. 2, gives the plot of variation of SD of natural frequencies ω^2 of a [90°] square lamina with variation in SD of longitudinal modulus E_{11}. Hence, only E_{11} is random, all other parameters are deterministic as SD of all other RVs are assumed to be zero. Figs. 3 , 4 and 5 show similar results for the other RVs $-E_{22}$ the transverse modulus, μ_{12} the Poisson's ratio, and G_{12} the shear modulus. The SD of natural frequencies are affected most by the change in SD of the longitudinal modulus E_{11}. The fundamental frequency is sensitive to changes in E_{11}, E_{22} and μ_{12}. We can see that the influence of SD of input RVs on SD of natural frequencies diminishes with increase in the mode for the above three cases. If we consider the magnitude of effect on the SD of natural frequencies by the SD of each of the input RVs – the order E_{11}, G_{12}, E_{22} and μ_{12} shows progressively decreasing influence.

Fig. 6 shows the influence of simultaneous change in the SD of all the input RVs on the SD of the natural frequencies. Here all the basic RVs are assumed to have the same ratio of

Table 1 Mean of primary random variables

Random variable	Graphite/Epoxy	Glass/Epoxy
E_{11}	181.0 GPa	53.78 GPa
E_{22}	10.3 GPa	17.93 GPa
μ_{12}	0.28	0.25
G_{12}	7.17 GPa	8.963 GPa

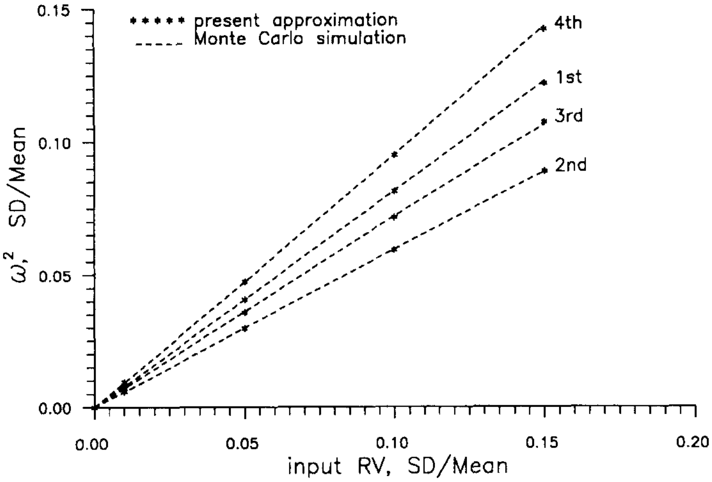

Fig. 1 Normalized SD of natural frequencies ω^2, plotted against SD of input RVs. The present approximation compared with results from a Monte Carlo simulation. [90°] laminate, AR=1.0. Graphite/Epoxy (Salim *et al.* 1994)

Table 2 Mean of natural frequencies ω^2, for different aspect ratios. [90°] lamina, all sides simply supported. Graphite/Epoxy (Hz^2)

AR	1st	2nd	3rd	4th
0.5	0.15044E-02	0.32505E-02	0.61606E-02	0.10235E-01
1.0	0.10679E-02	0.15044E-02	0.22319E-02	0.32505E-02
2.0	0.95878E-03	0.10679E-02	0.12498E-02	0.15044E-02

Table 3 Mean of natural frequencies ω^2, for different stacking sequences. Square plate, all sides simply supported. Graphite/Epoxy (Hz^2)

Stacking sequence	1st	2nd	3rd	4th
$[90°=0°]_s$	0.959E-03	0.139E-02	0.212E-02	0.314E-02
[45°]	0.123E-02	0.309E+00	0.393E+00	0.411E+00
[60°]	0.130E-02	0.361E+00	0.402E+00	0.431E+00
$[45°=0°]_s$	0.110E-02	0.277E+00	0.352E+00	0.369E+00

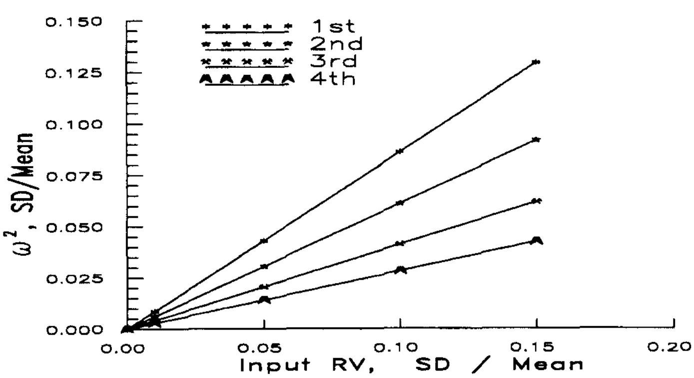

Fig. 2 [90°] laminate, AR=1, all sides simply supported, sensitivity of SD of normalized natural frequencies ω^2 to SD of input RV E_{11}. SD of all other input RVs kept zero. Graphite/Epoxy

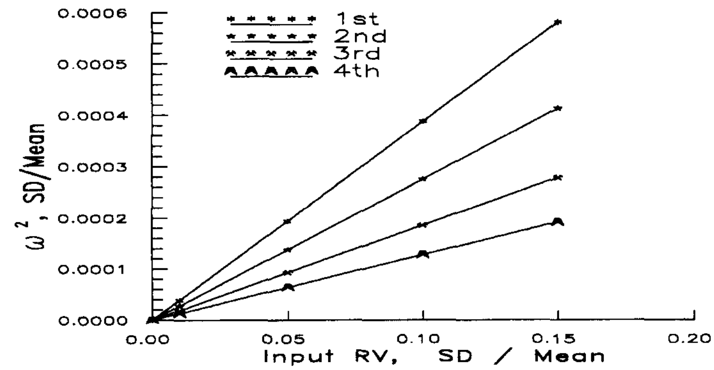

Fig. 3 [90°] laminate, AR=1, all sides simply supported, sensitivity of SD of normalized natural frequencies ω^2 to SD of input RV E_{22}. SD of all other input RVs kept zero. Graphite/Epoxy

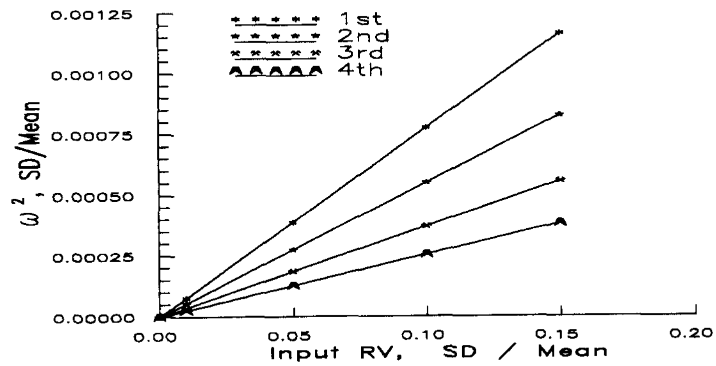

Fig. 4 [90°] laminate, AR=1, all sides simply supported, sensitivity of SD of normalized natural frequencies ω^2 to SD of input RV μ_{12}. SD of all other input RVs kept zero. Graphite/Epoxy

SD to mean. The results are for a (90°) lamina with aspect ratio 0.5. All the four sides are assumed to be simply supported. The first natural frequency is affected least. Higher and higher modes are progressively affected more by the change in SD of input RVs.

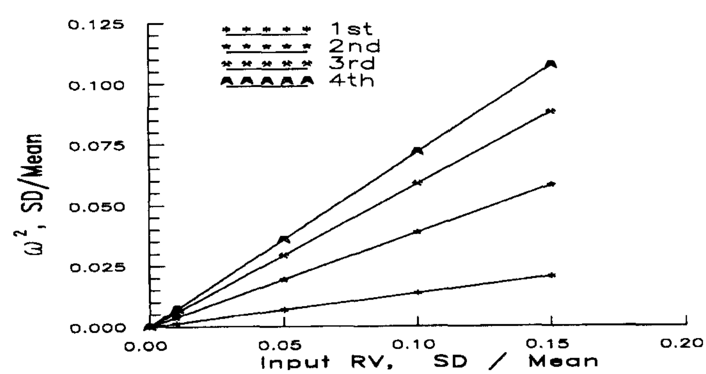

Fig. 5 [90°] laminate, AR=1, all sides simply supported, sensitivity of SD of normalized natural frequencies ω^2 to SD of input RV G_{12}. SD of all other input RVs kept zero. Graphite/Epoxy

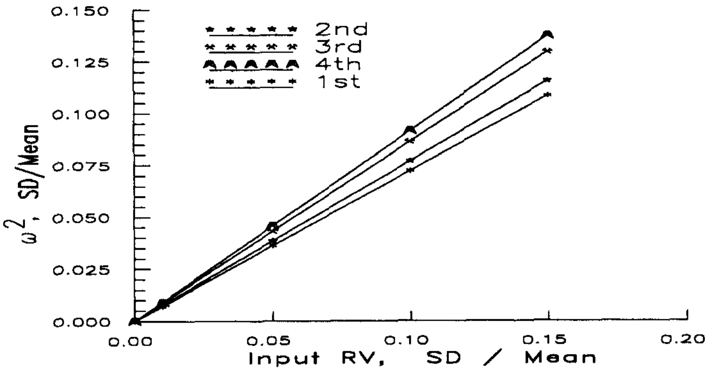

Fig. 6 [90°] laminate, AR=0.5, all sides simply supported. Variation of SD of normalized natural frequencies ω^2 with SD of input RVs. Graphite/Epoxy

The influence of change in aspect ratio on the SD of natural frequencies is shown by Fig. 7. Here all the input RVs are assumed to have a SD equal to 10% of their mean value. The results are for a [90°] lamina. The nature of variation of the SD of frequencies at low aspect ratios is strongly dependent on the mode we are considering. But the result seems to reach plateau as we approach higher aspect ratios. At aspect ratios above 2.0, the SD of first mode frequency is the largest, with gradually decreasing values for the higher modes compared to the first mode. At low aspect ratios this pattern is not present.

Fig. 8 illustrates the influence of change in aspect ratio on the SD of fundamental frequency at different SDs of input RVs, with all of them having the same SD to Mean ratio. When the SD of input RVs is low the change in aspect ratio doesn't seem to affect the SD of natural frequencies ω^2. As the SD of input RVs is increased, the influence of change in aspect ratio increases. The change in aspect ratio doesn't seem to affect the SD of ω^2 beyond a limit, depending on the SD of input RVs.

Results for the case of a $[90°=0°]_s$ square plate is shown in Fig. 9 for the first four frequencies.

Figs. 10 and 11 show the results for [45°] and $[45°=0°]_s$ laminates. The plots for different modes are closely packed together in both the cases. The 0° layers in the second case doesn't seem to have much influence on the output SD.

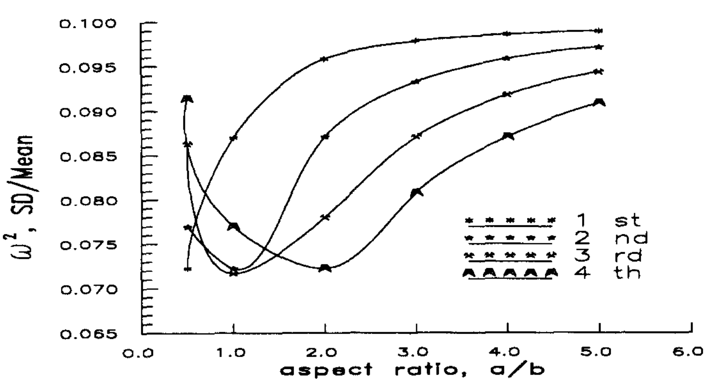

Fig. 7 [90°] laminate, AR=1, all sides simply supported. Variation of SD of normalized natural frequencies ω^2 with change in aspect ratio, for different modes. SD of input RVs 10% of mean. Graphite/Epoxy

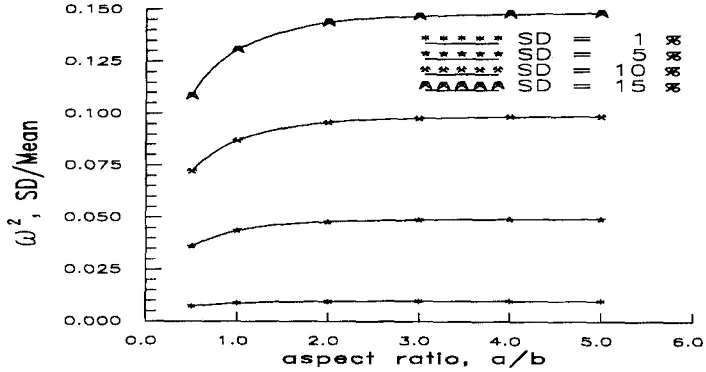

Fig. 8 [90°] laminate, all sides simply supported. Variation of SD of normalized fundamental frequency ω^2 with change in aspect ratio, for different SDs of input RVs. Graphite/Epoxy

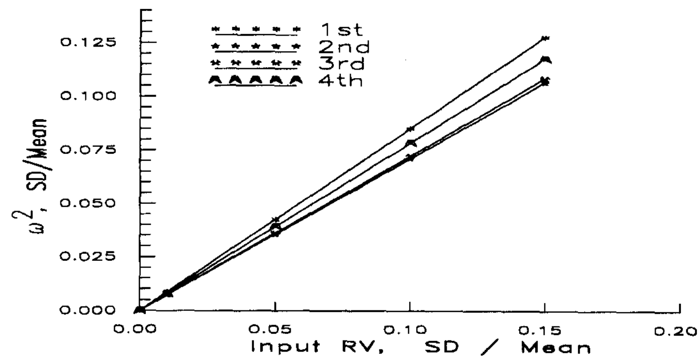

Fig. 9 [90°=0°]$_s$ laminate, AR=1, all sides simply supported. Variation of SD of normalized natural frequencies ω^2 with SD of input RVs. Graphite/Epoxy

Results for [60°] laminate configuration is shown in Fig. 12 for a square plate.

Some results for a typical Glass/Epoxy composite are now presented. Table 4 gives mean values of natural frequencies for a [90°] lamina, simply supported along all the four edges. Fig. 13 gives the variation of SD of natural frequencies with the SD of input RVs for an

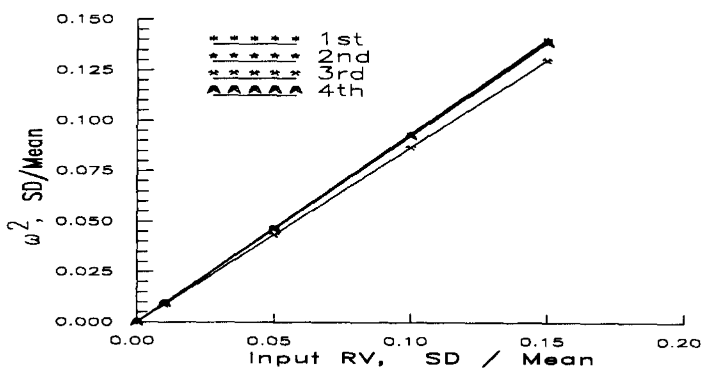

Fig. 10 [45°] laminate, AR=1, all sides simply supported. Variation of SD of normalized natural frequencies ω^2 with SD of input RVs. Graphite/Epoxy

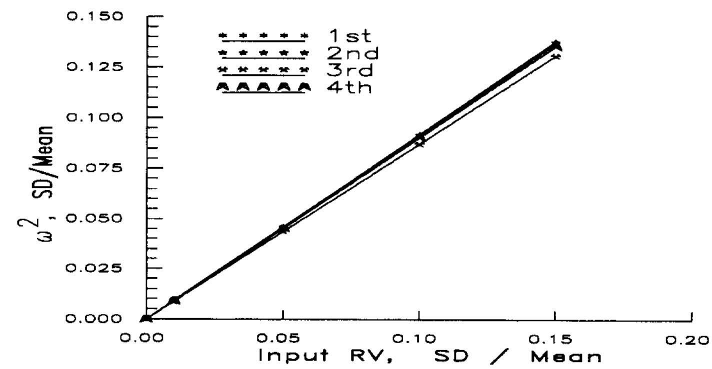

Fig. 11 [45°=0°]$_s$ laminate, AR=1, all sides simply supported. Variation of SD of normalized natural frequencies ω^2 with SD of input RVs. Graphite/Epoxy

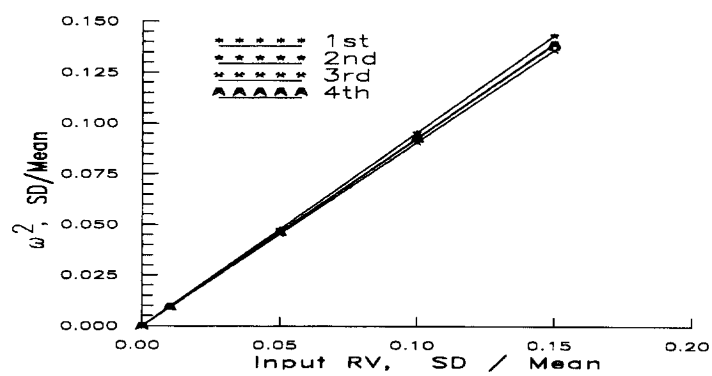

Fig. 12 [60°] laminate, AR=1, all sides simply supported. Variation of SD of normalized natural frequencies ω^2 with SD of input RVs. Graphite/Epoxy

aspect ratio of 0.5. The results are for a [90°] lamina, simply supported on all the four edges. The magnitude of SD of the natural frequencies are comparable to those for a similar Graphite/Epoxy configuration. The rate of change of the SD of the frequencies with the input parameters is different in these two cases.

Influence of the change in aspect ratio of the plate on the SD of natural frequencies is shown by Fig. 14, for different levels of SD of input RVs. The result is for a [90°] lamina with all the sides simply supported. At low levels of SD of input RVs the change in aspect ratio has practically no effect on the SD of first natural frequency. At higher levels of SD of input RVs the response SD dips to the lowest value at aspect ratio 1.0, to to again increase to higher values. All the response curves ultimately reach a plateau and further increase in aspect

Table 4 Mean of natural frequencies ω^2, for different aspect ratios. [90°] lamina, all sides simply supported. Glass/Epoxy. (Hz^2)

AR	1st	2nd	3rd	4th
0.5	0.10062E-02	0.31890E-02	0.68269E-02	0.73687E-02
1.0	0.46055E-03	0.10062E-02	0.19157E-02	0.31890E-02
2.0	0.32412E-03	0.46055E-03	0.68792E-03	0.10062E-02

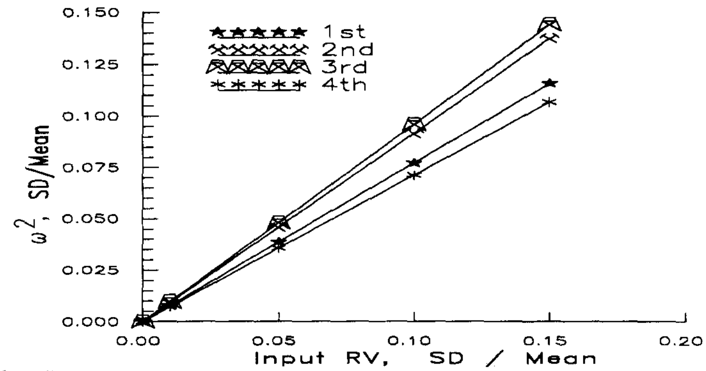

Fig. 13 Test case Glass/Epoxy composite. [90°] laminate, AR=0.5, all sides simply supported. Variation of SD of normalized natural frequencies ω^2 with SD of input RVs

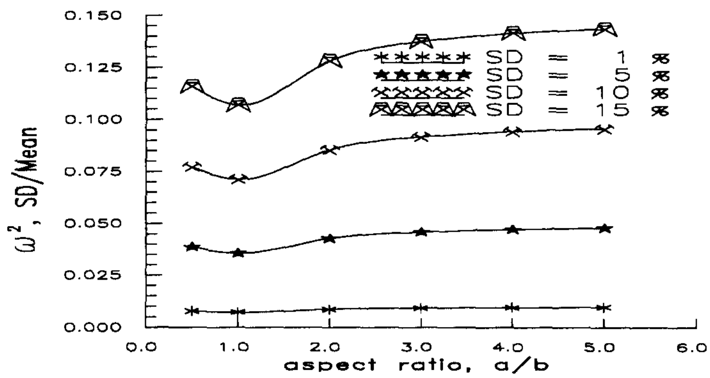

Fig. 14 Test case glass/epoxy composite. [90°] laminate, all sides simply supported. Variation of SD of normalized fundamental frequency ω^2 with change in aspect ratio, for different SDs of input RVs

ratio doesn't seem to have much effect on the SD of first natural frequency.

5. Conclusions

The perturbation approximation gives sufficiently accurate results for the problem under consideration. In general the SD of natural frequencies are affected most by the change in E_{11}. The nature of influence of the input RVs are strongly dependent on the mode of vibration and the extent of randomness of the variable under consideration. The nature of variation of SD of normal mode frequencies is strongly dependent on the mode at low aspect ratios. Change in aspect ratio beyond a limit doesn't seem to affect the SD of natural frequencies. At low levels of SD of input RVs change in aspect ratio, fiber orientation etc. doesn't seem to have much effect on the SD of natural frequencies. SD of natural frequencies have a maximum value for a laminate with 45° fiber orientation. The outer plies of laminates have a dominating influence on the SD of natural frequencies. In general the results show a linear behavior for the change in SD of natural frequencies with the SD of the input RVs. The rate and order of change of natural frequencies with change in SD of input RVs is strongly dependent on the stacking sequence and the boundary conditions.

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