Structural Engineering and Mechanics, Vol. 6, No. 4 (1998) 457-466
DOI: http://dx.doi.org/10.12989/sem.1998.6.4.457

Elastodyamic analysis of torsion of shaft of revolution by line-loaded integral equation method*

Tian Quan Yun†

Department of Mechanics, South China University of Technology, Guangzhou, 510641, China

Abstract. The dynamic response of an elastic torsion shaft of revolution is analysed by the Line-Loaded Integral Equation Method (LLIEM). A "Dynamic Point Ring Couple" (DPRC) is used as a fictitious fundamental load and is distributed in an elastic space along the axis of the shaft outside the shaft occupation. According to the boundary condition, our problem is reduced to a 1-D Fredholm integral equation of the first kind, which is simpler for solving than that of a 2-D singular integral equation of the same kind obtanied by Boundary Element Method (BEM), for steady periodically varied loading. Numerical example of a shaft with quadratic generator under sinusoidal type of torque is given. Formulas for stresses and dangerous frequency are mentioned.

Key words: torsion of shaft of revolution; elastodynamic analysis; the line-loaded integral equation method.

1. Introduction

The boundary element method (BEM) plays an important role in modern analysis, since it is better than the finite element method (FEM) for many problems. However, BEM is not the best for some problems. Many static problems have been analysed by LLIEM, for example, Yun (1979) solved the torsion problem of shaft of revolution; Yun (1981) analysed axially loaded pile embedded in an elastic half space, Yun *et al.* (1981) analysed ellipsoid compressed by two axial concentrated forces at two ends; Yun (1988) analysed the torsion problem of rigid circular shaft of varying diameter embedded in an elastic half space; Yun (1990) analysed the torsion problem of elastic shaft of revolution embedded in an elastic half space; Yun (1991) analysed rigid sloping pile under arbitrary loads; Yun and Su (1992) analysed a shaft (elastic cylindrical shell) with step-varied thickness, and/or with hinged supports, embedded in a granular half space under the action of gravity; Yun and Li (1995) analysed the in-plane-hinge-jointed rigid sloping piles. These examples show that LLIEM is efficient and simple in calculation. The major advantage of LLIEM over BEM is that LLIEM reduces a problem to a 1-D, non-singular integral equation (s), while BEM reduces the same problem to 2-D, singular integral equation (s) of the same kind. However, LLIEM has not been used for dynamic problem yet, this paper,

*The project supported by Natural Science Foundation of Guangdong Province of China.
† Professor

presented at WCCM 3 (Yun 1994), extends the application of LLIEM to elastodynamic analysis of torsion shaft of revolution.

A general elastodynamic problem is a 4-D (three spatial dimensions plus time) problem. There are basically two approaches (direct and indirect methods) for elastodynamic analysis. In the direct method, the problem is solved directly in real time space while the indirect method the problem is firstly transformed and solved in an integral transformed space, and then the inverse transformation is taken to return the problem to its original space. Usually, the indirect method is recommender for the analysis of steady state dynamic problem due to its quasi-state manner. The indirect method usually has troubles in getting a solution of inverse integral transformation. The direct method usually spends more computation time since it repe· the calculation in each time domain, but it is easy to understand and are widely acce;ti by engineers.

There is no previous paper concerned with elastodynamic analysis of torsion sh of revolution. The closest type of problem, in literature, related to our problem, is that dealin vith elastodynamic analysis of timedomain BEM (Cheung, Lei & Tham 1993, Tham, Cheung & Lei 1994). However, these analyses are concerned with transient dynamic response of piles and half space, not for steady varied loading and not for shaft of revolution. In this paper, a steady varied loading problem of torsion shaft of revolution is analysed by LLIEM with both advantages of the direct method (e.g., easy to understand, without making integral transform and its inverse transform) and indirect method (less computation time, like a static problem).

In Sec. 2, as a fictitious fundamental load, the solution of DPRC at origin of an elastic space is derived, and its property is mentioned. In Sec. 3, the integral equation of our problem is derived, and the formulas of stress and dangerous frequency, which are interested to engineers, are obtained. Finally, numerical example of a shaft with quadratic generator under sinusoidal type torque is given.

2. The solution of "Dynamic Point Ring Couple" (DPRC) at origin

2.1. The solution of "Impulse Point Ring Couple" (IPRC) on z=0 plane, at origin and t=0, in an elastic space

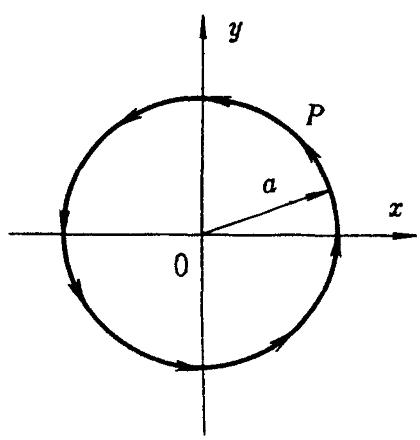

Fig. 1 An IPRC $M_0\delta(0)$ at origin. $M_0=\lim\ 2\pi a^2 P\ (a\rightarrow 0)$

Fig. 1 shows an impulsive loading with uniform intensity $P\cdot\delta(0)$ suddenly applied at time $t=0$, on a circle of $z=0$ plane, with radius a and centered at origin of an elastic space, along the tangent of circumference. The limit of this loading system $M_0\delta(0)$ for $a \longrightarrow 0$ is simply called an IPRC $M_0\delta(0)$ at origin, where

$$M_0=\lim 2\pi a^2 P \qquad (a\rightarrow 0) \tag{1}$$

The i-th component of displacement field at point x and time t due to a unit concentrated impulse force acting at a point x_1 and time $t=0$ in the direction of the x_j-axis has been found (Eringen 1975) and is listed:

$$u_{ij}(x,t;x_1,0)=\frac{1}{4\pi m}\left\{\frac{t}{R^3}\left[\frac{3x_ix_j}{R^2}-\delta_{ij}\right]\left[H\left(t-\frac{R}{c_1}\right)-H\left(t-\frac{R}{c_2}\right)\right]\right.$$
$$\left.+\frac{x_ix_j}{R^3}\left[\frac{\delta(t,R/c_1)}{c_1^2}-\frac{\delta(t,R/c_2)}{c_2^2}\right]+\frac{\delta_{ij}\,\delta(t,R/c_2)}{Rc_2^2}\right\} \tag{2}$$

where $R=|x-x_1|$, $x_i=RR_{,i}$, $x_j=RR_{,j}$, P-wave speed c_1, S-wave speed c_2 are:

$$c_1^2=(\lambda+2G)/m=2G(1-\nu)/[m(1-2\nu)], \qquad c_2^2=G/m$$

λ, G are Lamé coefficients, m=density, ν=Poisson's ratio, H=Heviside unit step function, δ= Dirac function, δ_{ij}=Kroncker delta.

The components of displacement and stress of a field point $N(r, z)$ in cylindrical coordinates at time t due to a IPRC at origin and time t=0 can be obtained using Eq. (2) by integration (see Yun and Gu 1993, for details) and are listed:

$$(U,V,W)=\left(0,\ \frac{M_0 r}{8\pi G R_0^3}\left(\delta_{20}+\frac{R_0}{c_2}\dot{\delta}_{20}\right),\ 0\right) \tag{3}$$

$$\sigma_r=\sigma_\theta=\sigma_z=\tau_{rz}=0$$

$$\tau_{\theta r}=-\frac{3M_0 r^2}{8\pi R_0^5}\left[\delta_{20}+\frac{R_0}{c_2}\dot{\delta}_{20}+\frac{1}{3}\frac{R_0^2}{c_2^2}\ddot{\delta}_{20}\right]$$

$$\tau_{\theta z}=-\frac{3M_0 rz}{8\pi R_0^5}\left[\delta_{20}+\frac{R_0}{c_2}\dot{\delta}_{20}+\frac{1}{3}\frac{R_0^2}{c_2^2}\ddot{\delta}_{20}\right] \tag{4}$$

where and $(\dot{\ })=\partial(\ )/\partial t$, $\delta_{20}=\delta(t-R_0/c_2)$, $R_0=(r^2+z^2)^{1/2}$, and L'Hospital rule, Laplace transformation and inverse Laplace transformation have been used in the integration and limit process.

2.2. The solution of a DPRC at origin

According to the property of δ-function, for arbitrary continuous function $F(t)$, we have

$$F(t)=\int_{-\infty}^{\infty} F(\tau)\,\delta(t-\tau)\,d\tau=\int_{-\infty}^{\infty} F(t-\tau)\,\delta(\tau)\,d\tau \tag{5}$$

by Eq. (5), we can obtain the components of displacement and stress of a field point $N(r, z)$ at time t due to a DPRC at origin and time τ, with time dependent magnitude $M_0F(\tau)$. For example, the components of displacement of a field point $N(r, z)$ at time t due to a DPRC at origin and time τ with intensity $M_0 \sin\omega\tau$ is: $(0 \leqq \omega\tau \leqq 2\pi)$

$$V=\frac{M_0 r}{8\pi G R_0^3}\int_{-\infty}^{\infty} \sin\omega(t-\tau)[\delta_{20}(\tau)+R_0 c_2^{-1}\,\dot{\delta}_{20}(\tau)]\,d\tau$$

$$=\frac{M_0 r}{8\pi G R_0^3}\{[\sin\omega(t-t_0)+\omega t_0\cos\omega(t-t_0)]H_{20}+t_0\sin\omega(t-t_0)\cdot\Delta\delta_{20}\} \tag{6}$$

where $H_{20}=H(t-t_0)$, $t_0=R_0/c_2$, $\Delta\delta_{20}=\lim_{\varepsilon\to 0}\{\delta(t+\varepsilon-t_0)-\delta(t-\varepsilon-t_0)\}$, ω=frequency. In the derivation of Eq. (6),

$$\int_{-\infty}^{\infty}=\int_{-\infty}^{t_0-\varepsilon}+\int_{t_0-\varepsilon}^{t_0+\varepsilon}+\int_{t_0+\varepsilon}^{\infty}$$

the formular of integral by parts and mid-value theorem of integration have been used. The last term involved $\Delta\delta_{20}$ belongs to the type of $(\infty-\infty)$, whether it would be infinite or finite, it just represents the response at $t=t_0$, and disappears for $t\neq t_0$. For a torsion problem of periodical loading, where the instant response is not so important comparing with the long lasting term H_{20}. Therefore, we neglect the instant terms in the following for simplifying, and list the components of displacement and stress

$$V=[M_0 r/(8\pi G R_0^3)][\sin\omega(t-t_0)+t_0\omega\cos\omega(t-t_0)]H_{20} \tag{7}$$

$$\tau_{\theta r}=-3M_0 r^2\phi(y_0, y)/[8\pi R_0^5], \qquad \tau_{\theta z}=-3M_0 rz\,\phi(y_0, y)/[8\pi R_0^5] \tag{8}$$

$$\phi(y_0, y)=A(y_0)\sin(y-y_1)\cdot H_{20} \tag{9}$$

$$A(y_0)=[(1-y_0^2/3)^2+y_0^2]^{1/2}, \qquad y=\omega t, \qquad y_0=\omega t_0, \qquad t_0=R_0/c_2 \tag{10}$$

$$y_1=tg^{-1}\{[(1-y_0^2/3)\sin y_0-y_0\cos y_0]/[(1-y_0^2/3)\cos y_0+y_0\sin y_0]\} \tag{11}$$

Two properties of the solution Eqs. (7) and (8) are mentioned:

1. For any instant, we have

$$\tau_{\theta r}/\tau_{\theta z}=r/z, \qquad \text{for } \forall t \tag{12}$$

Eq. (12) shows that a family of zero stressed cone surface with the same apex at origin and the z-axis as its symmetrical axis appears in the elastic space at any time.

2. The solution Eqs. (7) and (8) satisfies the differential equation of motion:

$$\partial\sigma_r/\partial r+(1/r)\,\partial\tau_{\theta r}/\partial\theta+\partial\tau_{rz}/\partial z+(\sigma_r-\sigma_\theta)/r=m\ddot{U}-F_r \tag{13a}$$

$$\partial\tau_{\theta r}/\partial r+(1/r)\,\partial\sigma_\theta/\partial\theta+\partial\tau_{\theta z}/\partial z+2\tau_{\theta r}/r=m\ddot{V}-F_\theta \tag{13b}$$

$$\partial\tau_{rz}/\partial r+(1/r)\,\partial\tau_{\theta z}/\partial\theta+\partial\sigma_z/\partial z+\tau_{rz}/r=m\ddot{W}-F_z \tag{13c}$$

Obviously, Eqs. (7) and (8) satisfy Eqs. (13a) and (13c) if the body force F is neglected. By Eq. (9), we have

$$\partial\phi(y_0, y)/\partial r = \psi\cdot r, \qquad \partial\phi(y_0, y)/\partial z = \psi\cdot z$$

$$\psi = [(\partial A/\partial y_0)\sin(y - y_1) + A\cos(y - y_1)(-\partial y_1/\partial y_0)][\omega/(c_2 R_0)]\cdot H_{20}$$

Substituting the above relationship, Eqs. (7) and (8) into Eq. (13b), we get $\partial\tau_{\theta r}/\partial r + \partial\tau_{\theta z}/\partial z + 2\tau_{\theta r}/r = -3M_0 r/(8\pi R_0^3) = m\ddot{V}$.

Since Eqs. (7) and (8) satisfy both the differential equation of motion in a cone and the zero stressed boundary condition on the surface of the twisted cone, therefore Eqs. (7) and (8) form the solution of the cone shaft twisted by $M_0 \sin\omega t$ at origin.

3. Formulation of the integral equation

Let S and ∂S be the body and boundary of a shaft of revolution respectively. The solution of dynamic torsion of shaft of revolution implies that the differential Eq. (13) is satisfied in $S\times t$ and the boundary condition of zero stress is satisfied on $\partial S\times t$, for $\forall t\in[0, \infty]$. Since Eq. (13) is always satisfied by any combination of the solution Eqs. (7) and (8) of DPRC distribution, therefore only the boundary condition of zero stress requires to be satisfied. In the following, a zero stressed surface of revolution with given generator will be constructed in an elastic space at any time due to DPRC distribution.

Suppose that the shaft of revolution is located in $z\in[0, b]$. Let the fictitious load DPRC with unknown intensity $x(s)$ $\sin\omega t$ to be distributed along the $s(=-z)-$axis in $[a, L](b\ll L<\infty)$. The reason for DPRC distribution outside the shaft occupation is to agree with the fact that no singularity occurs at points of the axis of symmetry of a torsion shaft. Usually, an inhomogeneous integral equation is easier to obtain a numerical solution than that of its homogeneous case. In order to reduce our problem to an inhomogeneous integral equation, analogous to an adding term of uniformly flow in the method of source-sink distribution in fluid mechanics, besides the above DPRC distribution, a single DPRC with intensity $C\cdot s_0^4$ sin ωt (C-constant) acting at $s=s_0$ ($s_0\gg b$) is added (Fig. 2). The stress at a point $N(r, z)$ in the elastic space and at time t, due to the above all DPRC, by Eq. (8), is

$$\tau_{\theta r} = -[3r^2/(8\pi)]\int_a^L R_s^{-5}\phi(y_s, y)x(s)\,ds \tag{14a}$$

$$\tau_{\theta r} = -[3r/(8\pi)][\int_a^L (z+s)\,\phi(y_s, y)R_s^{-5}x(s)\,ds - C\cdot\phi(y_{s_0}, y)] \tag{14b}$$

where $R_s=[r^2+(z+s)^2]^{1/2}$, $R_{s0}=[r^2+(z+s_0)^2]^{1/2}$, $y_s=\omega t_s$, $y_{s0}=\omega t_{s0}$, $t_s=R_s/c_2$, $t_{s0}=R_{s0}/c_2$, y, y_s, $y_{s0}\in[0, 2\pi]$, $\phi(y_s, y)$ and $\phi(y_{s0}, y)$ are the same expression as $\phi(y_0, y)$ shown in Eq. (9) but instead of y_0 by y_s and y_{s0} respectively. And s_0 ($L>s_0\gg b$) is chosen in the following.

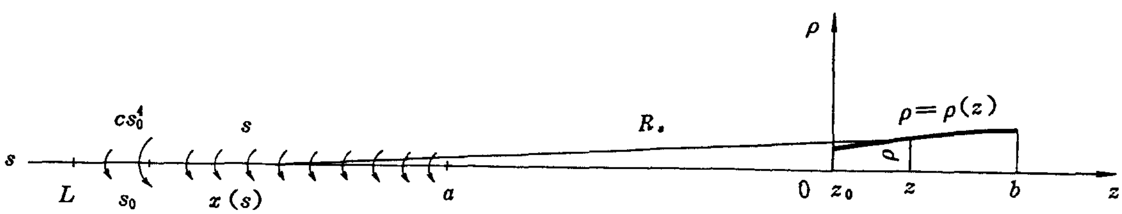

Fig. 2 DPRC distributed in $[a, L]$ which produces a zero stressed surface of revolution $\rho=\rho(z)$ in the elastic space at any time

By stress analysis, the normal direction n of the zero stressed plane in a small cylindrical element of a field point $N(r, z)$ is determined by

$$tg\,\alpha=\tau_{\theta r}/\tau_{\theta z}$$

where $\alpha+\pi/2$ is the angle between n and the z-axis. Every element of points has such a zero stressed plane, therefore a family of zero stressed surfaces of revolution occurs in the elastic space. One of them passing through $N(r, z)$ is constructed to coincide with the given generator $\rho=\rho(z)$, where the derivative $\rho'=d\rho/dz$ is equal to

$$\rho'=d\rho/dz=\tau_{\theta r}/\tau_{\theta z} \tag{15}$$

Substituting Eq. (14) into Eq. (15), we have

$$C\rho'\phi(y_{s0}, y)=\int_a^L[\rho-\rho'(z+s)]\,\phi(y_s, y)R_s^{-5}x(s)\,ds \tag{16}$$

By the theorem of integral mean value of function, there exists a $\phi(y_{s0}, y)$ $(\phi(y_{sa}, y)\leqq\phi(y_{s0}, y)\leqq\phi(y_{sL}, y))$ such that

$$\phi(y_{s0}, y)\int_a^L[\rho-\rho'(z+s)]R_s^{-5}x(s)\,ds=\int_a^L[\rho-\rho'(z+s)]R_s^{-5}\phi(y_s, y)x(s)\,ds \tag{17}$$

where $y_{sa}=\omega R_{sa}/c_2$, $R_{sa}=[\rho^2+(z+a)^2]^{1/2}$, and so on.

Now, s_0 is chosen by Eq. (17), and substituting Eq. (17) into Eq. (16), we have

$$C\rho'=\int_a^L[\rho-\rho'(z+s)]R_s^{-5}x(s)\,ds \tag{18}$$

Since constant C and generator $\rho=\rho(z)$ are given (without loss of generality, let C=1), then, Eq. (18) is a Fredholm integral equation of the first kind. Obviously, Eq. (18) is independent of time. This means that the zero stressed surface of revolution with generator $\rho=\rho(z)$ appears in the elastic space and keeps unchange at any time. This characteristic coincides with the zero stressed boundary condition of dynamic torsion of shaft of revolution. Thus the above solution can be regarded as the solution of dynamic torsion of shaft of revolution. Eq. (18) also shows that the solution $x(s)$, like a static case, is wholly determined by the given generator.

Once $x(s)$ of Eq. (18) has been solved, the stresses can be calculated by Eq. (14). The relation between $x(s)$ and the given dynamic torque $M_T(z_0)=T_0\sin\omega(t-t_i)$ can be obtained by

$$T_0\sin\omega(t-t_i)=M_T(z_0)=2\pi\int_0^{\rho(z_0)}\tau_{\theta z}r^2dr \tag{19}$$

Substituting Eq. (14b) with mid-value $\phi(y_{s0}, y)$ into Eq. (19), and again using the theorem of integral mean value of function, there exists a $r_m(0<r_m<\rho)$ *in* $z=z_0$ such that

$$\phi(y_{m0}, y)K(z_0)=\int_0^{\rho(z_0)}\Big[\int_a^L(z_0+s)R_s^{-5}x(s)\,ds-c\Big]r^3\phi(y_{s0}, y)\,dr \tag{20}$$

then, we have

$$T_0\sin\omega(t-t_i)=-(3/4)A\,(y_{m0})K(z_0)\sin(y-y_{1m}) \tag{21}$$

where $y_{m0}=\omega t_{m0}$, $t_{m0}=R_{m0}/c_2$, $R_{m0}=[r_m^2+(z_0+s_0)^2]^{1/2}$,

$$K(z_0)=\int_0^{\rho(z_0)}\Big[\int_a^L(z_0+s)R_s^{-5}x(s)\,ds-c\Big]r^3dr \tag{22}$$

$A(y_{m0})$ and y_{1m} are the same expressions as Eqs. (10) and (11), instead of y_0 by y_{m0}, respectively. From Eq. (21), we have

$$T_0=-(3/4)A(y_{m0})K(z_0) \tag{23}$$

$$t_i=y_{1m}/\omega \tag{24}$$

The maximum stress is of interest to engineers. Suppose that $2\rho(z_0)$ is the maximum diameter of the shaft, then, the maximum stresses of the shaft, by Eq. (14), are:

$$\tau_{\theta z}(\rho, z_0)=-(3\rho/(8\pi))\, f_1(\rho)\, \phi(y_{s0}, y) \tag{25a}$$

$$\tau_{\theta r}(\rho, z_0)=-(3\rho^2/(8\pi))\, f_2(\rho)\, \phi(y_{s0}, y) \tag{25b}$$

$$f_1(\rho)=\int_a^L (z_0+s)R_{sp}^{-5}\, x(s)\, ds-C, \quad R_{sp}=[\rho^2+(z_0+s)^2]^{1/2}, \quad f_2(\rho)=\int_a^L R_{sp}^{-5}\, x(s)\, ds.$$

Although $x(s)$ is independent of dynamic factor, but stress varies with frequence ω and time t. The so-called "dangerous" frequency ω_d and time t_d, at which $\tau_{\theta z}(\rho, z_0)$ and $\tau_{\theta r}(\rho, z_0)$ attend to maximum, can be found by solving the following optimization problem (O.P.):

O.P.: Find ω and t such that

Max $\tau_{\theta z}(\rho, z_0)$ and $\tau_{\theta r}(\rho, z_0)$ of Eq. (25)

s.t. Eq. (23), i.e., $A(y_{m0})=-(4/3)T_0/K(z_0)=C_1$

This is a constrainted O.P., it can be solved as an un-constrainted O.P. by the method of Lagrange multiplier. Let

$$\Psi=\phi(y_{s0}, y)+\beta[A(y_{m0})-C_1] \tag{26}$$

where β is the Lagrange multiplier. Then

$$\partial\Psi/\partial\omega=(\partial\Psi/\partial y_{s0})(\partial y_{s0}/\partial\omega)=0 \tag{27}$$

$$\partial\Psi/\partial t=(\partial\Psi/\partial y)(\partial y/\partial t)=0 \tag{28}$$

$$\partial\Psi/\partial\beta=0 \tag{29}$$

we get:

$$y-y_1=\pi/2 \tag{30}$$

$$\beta=-t_{s0}/t_{m0} \tag{31}$$

$$A(y_{m0})=C_1 \tag{32}$$

From Eq. (32),

$$T_0=-(3/4)K(z_0)A(y_{m0}), \qquad (0\leqq y_{m0}\leqq 2\pi) \tag{33}$$

Eq. (33) shows that the amplitude of dynamic torque T_0 related to $y_{m0}=\omega_d t_{m0}$, by Eq. (10), where

$$\omega_d=y_{m0}/t_{m0}=2\pi/t_{m0} \tag{34}$$

$$T_0=T_{0max}=-(3/4)K(z_0)A(2\pi) \tag{35}$$

From Eq. (30), we get $t_d=t_1+t_{m0}/4$, however, t_d is not important, while ω_d is interesting to engineers. Eq. (34) shows that the dangerous frequency related to $t_{m0}=R_{m0}/c_2$, which relates to the given generator $\rho=\rho(z)$.

4. Numerical solution and example

Eq. (18) is rewritten simply as

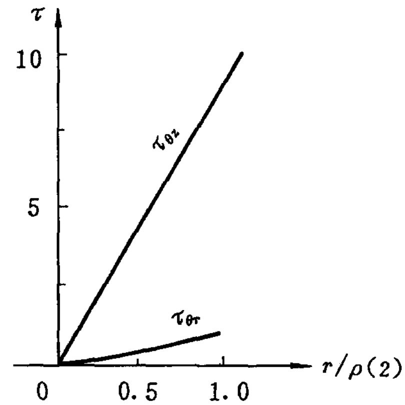

Fig. 3 The distribution of stresses on cross section z=2 at t=300 sec

$$\int_a^L K(z,s)x(s)\,ds = F(z) \tag{36}$$

$$K(z,s)=[\rho-\rho'(z+s)]R_s^{-5}, \qquad F(z)=C\rho'$$

Eq. (36) is solved by its discrete form, i.e.,

$$\sum_{j=1}^{n} K_{ij}\,x_j = F_i, \qquad i,j=1,2,\cdots,n.$$

$$x_j = x(s_j), \qquad F_i = F(z_i), \qquad K_{ij}=\int_{(j-1)\Delta L}^{j\cdot\Delta L} K(z_i,s)\,ds,$$

$$\Delta L=(L-a)/n, \qquad s_j = j\cdot\Delta L, \qquad z_i = i\cdot(b-z_0)/n. \tag{37}$$

Once x_j has been solved from Eq. (37), the stress and the corresponding torque $M_T(z_0)$ can be numerically calculated by Eq. (14) and Eq. (19), where the integral is replaced by summation, respectively.

Numerical example of a dynamic torsion shaft of revolution with generator $\rho=\rho(z)=\sqrt{1+0.1z}$ located in z_0=0, b=5 as shown in Fig. 2. In this example, $m=7.8\times10^3$ (Pa.sec^2/M^2), $G=8.0\times10^{10}$ (Pa), a=10 (M), L=20 (M), n=20 are used.

The distributions of stresses on different cross sections, at different time, due to dynamic torque $M_T(0)$ at z_0=0 are plotted and shown to be similar in pattern. A typical pattern of distribution of stresses on cross section z=2 (M) is presented in Fig. 3. In which the stress $\tau_{\theta z}$ is almost linear and $\tau_{\theta r}$ is obviously far less than $\tau_{\theta z}$. This result approaches to that of a torsion

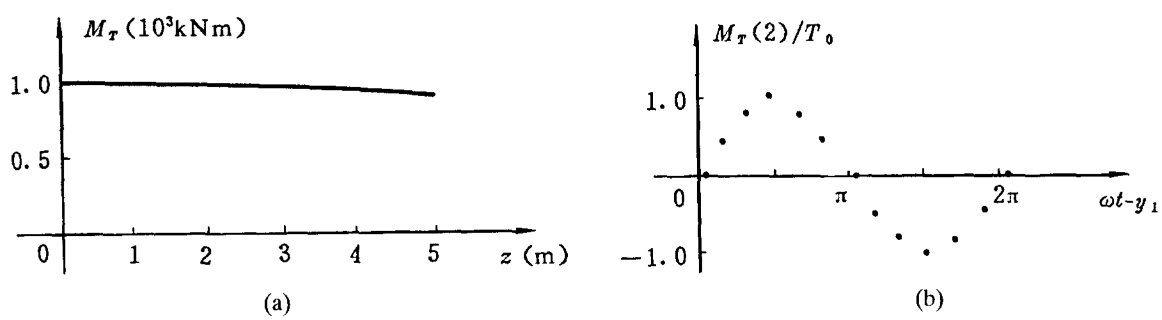

Fig. 4 (a) Torque of different cross section at t=300 sec. (b) Torque of cross section z=2 at different time cylinder and is expected, since the shaft $\rho=\sqrt{1+0.1z}$ is slowly varied in diameter. Fig. 4 shows the torque of different cross sections at the same time and the torque of the same cross section at different time.

The ratio of maximum amplitude of dynamic torque T_{0max} due to dangenous frequency ω_d of Eq. (34), to its minimum amplitude T_{0s} due to $\omega=0$ (the static case), by Eqs. (35) and (23), is

$$T_{0max}/T_{0s} = (1 + y_{m0}^2/3 + 4_{m0}^4/9)^{1/2} \approx y_{m0}^2/3 = (2\pi)^2/3 = 13.0 \tag{38}$$

This result shows that the effect of dynamic action is of significance.

5. Conclusions

The problem of dynamic torsion of shaft of revolution, twisted by end torque $T_0 \sin\omega(t-t_i)$, is studied by LLIEM with both advantages of the direct method (e.g., easy to understand, without making integral transform and its inverse transform) and indirect method (less computation time, like a static problem). In which, a zero stressed surface of revolution with given generator $\rho=\rho(z)$ can be constructed to appear in an elastic space at any time by DPRC, with unknown intensity $(x(s)+C\cdot s_0^4)\sin\omega t$, distributing on $[a,\ L](L>a\gg b)$ along the $s(=-z)$-axis outside the shaft occupation. Then, the problem is reduced to a Fredholm integral equation of the first kind and can be solved by known methods.

The advantage of LLIEM over BEM for dynamic analysis is the obtained integral equation is 1-D and non-singular, while the BEM needs to treat singularity (e.g., $0(1/r)$ and $0(1/r^2)$, see, Y. K. Cheung *et al.* 1993) by special method.

Acknowledgements

The author sincerely thanks to reviewer for suggestions of improvements of the paper.

References

Cheung, Y.K., Lei, Z.X. and Tham, L.G. (1993), "Application of time-domain BEM to elastodynamics analysis", *Computational Mechanics,* **1**, Valliappan, Pulmano & Tin-Loi (eds), Balkema, Rotterdam, 61-66.

Eringen, A.C. and Suhubi, E.S. (1975), *Elastodynamics,* Academic Press, New York.

Tham, L.G., Cheung, Y.K. and Lei, Z.X. (1994), "Torsional dynamic analysis of single piles by time-domain BEM", *J. Sound Vib.,* **174**(4), 505-519.

Yun, T.Q. (1979), "An integral equation method for solving the torsion problem of bodies of revolution", J.H.I.T., No. 1, 82-97.

Yun, T.Q. (1981), "Pile analysis by simple integral equation method", *Appl. Math. & Mech.,* **2**(3), 331-349.

Yun, T.Q., Shao, U.C. and Qiu, C.Q. (1981), "Analysis of ellipsoid compressed by two axial concentrated forces at two ends", *Appl. Math. & Mech.,* **2**(6), 699-710.

Yun, T.Q. (1988), "Torsion of rigid circular shaft of varying diameter embedded in an elastic half space", *Appl. Math. & Mech.,* **9**(5), 449-453.

Yun, T.Q. (1990), "Torsion of elastic shaft of revolution embedded in an elastic half space", *Appl. Math. & Mech.,* **11**(6), 489-498.

Yun, T.Q. (1991), "Analysis of sloping rigid pile under arbirtary load by Line-Loaded Integral Equation Method", *Computational Mechanics, Proc. of APCOM*, Hong Kong, Cheung, Lee, & Leung (eds), Balkema, Rotterdam, **1**, 845-850.

Yun, T.Q. and Su, C. (1992), "Analysis of shaft embedded in granular half space by the line-load integral equation method", *Int. J. Computers & Structures,* **43**(4), 729-735.

Yun, T.Q. and Gu, H.N. (1993), "The solution of dynamic point-ring-couple in an elastic space and its properties", *Appl. Math. & Mech.,* **14**(10), 845-852.

Yun, T.Q. (1994), "Application of LLIEM to elastodynamic analysis of torsion shaft of revolution", *Proceedings of the Third World Congress on Computational Mechanics (WCCM)*, Chiba, Japan, **1**, 157-158.

Yun, T.Q. and Li, L. (1995), "Analysis of in-plane-hinge jointed rigid sloping piles", *Int. J. Computers & Structures,* **55**(2), 341-346.