Structural Engineering and Mechanics, Vol. 6, No. 4 (1998) 411-425
DOI: http://dx.doi.org/10.12989/sem.1998.6.4.411

New stability equation for columns in unbraced frames

Hesham S. Essa†

Department of Civil Engineering, Ain Shams University, Cairo, Egypt

Abstract. The effective length factor of a framed column may be determined by means of the alignment chart procedure. This method is based on many unrealistic assumptions, among which is that all columns have the same stiffness parameter, which is dependent on the length, axial load, and moment of inertia of the column. A new approximate method is developed for the determination of effective length factors for columns in unbraced frames. This method takes into account the effects of inelastic column behaviour, far end conditions of the restraining beams and columns, semi-rigid beam-to-column connections, and differentiated stiffness parameters of columns. This method may be implemented on a microcomputer. A numerical study was carried out to demonstrate the extent to which the involved parameters affect the K factor. The beam-to-column connection stiffness, the stiffness parameter of columns, and the far end conditions of restraining members have a significant effect on the K factor of the column under investigation. The developed method is recommended for design purposes.

Key words: unbraced frames; stability; design; columns; effective length factor; inelastic behaviour; concrete frames; steel frames; buckling.

1. Introduction

Most concrete and steel building design standards recommend the alignment chart procedure for the determination of the effective length factors of columns in both braced and unbraced frames. The shortcomings of this simple graphical procedure are due to some limitations imposed on its original derivation. These limitations are attributed to many assumptions such as rigid connections, the elastic behaviour of all members, and that the stiffness parameter is constant for all columns. However, these assumptions may be violated by the real configuration of the structure, leading to a design which may be overly conservative or even unsafe, as was noted by many investigators such as Goncalves (1992), Duan and Chen (1988), Bridge and Fraser (1987), Fraser (1983), and Yura (1971). Goncalves (1992) presented a new approximate method that overcomes such limitations for determining effective length factor for columns in braced frames. A graphical procedure was developed by Bridge and Fraser (1987) to consider the influence of differentiated stiffness parameters of the connected members in braced frames. Duan and Chen (1988) proposed a simple modification of the alignment chart procedure in order to take into account the effect of the boundary conditions of top and bottom columns for braced frames. Fraser (1983) observed in braced frames that if the stiffness parameter of some of the attached columns is larger than the one of the column being analyzed, the effective length factor increases significantly over the value predicted by the alignment chart. Yura (1971)

† Assistant Professor

proposed an iterative procedure to determine the effective length factor in the inelastic range of column behaviour.

Essa (1997) presented a new design model used to derive expressions for the effective length factors for columns in unbraced frames, taking into account the effects of boundary conditions at the far ends of the columns above and below the column in question. A simple modification of the alignment chart procedure has been suggested to take such effect into consideration.

In this paper, a design approach is developed to predict the effective length factor for columns in unbraced frames taking into account the inelastic behaviour, semi-rigid connections, far end conditions, and differentiated stiffness parameters of the connected columns. Also, some numerical results are presented to gain some insight into the extent to which the involved variables affect the effective length factor. Finally, some important limitations are introduced on using the alignment chart procedure for determining the K factor to insure conservative design.

2. Modified slope-deflection equations

Consider a typical column member AB, as depicted in Fig. 1, over which an axial load P is applied. Using the slope-deflection equation procedure and taking into account the inelastic member behaviour (Goncalves 1992), the moments at ends A and B are given as

$$M_A = \frac{EI\eta}{L}\left[C\,\theta_A + S\,\theta_B - (C+S)\frac{\Delta}{L}\right] \tag{1a}$$

$$M_B = \frac{EI\eta}{L}\left[C\,\theta_B + S\,\theta_A - (C+S)\frac{\Delta}{L}\right] \tag{1b}$$

where C and S are the stability functions given by

$$C = \alpha L \,\frac{\sin\alpha L - \alpha L \cos\alpha L}{2 - 2\cos\alpha L - \alpha L \sin\alpha L} \tag{2a}$$

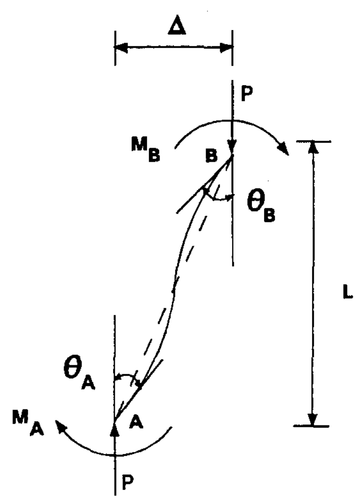

Fig. 1 Deformed configuration of a typical column member

$$S = \alpha L \frac{\alpha L - \sin\alpha L}{2 - 2\cos\alpha L - \alpha L \sin\alpha L} \tag{2b}$$

in which

$$\alpha = \sqrt{\frac{P}{EI\eta}} \tag{3}$$

and

$$\eta = \frac{E_t}{E} \tag{4}$$

where I=moment of inertia of the cross section about the axis perpendicular to the plane of buckling; L=length of the column; E=modulus of elasticity; and E_t=tangent modulus. When the axial load P is zero, it can be shown that C=4 and S=2.

Several methods can be used to evaluate the parameter $\eta=E_t/E$. Goncalves (1992) used the column-strength curve given by the Structural Stability Research Council (SSRC) to obtain the following expression

$$\eta = 1 \qquad \text{if } \lambda \geq \lambda_c \tag{5a}$$

$$\eta = \left(\frac{\lambda}{\lambda_c}\right)^2 \left[2 - \left(\frac{\lambda}{\lambda_c}\right)^2\right] \qquad \text{if } \lambda \geq \lambda_c \tag{5b}$$

where

$$\lambda_c = \sqrt{\frac{2\pi^2 E}{F_y}} \tag{6}$$

is the slenderness ratio separating elastic and inelastic buckling; F_y=yield stress; and

$$\lambda = \frac{KL}{r} \tag{7}$$

is the slenderness ratio of the member; K=effective length factor; and r=radius of gyration of the cross section.

3. Generalized stability functions

The procedure given by Goncalves (1992) is used herein to develop expressions for the generalized stability functions of a beam member, which take into account its boundary conditions. Neglecting the axial force in the member, the values of stability functions C and S are given as 4 and 2, respectively. If both ends A and B, of the beam are not rigidly but flexibly connected to other members (Fig. 2), the slope-deflection equation takes the form

$$M_A = \frac{EI}{L}[D\theta_A + T\theta_B] \tag{8a}$$

$$M_B = \frac{EI}{L}[D\theta_B + T\theta_A] \tag{8b}$$

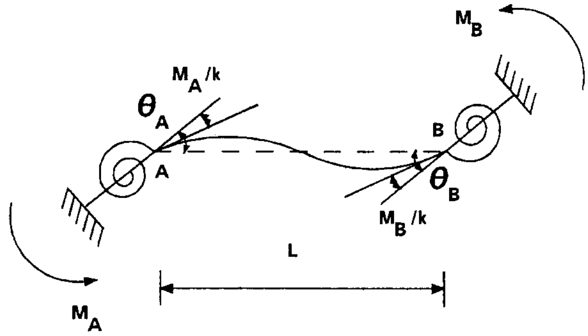

Fig. 2 Beam member with semi-rigid connections

where

$$D = \frac{4\left(1+\dfrac{3}{\kappa}\right)}{\left(1+\dfrac{6}{\kappa}\right)\left(1+\dfrac{2}{\kappa}\right)} \tag{9a}$$

$$T = \frac{2}{\left(1+\dfrac{6}{\kappa}\right)\left(1+\dfrac{2}{\kappa}\right)} \tag{9b}$$

are called generalized stability functions; and

$$\kappa = \frac{k}{\left(\dfrac{EI}{L}\right)} \tag{9c}$$

is a dimensionless joint stiffness coefficient; and k=joint stiffness. It is assumed that joints behave linearly and that both ends of the beam have the same connection stiffness.

If end A is flexibly connected to other members and the other end B has either hinged or fixed boundary condition, the expressions for end moments are given by

$$M_A = \frac{EI}{L}\left[4\theta_A - \frac{4M_A}{k} + 4\theta_B\right] \tag{10a}$$

$$M_B = \frac{EI}{L}\left[4\theta_B - \frac{2M_A}{k} + 2\theta_A\right] \tag{10b}$$

Solving these equations for end moments, the slope-deflection equation may be written as

$$M_A = \frac{EI}{L}\left[D\,\theta_A + T\,\theta_B\right] \tag{11a}$$

$$M_B = \frac{EI}{L}\left[H\,\theta_B + T\,\theta_A\right] \tag{11b}$$

where D, T, and H are generalized stability functions, given by

$$D = \frac{4}{1 + \dfrac{4}{\kappa}} \tag{12a}$$

$$T = \frac{2}{1 + \dfrac{4}{\kappa}} \tag{12b}$$

$$H = \frac{4 + \dfrac{12}{\kappa}}{1 + \dfrac{4}{\kappa}} \tag{12c}$$

It should be noted that when the beam far end B is fixed, the rotation θ_B is set as zero, whereas the end moment M_B vanishes when the end B is hinged.

4. General effective length factor equation

The model used for the determination of the value of K, the effective length factor, for a framed column subjected to sidesway is shown in Fig. 3. The column in question is denoted by $C2$ in the Figure. The assumptions used for this model are:

1. All members are prismatic.
2. The axial forces in all girders are negligible.
3. Adjacent columns reach their respective critical loads simultaneously.
4. The near ends of all girders are flexibly connected to other members.

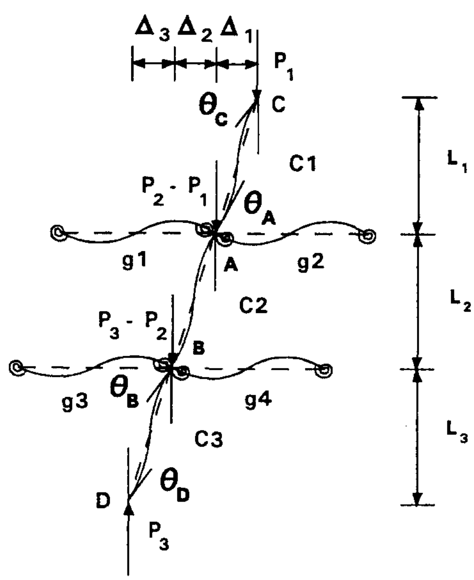

Fig. 3 Subassemblage model for K-factor in unbraced frames

5. The boundary conditions at the far ends of girders may be flexibly connected, hinged or fixed.
6. The boundary conditions at the far ends of columns $C1$ and $C3$ may be rigid, hinged or fixed.
7. Story drifts are generally different in magnitude.
8. The stiffness parameter $L\sqrt{P/EI}$ may be different from one column to another.

By applying the slope-deflection equation approach to the subassemblage shown in Fig. 3, it can be shown that the equation for the effective length factor K of column $C2$ can be solved from

$$\det \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{vmatrix} = 0 \tag{13}$$

where

$$a_{11}=K_{c2}C_2+K_{c1}(C_1+R_1S_1)+K_{g1}(D_1+R_7T_1)+K_{g2}(D_2+R_8T_2) \tag{14}$$
$$a_{12}=K_{c2}S_2+K_{c1}R_2S_1 \tag{15}$$
$$a_{13}=-K_{c1}(C_1+S_1(1-R_3)) \tag{16}$$
$$a_{15}=a_{23}=a_{34}=a_{35}=a_{43}=a_{45}=a_{53}=a_{54}=0 \tag{17}$$
$$a_{21}=K_{c2}S_2+K_{c3}R_4S_3 \tag{18}$$
$$a_{22}=K_{c2}C_2+K_{c3}(C_3+R_5S_3)+K_{g3}(D_3+R_9T_3)+K_{g4}(D_4+R_{10}T_4) \tag{19}$$
$$a_{24}=-a_{41}=-a_{42}=a_{14}=-K_{c2}(C_2+S_2) \tag{20}$$
$$a_{25}=-K_{c3}(C_3+S_3(1-R_6)) \tag{21}$$
$$a_{31}=-K_{c1}(C_1+S_1)(1+R_1) \tag{22}$$
$$a_{32}=-K_{c1}R_1(C_1+S_1) \tag{23}$$
$$a_{33}=K_{c1}(C_1+S_1)(2-R_3)-\pi^2K_{c2}L_1P_1/L_2K^2P_2 \tag{24}$$
$$a_{44}=-2K_{c2}(C_2+S_2-\pi^2/2K^2) \tag{25}$$
$$a_{51}=-K_{c3}R_4(C_3+S_3) \tag{26}$$
$$a_{52}=-K_{c3}(C_3+S_3)(1+R_5) \tag{27}$$
$$a_{55}=K_{c3}(C_3+S_3)(2-R_6)-\pi^2K_{c2}L_3P_3/L_2K^2P_2 \tag{28}$$

in which R_1 through R_{10} are the far end coefficients given in Table 1; K_{c1}, K_{c2}, and K_{c3} are the bending stiffnesses of columns $C1$, $C2$ and $C3$, respectively; L_1, L_2, and L_3 are the lengths of columns $C1$, $C2$ and $C3$, respectively; K_{g1}, K_{g2}, K_{g3} and K_{g4} are the bending stiffnesses of girders $g1$, $g2$, $g3$ and $g4$, respectively; P_1, P_2, and P_3 are the axial forces in columns $C1$, $C2$ and $C3$, respectively; D_1, T_1, D_2, T_2, D_3, T_3, D_4 and T_4 are the generalized stability functions evaluated by Eq. (9) or (12) for girders $g1$, $g2$, $g3$ and $g4$, respectively; and C_1, S_1, C_2, S_2, C_3 and S_3 are the usual C and S stability functions, evaluated for columns $C1$, $C2$ and $C3$, respectively. For the i-th column (Ci), the value of the stiffness parameter, α_i, required to determine the stability functions C_i and S_i from Eq. (2), is obtained as

$$\alpha_i = \frac{\pi}{KL_2}\sqrt{\frac{P_iI_2\eta_2}{P_2I_i\eta_i}} \tag{29}$$

in which I_i and I_2 are the moments of inertia of columns Ci and $C2$, respectively; P_i and P_2 are

Table 1 Far end coefficients

Far and condition	Column $C1$			Column $C3$			$g1$	$g2$	$g3$	$g4$
	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9	R_{10}
Fixed	0	0	0	0	0	0	0	0	0	0
Rigid (Flexible)	0	1	0	1	0	0	1	1	1	1
Hinged	$-\frac{S_1}{C_1}$	0	$1+\frac{S_1}{C_1}$	0	$-\frac{S_3}{C_3}$	$1+\frac{S_3}{C_3}$	$-\frac{T_1}{H_1}$	$-\frac{T_2}{H_2}$	$-\frac{T_3}{H_3}$	$-\frac{T_4}{H_4}$

the axial loads in columns Ci and $C2$, respectively; and η_i and η_2 are the ratios of tangent modulus to elastic modulus of columns Ci and $C2$, respectively. To solve the stability equation Eq. (13), a simple algorithm based on the bisection method with linear interpolation was implemented on a personal microcomputer.

The first two rows of Eq. (13) are obtained by considering the equilibrium of joints A and B, respectively. Rows 3, 4, and 5 of Eq. (13) are determined by writing the story shear equilibrium conditions for columns $C1$, $C2$, and $C3$, respectively. By making use of the far end coefficients, R's, Eq. (13) is applicable for all possible far end conditions. The values of R's given in Table 1 are used to enforce the following conditions:

a) If the far end is hinged, the moment is zero at that end.

b) If the far end is fixed, the rotation is zero at that end.

c) If the far end of column $C1$ (or $C3$) is rigid, the far end rotation is taken as $\theta_C=\theta_B$ (or $\theta_D=\theta_A$).

d) If the far end of a beam is rigid, the rotations at the near and far ends are equal in magnitude and direction.

After applying the above conditions, the only five unknowns involved in establishing Eq. (13) are θ_A, θ_B, Δ_1/L_1, Δ_2/L_2 and Δ_3/L_3. Because the value of the stiffness parameter, $L\sqrt{P/EI}$, is different from one column to another, different stability functions are used for columns. Assuming that the load history is such that the ratio P_1/P_2 remains constant for columns C_1 and C_2, it can be shown that $K_{of\,C1}{}^2=K^2L_2P_2K_{c1}/L_1P_1K_{c2}$ and the expression for a_{33} may be obtained as given by Eq. (12). Eq. (16) may be similarly derived.

In cases of leaned column frames and when the flexural stiffnesses of the columns in a story differ significantly, a story-based procedure may be used (LeMessurier 1977, Lui 1992, Hajjar and White 1994). Alternatively, the preliminary K factors may be obtained first for all columns in that story using Eq. (13) and then the simplified equation proposed by LeMessurier (1977) given as:

$$K=\sqrt{\frac{\pi^2 EI_2\Sigma P}{P_2L_2^2\Sigma P_{cr}}} \tag{30}$$

may be applied, where ΣP_{cr}=the sum of elastic critical loads of all non-leaning columns in the story evaluated using the preliminary K factors, P_2=axial load in the column under design, and ΣP=the total gravity load on the story.

5. Parametric study

In developing the alignment chart, it was assumed that the stiffness parameters, $L\sqrt{P/EI}$, of

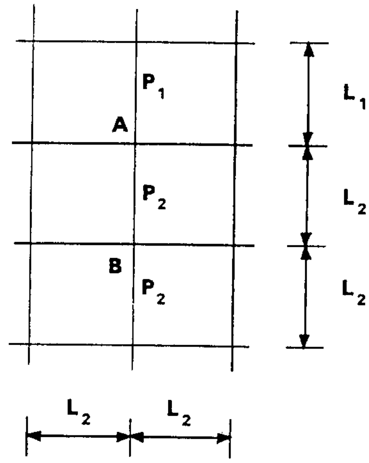

Fig. 4 Unbraced multistory frame

all columns are identical, where L=length of column, P=axial load in column, and I=moment of inertia of column. Also, it was assumed that the far end conditions of top and bottom columns as well as the beam-to-column connections are rigid. However, in the real configuration of a structure, the stiffness parameter may be different from one column to another; the beam-to-column connections are semi-rigid; and the hinged as well as the fixed conditions can be encountered at the far ends of top and bottom columns. To demonstrate the extent to which all of these parameters affect the effective length factor of columns in unbraced frames, the structural model shown in Fig. 4 is considered to investigate the stability of column AB. Initially, the frame members are composed of $W8\times 58$ shapes from $A36$ steel with F_y=250 MPa and E=250, 800 MPa. All members have the same length (L_1=L_2=4 m). The axial loads in columns are equal (P_1=P_2). The far end conditions of top and bottom columns as well as the beam-to-column connections are assumed to be rigid.

5.1. Effect of column loads

The axial loads in columns do not have any contribution in the alignment chart procedure. However, many investigators (Goncalves 1992, Bridge and Fraser 1987 and Fraser 1983) have indicated that the effect of axial loads in columns is significant for the determination of the K factor in braced frames.

To investigate the influence of the axial load in the attached columns on the K factor, the axial load P_2 in column AB was kept equal to that in bottom column; while the axial load P_1 in top column was varied such that $0 \leq P_1 / P_2 \leq 2.0$. The moments of inertia and the lengths of all columns were kept constant. The plot of the K factors, as obtained from the elastic and inelastic models and from the alignment chart, versus the axial load ratio P_1 / P_2 is shown in Fig. 5. The effective length factor obtained by means of the alignment chart procedure is conservative only when $P_1 / P_2 < 1.0$. On the other hand, when this condition is not fulfilled, the stiffness parameter of the upper column exceeds that of column AB and the K factors determined from the elastic analysis may be significantly larger than those given by the alignment chart. Fig. 5 also shows that the K factor obtained from an inelastic analysis is usually smaller than those obtained from inelastic analysis, indicating that as long as inelasticity is confined to the columns and all

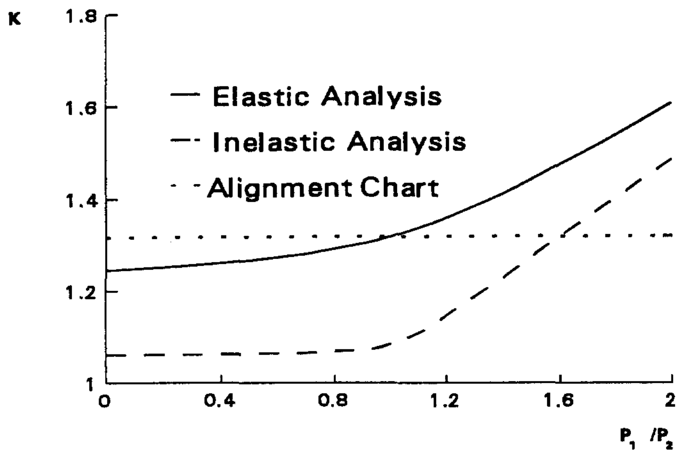

Fig. 5 Effect of top column load on K-factor

restraining beams remain elastic, the use of elastic K factor will usually result in a conservative design. This observation was noted by many investigators such as Yura (1971), Chapius and Galambos (1982), and Chen and Lui (1991).

5.2. Effect of column lengths

To study the effect of column lengths, the elastic and inelastic K factors were determined for column AB in Fig. 4. The length L_2 of column AB as well as the bottom column was kept equal to 4 m, whereas the length L_1 of top column was varied from 2 m to 8 m. The moments of inertia and the axial loads of all columns were kept constant. In Fig. 6, the K factors, as obtained from the elastic and inelastic models and from the alignment chart, are plotted as functions of the ratio L_1/L_2. The K factors determined from elastic and inelastic models grow up when the ratio L_1/L_2 increases. On the other hand, as the ratio L_1/L_2 increases there is a reduction in the K factor determined from the alignment chart. When the length ratio $L_1/L_2>1.0$, the stiffness parameter of the upper column exceeds that of column AB and the effective length factor deduc-

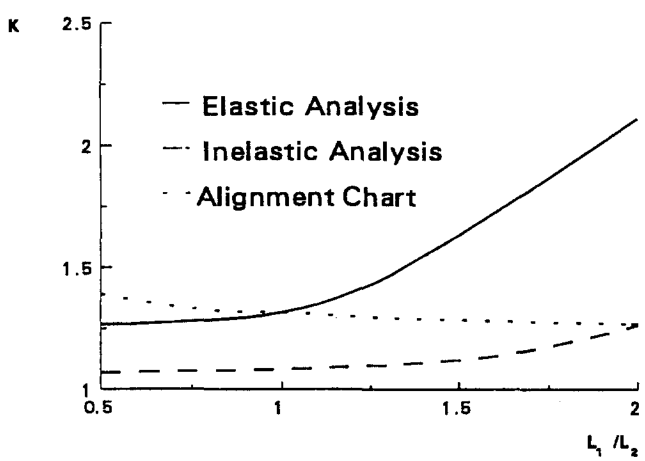

Fig. 6 Effect of top column length on K-factor

ed from the elastic model may be significantly larger than the value predicted by the alignment chart.

In the alignment chart procedure the relative joint bending stiffness ratios at the A-th and B-th ends of the column are used to determine the K factor. Since the column length is involved in evaluating the bending stiffness ratios, one can get a conclusion that the effect of column length is fully accounted for. However, this represents a misleading conclusion since, as shown in Fig. 6, the results obtained from elastic analysis and from the alignment chart are not identical.

5.3. Effect of column cross sections

With the purpose of assessing the influence of top and bottom column cross sections on the effective length factor, column AB in Fig. 4 was analyzed. The $W8\times58$ shape (I=9,490 cm^4, r= 9.28 cm) of the top column was progressively substituted by several shapes from the $W8\times18$ (I= 2,576 cm^4, r=8.71 cm) shape to the W12$\times$72 (I=24,800 cm^4, r=13.51 cm) shape. The lengths and the axial loads of all columns were kept constant. The results are shown in Fig. 7, in which the K factors, determined from the elastic and inelastic analyses as well as from the alignment chart, are plotted versus the ratio I_1/I_2, where I_1 and I_2 are the moments of inertia of the top column and the $W8\times58$ shape, respectively. The K factors obtained from the elastic analysis are significantly different from those determined by means of the alignment chart procedure. Also, it may be observed that there are some dramatic changes in the inelastic K factors. This phenomenon may be attributed to the fact that two cross sectional parameters are involved in Fig. 7, the moment of inertia and the radius of gyration.

In order to investigate separately the effect of moment of inertia on the K factor, the $W8\times58$ shape of the top column was replaced by several hypothetical cross sections which were chosen such that the radius of gyration was kept the same as that of $W8\times58$ but the moment of inertia assumed different values. The results are presented graphically in Fig. 8, where the variations in the K factors, obtained from the elastic and inelastic models and from the alignment chart, are plotted as functions of the ratio I_1/I_2. It is obvious that the K factors determined from the elastic model are slightly less than those given by the alignment chart. For relatively small values of I_1/I_2, the K factors predicted by the elastic and inelastic analyses are identical. Observe that for the

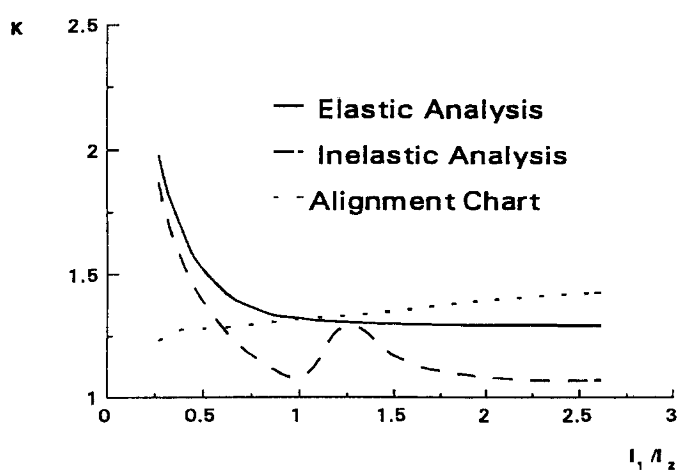

Fig. 7 Effect of top column cross section on K-factor

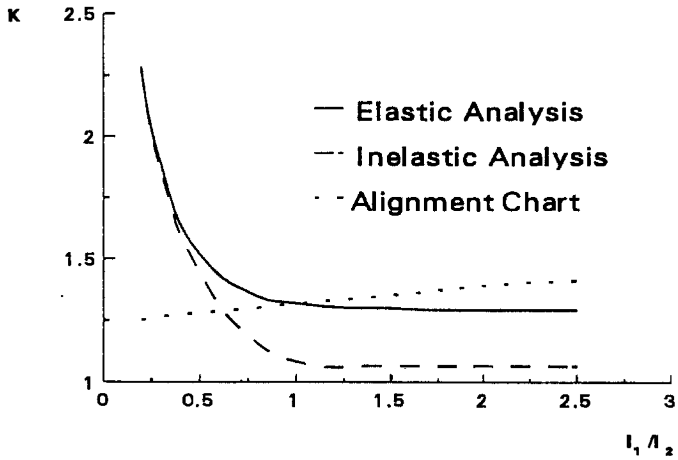

Fig. 8 Effect of top column inertia on K-factor

elastic analysis as well as the inelastic one, the K factor is almost constant when $I_1/I_2>1$.

Finally, the separate effect of the radius of gyration on the K factor was investigated. To this end, the $W8\times58$ shape of the top column was progressively substituted by several hypothetical cross sections which were selected such that the moment of inertia was kept the same as that of the $W8\times58$ shape but the radius of gyration assumed different values. In Fig. 9, the elastic and inelastic K factors are plotted against the ratio of r_1/r_2, where r_1 and r_2 are the radii of gyration of the upper column and column AB, respectively. As can be observed from Fig. 9, the use of elastic K factor does not necessarily result in a conservative design. It should be noted that the K factors determined from the alignment chart are identical to those given by the elastic analysis.

5.4. Effect of flexible beam-to-column connections

To demonstrate the effect of the flexibility of beam-to-column connections on the K factor,

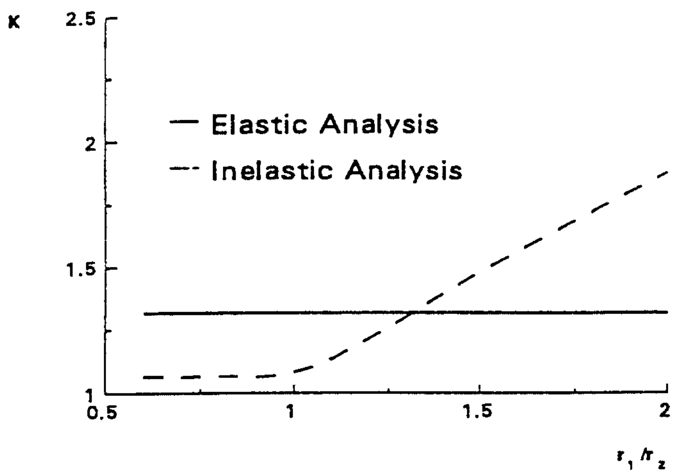

Fig. 9 Effect of top column radius of gyration on K-factor

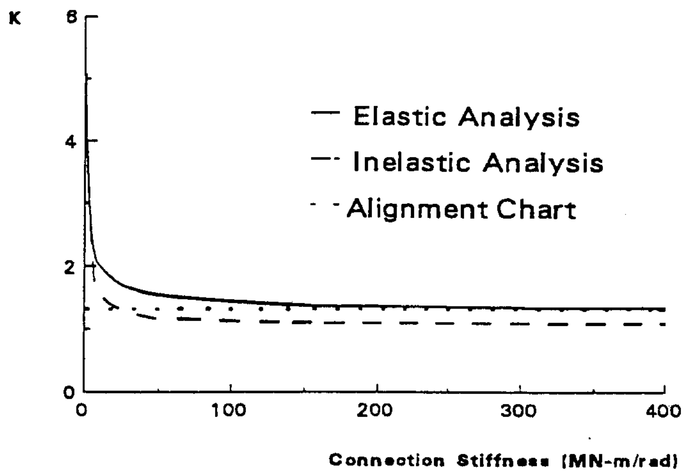

Fig. 10 Effect of connection stiffness on K-factor

the connection stiffnesses of the four beam-to-column joints attached to column AB (Fig. 4) were varied from zero to 400 MN-m/rad which represents a realistic range for the initial tangent stiffness, k, determined experimentally for various types of the connections usually used in building frames (Ackroyd and Gerstle 1983). The lengths, axial loads, and moments of inertia of all columns were kept constant. The graph in Fig. 10 shows the variation of elastic, inelastic, and alignment chart K factors with the increase in the connection stiffness. For relatively stiff connections ($k>70$ MN-m/rad) the elastic and inelastic K factors are relatively stable and for a narrow range of connection stiffness, a slight increase in joint stiffness results in a significant reduction in the K factor. This clearly indicates that in cases where the choice of connection size and type in unbraced frames is not strictly regulated by design standards, the buckling capacity of columns can be significantly increased by selecting a stiffer connection. It should be noted from Fig. 10 that the case of $k=0$ (hinged connection) actually corresponds to an unstable mechanism, resulting in an infinite value for both the elastic and inelastic K factors.

5.5. Effect of far end conditions of attached columns

Duan and Chen (1989) and Essa (1997) proposed a method to include the effect of the boundary conditions at the far ends of the attached columns into the calculation of the K factor using the alignment chart. However, the assumptions used in the development of the method are similar, to some extent, to the assumptions of the alignment chart, therefore, the shortcomings and pitfalls are similar.

In order to measure the effect of far end conditions of top and bottom columns on the K factor, the moments of inertia of the four girders attached to column AB (Fig. 4) were progressively modified by substituting the $W8\times58$ shape with several W shapes. The loads, cross sections, and lengths of all colums were kept identical. Fig. 11 shows plots of the elastic K factors versus the inertia ratio I_g/I_c, where I_g and I_c are the moments of inertia of the girder and column, respectively, for three different boundary conditions of the top and bottom columns: hinged, fixed, and rigid. For the case of rigid far end conditions, the effective length factors obtained by means of the stability equation developed herein are identical to those predicted by the alignment chart procedure. It is obvious that the case of hinged far end

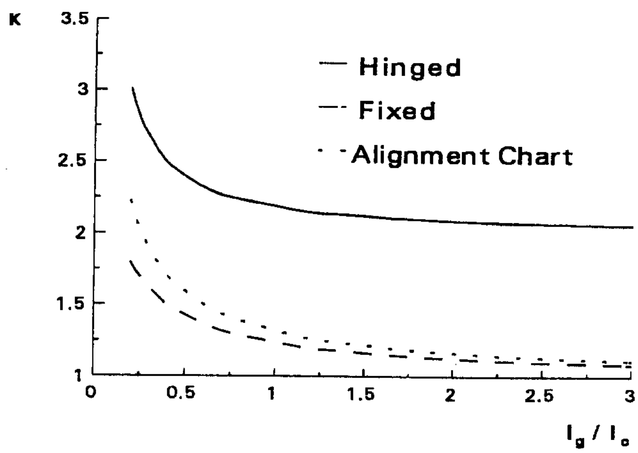

Fig. 11 Effect of far end condition on *K*-factor

conditions results in *K* factors which are considerably higher than those given by the other two boundary conditions. It is interesting to note that the *K* factors predicted by the alignment chart, which correspond to rigid far end conditions, are almost similar to those corresponding to fixed far end conditions.

5.6. Numerical example

Fig. 12 shows the scheme of a portion of a multistory rectangular frame. The far ends of upper column as well as of all beams are assumed rigid whereas a hinged base is assumed for the bottom column. The axial loads in the beams are negligible. The relative lengths, inertia, and axial load for all members are as indicated in Fig. 12. Table 2 gives the *K* factor of column *AB* obtained by using the method developed in this paper and compares it with the results deduced from the alignment chart procedure and from Essa and Hekal (1997). The iterative method proposed by Essa and Hekal is based on using the exact member stiffnesses to determine the *G* factors used in the context of the alignment chart procedure.

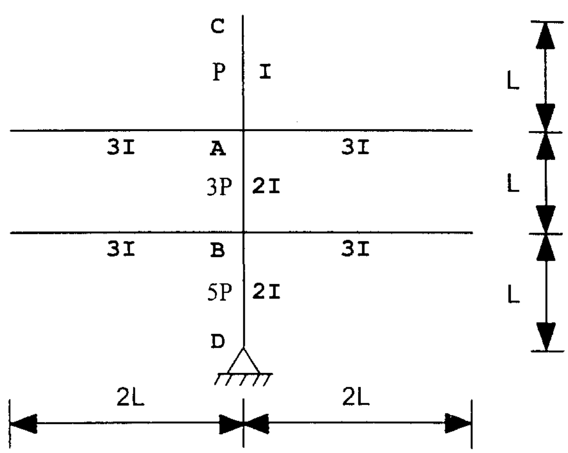

Fig. 12 Scheme of subassemblage for numerical example

Table 2 Results for numerical example

Alignment chart	Essa and Hekal (1997)	Present paper
1.38	2.80	2.85

6. Conclusions

The alignment chart procedure is applicable in predicting the K factors of unbraced frames only if the stiffness parameters of the attached columns are identical to that of the column being analyzed. If the stiffness parameter of some of the attached columns is larger than that of the column in question, the effective length factor increases significantly because such a column will disturb rather than restrain the column being analyzed.

The far end conditions of the columns above and below the column under investigation have a significant effect on the K factor of the column being considered. The alignment chart yields an effective length factor that is almost similar to that corresponding to fixed far end conditions.

In cases where the choice of connection size and type in unbraced frames is not strictly regulated by code provisions, the stability of columns is greatly enhanced by using a stiffer beam-to-column connection.

A new approximate method that overcomes the limitations imposed on the development of the alignment chart is presented. This method incorporates the effects of inelastic behaviour, far end conditions of the attached members, differentiated stiffness parameter, and flexible beam-to-column connections. This method may be implemented on a microcomputer. Because of its simplicity and generality, this method is recommended for design purposes instead of the alignment chart.

References

Ackroid, M.H. and Gerstle, K.H. (1983), "Elastic stability of flexibly connected frames.", *J. Struct. Engrg., ASCE,* **109**(1), 241-245.

Bridge, R.Q. and Fraser, D. (1987), "Improved G-factor method for evaluating effective lengths of columns", *J. Struct. Engrg., ASCE,* **113**(6), 1341-1356.

Chapius, J. and Galambos, T.V. (1982), "Restrained crooked aluminum columns", *J. Struct. Div., ASCE,* **108**(3), 511-524.

Chen, W.F. and Lui, E.M. (1991), *Stability Design of Steel Frames,* CRC Press.

Duan, L. and Chen, W.F. (1988), "Effective length factor for columns in braced frames", *J. Struct. Engrg., ASCE,* **114**(10), 2357-2370.

Duan, L. and Chen, W.F. (1989), "Effective length factor for columns in unbraced frames", *J. Struct. Engrg., ASCE,* **115**(1), 149-165.

Essa, H.S. (1997), "Stability of columns in unbraced frames", *J. Struct. Engrg., ASCE,* **123**(7).

Essa, H.S. and Hekal, S.A. (1997), "Improved G factor method for columns in unbraced frames", *Scientific Bulletin,* Faculty of Engrg., Ain Shams University, Cairo, Egypt, Accepted for publication.

Fraser, D. (1983), "Evaluation of effective length factor in braced frames", *Canadian J. Civ. Engrg.,* **10**, 18-26.

Goncalves, R. (1992), "New stability equation for columns in braced frames", *J. Struct. Engrg., ASCE,* **118**(7), 1853-1870.

Hajjar, J.F. and White, D.W. (1994), "The accuracy of column stability calculations in unbraced frames and the influence of columns with effective length factor less than one", *AISC Eng. J.,* **31**(3), 81-97.

LeMessurier, W.J. (1977), "A practical method of second order analysis", *Part 2-Rigid frames, Eng. J.*, **14**(2), 49-67.
Lui, E.M. (1992), "A novel approach for K factor determination", *AISC Eng. J.*, **29**(4), 150-159.
Yura, J.A. (1971), "The effective length of columns in unbraced frames", *AISC Eng. J.*, **8**(2), 37-42.

Notations

The following symbols are used in this paper:

C_i	=stability function of the i-th column;
D_i	=generalized stability function of the i-th girder;
E	=elastic modulus;
E_t	=tangent modulus;
F_y	=yield stress;
H_i	=generalized stability function of the i-th girder;
I	=moment of inertia;
k	=beam-to-column connection stiffness;
K	=effective length factor of the column under design;
K_i	=bending stiffness of member i;
L	=length of member;
L_2	=length of the column under design;
M	=bending moment;
P_i	=axial load in the i-th column;
r	=radius of gyration;
$R_1, R_2, \cdots, R_{10}$	=far end coefficients;
S_i	=stability function of the i-th column;
T_i	=generalized stability function of the i-th girder;
α_i	=stiffness parameter of the i-th column;
Δ	=story drift;
η	=ratio of tangent modulus to elastic modulus;
κ	=dimensionless joint stiffness coefficient;
λ	=slenderness ratio;
λ_c	=slenderness ratio separating elastic and inelastic buckling;
ΣP	=total gravity load on a story;
ΣP_{cr}	=sum of the elastic critical loads of all columns in a story; and
θ	=joint rotation.