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Control of buildings using single and multiple tuned liquid column dampers

C.C. Chang†, C.T. Hsu‡ and S.M. Swei‡†

Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong

Abstract. Some design formulas and design procedures for single and multiple tuned liquid column dampers (TLCDs) are proposed in this study. Previous studies show that if the properties of the TLCD system are properly selected then the TLCD could be as effective as the traditional tuned mass dampers. In addition, the TLCD system offers advantages such as flexibility in terms of installation, little maintenance required, and potentials for multiple usage, etc., which are incomparable by other mechanical types of dampers. In this paper, a set of optimal properties such as length and head loss of a TLCD system are derived under the assumption that the building vibrates in a dominate mode and is subjected to Gaussian white noise excitation. A design procedure for a single TLCD system will be illustrated and discussed. Due to the nonlinearity in the damping term, the TLCD system is sensitive to the loading intensity. This loading sensitivity could limit the application range of the TLCD system. It will be shown in this paper that such a nonlinear effect can be reduced by using multiple TLCDs. As a demonstrative example, the control effects on a flexible building modeled as a single degree-of-freedom system subjected to white noise excitation will be analyzed and discussed using single or multiple TLCDs.

Key words: control of buildings; TLCD; MTLCD; TMD; design procedure.

1. Introduction

One of the biggest challenges the structural engineers facing today is to find effective and better means of protecting structures and their contents from the damaging effects of environmental hazards such as earthquakes and wind loadings. A possible solution is to introduce more conservative designs so that structures such as buildings are more able to cope with external loads of unexpected magnitudes and characteristics. An alternative is to make structures perform more favorably to the external loading conditions by installing devices of various types at proper locations. This latter approach has led to passive/active structural control research and has opened up a new field of investigation in the last two decades.

Some of the key devices developed along this direction include, the tuned mass damper (TMD) (Luft 1979), the active mass damper (AMD) (Koizumi, *et al.* 1989), the active tuned mass damper (ATMD) (Abdel-Rohman 1984), the active tendon (Soong, *et al.* 1991), the active variable stiffness (AVS) (Kobori, *et al.* 1991), and also the tuned liquid damper (TLD) (Kareem and Sun 1987 and Fujino, *et al.* 1988), and the tuned liquid column damper (TLCD) (Sakai, *et al.* 1989).

As compared to other types of devices, the ones using water sloshing mechanism (TLD and

† Assistant Professor
‡ Senior Lecturer
‡† Formally Visiting Scholar

TLCD) provide some unique advantages such as low cost, easy installation and easy adjustment of natural frequency, etc. (Fujino and Sun 1993), which are unmatched by other control devices. Just like other passive tuning devices, the TLD and TLCD are capable of enhancing the structural damping by a few percents of the critical damping, which makes these devices ideal candidates for improving comfort of the occupants residing in some lightly-damped flexible structures.

The TLD works by absorbing and dissipating energy through the sloshing of shallow liquid inside a container. Extensive parametric studies on a TLD using a circular container were conducted by Fujino, *et al.* (1988). The interaction of a TLD and a structure was studied by Chaiseri, *et al.* (1989) from both theoretical and experimental viewpoints. Analogy between the TLD and the TMD was reported from Fujino, *et al.* (1990) by calculating virtual mass and virtual dashpot of the TLD attached to an undamped linear single-degree-of-freedom (SDOF) structure. The TLD has been successfully applied onto some practical structures, such as, the Sakitama Bridge (Ueda, *et al.* 1992) and the Shin Yokohama Prince (SYP) Hotel in Yokohama, Japan (Wakahara, *et al.* 1992). It was shown that a reduction up to fifty percents of wind-induced response can normally be achieved for the tall building equipped with TLDs. More recently, the effects of using multiple TLDs (MTLDs) were conducted by Fujino and Sun (1993). They showed that the efficiency of a TLD can be improved by using MTLDs with a proper frequency bandwidth. Also, Koh, *et al.* (1995) investigated the combined use of TLDs which were tuned to different vibration frequencies of a multi-degree-of-freedom structure. They demonstrated the advantages of having TLDs tuned to several vibration modes of the structure. Full-scale measurements of four buildings were conducted by Tamura, *et al.* (1995) to verify the efficiency of the TLD under wind excitations.

On the other hand, the TLCD achieves its vibration suppression effect through the motion of liquid mass in a tube-like container. The damping effect is a result of loss of hydraulic pressure due to the orifice installed inside the tube (Xu, *et al.* 1992). As compared to TLD, the TLCD attracts less amount of attention. Xu, *et al.* (1992) investigated the possibility of applying a TLCD and a tuned liquid column/mass damper (TLCMD) in reducing the along-wind response of wind-sensitive structures. Also, a mathematical model for numerical analysis of wind-induced stochastic response of structures equipped with a TLCD was proposed by Sun, *et al.* (1993). They showed that the TLCD has significant practical advantages if the parameters of the dampers are properly selected. Balendra, *et al.* (1995) studied the effectiveness of TLCD in controlling the wind-induced vibration of towers and found that the opening ratio of the TLCD needs to be varied between 0.5 and 1.0 with smaller ratios for shorter towers. More recently, a conceptual development of hybrid liquid column dampers was presented by Kareem (1994) and Haroun, *et al.* (1996) where a variable orifice and a pressure control mechanism were used to improve the effectiveness of TLCD.

In this study, some design formulas and design procedures for single and multiple tuned liquid column dampers (STLCD and MTLCD) are proposed. A set of optimal properties such as length and head loss of a STLCD are derived under the assumption that the building vibrates in a dominate mode and is subjected to Gaussian white noise excitation. A design procedure for this STLCD system will be illustrated and discussed. Due to the nonlinearity in the damping term, the STLCD system is sensitive to the loading intensity. This loading sensitivity could limit the applicable range of the STLCD system. It will be shown that such a nonlinear effect can be reduced by using MTLCD with the frequency bandwidth set to the frequency difference of the STLCD-structure combined system. As a demonstrative example, a flexible building modeled as a SDOF system equipped with STLCD and MTLCD will be analyzed and discussed.

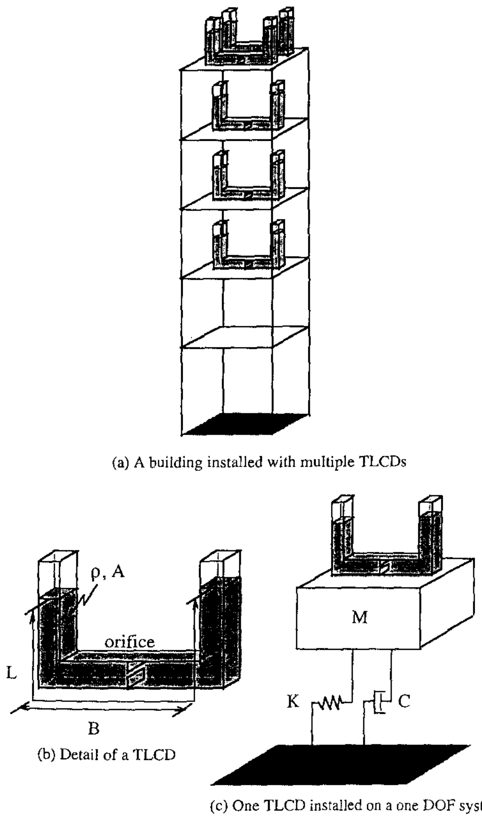

Fig. 1 Modeling of TLCDs installed on a building.

2. Formulation

The equations of motion for a building modeled using n degrees of freedom and installed with m TLCDs (see Fig. 1a) can be written as,

$$\boldsymbol{M}\ddot{\boldsymbol{y}}+\boldsymbol{C}\dot{\boldsymbol{y}}+\boldsymbol{K}\boldsymbol{y}=\boldsymbol{f}-\sum_{j=1}^{m}\rho_j A_j B_j \boldsymbol{D}_j \ddot{x}_j-\sum_{j=1}^{m}\rho_j A_j L_j \boldsymbol{D}_j \boldsymbol{D}_j^T \ddot{\boldsymbol{y}} \tag{1}$$

where $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are the mass, damping and stiffness matrices of the building, respectively; $\boldsymbol{y}$ is the displacement vector of the building; $\boldsymbol{f}$ is the external force vector; and ρ_j, A_j, B_j, $\ddot{x}_j$, and $\boldsymbol{D}_j$ are the mass density, cross section area, width, relative acceleration, and location vector of the jth TLCD, respectively. The superscript T represents the matrix transpose. The equation of motion for the jth TLCD can be written as (Saoka, *et al.* 1988 and Xu, *et al.* 1992)

$$\rho_j A_j L_j \ddot{x}_j+\frac{1}{2}\rho_j A_j h_j |\dot{x}_j| \dot{x}_j+2\rho_j A_j g x_j=-\rho_j A_j B_j \boldsymbol{D}_j^T \ddot{\boldsymbol{y}} \tag{2}$$

where L_j is the total length of the TLCD; h_j is the head loss coefficient determined by the opening ratio of the TLCD's orifice; g is the gravity constant. Noted that Eq. (2) is a nonlinear equation due to the presence of a nonlinear liquid damping term. The equation can be approximated using the equivalent linearization method assuming $\dot{x}_j$ to be a zero-mean stationary Gaussian process as (Caughey 1963, Wen 1980, Xu, *et al.* 1992 and Sun, *et al.* 1993),

$$m_j\ddot{x}_j+c_j\dot{x}_j+k_jx_j=-m_j\alpha_j\boldsymbol{D}_j^T\ddot{\boldsymbol{y}} \tag{3}$$

where m_j, c_j, k_j and α_j are the mass, equivalent damping coefficient, stiffness and the width-length ratio of the jth TLCD, respectively, and can be expressed as,

$$m_j=\rho_jA_jL_j \tag{4}$$

$$c_j=\sqrt{\frac{2}{\pi}}\rho_jA_jh_j\sigma_{\dot{x}_j} \tag{5}$$

$$k_j=2\rho_jA_jg \tag{6}$$

$$\alpha_j=\frac{B_j}{L_j} \tag{7}$$

where $\sigma_{\dot{x}_j}$ is the standard deviation of the TLCD's liquid level velocity. Rewriting (1) and (3) in the frequency domain by letting

$$x_j=X_je^{i\omega t},\ \boldsymbol{y}=\boldsymbol{Y}e^{i\omega t},\ \boldsymbol{f}=\boldsymbol{F}e^{i\omega t} \tag{8}$$

gives

$$(-\boldsymbol{M}\omega^2+i\boldsymbol{C}\omega+\boldsymbol{K})\boldsymbol{Y}=\boldsymbol{F}+\sum_{j=1}^{m}m_j\alpha_j\omega^2\boldsymbol{D}_jX_j+\sum_{j=1}^{m}m_j\omega^2\boldsymbol{D}_j\boldsymbol{D}_j^T\boldsymbol{Y} \tag{9}$$

$$(-m_j\omega^2+ic_j\omega+k_j)X_j=m_j\alpha_j\omega^2\boldsymbol{D}_j^T\boldsymbol{Y} \tag{10}$$

Solving for X_j from Eq. (10) and substituting it into Eq. (9) give the following two equations, respectively,

$$X_j=\frac{m_j\alpha_j\omega^2}{-m_j\omega^2+ic_j\omega+k_j}\boldsymbol{D}_j^T\boldsymbol{Y} \tag{11}$$

$$\left\{-\left[\boldsymbol{M}+\sum_{j=1}^{m}m_j\boldsymbol{D}_j\boldsymbol{D}_j^T\right]\omega^2+i\boldsymbol{C}\omega+\boldsymbol{K}-\sum_{j=1}^{m}\frac{m_j^2\alpha_j^2\omega^4\boldsymbol{D}_j\boldsymbol{D}_j^T}{-m_j\omega^2+ic_j\omega+k_j}\right\}\boldsymbol{Y}=\boldsymbol{F} \tag{12}$$

Assume that the building vibrates in the first mode predominately and the TLCDs are designed to control the first mode of the building only, then

$$\boldsymbol{Y}\approx\boldsymbol{\Phi}_1\eta_1(\omega) \tag{13}$$

where $\boldsymbol{\Phi}_1$ is the first mode shape vector and η_1 is the first modal amplitude in the frequency domain. Pre-multiplying Eq. (12) by the transpose of $\boldsymbol{\Phi}_1$ and assuming that the damping matrix is orthogonal to the mode shape, the modal amplitude can be derived as

$$\eta_1=\frac{F_1}{N_1(\omega)-\sum_{j=1}^{m}P_jH_j(\omega)} \tag{14}$$

where

$$N_1(\omega)=-\left(M_1+\sum_{j=1}^{m}m_jP_j\right)\omega^2+iC_1\omega+K_1 \tag{15}$$

$$P_j = (\boldsymbol{D}_j^T \boldsymbol{\Phi}_j)^2 \tag{16}$$

$$H_j(\omega) = \frac{m_j^2 \alpha_j^2 \omega^4}{-m_j \omega^2 + ic_j \omega + k_j} \tag{17}$$

$$M_1 = \boldsymbol{\Phi}_1^T \boldsymbol{M} \boldsymbol{\Phi}_1,\ C_1 = \boldsymbol{\Phi}_1^T \boldsymbol{C} \boldsymbol{\Phi}_1,\ K_1 = \boldsymbol{\Phi}_1^T \boldsymbol{K} \boldsymbol{\Phi}_1,\ F_1 = \boldsymbol{\Phi}_1^T \boldsymbol{F} \tag{18a-d}$$

Substituting Eqs. (13) and (14) into Eq. (11) gives

$$X_j = \frac{H_j(\omega) \boldsymbol{D}_j^T \boldsymbol{\Phi}_1}{m_j \alpha_j \omega^2 \left[N_1(\omega) - \sum_{j=1}^{m} P_j H_j(\omega) \right]^2} F_1 \tag{19}$$

The power spectral density functions $S_{\eta_1 \eta_1}$, $S_{X_j X_j}$, and $S_{\dot{X}_j \dot{X}_j}$ can then be derived as

$$S_{\eta_1 \eta_1} = \frac{S_{F_1 F_1}}{\left[N_1(\omega) - \sum_{j=1}^{m} P_j H_j(\omega) \right]^2} \tag{20}$$

$$S_{X_j X_j} = \frac{H_j^2(\omega)}{m_j^2 \alpha_j^2 \omega^4} P_j S_{\eta_1 \eta_1} \tag{21}$$

$$S_{\dot{X}_j \dot{X}_j} = \frac{H_j^2(\omega)}{m_j^2 \alpha_j^2 \omega^2} P_j S_{\eta_1 \eta_1} \tag{22}$$

The variances $\sigma_{\eta_1}{}^2$, $\sigma_{X_j}{}^2$, and $\sigma_{\dot{X}_j}{}^2$ can be calculated using the following expressions,

$$\sigma_{\eta_1}{}^2 = \int_{-\infty}^{\infty} S_{\eta_1 \eta_1} d\omega \tag{23}$$

$$\sigma_{X_j}{}^2 = \int_{-\infty}^{\infty} S_{X_j X_j} d\omega \tag{24}$$

$$\sigma_{\dot{X}_j}{}^2 = \int_{-\infty}^{\infty} S_{\dot{X}_j \dot{X}_j} d\omega \tag{25}$$

It is noted that the TLCDs' damping coefficients c_j's are embedded in the expressions of $S_{\eta_1 \eta_1}$, $S_{X_j X_j}$, and $S_{\dot{X}_j \dot{X}_j}$ which are functions of $\sigma_{\dot{X}_j}$'s as suggested in Eq. (5), so an iterative procedure is required to calculate these variances.

3. Design procedure for a single TLCD (STLCD)

Assume that only one TLCD is installed on the building at location z_1. The sub-indices of the building's first modal properties and the TLCD's properties are omitted in the following derivation for the sake of simplicity. The dominate mode shape of the building is scaled proportionally so that

$$P_1 = 1 \tag{26}$$

Under the Gaussian white noise type of excitation, the variance of the first modal amplitude can be expressed in closed form as (Crandal and Mark 1973)

$$\sigma_\eta^2=\pi S_0\left[\frac{b_0^2}{a_0}(a_2a_3-a_1a_4)+a_3(b_1^2-2b_0b_2)+a_1(b_2^2-2b_1b_3)\right.$$
$$\left.+\frac{b_3^2}{a_4}(a_1a_2-a_0a_3)\right]\Big/\left[a_1(a_2a_3-a_1a_4)-a_0a_3^2\right] \tag{27}$$

where

$$a_0=Kk,\ a_1=Kc+kC,\ a_2=(M+m)k+mK+Cc,\ a_3=(M+m)c+mC,$$
$$a_4=(M+m)m-m^2\alpha^2,\ b_0=k,\ b_1=c,\ b_2=m,\ b_3=0 \tag{28a-i}$$

By neglecting structural damping ($C=0$) and assuming $\sigma_{\ddot{x}}$ to be a constant, the optimal values of length L and damping coefficient c can be derived by differentiating σ_η^2 with respect to L and c and setting them equal to zero, respectively.

$$\frac{\partial\sigma_\eta^2}{\partial L}=0 \tag{29}$$

$$\frac{\partial\sigma_\eta^2}{\partial c}=0 \tag{30}$$

Solving these two equations, under the condition that K is much larger than k, gives

$$L_{opt}=\frac{2gM}{K} \tag{31}$$

$$c_{opt}=m\left(\mu\alpha^2\omega_t^2+\omega_s^2+\frac{\omega_t^4}{\omega_s^2}-2\omega_t^2\right)^{1/2} \tag{32}$$

where the mass ratio μ, the frequency of the TLCD ω_t and the frequency of the structure ω_s are defined as

$$\mu=\frac{\rho AL}{M} \tag{33}$$

$$\omega_t=\sqrt{\frac{2g}{L}} \tag{34}$$

$$\omega_s=\sqrt{\frac{K}{M}} \tag{35}$$

Eq. (31) suggests that the optimal length of the TLCD is when the TLCD's frequency equals to that of the building's frequency ($\omega_t=\omega_s$). Using this relation in Eq. (32) gives

$$c_{opt}=m\,\omega_t\,\alpha\sqrt{\mu} \tag{36}$$

which results in the following optimal damping ratio for the TLCD

$$\xi_{opt}=0.5\,\alpha\sqrt{\mu} \tag{37}$$

If the standard deviation of the TLCD's liquid level velocity can be calculated, then the optimal head loss for the TLCD can be derived by equating (5) to (36) as

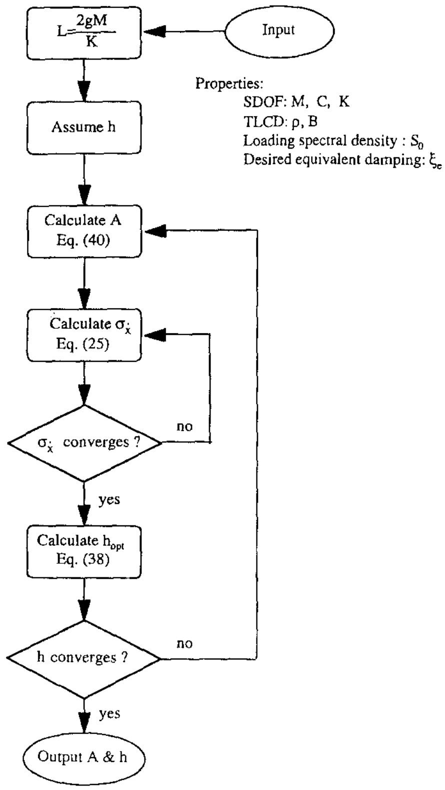

Fig. 2 Design procedure for a single TLCD installed on a building modeled as a one degree-of-freedom system.

$$h_{opt}=\sqrt{\frac{\pi m K}{2M^2}}\,\frac{B}{\sigma_{\dot{x}}} \tag{38}$$

The effectiveness of the TLCD can be evaluated in terms of the increase in the effective damping ratio ξ_e which is defined as the viscous damping required for the building to sustain the same magnitude of response under the same intensity of excitation (Sun, *et al.* 1993).

$$\xi_e=\frac{\pi S_0}{2\omega_s^3 M^2\sigma_\eta^2} \tag{39}$$

Substituting Eq. (27) into (39), the cross section area A of the TLCD required to achieve the equivalent damping ratio of ξ_e can be derived as

$$A=\frac{L(M+m)}{B^2\rho}-\left\{2\xi_e\omega_s^3M^2\left[\frac{b_0^2}{a_0}\,a_2a_3+a_3(b_1^2-2b_0b_2)+a_1b_2^2\right]-a_1a_2a_3+a_0a_3^2\right\}$$
$$\Big/\left[B^2\rho\left(2\xi_e\omega_s^3M^2\frac{b_0^2}{a_0}\,a_1-a_1^2\right)\right] \tag{40}$$

Substituting the two optimal values L_{opt} and h_{opt} into Eq. (40), the smallest cross section needed to produce the effective damping ratio ξ_e for the building can be obtained. It is noted that some parameters, such as a_i's and b_i's in Eq. (40), depend on the value of $\sigma_{\dot{x}}$ which is an implicit function of A. An iterative procedure is again needed to solve for A. Fig. 2 shows a simple procedure to design the TLCD's cross section area A and head loss h based on some given parameters.

4. Design of multiple TLCDs (MTLCD)

Assume that now m TLCDs are installed at a same location on the building. To simplify the analysis, it is further assumed that m is an odd number and the properties of these TLCDs are the same except the lengths are different. It has been shown by Abé and Fujino (1994) and similarly by Kareem and Kline (1995) for the case of multiple tuned mass dampers (MTMD) that, if properly designed, the MTMD can be much more robust than a conventional single TMD while keeping more or less the same efficiency. Also, the frequency bandwidth of the MTMD should be an appropriate value to achieve the best result. Abé and Fujino (1994) further determined a proper MTMD frequency bandwidth by letting the modal amplitude of the central TMD of the central mode equal to that of the outside TMD of the lowest outside mode. Since the TLCD is nonlinear in nature, the perturbation approach used by Abé and Fujino (1994) becomes too complicated if not impossible to be adopted. In this study, a simple guideline on choosing the proper MTLCD frequency bandwidth is proposed.

Assumes that the frequencies of these m number of TLCDs, ω_{t_j}, are equally distributed with frequency spacing of $\omega_s \beta$ and centered at the structural dominant frequency ω_s.

$$\omega_{t_j} = \sqrt{\frac{2g}{L_j}} = \omega_s(1+j\beta) \qquad j = -\frac{m-1}{2}, \; \cdots, \; \frac{m-1}{2} \tag{41}$$

The non-dimensional frequency bandwidth of the MTLCD, ΔR, is found as

$$\Delta R = (m-1)\beta \tag{42}$$

Considering the following two observations for the case of MTMD: (i) the effect of the MTMD is to make the dynamic characteristics of the structure more robust which, equivalently speaking, is to flatten the sharp peaks of the structural frequency response function; (ii) the lowest and the highest frequencies of the TMDs are very close to the outer two frequencies of the MTMD-structure combined system which justify the perturbation method used by Abé and Fujino (1994). Since the TMD and the TLCD work in a similar way, it is expected that these two observations would still hold for the TLCD. Just as the case of MTMD, the overall efficiency of a well designed MTLCD system should be more or less the same as an optimal STLCD except that the MTLCD provides more robust performance. As a result, this well designed MTLCD should lower the two peaks (while raise the valley) of the combined STLCD-structure's frequency response function. The frequency difference between the two peaks of the STLCD-structure frequency response function then becomes a good indication of how wide the frequency bandwidth of the MTLCD should be. The effects of installing an optimal STLCD and a MTLCD on the structural frequency response function can be schematically shown in Fig. 3. The two peak frequencies of the structure with an optimal STLCD can be calculated to be $\omega_s(1+\alpha\sqrt{\mu}/2)$ and $\omega_s(1-\alpha\sqrt{\mu}/2)$, which are accurate to the first order. If we let

$$\Delta R = \frac{\omega_s(1+\alpha\sqrt{\mu}/2) - \omega_s(1-\alpha\sqrt{\mu}/2)}{\omega_s} = \alpha\sqrt{\mu} \tag{43}$$

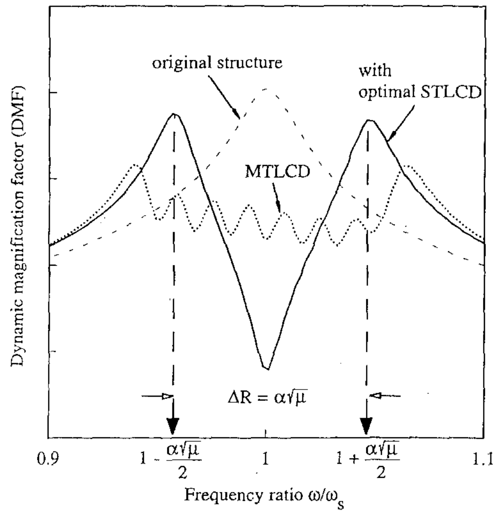

Fig. 3 The effects of STLCD and MTLCD on the structural frequency response.

then the frequency response function could be flatten between the two peaks as seen from Fig. 3. Eq. (42) to Eq. (43), the frequency spacing is found as

$$\beta=\frac{\alpha\sqrt{\mu}}{m-1} \tag{44}$$

The length of each TLCD can then be found as

$$L_j=\frac{2g}{\omega_s^2\left(1+j\dfrac{\alpha\sqrt{\mu}}{m-1}\right)^2} \qquad j=-\frac{m-1}{2},\cdots,\frac{m-1}{2} \tag{45}$$

5. Numerical examples

To verify the proposed TLCD design procedure and to study the effectiveness of STLCD and MTLCD on suppression excessive building vibration, a flexible building modeled as a SDOF system subjected to Gaussian white noise excitation was used as an example. The mass, stiffness, and damping constants of the SDOF system were assumed as 4.61×10^7 N-s^2/m, 5.83×10^7 N/m, and 1.04×10^6 N-s/m (1% damping ratio), respectively. These properties represented the first mode of a 75-story flexible skyscraper (Tsijiuchi, *et al.* 1991). The frequency of this SDOF system was 0.179 Hz. It was assuming that the liquid inside the TLCDs was water with density of 997 N-s/m^4 and the width of the TLCDs was assumed to be 12 m.

5.1. Single tuned liquid column damper

If only one TLCD was to be installed on this SDOF system, the optimal length of the STLCD was calculated to be 15.51 m which was the length that made the frequency of the TLCD equal to that of the SDOF system. Assuming that the SDOF system was subjected to a Gaussian

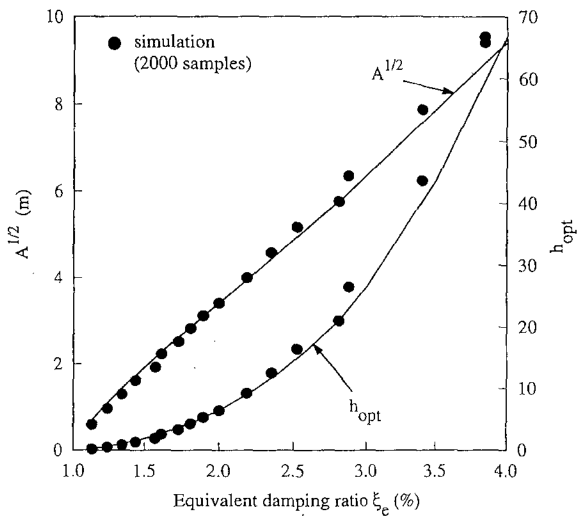

Fig. 4 Optimal values of head loss and cross section area of the TLCD as functions of desired equivalent damping ratio ξ_e.

white noise excitation with spectral intensity $S_o=7.73\times10^9$ N^2-s, the displacement standard deviation of this SDOF without any TLCD installed was calculated to be 2 cm.

Fig. 4 shows the area A and the optimal head loss h_{opt} of the TLCD calculated as functions of the desired equivalent damping ratio ξ_e. It is seen that the square root of A increases quite linearly as ξ_e increases from 1 to 4%. On the other hand, h_{opt} increases parabolically from approximately 0 to 70 as ξ_e increases. This result shows that to increase the effective damping of the SDOF system, the optimal values of A and h have to be increased simultaneously. Monte Carlo simulation incorporating the Newmark integration was also performed using Eqs. (1) and (2) to verify the accuracy of the equivalent linearization approach adopted in Eq. (3). The Newmark parameters were assumed as $\beta=0.25$ and $\gamma=0.5$, which guaranteed unconditional stability. The time increment was assumed as 0.1 sec. For each time step, the Newton-Raphson iterative procedure was adopted to ensure equilibrium (Bathe 1982). Two thousand samples were used to generate mean equivalent damping ratio from a given set of A and h_{opt}. Good agreement between the results using Monte Carlo simulation and numerical calculation are seen. This suggests that under the range of ξ_e considered (1 to 4%), the error introduced by using the equivalent linearization method is not significant.

Fig. 5 shows the cross section area of the TLCD required when the head loss h is not optimal. Five values of head loss $h=10$, 20, 30, 40, and 50 were used for comparison. It is seen that when the head loss is not optimal, the cross section areas required to produce the same level of ξ_e are always larger than that using optimal head loss.

It is well known that another form of damper, the so-called tuned mass damper (TMD) is quite effective in suppressing wind-induced or earthquake-induced structural vibration. It will be interesting to compare the efficiency of a TLCD and a TMD under the same condition. The stiffness and damping coefficients for a TMD system to control wind-induced vibration has been derived by, among others, Luft (1979) as,

$$k=\frac{\mu}{1+1.5\mu}K \tag{46}$$

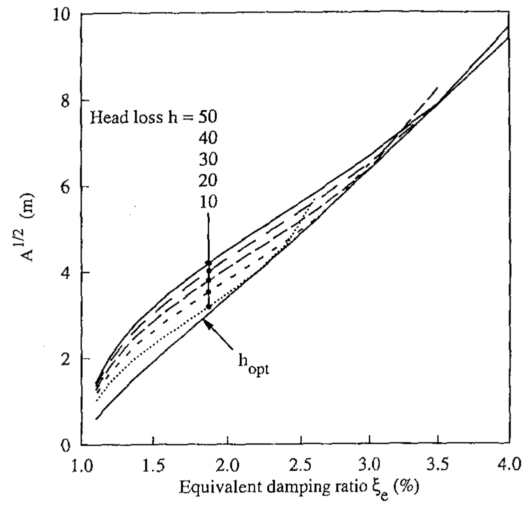

Fig. 5 Cross section area A as a function of the equivalent damping ratio ξ_e for different values of head loss.

$$c=\mu^{3/2}\left(\frac{1-0.75\mu}{1+1.5\mu}\right)^{1/2}\sqrt{MK} \tag{47}$$

where $\mu=m/M$ is the mass ratio.

Fig. 6 shows the comparison between the control efficiency of a TMD system and a TLCD system with various values of head loss h. It is seen that for the same mass ratio m, the equivalent damping ratio of the SDOF system installed with the TMD is larger than the system installed with the TLCD. On the other hand, the standard deviation of the TMD stroke is almost equal to the standard deviation of the optimal TLCD water level displacement. It is clearly seen that the TMD provides a slightly better control performance than the TLCD. This is probably due to the fact that only horizontal portion of the TLCD is physically interacting with the structure as can be seen from the right hand side of (2). This suggests that when designing a TLCD, the width-length ratio a should be taken as large as possible to provide a better performance. Although the TLCD is less efficient than the TMD in general, it is still a viable alternative as far as costs, maintenance, and space limitation are concerned.

Since the equation of motion for the TLCD is nonlinear, the optimal head loss hopt will change as the loading intensity varies. Fig. 7 shows the optimal head loss h_{opt} as a function of the loading spectral intensities S_o for different values of equivalent damping ratio. It is seen that h_{opt} decreases significantly as S_o increases which implies that if a constant head loss is used then the performance of the STLCD might vary as the loading intensity changes. The results suggest that to render an optimal control efficiency, it might be necessary to adaptively adjust the head loss of the TLCD as the wind load spectral density varies as suggested by Kareem (1994) and Haroun, *et al.* (1996).

Fig. 8 shows the TLCD performance sensitivity for three design cases, case 1: $A=11.6$ m^2, $h=6.5$; case 2: $A=40.4$ m^2, $h=26.5$; and case 3: $A=88.5$ m^2, $h=66.8$. These three cases were designed based on $S_o=7.73\times10^9$ N^2-s $[\sigma_y=(\pi S_o/CK)^{1/2}=2$ cm$]$ and the desired equivalent damping ratio ξ_e equals to 2%, 3% and 4%, respectively. Sakai, *et al.* (1989) and Balendra, *et al.* (1995) studied experimentally the relation between opening ratio of the orifice and the head

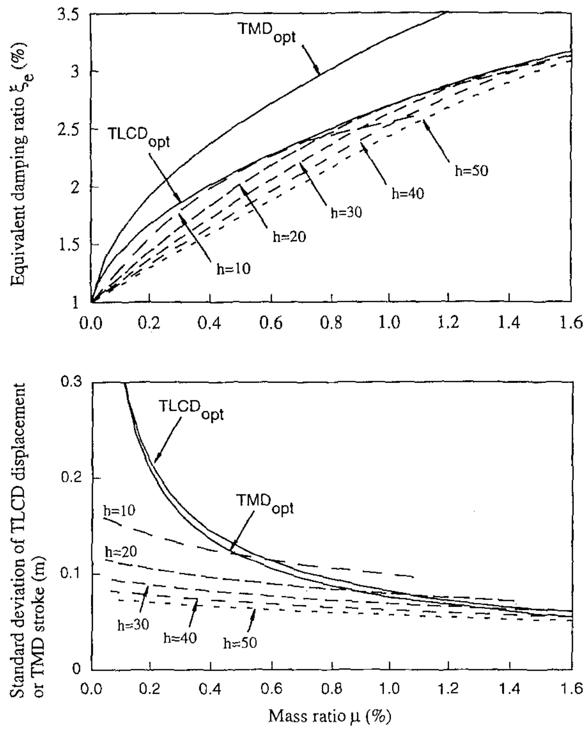

Fig. 6 Comparison of the efficiency of an optimal TMD and a TLCD with various values of head loss h.

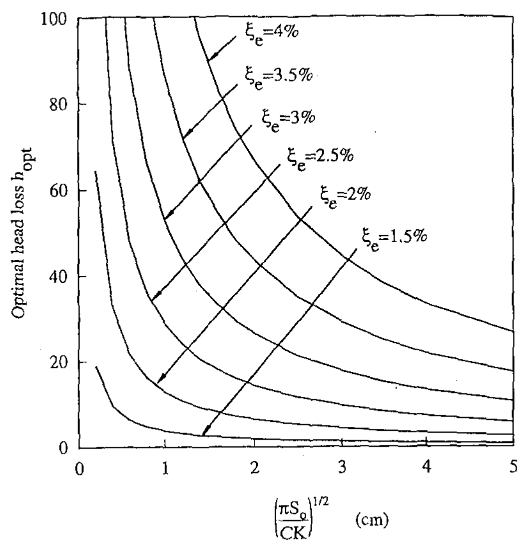

Fig. 7 Optimal head loss h_{opt} as a function of the loading spectral intensity S_o for different values of equivalent damping raio.

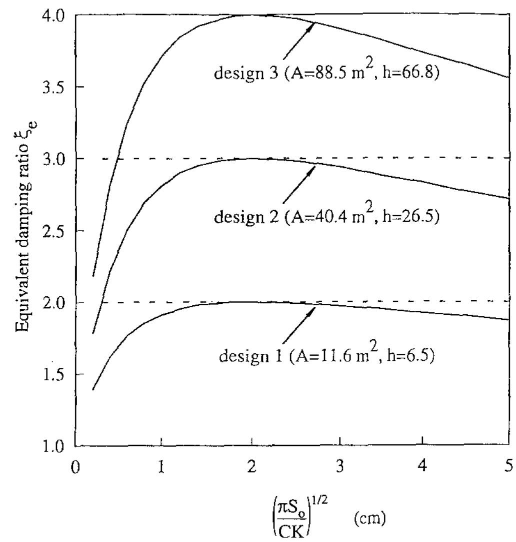

Fig. 8 Sensitivity of TLCD performance for three design cases.

loss coefficient. Based on the results by Balendra, *et al.* (1995), the opening ratios for cases 1, 2, and 3 are approximately equal to 80%, 45% and 20%, respectively. It is seen that as S_0 deviates from the design value, the equivalent damping ratio ξ_e all decreases from the desired values. It is further noted that significant reduction in ξ_e is seen when S_0 is smaller than the design value, this indicates that the TLCD might not be effective when the intensity of loading is smaller than the design value.

5.2. Multiple tuned liquid column dampers

Assumed that if this SDOF system was to be controlled using 3, 7, and 11 TLCDs, respectively. Fig. 9 shows the effect of the MTLCD's frequency bandwidth on the equivalent damping ratio of the structure for the three design cases presented in Fig. 8. The frequency differences of the STLCD-structure combined system were calculated to be 0.048, 0.09, and 0.133 for case 1, 2, and 3, respectively. It is seen that, for all three cases, the equivalent damping ratio ξ_e remains almost unchanged for the range of bandwidth smaller than the frequency differences of the STLCD-structure combined system. Once the bandwidths exceed these frequency differences, the equivalent damping ratio starts to decrease. The results suggest that the frequency difference of the STLCD-structure combined system is a good indicative value for the bandwidth of the MTLCD.

The performance of the MTLCD as a function of loading intensity for frequency bandwidth $\Delta R=5\%$, 9%, and 15%, respectively, is plotted in Fig. 10 for design case 2 only. It is seen that when $\Delta R=5\%$, the performance of the MTLCD is almost the same as that of a STLCD. When ΔR equals to the suggested value of 9%, the performance of the MTLCD improves significantly at the low intensity region while keeping almost the same as that of the STLCD at the high intensity region. As ΔR increases further to 15%, it shows that the performance of the MTLCD deteriorates noticeably at the high loading intensity region and improves at the low loading intensity region. Also, for $\Delta R=15\%$, it is seen that the number of TLCDs used starts to affect

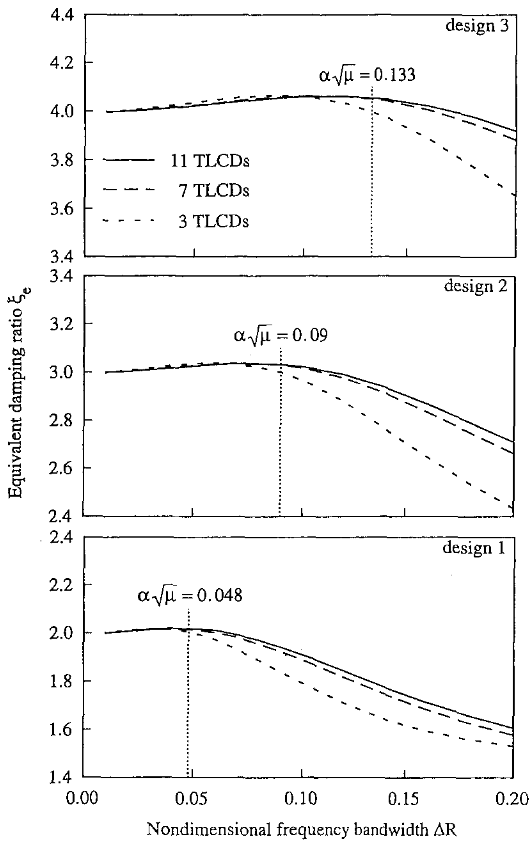

Fig. 9 The effect of the frequency bandwidth of the MTLCD on the structural equivalent damping ratio.

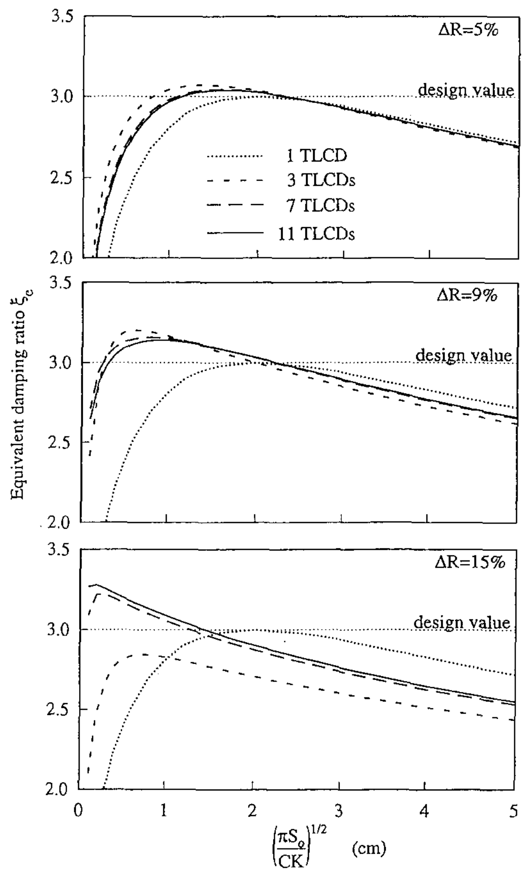

Fig. 10 Performance of MTLCD as a function of loading intensity S_o for different values

the performance of MTLCD. The results shown in this figure demonstrate that the loading sensitivity which occurs in the STLCD system due to the nonlinear damping term can be mitigated if a MTLCD system with a properly selected bandwidth ($\Delta R = 9\%$) is used.

Fig. 11 shows the frequency response for the SDOF system equipped with a STLCD or a MTLCD for three levels of loading intensity $\sigma_y = 0.1$, 0.5, and 2 cm, respectively. It is seen that the frequency response functions for the MTLCD with 3, 7 or 11 TLCDs are in general smoother than that of a STLCD system for the three loading intensities considered. This result shows that the performance of the MTLCD is more robust than that of the STLCD.

6. Concluding remarks

A set of optimal properties such as the length and the head loss of a STLCD system were derived under the assumption that the building vibrates in a dominate mode and subjected to Gaussian white noise excitation. A design procedure for a STLCD system and some guidelines for designing a MTLCD system are also discussed.

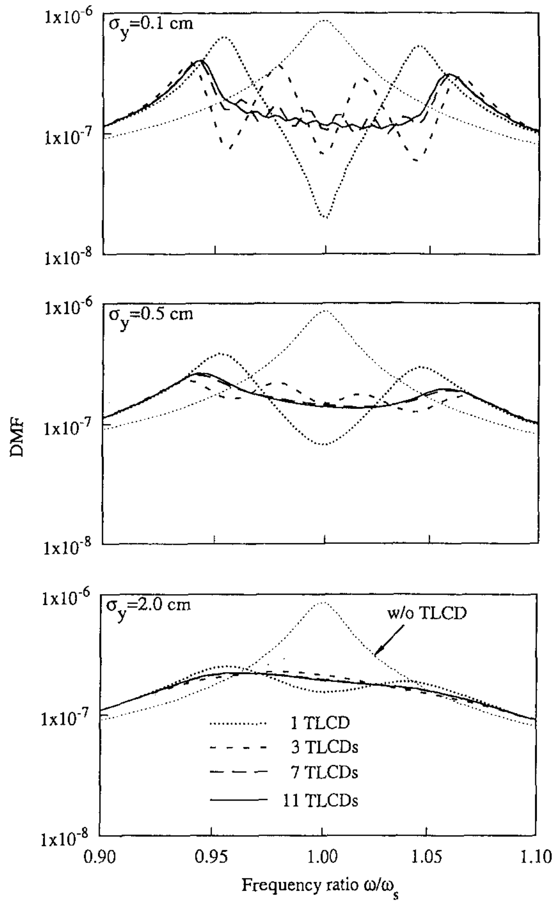

Fig. 11 Frequency response for the SDOF system equipped with STLCD or MTLCD.

The results show that due to the nonlinearity in the damping term, for a given set of properties, the performance of a STLCD system would vary as the loading intensity changes. The results further suggest that the STLCD system might not be effective when the actual loading intensity is smaller than the designed intensity. It is also shown that the MTLCD system designed with the frequency bandwidth equals to the frequency difference of the STLCD-structure combined system can improve the effectiveness at the low loading intensity region and provides a more robust performance when compared to the STLCD system.

The performance of a TLCD system is in general slightly inferior to that of a TMD system due to the fact that only portion of its mass is interacting with the structure. The TLCD system, however, remains a good alternative due to its unique advantages such as low cost, easy installation, and easy maintenance and adjustment.

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