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Dispersion of shear wave in a pre-stressed hetrogeneous orthotropic layer over a pre-stressed anisotropic porous half-space with self-weight

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Abstract. The purpose of this study is to illustrate the propagation of the shear waves (SH-waves) in a pre-stressed hetrogeneous orthotropic media overlying a pre-stressed anisotropic porous half-space with self weight. It is considered that the compressive initial stress, mass density and moduli of rigidity of the upper layer are space dependent. The proposed model is solved to obtain the different dispersion relations for the SH-wave in the elastic-porous medium of different properties. The effects of compressive and tensile stresses along with the heterogeneity, porosity, Biot's gravity parameter on the dispersion of SH-wave are shown numerically. The wave analysis further indicates that the technical parameters of upper and lower half-space affect the wave velocity significantly. The results may be useful to understand the nature of seismic wave propagation in geophysical applications and in the field of earthquake and material science engineering.

Keywords: SH-wave; Biot's parameter; porous; orthotropic; heterogeneity; initial stress

1. Introduction

A disturbance, confined to a bounded part of a medium, which propagates with a finite speed to other parts of the medium forms the basis for the study of wave propagation. Wave propagation manifests in forms that are familiar in everyday life such as acoustic waves from musical instruments, water waves breaking on a coastline or elastic displacements in the Earth. At least, there are two types of waves that can propagate in an elastic material, shear waves and pressure waves. Both these waves are governed by the same wave equation, Whitham (1974), The elastic wave equation explains the vibrations in plates and beams. Also, disturbances due to by seismic events in the Earth can be described by these wave equations. The description of the wave fields resulting from an initial configuration or time dependent forces is a valuable tool when gaining insight into the effects of the layering of the Earth, the propagation of earthquakes or the behavior of underwater sound. The shear wave (SH-wave) is called a Love wave after its finder Love (1911)

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and accounts for the significantly damage causing effects of aftershocks of earthquakes. In general, the disturbance that generates propagating waves can either manifest in an initial state or as a time dependent forcing, the geometry of the underlying domain can have a complex structure with curved boundaries, and the media may have discontinuous parameters with the discontinuity taking place along a non-planar interface. The complexity of boundary conditions, transient time behavior, geometries and material properties make the description of the resulting wave field hard, if not impossible, to express using mathematical analysis. For this reason, the authors abandon the ambition to seek exact solutions for the propagation of shear wave in an elastic-porous medium of different properties. SH-type shear waves transmit in the layer and weekend along the thickness of the half-space such that wave particles vibrate parallel to the direction of propagation. Further, the Earth is under the high initial stresses, therefore initial stresses play a significant role in the propagation of these seismic waves. The complex structure of the Earth has various types of layers. The porous layer of the Earth has the amazing properties like anisotropy and heterogeneity and these pores contain water or gas or oil, the one of these three is responsible for the rock to be saturated with.

The earth is orthotropic i.e. its mechanical properties are, in general, different along each axis. The development of initial stresses in the elastic solid half-space (the Earth) arise due to many reasons, such as gravity variations, the distinction of temperature, process of extinguish shot pinning and frosty working, moderate process of creep and different internal forces. These initial stresses of earth induce great impact on SH-waves during propagation and have great impact on the mechanical riposte of the materials. The concept of initial stresses has important significance in engineering structures, geomechanics and in the research of soft living tissues. It is therefore of great attraction to investigate the influence of initial stresses on the elastic wave propagation. Due to large applications, pre-stressed SH-waves in different media tempt researcher's interests even nowadays. Watanabe and Payton (2002) discussed SH- waves in a cylindrically monoclinic material with Green's function. Chattopadhyay *et al.* (2010, 2012) used Green's function technique to study propagation of SH-waves and heterogeneity on the SH-waves in viscoelastic half-spaces. Also, Chattopadhyay *et al.* (2014) discussed the influence of heterogeneity and reinforcement on propagation of a crack due to SH-waves. Gupta and Gupta (2013) studied the effect of initial stress on wave motion in an anisotropic fiber reinforced thermoelastic medium. Sahu *et al.* (2014) showed the effect of gravity on shear waves in a heterogeneous fiber-reinforced layer placed over a half-space. Kundu *et al.* (2014) analyzed SH-wave in initially stressed orthotropic homogeneous and a heterogeneous half space.

The layered structure of the Earth is very complex containing different types of layers including elastic, viscoelastic or porous layer. The porous layer of earth has amazing properties such as heterogeneity, anisotropy, and initial stress. Generally the pores of the porous rock may contain gas or oil or water and layer will be saturated either with gas or any one of these. The fluid saturated porous medium affects the propagation of torsional surface waves and it is a considerable root of attenuation. A large number of problems in seismology can be explained by representing earth as a fluid saturated porous layered structure with mechanical properties and finite thickness. In fact the study of torsional waves in heterogeneous fluid saturated porous layered media has been of central interest to geophysicists until recently. In order to understand the underground response of seismic wave propagation toward material properties and initial stresses of the Earth, researchers and seismologists generally prefer porous rock models with various heterogeneities in semi-infinite domains. These initial stresses influence elastic wave propagation more prominently. The propagation of SH-wave in a fluid saturated anisotropic porous media has received prime attention in the field of earthquake engineering and applied informatics. Many researchers have discussed the elastic properties of porous media. The propagation of Love type waves with irregular boundary in a porous layer has been discussed by Chattopadhyay and De (1983), Dey and Gupta (1987) investigated wave propagation in void medium. Chattaraj *et al*. (2013) studied Love wave propagation pre-stressed porous layer lying between two isotropic half-spaces and studied the effect of anisotropy and porosity on Love wave phase velocity. Gupta *et al*. (2013) presented a technical note on the propagation of Love wave in porous layer. The Earth's gravitational force affects the seismic wave propagation. The hydrostatic stresses in the gravitational half-space play an important role to analyze the static and dynamic problems of the Earth. Ghorai *et al*. (2010) discussed Love wave propagation in a porous layer overlying a gravitational half-space. Abd-Alla *et al*. (2013) investigated the effect of various parameters such as fibre-reinforcement, anisotropy and gravity of the elastic media on surface waves.

In this paper, the dispersion of shear waves in a pre-stressed hetrogeneous orthotropic layer over pre-stressed anisotropic porous half-space under gravity has been investigated. The influence of, porosity, initial stress and gravity parameter on the shear wave propagation has been discussed graphically. The obtained dispersion equation is in perfect agreement with the standard results investigated by other relevant researchers in the absence of heterogeneity, porosity, stress and gravity parameters.

2. Formulation of the problem

In this model, a heterogeneous orthotropic media under initial stress P of thickness H overlying an anisotropic gravitational porous half-space under stress P' is considered as shown in Fig. 1.

The direction of propagation of SH-wave is considered to be along x-axis and z-axis is positively in downwards. Let u,v,w be the displacements along x, y and z-axis, respectively. Let the compressive initial stress, mass density and moduli of rigidity of the upper layer are $P=P_0(1-\cos az)$, $\rho_1=\rho_0(1-\cos az)$ and $E_i=Q_0(1-\cos az)$ respectively, where P_0,ρ_0 and Q_0 are the

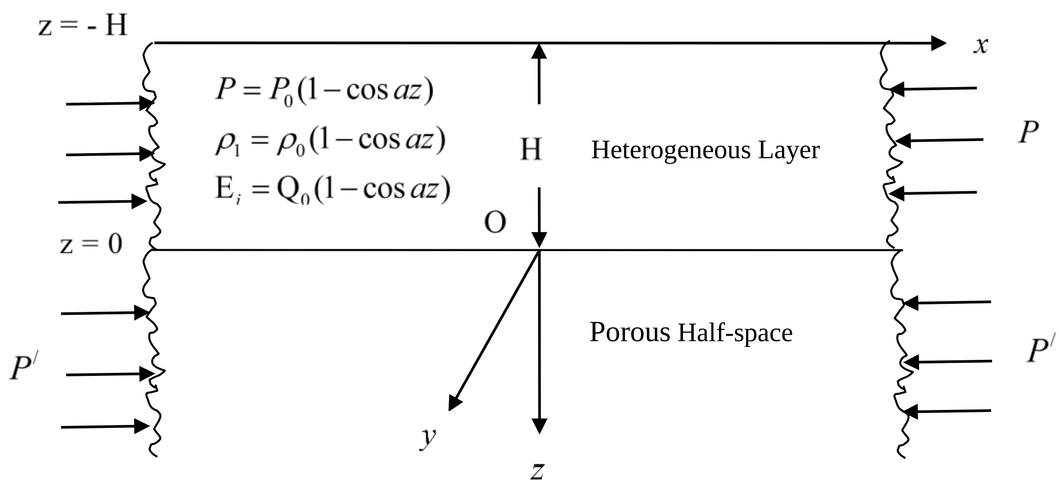

Fig. 1 Geometry of the problem

values of initial stress, mass density and moduli of rigidity at z=0. Here $'a'$ is heterogeneous parameter of the upper layer and having dimension that is inverse of length.

3. Solution of the problem

3.1 Solution for the upper layer

It is considered that the upper layer is hetrogeneous orthotropic in nature and under the compressive initial stress P. Let u_1,v_1 and w_1 be the displacements along x, y and z-axis, respectively. The SH-wave propagation is considered to be along x-axis, so the equation of motion for the orthotropic elastic medium under the compressive initial stress P in the absence of body forces are (Biot 1965)

$$\begin{cases} \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} - P\left(\frac{\partial \Omega_z}{\partial y} - \frac{\partial \Omega_y}{\partial z}\right) = \rho_1 \frac{\partial^2 u_1}{\partial t^2} \\ \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} - P\left(\frac{\partial \Omega_z}{\partial x}\right) = \rho_1 \frac{\partial^2 v_1}{\partial t^2} \\ \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} - P\left(\frac{\partial \Omega_y}{\partial x}\right) = \rho_1 \frac{\partial^2 w_1}{\partial t^2} \end{cases} \tag{1}$$

where

$$\Omega_x = \frac{1}{2}\left(\frac{\partial w_1}{\partial y} - \frac{\partial v_1}{\partial z}\right), \quad \Omega_y = \frac{1}{2}\left(\frac{\partial u_1}{\partial z} - \frac{\partial w_1}{\partial x}\right), \quad \Omega_z = \frac{1}{2}\left(\frac{\partial v_1}{\partial x} - \frac{\partial u_1}{\partial y}\right) \tag{2}$$

τ_{xx}, τ_{xy}, τ_{xz}, τ_{xy}, τ_{yy}, τ_{xz}, τ_{zx}, τ_{zy}, and τ_{zz} are the incremental stress components, u_1,v_1 and $w_1$are the components of the displacement vector in the upper layer, $\rho_1$is the density of the layer. Here, Ω_x, Ω_y and Ω_z are the rotational components in the upper half-space.

The incremental stress-strain relations are

$$\begin{cases} \tau_{xx} = \mathrm{N}_{xx} e_{xx} + \mathrm{N}_{xy} e_{yy} + \mathrm{N}_{xz} e_{zz} \\ \tau_{xy} = 2\mathrm{E}_z e_{xy} \\ \tau_{yy} = \mathrm{N}_{yx} e_{xx} + \mathrm{N}_{yy} e_{yy} + \mathrm{N}_{yz} e_{zz} \\ \tau_{yz} = 2\mathrm{E}_x e_{yz} \\ \tau_{zz} = \mathrm{N}_{zx} e_{xx} + \mathrm{N}_{zy} e_{yy} + \mathrm{N}_{zz} e_{zz} \\ \tau_{zx} = 2\mathrm{E}_y e_{zx} \end{cases} \tag{3}$$

where N_{xx}, N_{xy}, N_{xz}, N_{yx}, N_{yy}, N_{yz}, N_{zx}, N_{zy}, and N_{zz} are the incremental normal elastic coefficients, E_x, E_y and E_z shear modulus along x, y and z axis respectively. The strain components e_{xy}, e_{xx}, e_{yy}, e_{yz}, e_{zx}, and e_{zz} are defined by

$$\begin{cases} e_{xy} = \frac{1}{2}\left(\frac{\partial v_1}{\partial x} + \frac{\partial u_1}{\partial y}\right), e_{yz} = \frac{1}{2}\left(\frac{\partial w_1}{\partial y} + \frac{\partial v_1}{\partial z}\right), e_{zx} = \frac{1}{2}\left(\frac{\partial v_1}{\partial z} + \frac{\partial w_1}{\partial x}\right) \\ e_{xx} = \left(\frac{\partial u_2}{\partial x}\right), e_{yy} = \left(\frac{\partial v_2}{\partial y}\right), e_{zz} = \left(\frac{\partial w_2}{\partial z}\right) \end{cases} \tag{4}$$

Using SH-wave conditions

$$u_1 = w_1 = 0 \text{ and } v_1 = v_1(x, z, t) \tag{5}$$

Using Eqs. (2)-(5), the equation of motion for the upper orthotropic half-space given by Eq. (1) becomes

$$\mathrm{E}_x \frac{\partial^2 v_1}{\partial z^2} + \left(\mathrm{E}_z - \frac{P}{2}\right)\frac{\partial^2 v_1}{\partial x^2} + \frac{\partial \mathrm{E}_z}{\partial x}\frac{\partial v_1}{\partial x} + \frac{\partial \mathrm{E}_x}{\partial z}\frac{\partial v_1}{\partial z} = \rho_1 \frac{\partial^2 v_1}{\partial t^2} \tag{6}$$

and the non-zero stress components are

$$\tau_{yx} = 2\mathrm{E}_z e_{xy} \text{ and } \tau_{yz} = 2\mathrm{E}_x e_{yz} \tag{7}$$

To solve Eq. (6) take the following substitution

$$v_1 = \mathrm{V(z)}\,\mathrm{e}^{ik(x-ct)} \tag{8}$$

where $\omega=kc$, c is phase velocity and k is wave number.
Using Eq. (8) in Eq. (6), the following equation is obtained

$$\mathrm{E}_x \frac{\mathrm{d}^2\mathrm{V}(z)}{\mathrm{d}z^2} + \frac{\partial \mathrm{E}_z}{\partial z}\frac{\mathrm{dV}(z)}{\mathrm{d}z} + ik\frac{\partial \mathrm{E}_x}{\partial z}\mathrm{V}(z) + k^2\left(\rho_1 c^2 + \frac{P}{2} - \mathrm{E}_z\right)\mathrm{V}(z) = 0 \tag{9}$$

The assumed inhomogeneities of the upper orthotropic layer are

$$\begin{cases} P = P_0(1-\cos az) \\ \rho_1 = \rho_0(1-\cos az) \\ \mathrm{E}_i = \mathrm{Q}_0(1-\cos az) \end{cases} \tag{10}$$

Using Eq. (10) in Eq. (9), the following equation is obtained

$$\frac{\mathrm{d}^2\mathrm{V}(z)}{\mathrm{d}z^2} + \frac{a\sin(az)}{1-\cos(az)}\frac{\mathrm{dV}(z)}{\mathrm{d}z} + k^2\left(\frac{c^2}{c_1^2} + \frac{P_0 - 2\mathrm{Q}_z}{2\mathrm{Q}_x}\right)\mathrm{V}(z) = 0 \tag{11}$$

where $c_1 = \sqrt{\frac{\mathrm{Q}_x}{\rho_0}}$ is the shear velocity of the upper layer.

To eliminate $\frac{\mathrm{dV}(z)}{\mathrm{d}z}$ put $\mathrm{V}(z) = \frac{q(z)}{\sqrt{1-\cos(az)}}$ in Eq. (11), the following equation is obtained

$$\frac{\mathrm{d}^2 q}{\mathrm{d}z^2}+\gamma^2 q=0 \tag{12}$$

where, $\gamma^2=k^2\left(\frac{a^2}{4k^2}+\frac{c^2}{c_1^2}+\frac{P_0-2\mathrm{Q}_z}{2\mathrm{Q}_x}\right)$

Therefore, the solution for Eq. (12) is given by

$$q(z)=\mathrm{A}\cos\gamma z+\mathrm{B}\sin\gamma z \tag{13}$$

where B and C are arbitrary constants.

Therefore, the displacement component for the upper hetrogeneous orthotropic layer can be written as

$$v_1=\frac{\mathrm{A}\cos\gamma z+\mathrm{B}\sin\gamma}{\sqrt{1-\cos(az)}}\mathrm{e}^{ik(x-ct)} \tag{14}$$

3.2 Solution for the porous half-space

Consider an anisotropic initially stressed porous half space. Neglecting the viscosity of water, the dynamic equations of motion in the porous half-space under the compressive initial stress P', in the absence of body forces are (Biot 1965)

$$\begin{cases}\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\sigma_{xy}}{\partial y}+\frac{\partial\sigma_{xz}}{\partial z}-P'\left(\frac{\partial\omega_z'}{\partial y}-\frac{\partial\omega_y'}{\partial z}\right)=\frac{\partial^2}{\partial t^2}\left(\rho_{11}u_1'+\rho_{12}U_x'\right)\\ \frac{\partial\sigma_{yx}}{\partial x}+\frac{\partial\sigma_{yy}}{\partial y}+\frac{\partial\sigma_{yz}}{\partial z}-P'\left(\frac{\partial\omega_z'}{\partial x}\right)=\frac{\partial^2}{\partial t^2}\left(\rho_{11}v_1'+\rho_{12}V_y'\right)\\ \frac{\partial\sigma_{zx}}{\partial x}+\frac{\partial\sigma_{zy}}{\partial y}+\frac{\partial\sigma_{zz}}{\partial z}-P'\left(\frac{\partial\omega_y'}{\partial x}\right)=\frac{\partial^2}{\partial t^2}\left(\rho_{11}w_1'+\rho_{12}W_z'\right)\\ \frac{\partial\sigma}{\partial x}=\frac{\partial^2}{\partial t^2}\left(\rho_{11}u_1'+\rho_{12}U_x'\right)\\ \frac{\partial\sigma}{\partial y}=\frac{\partial^2}{\partial t^2}\left(\rho_{11}v_1'+\rho_{12}V_y'\right)\\ \frac{\partial\sigma}{\partial z}=\frac{\partial^2}{\partial t^2}\left(\rho_{11}w_1'+\rho_{12}W_z'\right)\end{cases} \tag{15a}$$

where, $\sigma_{ij}(i,j=x,y,z)$ are the incremental stress components, (u_1',v_1',w_1') are the components of the displacement vector of the solid, (U_x',V_y',W_z') are the components of the displacement vector of the liquid and σ is the stress vector due to the liquid. This stress vector σ is related to the fluid pressure p by the relation $\sigma=-fp$, where f is porosity of the layer. The angular components $\omega_x',\omega_y',\omega_z'$ are given by

$$\omega_x'=\frac{1}{2}\left(\frac{\partial w_1'}{\partial y}-\frac{\partial v_1'}{\partial z}\right),\omega_y'=\frac{1}{2}\left(\frac{\partial u_1'}{\partial z}-\frac{\partial w_1'}{\partial x}\right),\omega_x'=\frac{1}{2}\left(\frac{\partial v_1'}{\partial x}-\frac{\partial u_1'}{\partial y}\right) \tag{15b}$$

The mass coefficients ρ_{11}, ρ_{12} and ρ_{22} are related to the densities ρ, ρ_s and ρ_w of the layer, solid and water, respectively, given by

$$\rho_{11}+\rho_{12}=(1-f)\rho_s,\ \ \rho_{12}+\rho_{22}=f\rho_w \tag{15c}$$

So that the mass density of the aggregate

$$\rho=\rho_{11}+2\rho_{12}+\rho_{22}=\rho_s+f(\rho_w-\rho_s) \tag{15d}$$

These mass coefficients also obey the following inequalities

$$\rho_{11}>0,\ \ \rho_{12}\le 0,\ \ \rho_{22}>0,\ \ \rho_{11}\rho_{22}-\rho_{12}{}^2>0 \tag{15e}$$

For Love-wave propagation, the stress-strain relations for the water saturated initially stressed anisotropic porous layer

$$\begin{cases}\sigma_{xx}=(I+P')e_{xx}+(I-2N+P')e_{yy}+(J+P')e_{zz}+K\Delta\\ \sigma_{yy}=(I-2N)e_{xx}+Ie_{yy}+Je_{zz}+K\Delta\\ \sigma_{zz}=Je_{xx}+Je_{yy}+De_{zz}+K\Delta,\\ \sigma_{xy}=2Ne_{xy},\sigma_{yz}=2Le_{yz},\sigma_{zx}=2Le_{zx}\end{cases} \tag{15f}$$

where $e_{ij}=\dfrac{1}{2}\left(u_{i,j}+u_{j,i}\right)$, $\Delta=div\,\vec{u}$ is the dilation, I, J, D, N and L are elastic constants for the medium; N and L are, in particular, shear moduli of the anisotropic layer in the x and z direction respectively. Further, K being the measure of coupling between the volume change of the solid and the liquid is a positive quantity.

The hydrostatic stresses in the gravitational half-space are given by

$$\sigma_{xx}=\sigma_{zz}=-dgz \tag{15g}$$

where d is the density of the lower half-space.

The components of body force are due to gravity g and are

$$x=0,\ y=0,\ z\ =\ g \tag{15h}$$

For the Love-waves propagating along the x-direction, having the displacement of particles along the y-direction, we have

$$\begin{cases}U'_x=0,\ U'_y=U'_y(x,z,t)\text{ and }U'_z=0\\ u'_1=0,\ u'_2=u'_2(x,z,t)\text{ and }u'_3=0\end{cases} \tag{15i}$$

These displacements will produce only the e_{xy} and e_{yz} strain components and the other strain components will be zero. Hence, the stress-strain relations useful in the problem are

$$\begin{cases}\sigma_{xy}=2Ne_{xy}=N\dfrac{\partial u'_2}{\partial x}\\ \sigma_{yz}=2Le_{yz}=L\dfrac{\partial u'_2}{\partial z}\\ \text{Also, }\dfrac{\partial}{\partial y}=0\end{cases} \tag{15j}$$

Therefore Eq. (15a) with the help of Eqs. (15g)-(15j) can be written as

$$\frac{\partial\sigma_{xy}}{\partial x}+\frac{\partial\sigma_{yz}}{\partial z}-P'\left(\frac{\partial\omega'_z}{\partial x}\right)-dg\omega'_{yz}-dgz\frac{\partial\omega'_{yz}}{\partial z}+dgz\frac{\partial\omega'_{xy}}{\partial x}=\frac{\partial^2}{\partial t^2}(\rho_{11}u'_2+\rho_{12}V'_y)$$
$$\frac{\partial\sigma}{\partial y}=\frac{\partial^2}{\partial t^2}(\rho_{12}u'_2+\rho_{22}V'_y)=0 \tag{16}$$

where ω'_{ij} are the angular components, which are defined as

$$\omega'_{xy}=-\frac{1}{2}\frac{\partial u'_2}{\partial x},\ \omega'_{yz}=+\frac{1}{2}\frac{\partial u'_2}{\partial z} \tag{17}$$

Eliminating U'_x (displacement of liquid part) from Eq. (16), the following equation is obtained

$$\frac{\partial\sigma_{xy}}{\partial x}+\frac{\partial\sigma_{yz}}{\partial z}-P'\left(\frac{\partial\omega'_z}{\partial x}\right)-dg\omega'_{yz}-dgz\frac{\partial\omega'_{yz}}{\partial z}+dgz\frac{\partial\omega'_{xy}}{\partial x}=d\frac{\partial^2u'_2}{\partial t^2} \tag{18}$$

where $d=\rho_{11}-\dfrac{\rho_{12}^2}{\rho_{22}}$.

Using Eq. (15j), Eq. (17) in Eq. (18), we have

$$\left(N-\frac{P'}{2}-\frac{dgz}{2}\right)\frac{\partial^2u'_2}{\partial x^2}+\left(L-\frac{dgz}{2}\right)\frac{\partial^2u'_2}{\partial z^2}-\left(\frac{dg}{2}\right)\frac{\partial u'_2}{\partial z}=d\frac{\partial^2u'_2}{\partial t^2} \tag{19}$$

In order to solve Eq. (19), we take

$$u'_2(x,z,t)=\xi(z)\,\mathrm{e}^{ik(x-ct)} \tag{20}$$

From Eq. (19) and Eq. (20), the following equation is obtained

$$\frac{\partial^2\xi(z)}{\partial z^2}-\frac{G\delta k}{2\left(1-\dfrac{G\delta kz}{2}\right)}\frac{\partial\xi(z)}{\partial z}-k^2\left(\frac{\left(\dfrac{N}{L}-\bar{\zeta}-\dfrac{G\delta kz}{2}\right)-\delta\dfrac{c^2}{c_2^2}}{\left(1-\dfrac{G\delta kz}{2}\right)}\right)\xi(z)=0 \tag{21}$$

where $c_2=\left[L/\rho\right]^{\frac{1}{2}}$ is the shear velocity in the lower half-space, $G=dg/Lk$ known as Biot's gravity parameter, $\bar{\zeta}=\dfrac{P'}{2L}$ is stress parameter, $\delta=\gamma_{11}-\gamma_{12}^2/\gamma_{22}$, $\gamma_{11}=\rho_{11}/\rho$, $\gamma_{12}=\rho_{12}/\rho$, $\gamma_{22}=\rho_{22}/\rho$, are the non-dimensional parameters for the material of the porous half-space, k is wave number.

Now substituting $\xi(z)=\chi(z)\left(1-\dfrac{G\delta kz}{2}\right)^{-\frac{1}{2}}$ in Eq. (21) to eliminating term $\dfrac{\partial\xi(z)}{\partial z}$, we obtain

$$\chi''(z)+k^2\left[\frac{G^2\delta^2}{16}\left(1-\frac{G\delta kz}{2}\right)^{-2}\left\{\left(\frac{N}{L}-\bar{\zeta}-\frac{G'\delta kz}{2}\right)-\delta\frac{c^2}{c_2^2}\right\}\left(1-\frac{G\delta k}{2}\right)^{-1}\right]\chi(z)=0 \tag{22}$$

Introducing the dimensionless quantities $\eta = \frac{4}{G\delta}\left(\frac{N}{L} - \bar{\zeta} - \frac{G\delta kz}{2}\right)$ and $\chi(z) = g(\eta)$ in Eq. (22), the following equation is obtained

$$\frac{d^2 g(\eta)}{dz^2} + \left(-\frac{1}{4} + \frac{R}{\eta} + -\frac{1}{4\eta^2}\right) g(\eta) = 0 \tag{23}$$

where, $R = \frac{1}{G}\left[\left(1 - \frac{N}{L} - \bar{\zeta}\right)\frac{1}{\delta} + \frac{c^2}{c_2^2}\right]$

Eq. (23) is the well known Whittaker's equation (Whittaker and Watson 1990).

The solution of Whittaker's Eq. (23) is given by

$$g(\eta) = \mathrm{C}\mathrm{W}_{\mathrm{R},0}(\eta) + \mathrm{D}\mathrm{W}_{-\mathrm{R},0}(-\eta) \tag{24}$$

where C and D are arbitrary constants and $W_{R,0}(\eta)$, $W_{-R,0}(-\eta)$ are the Whittaker function.

Now Eq. (24) satisfying the condition $\lim_{z\to\infty} u_2' \to 0$ i.e., $\lim_{z\to\infty} g(\eta) \to 0$ as $\eta \to 0$ may be taken as

$$g(\eta) = \mathrm{C}\mathrm{W}_{-\mathrm{R},0}(-\eta) \tag{25}$$

On solving Whittaker's function up to second degree term, displacement for the SH-wave in the lower layer is

$$u_2'(x,z,t) = \mathrm{C}\left(-\frac{4}{G\delta}\right)^{-R}\left(1 - \frac{G\delta kz}{2}\right)^{-R-\frac{1}{2}} \mathrm{e}^{\frac{2}{G\delta}\left(1-\frac{G\delta kz}{2}\right)} \left\{1 + \frac{G\delta}{4}\frac{\left(R+\frac{1}{2}\right)^2}{1-\frac{G\delta kz}{2}}\right\} e^{ik(\mathrm{x}-\mathrm{ct})} \tag{26}$$

4. Boundary conditions

The displacement components and stress components are continuous at $z=-H$, and at $z=0$, therefore the geometry of the problem leads to the following conditions

At $z=-H$, the stress component $\tau_{yz}=0$.

At $z=0$, the stress component of the layer and half space is continuous, i.e., $\tau_{yz}=\sigma_{yz}$.

At $z=0$, the velocity component of both the layer is continuous, i.e., $u_2=u_2'$.

5. Dispersion relation

The dispersion relation for SH-wave can be obtained by using boundary conditions given in section 4. Therefore, the displacement for the SH-wave in the half-space using boundary conditions (1), (2) and (3) in Eq. (7), Eq. (14), Eq. (15j) and Eq. (26)

$$\mathrm{A}\left[\gamma\cos(\gamma\mathrm{H}) - \frac{\mathrm{a}\sin(a\mathrm{H})\sin(\gamma\mathrm{H})}{2(1+\cos(a\mathrm{H}))}\right] + \mathrm{B}\left[\gamma\sin(\gamma\mathrm{H}) + \frac{\mathrm{a}\sin(a\mathrm{H})\cos(\gamma\mathrm{H})}{2(1+\cos(a\mathrm{H}))}\right] = 0 \tag{27}$$

$$\mathrm{A}{=}\mathrm{C}\left(-\frac{4}{G\delta}\right)^{-R}\mathrm{e}^{\frac{2}{G\delta}}\left\{1+\frac{G\delta}{4}\left(R+\frac{1}{2}\right)^{2}\right\} \tag{28}$$

$$\mathrm{A}\left(-\frac{a}{2}\right)+\mathrm{B}(\gamma)=\mathrm{C}\left(\frac{L}{\mathrm{Q_x}}\right)k\left(-\frac{4}{G\delta}\right)^{-R}e^{\frac{2}{G\delta}}\left[\left\{1+\frac{G\delta}{4}\left(R+\frac{1}{2}\right)^{2}\right\}\left\{\frac{G\delta}{4}\left(R+\frac{1}{2}\right)-1\right\}+\frac{G^2\delta^2}{8}\left(R+\frac{1}{2}\right)^{2}\right] \tag{29}$$

Now eliminating *A*, *B* and *C* from the Eq. (27), Eq. (28) and Eq. (29), we obtain

$$\begin{vmatrix} \left[\gamma\cos(\gamma \mathrm{H})-\dfrac{\mathrm{a}\sin(a\mathrm{H})\sin(\gamma\mathrm{H})}{2(1+\cos(a\mathrm{H}))}\right] & \left[\gamma\sin(\gamma \mathrm{H})+\dfrac{\mathrm{a}\sin(a\mathrm{H})\cos(\gamma\mathrm{H})}{2(1+\cos(a\mathrm{H}))}\right] & 0 \\ 1 & 0 & -\left(-\dfrac{4}{G\delta}\right)^{-R}\mathrm{e}^{\frac{2}{G\delta}}\left\{1+\dfrac{G\delta}{4}\left(R+\dfrac{1}{2}\right)^{2}\right\} \\ \left(-\dfrac{a}{2}\right) & (\gamma) & -\left(\dfrac{L}{\mathrm{Q_x}}\right)k\left(-\dfrac{4}{G\delta}\right)^{-R}e^{\frac{2}{G\delta}}\left[\left\{1+\dfrac{G\delta}{4}\left(R+\dfrac{1}{2}\right)^{2}\right\}\left\{\dfrac{G\delta}{4}\left(R+\dfrac{1}{2}\right)-1\right\}+\dfrac{G^2\delta^2}{8}\left(R+\dfrac{1}{2}\right)^{2}\right] \end{vmatrix}=0 \tag{30}$$

On solving Eq. (30), the following equation is obtained

$$\tan\left[kH\sqrt{\frac{a^2}{4k^2}+\frac{c^2}{c_1^2}+\frac{P_0-2\mathrm{Q}_z}{2\mathrm{Q}_x}}\right]=\frac{\left(\dfrac{\mathrm{a}}{2k}\right)\dfrac{\sin(a\mathrm{H})}{(1+\cos(a\mathrm{H}))}-\left(\dfrac{L}{\mathrm{Q}_x}\dfrac{F_1}{F_1k}+\dfrac{\mathrm{a}}{2k}\right)\sqrt{\dfrac{a^2}{4k^2}+\dfrac{c^2}{c_1^2}+\dfrac{P_0-2\mathrm{Q}_z}{2\mathrm{Q}_x}}}{\dfrac{a^2}{4k^2}+\dfrac{c^2}{c_1^2}+\dfrac{P_0-2\mathrm{Q}_z}{2\mathrm{Q}_x}+\left(\dfrac{\mathrm{a}}{2k}\right)\dfrac{\sin(a\mathrm{H})}{(1+\cos(a\mathrm{H}))}\left(\dfrac{L}{\mathrm{Q}_x}\dfrac{F_1}{F_1k}+\dfrac{\mathrm{a}}{2k}\right)} \tag{31}$$

where

$$F_1=\left(-\frac{4}{G\delta}\right)^{-R}\mathrm{e}^{\frac{2}{G\delta}}\left\{1+\frac{G\delta}{4}\left(R+\frac{1}{2}\right)^{2}\right\}$$

$$F_2=k\left(-\frac{4}{G\delta}\right)^{-R}e^{\frac{2}{G\delta}}\left[\left\{1+\frac{G\delta}{4}\left(R+\frac{1}{2}\right)^{2}\right\}\left\{\frac{G\delta}{4}\left(R+\frac{1}{2}\right)-1\right\}+\frac{G^2\delta^2}{8}\left(R+\frac{1}{2}\right)^{2}\right]$$

Eq. (31) is the dispersion equation of SH-wave propagation in a hetrogeneous layer overlying a gravitational porous half-space under initial stresses.

Particular Cases

If the layer is non-porous then $f\to 0$ and $\rho_s\to\rho$ which leads to $\gamma_{11}+\gamma_{12}\to 1$ and $\gamma_{12}+\gamma_{22}\to 0$, which leads to $\gamma_{11}-\dfrac{\gamma_{12}^2}{\gamma_{22}}\to 1$ or $\delta\to 1$. If the layer is porous then $f\to 1$, then $\rho_w\to\rho$, the liquid becomes fluid $\gamma_{11}-\dfrac{\gamma_{12}^2}{\gamma_{22}}\to 0$ or $\delta\to 0$, which means shear waves do not exit. Hence, for porous layer $0<f<1$ corresponds to $0<\delta<1$.

Case-1

If upper layer is stress free then $P_0\to 0$, Eq. (31) reduces to

$$\tan\left[kH\sqrt{\frac{a^2}{4k^2}+\frac{c^2}{c_1^2}-\frac{\mathrm{Q}_z}{\mathrm{Q}_x}}\right]=\frac{\left(\dfrac{\mathrm{a}}{2k}\right)\dfrac{\sin(a\mathrm{H})}{(1+\cos(a\mathrm{H}))}-\left(\dfrac{L}{\mathrm{Q}_x}\dfrac{F_1}{F_1k}+\dfrac{\mathrm{a}}{2k}\right)\sqrt{\dfrac{a^2}{4k^2}+\dfrac{c^2}{c_1^2}-\dfrac{\mathrm{Q}_z}{\mathrm{Q}_x}}}{\dfrac{a^2}{4k^2}+\dfrac{c^2}{c_1^2}-\dfrac{\mathrm{Q}_z}{\mathrm{Q}_x}+\left(\dfrac{\mathrm{a}}{2k}\right)\dfrac{\sin(a\mathrm{H})}{(1+\cos(a\mathrm{H}))}\left(\dfrac{L}{\mathrm{Q}_x}\dfrac{F_1}{F_1k}-\dfrac{\mathrm{a}}{2k}\right)}\tag{32}$$

Eq. (32) is the dispersion equation of SH-wave propagation in a hetrogeneous layer overlying a pre-stressed gravitational porous half-space.

Case-2

For SH-wave propagation in a hetrogeneous layer over a porous half-space free under gravity free from initial stresses, then $P_0\to 0$ and $\zeta\to 0$, therefore, Eq. (31) reduces to

$$\tan\left[kH\sqrt{\frac{a^2}{4k^2}+\frac{c^2}{c_1^2}-\frac{\mathrm{Q}_z}{\mathrm{Q}_x}}\right]=\frac{\left(\dfrac{a}{2k}\right)\dfrac{\sin(a\mathrm{H})}{(1+\cos(a\mathrm{H}))}-\left(\dfrac{L}{\mathrm{Q}_x}\dfrac{F_1'}{F_1'k}+\dfrac{a}{2k}\right)\sqrt{\dfrac{a^2}{4k^2}+\dfrac{c^2}{c_1^2}-\dfrac{\mathrm{Q}_z}{\mathrm{Q}_x}}}{\dfrac{a^2}{4k^2}+\dfrac{c^2}{c_1^2}-\dfrac{\mathrm{Q}_z}{\mathrm{Q}_x}+\left(\dfrac{a}{2k}\right)\dfrac{\sin(a\mathrm{H})}{(1+\cos(a\mathrm{H}))}\left(\dfrac{L}{\mathrm{Q}_x}\dfrac{F_1'}{F_1'k}+\dfrac{a}{2k}\right)}\tag{33}$$

where

$$F_1'=\left(-\frac{4}{G\delta}\right)^{-R}\mathrm{e}^{\frac{2}{G\delta}}\left\{1+\frac{G\delta}{4}\left(R'+\frac{1}{2}\right)^2\right\}$$

$$F_2'=k\left(-\frac{4}{G\delta}\right)^{-R}e^{\frac{2}{G\delta}}\left[\left\{1+\frac{G\delta}{4}\left(R'+\frac{1}{2}\right)^2\right\}\left\{\frac{G\delta}{4}\left(R'+\frac{1}{2}\right)-1\right\}+\frac{G^2\delta^2}{8}\left(R'+\frac{1}{2}\right)^2\right]$$

and $R'=\dfrac{1}{G}\left[\left(1-\dfrac{N}{L}\right)\dfrac{1}{\delta}+\dfrac{c^2}{c_2^2}\right]$

Case-3

If upper layer is isotropic and stress free then $Q_x=Q_z=\mu_1$, $P_0\to 0$, Eq. (31) reduces to

$$\tan\left[kH\sqrt{\frac{a^2}{4k^2}+\frac{c^2}{c_1^2}-1}\right]=\frac{\left(\dfrac{\mathrm{a}}{2k}\right)\dfrac{\sin(a\mathrm{H})}{(1+\cos(a\mathrm{H}))}-\left(\dfrac{L}{\mu_1}\dfrac{F_1}{F_1k}+\dfrac{\mathrm{a}}{2k}\right)\sqrt{\dfrac{a^2}{4k^2}+\dfrac{c^2}{c_1^2}-1}}{\dfrac{a^2}{4k^2}+\dfrac{c^2}{c_1^2}-1+\left(\dfrac{\mathrm{a}}{2k}\right)\dfrac{\sin(a\mathrm{H})}{(1+\cos(a\mathrm{H}))}\left(\dfrac{L}{\mu_1}\dfrac{F_1}{F_1k}+\dfrac{\mathrm{a}}{2k}\right)}\tag{34}$$

Eq. (32) is the dispersion equation of SH-wave propagation in a hetrogeneous isotropic layer overlying a pre-stressed gravitational porous half-space.

Case-4

For SH-wave propagation in a hetrogeneous isotropic layer over a porous half-space free under gravity free from initial stresses, then $Q_x=Q_z=\mu_1$, $P_0\rightarrow 0$ and $\zeta\rightarrow 0$, therefore, Eq. (31) reduces to

$$\tan\left[kH\sqrt{\frac{a^2}{4k^2}+\frac{c^2}{c_1^2}-1}\right]=\frac{\left(\frac{a}{2k}\right)\frac{\sin(a\mathrm{H})}{(1+\cos(a\mathrm{H}))}-\left(\frac{L}{\mu_1}\frac{F_1'}{F_1'k}+\frac{a}{2k}\right)\sqrt{\frac{a^2}{4k^2}+\frac{c^2}{c_1^2}-1}}{\frac{a^2}{4k^2}+\frac{c^2}{c_1^2}-1+\left(\frac{a}{2k}\right)\frac{\sin(a\mathrm{H})}{(1+\cos(a\mathrm{H}))}\left(\frac{L}{\mu_1}\frac{F_1'}{F_1'k}+\frac{\mathrm{a}}{2k}\right)} \tag{35}$$

where

$$F_1'=\left(-\frac{4}{G\delta}\right)^{-R}\mathrm{e}^{\frac{2}{G\delta}}\left\{1+\frac{G\delta}{4}\left(R'+\frac{1}{2}\right)^2\right\}$$

$$F_2'=k\left(-\frac{4}{G\delta}\right)^{-R}e^{\frac{2}{G\delta}}\left[\left\{1+\frac{G\delta}{4}\left(R'+\frac{1}{2}\right)^2\right\}\left\{\frac{G\delta}{4}\left(R'+\frac{1}{2}\right)-1\right\}+\frac{G^2\delta^2}{8}\left(R'+\frac{1}{2}\right)^2\right]$$

and $R'=\frac{1}{G}\left[\left(1-\frac{N}{L}\right)\frac{1}{\delta}+\frac{c^2}{c_2^2}\right]$

Case-5

For SH-wave propagation in homogeneous layer overlying a gravitational porous half-space under initial stresses, $a\rightarrow 0$

$$\tan\left[kH\sqrt{\frac{c^2}{c_1^2}+\frac{P_0-2\mathrm{Q}_z}{2\mathrm{Q}_x}}\right]=-\frac{\left(\frac{L}{\mathrm{Q}_x}\frac{F_1}{F_1k}\right)\sqrt{\frac{c^2}{c_1^2}+\frac{P_0-2\mathrm{Q}_z}{2\mathrm{Q}_x}}}{\frac{c^2}{c_1^2}+\frac{P_0-2\mathrm{Q}_z}{2\mathrm{Q}_x}} \tag{36}$$

Case-6

For SH-wave propagation in a hetrogeneous pre-stressed layer overlying a pre-stressed porous half-space free from gravity then $G\rightarrow 0$, Eq. (31) reduces to

$$\tan\left[kH\sqrt{\frac{a^2}{4k^2}+\frac{c^2}{c_1^2}+\frac{P_0-2\mathrm{Q}_z}{2\mathrm{Q}_x}}\right]=\frac{\left(\frac{\mathrm{a}}{2k}\right)\frac{\sin(a\mathrm{H})}{(1+\cos(a\mathrm{H}))}-\left(\frac{L}{\mathrm{Q}_x}F_3+\frac{\mathrm{a}}{2k}\right)\sqrt{\frac{a^2}{4k^2}+\frac{c^2}{c_1^2}+\frac{P_0-2\mathrm{Q}_z}{2\mathrm{Q}_x}}}{\frac{a^2}{4k^2}+\frac{c^2}{c_1^2}+\frac{P_0-2\mathrm{Q}_z}{2\mathrm{Q}_x}+\left(\frac{\mathrm{a}}{2k}\right)\frac{\sin(a\mathrm{H})}{(1+\cos(a\mathrm{H}))}\left(\frac{L}{\mathrm{Q}_x}F_3+\frac{\mathrm{a}}{2k}\right)} \tag{37}$$

where $F_3=\frac{1}{2}\left(1-\frac{N}{L}-\frac{P'}{2L}+\delta\frac{c^2}{c_2^2}\right)-1$

Case-7

For SH-wave propagation in a hetrogeneous pre-stressed layer overlying a pre-stressed non-porous half-space free from gravity then $\delta\rightarrow 0$, $G\rightarrow 0$, Eq. (31) reduces to

$$\tan\left[kH\sqrt{\frac{a^2}{4k^2}+\frac{c^2}{c_1^2}+\frac{P_0-2\mathrm{Q}_z}{2\mathrm{Q}_x}}\right]=\frac{\left(\frac{\mathrm{a}}{2k}\right)\frac{\sin(a\mathrm{H})}{(1+\cos(a\mathrm{H}))}-\left(\frac{L}{\mathrm{Q}_x}F_3'+\frac{\mathrm{a}}{2k}\right)\sqrt{\frac{a^2}{4k^2}+\frac{c^2}{c_1^2}+\frac{P_0-2\mathrm{Q}_z}{2\mathrm{Q}_x}}}{\frac{a^2}{4k^2}+\frac{c^2}{c_1^2}+\frac{P_0-2\mathrm{Q}_z}{2\mathrm{Q}_x}+\left(\frac{\mathrm{a}}{2k}\right)\frac{\sin(a\mathrm{H})}{(1+\cos(a\mathrm{H}))}\left(\frac{L}{\mathrm{Q}_x}F_3'+\frac{\mathrm{a}}{2k}\right)} \tag{38}$$

where $F_3'=\frac{1}{2}\left(1-\frac{N}{L}-\frac{P'}{2L}+\frac{c^2}{c_2^2}\right)-1$

Case-8

For SH-wave propagation in a hetrogeneous pre-stressed layer overlying a pre-stressed isotropic non-porous half-space free from gravity then $N=L=\mu_2$, $\delta\to 1$, $G\to 0$, Eq. (31) reduces to

$$\tan\left[kH\sqrt{\frac{a^2}{4k^2}+\frac{c^2}{c_1^2}+\frac{P_0-2\mathrm{Q}_z}{2\mathrm{Q}_x}}\right]=\frac{\left(\frac{\mathrm{a}}{2k}\right)\frac{\sin(a\mathrm{H})}{(1+\cos(a\mathrm{H}))}-\left(\frac{\mu_2}{\mathrm{Q}_x}F_4+\frac{\mathrm{a}}{2k}\right)\sqrt{\frac{a^2}{4k^2}+\frac{c^2}{c_1^2}+\frac{P_0-2\mathrm{Q}_z}{2\mathrm{Q}_x}}}{\frac{a^2}{4k^2}+\frac{c^2}{c_1^2}+\frac{P_0-2\mathrm{Q}_z}{2\mathrm{Q}_x}+\left(\frac{\mathrm{a}}{2k}\right)\frac{\sin(a\mathrm{H})}{(1+\cos(a\mathrm{H}))}\left(\frac{\mu_2}{\mathrm{Q}_x}F_4+\frac{\mathrm{a}}{2k}\right)} \tag{39}$$

where $F_4=\frac{1}{2}\left(-\frac{P'}{2\mu_2}+\frac{c^2}{c_2^2}\right)-1$

Case-9

For SH-wave propagation in a hetrogeneous layer overlying a pre-stressed isotropic non-porous half-space free from gravity then $N=L=\mu_2$, $\delta\to 1$, $P_0\to 0$, $G\to 0$, Eq. (31) reduces to

$$\tan\left[kH\sqrt{\frac{a^2}{4k^2}+\frac{c^2}{c_1^2}-\frac{\mathrm{Q}_z}{\mathrm{Q}_x}}\right]=\frac{\left(\frac{\mathrm{a}}{2k}\right)\frac{\sin(a\mathrm{H})}{(1+\cos(a\mathrm{H}))}-\left(\frac{\mu_2}{\mathrm{Q}_x}F_4+\frac{\mathrm{a}}{2k}\right)\sqrt{\frac{a^2}{4k^2}+\frac{c^2}{c_1^2}-\frac{\mathrm{Q}_z}{\mathrm{Q}_x}}}{\frac{a^2}{4k^2}+\frac{c^2}{c_1^2}-\frac{\mathrm{Q}_z}{\mathrm{Q}_x}+\left(\frac{\mathrm{a}}{2k}\right)\frac{\sin(a\mathrm{H})}{(1+\cos(a\mathrm{H}))}\left(\frac{\mu_2}{\mathrm{Q}_x}F_4+\frac{\mathrm{a}}{2k}\right)} \tag{40}$$

where $F_4=\frac{1}{2}\left(-\frac{P'}{2\mu_2}+\frac{c^2}{c_2^2}\right)-1$

Case-10

For homogeneous layer over a homogeneous non-porous half space free from gravity and stresses then $G\to 0$, $\zeta\to 0$, $N=L=\mu_2$, $\delta\to 1$, $Q_x=Q_z=\mu_1$, $P_0\to 0$, $a\to 0$ therefore, Eq. (31) reduces to

$$\tan\left[kH\sqrt{\frac{c^2}{c_1^2}-1}\right]=\frac{\mu_2}{\mu_1}\frac{1-\frac{1}{2}\frac{c^2}{c_2^2}}{\sqrt{\frac{c^2}{c_1^2}-1}} \tag{41}$$

On approximation Eq. (40) gives

$$\tan\left[kH\sqrt{\frac{c^2}{c_1^2}-1}\right]=\frac{\mu_2}{\mu_1}\frac{\sqrt{1-\frac{c^2}{c_2^2}}}{\sqrt{\frac{c^2}{c_1^2}-1}} \tag{42}$$

Eq. (42) is the classical dispersion equation for SH-waves propagating in a homogenous isotropic elastic medium over isotropic homogeneous elastic medium given by Love (1911) and Ewing *et al.* (1957),

5. Numerical analysis

To examine the effect of thickness of the layer (kH), heterogeneity parameter $\left(\bar{\psi} = \frac{\mathrm{a}}{2k}\right)$ of upper layer, stress parameters $\left(\bar{\eta} = \frac{2\mathrm{Q}_x}{P_0 - 2\mathrm{Q}_z} \text{ and } \bar{\zeta} = \frac{P'}{2L}\right)$ of the considered geometry, porosity (δ), Biot's gravity parameter (G) and other dimensionless parameters $\left(\bar{\varsigma} = \frac{N}{L}, \bar{\xi} = \frac{L}{\mathrm{Q}_x}, \bar{\beta} = \frac{c_1}{c_2}\right)$ on phase velocity (c/c_1), the following numerical values are considered (given in Table 1), The material parameters for this model are taken in Table 2 used by Chattaraj and Samal (2013), Fig. 2 describes the effect of dimensionless thickness (kH) of the inhomogeneous layer on phase velocity (c/c_1) under porosity (δ). It is clear that the phase velocity gradually decreases with the increment in thickness of the layer. However when the porosity (δ) is increased the following observations are made:

1. As the porosity (δ) increases, the dimensionless phase velocity (c/c_1) decreases at a particular value of dimensionless thickness (kH).
2. The curves are plotted for the values of porosity δ=0.0, 0.3, 0.5, 0.7 and 0.9, it is clear that the curves are going away from each other as porosity increases, it implies that porosity dominates at large values of dimensionless thickness (kH).

Fig. 3 depicts the effect of dimensionless thickness (kH) of the heterogeneous layer on phase velocity (c/c_1) under heterogeneity parameter $\left(\bar{\psi} = \frac{\mathrm{a}}{2k}\right)$. Again, it is clear that the phase velocity gradually decreases with the increment in thickness of the heterogeneous layer.

Table 1 Values of parameters for figures

	Fig. 2	Fig. 3	Fig. 4	Fig. 5	Fig. 6	Fig. 7	Fig. 8	Fig. 9	Fig. 10
δ	–	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
$\bar{\psi}$		–							
G	0.5	0.5	0.5	0.5	–	0.5	0.5	0.5	0.5
kH	6	6	–	6	6	6	6	6	6
$\bar{\varsigma}$	0.3	0.3	0.3	–	0.3	0.3	0.3	0.3	0.3
$\bar{\eta}$	1.5	1.5	1.5	1.5	1.5	–	1.5	1.5	1.5
$\bar{\xi}$	0.3	0.3	0.3	0.3	0.3	0.3	–	0.3	0.3
$\bar{\beta}$	0.7	0.7	0.7	0.7	0.7	0.7	0.7	–	0.7
$\bar{\zeta}$	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	–

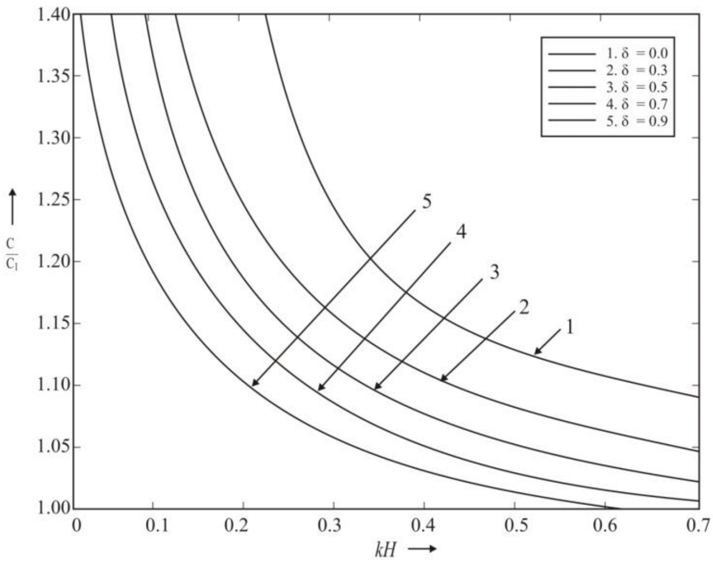

Fig. 2 Dimensionless phase velocity c/c_1 against dimensionless wave number *kH* for values of δ=0.0, 0.3, 0.5, 0.7 and 0.9

Table 2 Data for anisotropic porous medium

Symbol	Numerical Value	Units
ρ	7800	kg/m^3
L	0.1167×10^{10}	N/m^2
ρ_{11}	1.7567×10^3	Kg/m^3
ρ_{12}	-0.001567×10^3	kg/m^3
ρ_{22}	0.19867×10^3	kg/m^3
f	0.34	----

However when the heterogeneity parameter $\left(\bar{\psi}\right)$ is increased the following observations are made:

1. As the heterogeneity increases, the dimensionless phase velocity (c/c_1) increases at a particular value of dimensionless thickness (*kH*).
2. The curves are plotted for the values of heterogeneity parameter $\bar{\psi}$ =0.0, 0.3, 0.5, 0.7 and 0.9, it is clear that the curves are going close from each other as heterogeneity parameter decreases, it implies that even if the heterogeneity parameter $\left(\bar{\psi}\right)$ dominates, phase velocity of SH-waves remain constant for the same frequency as the curves are collimating at a single point.

Fig. 4 shows the effect of dimensionless phase velocity (c/c_1) against Biot's gravity parameter (*G*) for different values *kH*. Various curves are plotted for the values of thickness *kH*=1, 3, 5, 7 and 9. However when the dimensionless thickness (*kH*) is increased the following effects are observed:

1. The dimensionless phase velocity decreases for the increasing value of *kH* at a particular wave number.

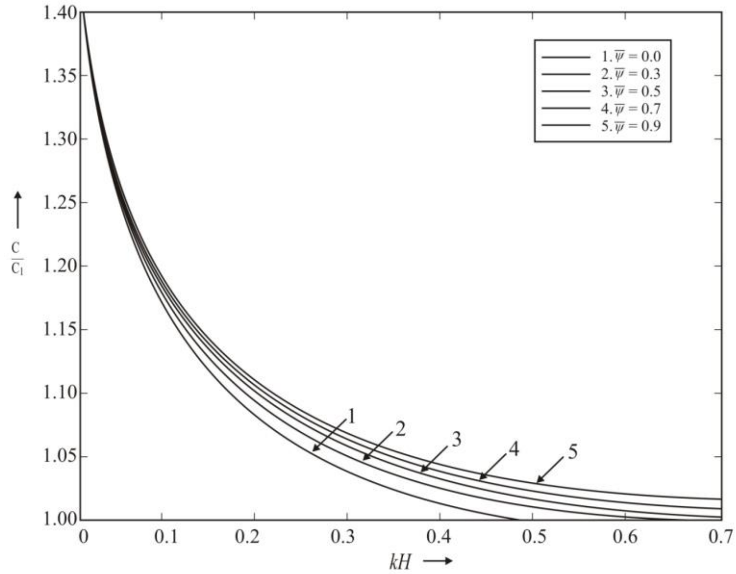

Fig. 3 Dimensionless phase velocity c/c_1 against dimensionless wave number kH for values of $\overline{\psi}$ =0.0, 0.3, 0.5, 0.7 and 0.9

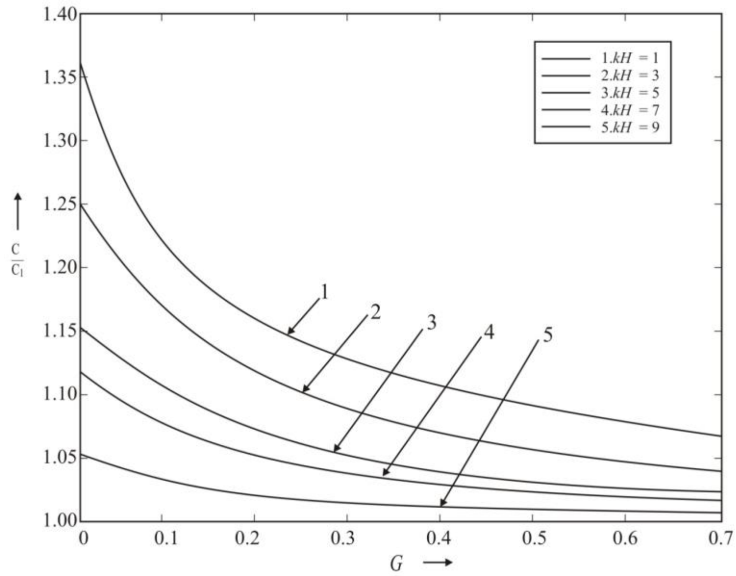

Fig. 4 Dimensionless phase velocity c/c_1 against G for values of kH=1,3,5,7 and 9

2. The curves are separated apart to each; from that we can conclude that the effect of dimensionless thickness (kH) has great impact on SH-wave for higher frequency.

Fig. 5 describes the effect of dimensionless ratio $\left(\overline{\varsigma} = \dfrac{N}{L}\right)$ on phase velocity (c/c_1). Various

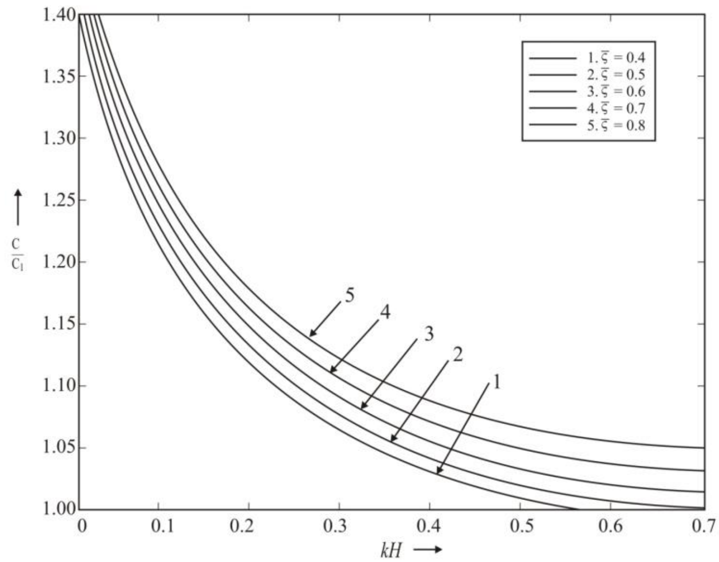

Fig. 5 Dimensionless phase velocity c/c_1 against dimensionless wave number *kH* for values of $\bar{\varsigma}$ =0.1, 0.3, 0.5, 0.7 and 0.9

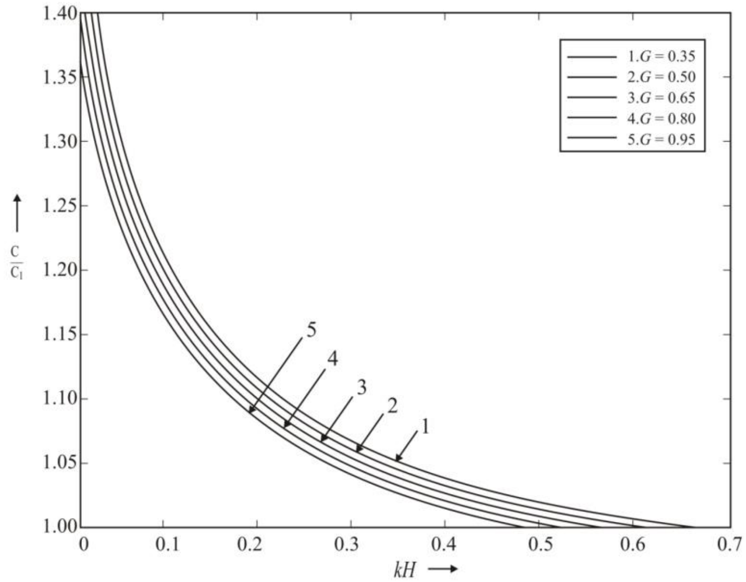

Fig. 6 Dimensionless phase velocity c/c_1 against dimensionless wave number *kH* for values of *G*=0.35, 0.50, 0.65, 0.80 and 0.95

curves are plotted for the values of $\bar{\varsigma}$ =0.0, 0.3, 0.5, 0.7 and 0.9. However when the dimensionless ratio $(\bar{\varsigma})$ is increased the following effects are observed:

1. The dimensionless phase velocity increases for the increasing value of '$\bar{\varsigma}$' at a particular

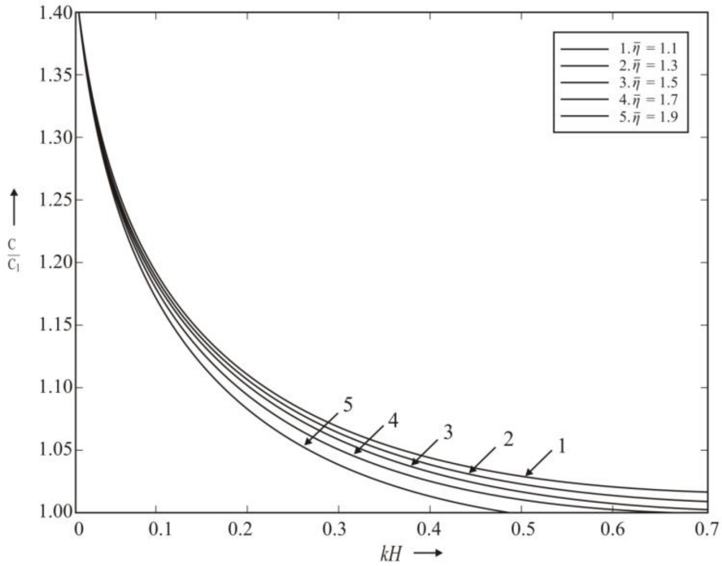

Fig. 7 Dimensionless phase velocity c/c_1 against dimensionless wave number kH for values of $\bar{\eta}$ =1.1, 1.3, 1.5, 1.7 and 1.9

wave number; it means that $\bar{\varsigma}$ parameter present in the medium gives the direct effect on the shear wave velocity.

2. The curves are moving away to each other for higher wave number (kH); from that we can conclude that the effect of dimensionless ratio $(\bar{\varsigma})$ has great impact on shear wave for higher frequency.

Fig. 6 depicts the effect of Biot's gravity parameter (G) on phase velocity (c/c_1). Various curves are plotted for the values of Biot's gravity parameter G=0.35, 0.55, 0.65, 0.75 and 0.85. However when the Biot's gravity parameter is increased the following effects are observed:

1. The dimensionless phase velocity decreases for the increasing value of Biot's gravity parameter 'G' at a particular wave number; it means that the SH- wave velocity is inversely proportional to gravity parameter present in the medium M_2.
2. The curves are equally apart from each other; this shows that the Biot's gravity parameter 'G' has great impact on shear wave.

Fig. 7 explains the effect of initial compression $\left(\bar{\eta} = \dfrac{2Q_x}{P_0 - 2Q_z}\right)$ present in the medium M_1. On phase velocity (c/c_1). Various curves are plotted for the values of stress parameter $\bar{\eta}$ =1.1, 1.3, 1.5, 1.7 and 1.9. However when the stress parameter '$\bar{\eta}$' is increased the following effects are observed:

1. The dimensionless phase velocity decreases for the increasing value of stress parameter '$\bar{\eta}$'at a particular wave number; it means that the shear wave velocity is inversely proportional to stress parameter '$\bar{\eta}$' present in the medium M_1.
2. The curves are getting closer to each other at a particular frequency; this shows that the stress

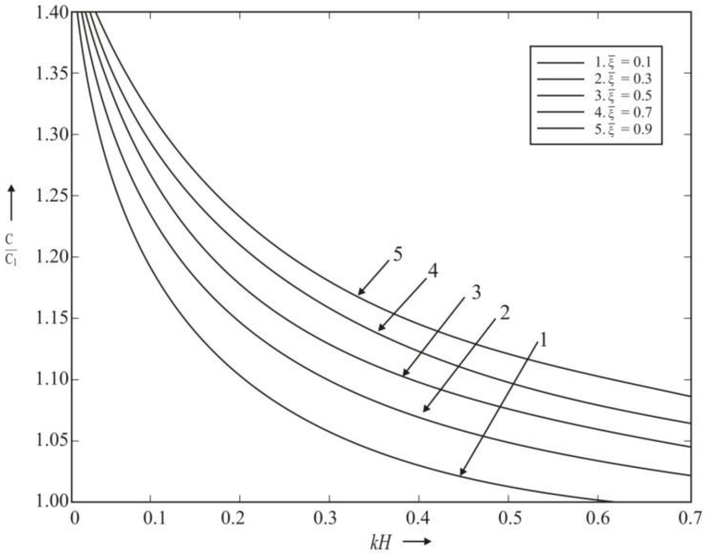

Fig. 8 Dimensionless phase velocity c/c_1 against dimensionless wave number *kH* for values of $\bar{\xi}$ =0.1, 0.3, 0.5, 0.7 and 0.9

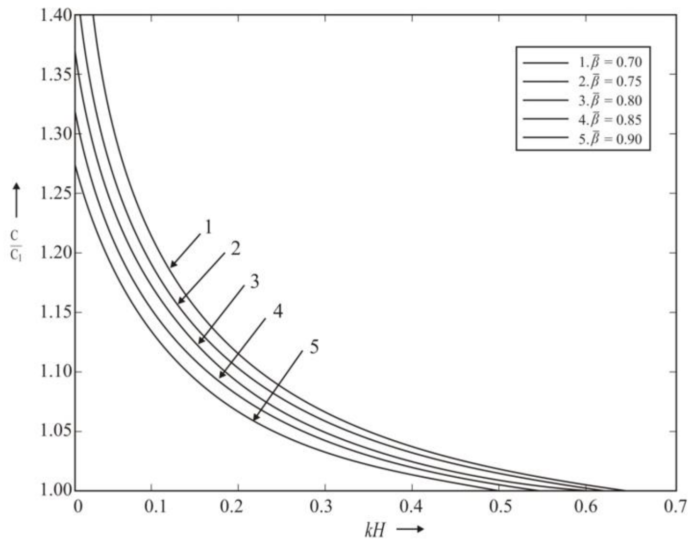

Fig. 9 Dimensionless phase velocity c/c_1 against dimensionless wave number *kH* for values of $\bar{\beta}$ =0.70, 0.75, 0.75, 0.80, 0.85 and 0.90

parameter '$\bar{\eta}$' has significant dominance for high values of wave number.

Fig. 8 explains the effect of directional rigidities $\left(\bar{\xi} = \frac{L}{Q_x}\right)$ on phase velocity (c/c_1). Various curves are plotted for the values of $\bar{\xi}$ =0.1, 0.3, 0.5, 0.7 and 0.9. However when the dimensionless

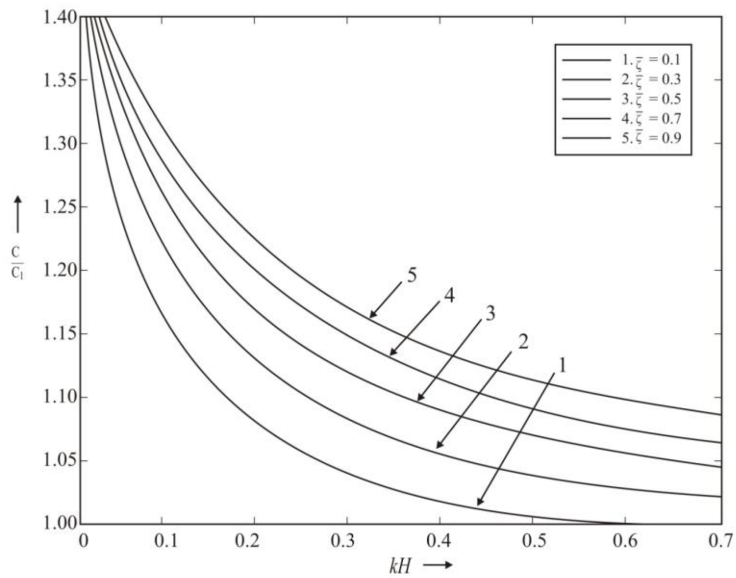

Fig. 10 Dimensionless phase velocity c/c_1 against dimensionless wave number kH for values of $\bar{\zeta}$ =0.1, 0.3, 0.5, 0.7 and 0.9

ratio $\left(\bar{\xi}\right)$ is increased the following effects are observed:

1. The dimensionless phase velocity increases for the increasing value of directional rigidities '$\bar{\xi}$' at a particular wave number; it means that directional rigidities $\bar{\xi}$ present in the medium gives the direct effect on the shear wave velocity.
2. The curves are moving away to each other for higher wave number(kH); from that we can conclude that the effect of directional rigidities $\left(\bar{\xi}\right)$ has great impact on shear wave for higher frequency.

Fig. 9 describes the effect of '$\bar{\beta} = \frac{c_1}{c_2}$'on phase velocity ($c/c_1$). Various curves are plotted for the values of inhomogeneity $\bar{\beta}$ =0.70, 0.75, 0.85, 0.80, 0.85 and 0.90. However when the velocity ratio '$\bar{\beta}$' is increased the following effects are seen:

1. The dimensionless phase velocity decreases for the increasing value of '$\bar{\beta}$' at a particular wave number; it means that '$\bar{\beta}$' gives the reverse effect on the shear wave velocity.
2. From these curves we can conclude that the parameter'$\bar{\beta}$' has significant effect on shear wave for lower frequency.

Fig. 10 explains the effect of initial stress parameter $\left(\bar{\zeta} = \frac{P'}{2L}\right)$ present in the medium M_2.on phase velocity (c/c_1). Various curves are plotted for the values of stress parameter $\bar{\zeta}$ =0.1, 0.3, 0.5, 0.7 and 0.9. However when the stress parameter '$\bar{\zeta}$' is increased the following effects are observed:

1. The dimensionless phase velocity decreases for the increasing value of stress parameter ‘ $\bar{\zeta}$ ’at a particular wave number; it means that the shear wave velocity is inversely proportional to stress parameter ‘ $\bar{\zeta}$ ’ present in the medium M_2.
2. The curves are getting closer to each other at a particular frequency; this shows that the stress parameter ‘ $\bar{\zeta}$ ’ has significant dominance for high values of wave number.

6. Conclusions

In this work, the dispersion of shear waves in a pre-stressed hetrogeneous orthotropic layer over pre-stressed anisotropic porous half-space under gravity has been investigated analytically. It has been observed that the phase velocity is larger for a porous initially stressed gravitational elastic half-space as compared to an initially stressed non-porous elastic half-space ($\delta \rightarrow 1$), It has been observed that on the removal of heterogeneity of layer, initial stress and porosity of the half-space, the derived dispersion equation reduces to Love wave dispersion equation thereby validates the solution of considered problem. Finally, on the basis of result developed, the following conclusions regarding the propagation of the SH-wave in a heterogeneous initially stressed elastic layer placed over anisotropic porous half-spaces under self weight can be drawn:

1. The SH-wave velocity increases with the decrease of wave number in all the cases.
2. SH-phase velocity decreases as the porosity of lower half-space increases, the wave velocity is inversely proportional to porosity.
3. Presence of compressive stress in the layer affects the SH-wave; the wave velocity is directly proportional to compressive stresses.
4. The presence of upper half heterogeneity has great impact on SH-wave and it gives the direct effect on the SH-wave velocity.
5. The SH- wave velocity is directly proportional to rigidity parameter present in the heterogeneous layer; it gives the significant effect on the SH-wave velocity.
6. The inhomogeneities in initial stress have great affect on the SH-wave velocity.
7. Presence of Biot’s gravity parameter in lower half-space greater the impact on SH-phase velocity. It decreases as Biot’s gravity parameter increases.
8. The compressive stress in the lower half-space affects the SH-wave; it is directly proportional to it.

From above discussion it can be concluded that in the presence of heterogeneity, initial stress, porosity, rigidity and gravity in the medium affect the seismic wave energy. The results may be useful to understand the nature of seismic wave propagation generated by artificial explosion (especially SH-waves) in the multilayered earth structure, material science engineering and in the field of earthquake engineering. Since the shear phase velocity is affected by various technical constants, the results of this paper may be helpful to design new structural materials for construction work.

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