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Uncertainty effects of soil and structural properties on the buckling of flexible pipes shallowly buried in Winkler foundation

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Abstract. The failure of civil engineering systems is a consequence of decision making under uncertain conditions. Generally, buried flexible pipes are designed for their transversal behavior to prevent from the important failure mode of buckling. However, the interaction effects between soil and pipe are neglected and the uncertainties in their properties are usually not considered in pipe design. In this regard, the present research paper evaluates the effects of these uncertainties on the uncertainty of the critical buckling hoop force of flexible pipes shallowly buried using the subgrade reaction theory (Winkler model) and First-Order Second-Moment (FOSM) method. The results show that the structural uncertainties of the studied pipes and those of the soil properties have a significant effect on the uncertainty of the critical buckling hoop force, and therefore taking into account these latter in the design of the shallowly flexible pipes for their buckling behavior is required.

Keywords: uncertainty; soil-structure interaction; flexible buried pipes; subgrade reaction modulus; critical uniform hoop force; FOSM

1. Introduction

Buried pipes have applications in water supply, sewerage, oil and natural gas transport and distribution, and leachate collection under landfills. These intricate networks consist of various types of pipe materials which are divided traditionally into two types: rigid and flexible, although the distinction between these two categories is blurring. Rigid pipe includes reinforced concrete, vitrified clay, and ductile iron. Flexible pipe includes steel, aluminum, fiberglass, and high density

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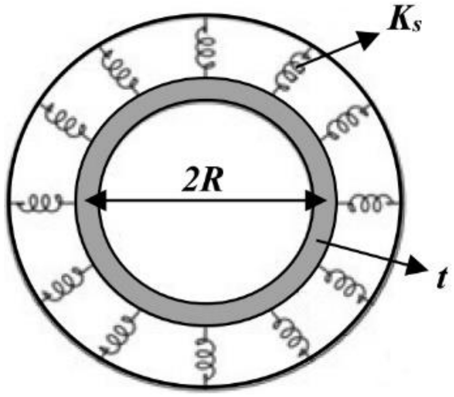


Fig. 1 Winkler model of buried pipe

polyethylene (Carrier 2005, Khan and Tee 2015, Terzi *et al.* 2015). Whatever the material type, the most common challenges for the safety of such infrastructure facilities is to prevent from the important transversal failure modes in order to provide some minimum service qualities. This can only be ensured through ensuring a reliable design including uncertainties due to external loads, pipe materials and surrounding soil properties (Mahdi and Katebi 2015, Alani *et al.* 2014). In fact, considering the flexible pipes, their design is controlled by either deflection or elastic buckling (Carrier 2005). Regarding this latter, two cases in which it controls the design of a flexible pipe: a) shallow cover with an internal vacuum pressure; and b) shallow cover, submerged in deep water, with atmospheric internal pressure (Carrier 2005). In this context, in our research paper we will focus on the first case, studying the uncertainties effects issued from the interaction between the surrounding soil and the shallowly buried flexible pipe on its transversal buckling behavior.

The interaction between flexible piping and backfill soil is rather complex (Terzi *et al.* 2015). The ground resistance to structural movements is a complex function of structural geometry, burial depth and soil properties. However, usually the ground support at the interface of pipeline structures is modeled using a series of elastic springs such as the case of Winkler theory (Fig. 1), wherein the spring's stiffness is expressed as a simple function of so-called the coefficient of soil reaction modulus K_s. This approach is an uncertain approximation because the influences of structural size and shape, burial depth, geometry of the backfill zone and the embankment soil condition are not considered (Moore *et al.* 1988).

In this regard, the current research paper focuses on the estimation of uncertainty of pipe geometry in terms of flexibility parameter S_f and uncertainty of soil features in terms of subgrade reaction modulus K_s based on four semi-empirical models in order to highlight their effects on the uncertainty of critical buckling hoop force N_{cr} of the flexible pipes shallowly buried in Winkler foundation. Additionally, for more representative results, the variation of N_{cr} is carried out as a function of burial depth h. To perform this task, the First-Order Second-Moment (FOSM) method is applied, in which the uncertainty of the most influential inherent item contributing in the definition of the subgrade reaction modulus or the pipe flexibility parameter is determined. The main outcomes obtained from the applied methods and models are discussed in this paper in view of consideration in the design of flexible pipes shallowly buried for more controlling the instability of the transversal behavior.

2. Origin of uncertainties

Generally, there are two types of uncertainties – random and epistemic uncertainties (Uzielli *et al.* 2008). Random uncertainty deals with the natural spatio-temporal variability of a parameter (Cho and Park 2010). On the other hand, epistemic uncertainty comprises statistical uncertainty, model uncertainty and measurement uncertainty that are due to approximate or insufficient knowledge (Denis *et al.* 2011, Fang *et al*. 2013). Indeed, idealized assumptions and simplifications on natural processes as Winkler theory (Fig. 1) are considered to ignore uncertainties, which are unavoidable in almost all engineering analysis and design problems. The approaches based on such assumptions and simplifications are usually deterministic. However, such assumptions and simplifications are not sufficient in many cases and mostly arise due to a) incompleteness of the available information/data, and, b) consideration of natural processes and phenomena, which are inherently random. Definite decision in such cases cannot be taken. However, the decisions are required even with the incomplete information/data and for the natural processes as well as the condition of uncertainty (Ang and Tang 1975).

In addtion to the uncertainties of the applied analytical model for modeling the soil-buried flexible pipe systems, the other sources of uncertainties can be presented mainly in the inherent properties of the soil as well as the mechanical and geometrical variability of the pipe structure. Various uncertainties can occur when natural variability of soil is included in the analysis (Elachachi *et al.* 2012). This variability depends on the way they are formed and the environmental changes that they are subjected to. The uncertainty in the mechanical properties of soils results from the spatial variability of the soil itself (Breysse *et al.* 2007, Cho and Park 2010, Denis *et al.* 2011, Elachachi *et al.* 2011, Elachachi *et al.* 2012, Imanzadeh 2013, Tani *et al.* 2013) and erroneous measurements in laboratories (Imanzadeh *et al.* 2013). Regarding the flexible buried pipe structure, the uncertainties in the external diameter and Young's modulus might occur during their construction.

3. Buckling theories

Buckling is a premature failure in which the pipe is not able to maintain its initial circular shape when the tangential compressive stress reaches a limit value and the pipe distorts unstably in buckling (Tee *et al.* 2013, Cheney 1963, 1971) analyzed the stability of a circular ring under plane stress condition, supported by elastic springs and acted upon by a uniform external pressure on the ring wall with constant magnitude and direction. The results showed that, for higher modes and at higher pressures, circumferential support of the ring forced it to buckle compared to the unsupported case (Leonards and Stetkar 1978). Corrosion-induced buckling failure mode has been studied for pipe reliability analysis (Tee and Khan 2014, Tee *et al.* 2014b) and risk-based life cycle cost optimization (Tee *et al.* 2014a). Meyerhof and Baikie (1963) extended plate-buckling theory developed by Timoshenko and Gere (1961) in order to estimate the buckling behavior of elastically supported cylindrical shells. Previously developed more rigorous shell buckling theories that considered soil support are used in the current design practice. Luscher (1966) formulated a semi-empirical solution for the critical, uniform, radially applied buckling pressure acting on the wall of a buried pipe. The elastic support in Luscher's model is modeled by elastic springs. In order to represent the boundary conditions at a culvert-soil interface more closely, Chelapati and Allgood (1972) attempted to derive a more rigorous theoretical solution for buckling of deeply buried culverts under a uniform external pressure and to verify the solution with extensively instrumented model tests (Leonards and Stetkar 1978).

A large number of studies have been reported concerning the stability of circular tube buried in elastic ground. Forrestal and Herrmann (1965) extended Cheney's analysis to consider cylindrical shells subjected to a uniform, constant-direction, external pressure on the shell wall and supported by a continuous, elastic medium rather than discrete springs. The theory of elastic buckling in the presence of initial stress and displacement is used in the general solution of the complicated boundary value problem. Forrestal and Herrmann's theory indicates that slip at the pipe-soil interface decreases the critical buckling pressure significantly (Forrestal and Herrmann 1965). Different analytical models for the study of soil-structure interaction on elastic soil are available (Deck and Singh 2012). The most common parameter for all of these models is subgrade reaction modulus (K_s). Numerous expressions or semi-empirical models are available to determine this modulus as a function of the studied applications. Meyerhof and Baikie (1963), Meyerhof (1968), Kloppel and Glock (1979) and Selvadurai (1985) used K_s in the design of buried pipes. This modulus is not an intrinsic parameter of soil. It depends on the soil properties and pipe's radius. The critical buckling displacement is strongly influenced by the soil's elastic modulus. If this modulus is increased ten times, the critical pressure is decreased by roughly the same order of magnitude.

4. Proposed methods and models

4.1 Soil-structure interaction model

The discrepancy between theory and experiment is largely due to the difficulty of assessing the representative elastic modulus of ground surrounding buried pipes (Moore 1989). Measurements of critical hoop force of the exposed element (buried pipes) seem to be further below theoretical predictions than reality. For analyzing buckling behavior of buried pipes, a harmonic disturbance to the unreformed structure is considered. Eqs. (1)-(2) illustrate that the radial w and circumferential v deflections are expressed as a function of circumferential coordinate θ (Moore 1989)

$$w = w_n.\cos(\mathrm{n}\,\theta) \tag{1}$$

$$v = v_n.\sin(\mathrm{n}\,\theta) \tag{2}$$

Where w_n and v_n are the harmonic coefficient of radial and circumferential displacement, respectively for a given buckling mode n ($n \geq 2$). Therefore, the critical hoop force N_{cr} (the force which makes the buried structure elastically unstable in its unreformed state) is considered as a function of harmonic number n, as well as ground and structural stiffness (Moore 1989). As already mentioned, the Winkler model has been used herein to characterize the stiffness of the ground in terms of series of independent springs resisting to the radial deformation of pipe structure. The uniform hoop force N leading to instability is given by Eq. (3) (Moore 1989)

$$N = (n^2 - 1).\frac{EI}{R^2} + \frac{K_s.R^2}{(n^2 - 1)} \tag{3}$$

The ground resists both inward and outward deformations and the circular pipe has radius R

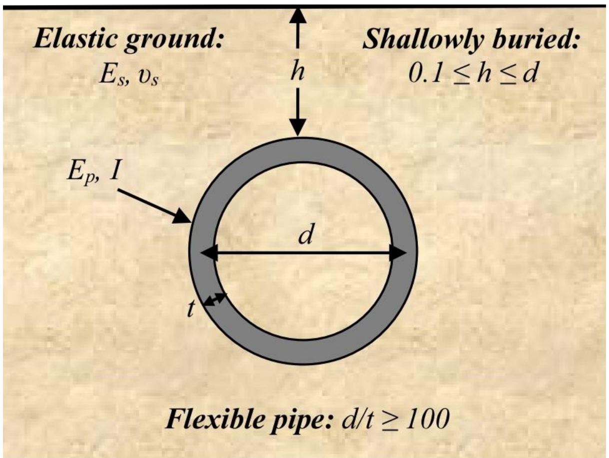


Fig. 2 Model of flexible pipe shallowly buried

Table 1 Expressions of K_s as a function of soil and geometrical parameters

N°	Investigator(s)	Suggested expression	Reference
1	Meyerhof and Baikie	$K_s = \frac{E_s}{d\left(1-v_s^2\right)}$	(Meyerhof and Baikie 1963)
2	Kloppel and Glock	$K_s = \frac{2E_s}{d\left(1+v_s\right)}$	(Kloppel and Glock 1979)
3	Selvadurai	$K_s = \frac{0.65E_s}{d\left(1-v_s^2\right)}$	(Selvadurai 1985)
4	Luscher	$K_s = \frac{E_s}{R\left(1+v_s\right)} . \left[\frac{1-\left(\frac{R}{R+h}\right)^2}{1+\left(\frac{R}{R+h}\right)^2 (1-2.v_s)} \right]$	(Luscher 1966)

and plain strain flexural rigidity *EI*. In the case study, the pipe is considered shallowly buried (Fig. 2) in which a long wavelength (n_{cr}=2) response is predicted (Moore 1989).

Due to the shallow burial depth that generally taken for the case of pipes of the Fig. 2, it is assumed that the coefficient of reaction is uniform in each given point in the surrounding soil. Within this analytical approach, it is supposed that the spatial variability of the soil does not significantly affect the transversal buckling behavior of the pipe and therefore, it is not considered hereinafter in the uncertainty analysis.

4.2 Modulus of soil reaction in different expressions

One of the most complex and sophisticated problems in geotechnical engineering is the assessment of numerical value of K_s. Table 1 presents the most commonly used semi-empirical models in buried pipe design for transversal direction which are chosen to determine the value of K_s (Sadrekarimi and Akbarzad 2009). This modulus depends mainly on the mechanical parameters

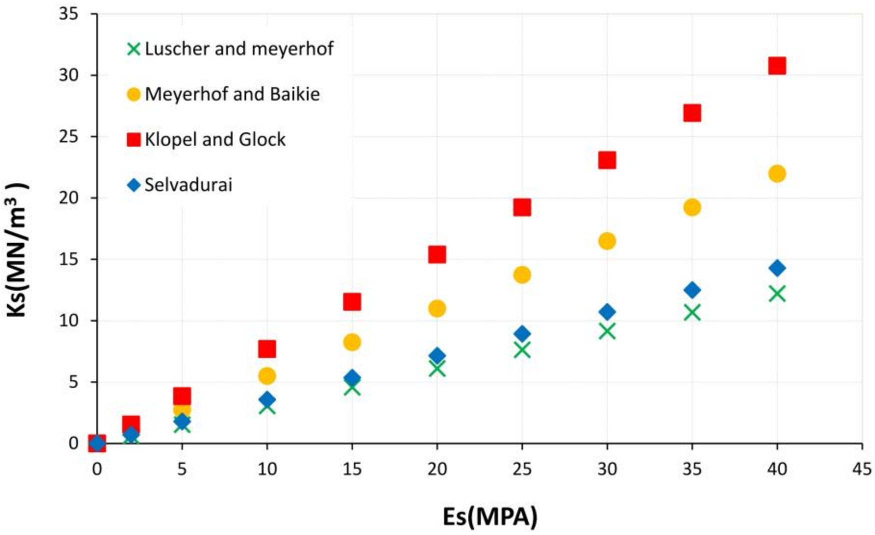


Fig. 3 Evolution of K_s as a function of soil Young's modulus E_s

of soil such as soil modulus (E_S) and soil Poisson's ratio (ν_S), as well as external diameter of pipe (*d*). As stated above, the expression of Luscher is applied herein which is deemed to be the best one introducing empirical correction to estimate an effective soil reaction modulus. Indeed, the burial close to the ground surface is considered using an equivalent soil cylinder with a thickness of equal to the average cover height *h*. To estimate the uncertainty of the reaction modulus and define the most influential parameters, the FOSM method is applied on the considered semi-empirical models (Table 1).

In order to compare the performance of these semi-empirical expressions, common dimensions of a pipe are considered: R=1 m; v_s=0.3; h=0.5 m and Young's modulus of soil varies between 2 MPa and 40 MPa. It can be concluded from Fig. 3 that the greatest values between 0 and 40 MN/m^3 of the calculated modulus K_s are found using the Kloppel and Glock (1979) model.

4.3 Modulus of pipe flexibility in standard expression

The pipe flexibility S_f is a function of geometrical parameters such as radius R and wall thickness of pipe t, as well as mechanical parameter E_p (Young's modulus of pipe) as presented in Eq. (4) (Leonards and Stetkar 1978)

$$S_f = \frac{E_p I}{R^3} \tag{4}$$

5. FOSM method

The greatest advantage of the FOSM method is its simplicity and no higher moments or distributional information on the system's basic variables are necessary. When the method is applied to engineering design problems, two theoretical and/or conceptual issues can be pointed

Table 2 Origin of uncertainties in soil and structural parameters and possible range of coefficient of variation for each parameter in the case of buried pipes (Imanzadeh *et al*. 2013a, b)

Parameter	Aleatory uncertainty	Epistemic uncertainty		Possible range of coefficient of variation (%)
	Natural variability	Measurement uncertainty	Construction uncertainty	
E_s	*	*	-	[5-50]
ν_s	*	*	-	[2-10]
d	-	-	*	[2-10]
E_p	-	-	*	[2-10]
t	-	-	*	[2-10]

E_s: Young's modulus of soil; ν_S: Poisson's ratio of soil; d: external diameter of pipe; E_p: Young's modulus of pipe and t: wall thickness of pipe

out (Imanzadeh *et al*. 2015).

- the relative non-accuracy of the first-order Taylor series approximation for strong non-linear problems.
- for engineering systems, the events of failure generally happen at extreme values rather than near the mean values (Imanzadeh *et al.* 2013a).

One of the possible ways to calculate the uncertainties in the absence of sufficient data is based on the published coefficients of variation (*CV*). Values of these coefficients of variation for certain geotechnical engineering parameters and in-situ tests (such as soil strength properties, soil index parameters, field measurements and laboratory tests) have been compiled by Harr (1977, 1987), Kulhawy *et al.* (1991, 1992), Lacasse and Nadim (1996, 1997, 2007), Phoon and Kulhawy (1999a, 1999b) and Duncan (2000). However, few data exist in the literatures concerning the value of the coefficient of variation for soil modulus. Phoon and Kulhawy (2005) determined the *CV* of soil modulus in sand from direct methods (pressure-meter test and dilatometer test) that was in the range of 15-70%. The *CV* of soil modulus in silt determined by standard penetration test blow counts was found to be in the range of 40-60% whereas the *CV* of soil modulus in clay was estimated to be the highest (up to 85%) derived from a correlation between the values of soil modulus and standard penetration tests (Phoon and Kulhawy 2005).

In the current study, the values of *CV* for soil modulus between 5% and 50% are considered. In the absence of sufficient data concerning the parameters associated to the soil (E_s, ν_S) and pipe radius (R), the ranges of *CV* can be relied on expert judgment. The origin of soil and structural parameters' uncertainties and the possible ranges of *CV* are presented in Table 2.

The calculation methods of coefficient of variation *CV* (ratio of standard deviation to mean) used in this study are based on the first order of the Taylor series (Harr 1987). The coefficient of variation of the studied function $f(x)$ is obtained from Eq. (5) (Imanzadeh *et al.* 2013a)

$$CV^2_{f(x)}(x_i) = \sum_i^n \left(\frac{\partial f(x)}{\partial x_i} \frac{\overline{x_i}}{\overline{f(x)}} \right)^2 CV^2_{xi} \tag{5}$$

where $CV_{f(x)}(x_i)$ is the coefficient of variation of $f(x)$ for the i^{th} input variable (x_i), CV_{x_i} is the coefficient of variation for i^{th} input variable, $\overline{x_i}$ is the mean of i^{th} input variables , $\overline{f(x)}$ is the mean of function $f(x)$ and n is the number of variable.

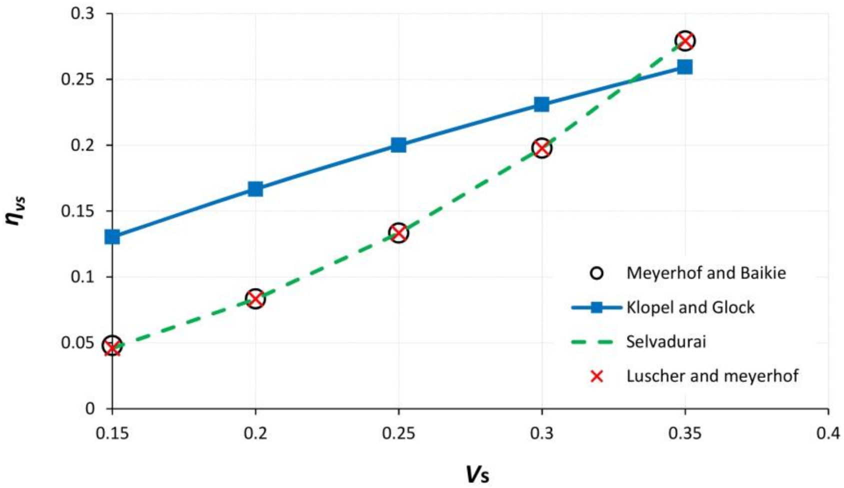


Fig. 4 Evolution of coefficient η_{vs} as a function of Poisson's ratio v_s

6. Results and discussion

6.1 Estimation of the influence of soil and structural parameters on CV_{Ks}

Soil subgrade reaction coefficient (K_S) is a function of soil properties (E_S, v_S) and pipe radius (R). Using the FOSM method (Imanzadeh *et al*. 2013a, b), the coefficient of variation of K_s can be obtained as a unique expression for the four semi-empirical models including coefficients of variation of soil and structural parameters with different weights as shown in Eq. (6)

$$CV_{K_s} = \left(\left(\eta_{E_s} .CV_{E_s} \right)^2 + \left(\eta_R .CV_R \right)^2 + \left(\eta_{V_s} .CV_{v_s} \right)^2 \right)^{0.5} \tag{6}$$

The values of η_{xi} with respect to E_s, R and v_s depend on each semi-empirical model. The CV_{Ks}, CV_{Es}, CV_R and CV_{Vs} are respectively the coefficients of variation in respect of K_s, E_s, R and v_s. By mean of Eq. (6), the influences of the variability of soil parameters, geometrical and mechanical properties of pipe on the reaction coefficient (K_s) are studied for each semi-empirical model using the FOSM and the results are presented as follows:

6.1.1 Effect of soil parameters

Generally, the uncertainty of soil Poisson's ratio is often predicted from expert judgment rather than a deterministic value. However, the uncertainty of this parameter is studied in this paper. Regarding the soil Poisson's ratio, the same expression for the coefficient η_{v_s} is obtained for the case of Meyerhof, Selvadurai and Luscher models (Eq. (7)). As shown in Fig. 4, considering the interval of Poisson's ratio from 0.15 to 0.35, the coefficient η_{v_s} varies from 0.05 to 0.28.

$$\eta_{v_s} = \frac{2v_s^2}{1-v_s^2} \tag{7}$$

For Kloppel's model (1979), the expression of the coefficient η_{v_s} is obtained as shown in Eq.

Table 3 Coefficient η_{χ_i} obtained for each parameter of semi-empirical models

η_{χ_i}	Semi-empirical models			
	Koppel	Meyerhof	Selvadurai	Luscher
η_{E_S}	1	1	1	1
η_{ν_s}	$\eta_{vs}=\frac{v_s}{1+v_s}$	$\eta_{vs}=\frac{2.v_s^2}{1-v_s^2}$	$\eta_{vs}=\frac{2.v_s^2}{1-v_s^2}$	$\eta_{vs}=\frac{2.v_s^2}{1-v_s^2}$
η_R	1	1	1	$f(R,h,v_s)$

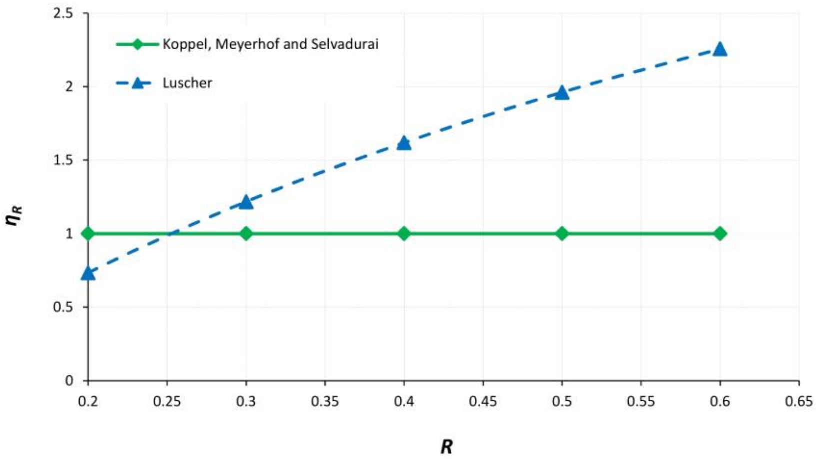


Fig. 5 Evolution of coefficient η_R as a function of pipe radius R

(8). For the same interval of Poisson's ratio from 0.15 to 0.35, the coefficient varies from 0.13 to 0.26 as shown in Fig. 4.

$$\eta_{v_s}=\frac{v_s}{1+v_s} \tag{8}$$

Similarly, by applying the same method (FOSM) but focusing on soil modulus (E_s) which is in linear relationship with subgrade reaction modulus (K_S) (Kloppel and Glock 1979, Meyerhof and Baikie 1963, Phoon and Kulhawy 1999b, Selvadurai 1985), for all the semi-empirical models as listed in Table 1, the obtained values of the corresponding coefficient η_{E_S} are all equal to 1 ($\eta_{E_S}=1$). Therefore, based on the above comments, regarding the effect of soil parameters (Poisson's ratio and soil modulus E_s), it can be concluded that the uncertainty of the latter is more significant in influencing on the uncertainty of subgrade reaction modulus K_s.

6.1.2 Effect of structural parameters

The pipe radius (R) appears in the four semi-empirical models. For the parameter R, a coefficient η_R=1 is obtained for Meyerhof, Kloppel and Selvadurai models for flexible pipes buried in Winkler foundation without taking into account the effect of burial depth. On the other hand, considering this effect in Luscher's model, the coefficient η_R is found as a function of R, h and v_s as shown in Eq. (9)

$$\eta_R = \frac{(8v_s - 8)R^3 + (6hv_s - 10)R^2 - 4h^2R - h^3}{(2R+h)\left((2v_s - 2)R^2 - 2hR - h^2\right)} \tag{9}$$

Fig. 5 shows the evolution of the coefficient η_R as a function of pipe radius (R) for the studied semi-empirical models using FOSM method. As shown in Fig. 5, the influence of the variability of R parameter on the K_s modulus is more important in Luscher's model due to the consideration of burial depth. For instance, when R=0.6 m, the coefficient η_R is equal to 2.25 for Luscher's model whereas this value is underestimated for other semi-empirical expressions (η_R=1).

6.1.3 Uncertainty estimation of K_s for each semi-empirical model

By applying the FOSM method, a same value of CV_{xi} is considered to estimate the variability of K_s. The coefficients η of the parameters which are listed in Table 3 are substituted into Eq. (6) to obtain the value of CV_{Ks}. For Kloppel and Glock's model, the expression is shown as follows

$$CV_{K_s} = \left(\left(CV_{E_s}\right)^2 + \left(CV_R\right)^2 + \left(\frac{v_s}{1+v_s} . CV_{v_s} \right)^2 \right)^{0.5} \tag{10}$$

For both Meyerhof's and Selvadurai's models, the value of CV_{Ks} can be calculated as follows

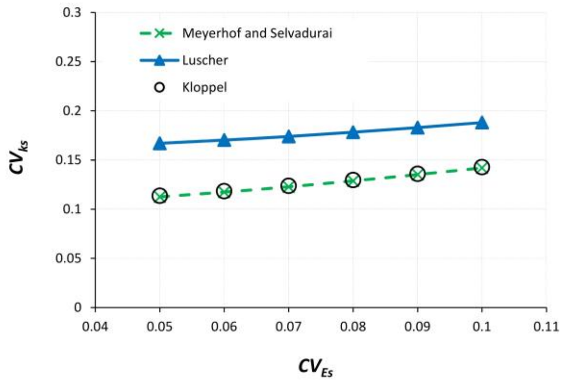
$$CV_{K_s} = \left(\left(CV_{E_s}\right)^2 + \left(CV_R\right)^2 + \left(\frac{2.v_s^{\,2}}{1-v_s^2} . CV_{v_s} \right)^2 \right)^{0.5} \tag{11}$$

For Luscher's model, the expression of CV_{ks} is given as follows

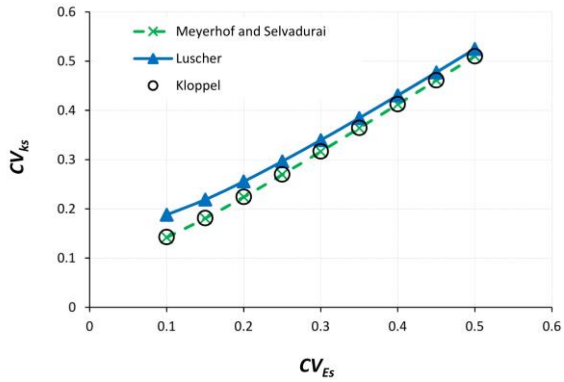
$$CV_{K_s} = \left(\left(CV_{E_s}\right)^2 + \left(f(R,h,v_s).CV_R \right)^2 + \left(\frac{v_s}{1+v_s} . CV_{v_s} \right)^2 \right)^{0.5} \tag{12}$$

The evolution of the coefficient of variation of K_s has been studied as a function of the coefficient of variation of E_s using the simplified semi-empirical expressions as shown in Eqs. (10)-(12). Fig. 6 is plotted based on the values of R=0.5 m, v_s=0.25 and the coefficients of variation of R and v_s are equal to 10%. As shown in Fig. 6, regardless of the value of CV_{Es}, the highest values of the CV_{Ks} are found in Luscher's model. As already indicated, this is mainly due to the additional consideration of the uncertainty of the burial depth h, in contrast to the other semi-empirical models which only take into account the uncertainty parameters of v_s, E_s and R. For the values of CV_{Es} less than or equal to 10% (which is the maximum value that could be attained by the CV of the other parameters as indicated in Table 2), the increase of the CV_{Es} leads to a gradual increase of the CV_{Ks} as shown in Fig. 6(a). This observation can be explained by the participation of the uncertainty effect of pipe radius R (CV_R) and wall thickness t (CV_t) on the uncertainty of subgrade reaction modulus K_s (CV_{Ks}). On the other hand, for the values of CV_{Es} greater than 10% as shown in Fig. 6(b), despite of the nonlinear expressions of the coefficient of variation of K_s (Eqs. (10)-(12)), almost a linear behavior is observed between CV_{ks} and CV_{Es} for all the models.

The results indicate that the influence of the uncertainty effect of pipe radius and soil Poisson's ratio (appear in Eqs. (10)-(12)) on the coefficient of variation of K_s can be negligible compared to the uncertainty effect of E_s. Moreover, the increase of CV_{Es} is corresponding to the same rate of



(a)



(b)

Fig. 6 Evolution of CV_{ks} as a function of CV_{Es}

increase of CV_{ks}, therefore it can be concluded that the coefficient of variation of K_s is almost equal to the coefficient of variation of E_s ($CV_{ks} \approx CV_{Es}$).

6.2 Estimation of the influence of structural parameters on the CV_{Sf}

Based on Eq. (4) and applying the FOSM method [6, 34], the influence of the uncertainty of both geometrical and mechanical properties of the pipe on its flexibility parameter S_f is carried out, where the corresponding coefficient of variation can be expressed as follows (Eq. (13))

$$CV_{sf} = \left(\left(\eta_{E_p} .CV_{E_p} \right)^2 + \left(\eta_t .CV_t \right)^2 + \left(\eta_R .CV_R \right)^2 \right)^{0.5} \tag{13}$$

Where S_f is pipe flexibility, η is the weight in respect of $E_{p,}$ t and R, respectively. The CV_{sf}, $CV_{Ep,}$ CV_t and CV_R are respectively the coefficients of variation in respect of S_f, $E_{p,}$ t and R. The values of the coefficient η are obtained as following: $\eta_R = \eta_t = 3$ and $\eta_{Ep} = 1$. Therefore, the Eq. (13) of the coefficient of variation of flexibility parameter S_f, can be simplified to

$$CV_{sf} = \left(\left(CV_{E_p} \right)^2 + \left(3.CV_t \right)^2 + \left(3.CV_R \right)^2 \right)^{0.5} \tag{14}$$

According to Table 2, the possible ranges of coefficient of variation of Young's module E_p, pipe radius and pipe thickness are all equal. However, based on the value of each η coefficient, it can be concluded that the influence of the uncertainty of pipe radius and pipe wall thickness on the flexibility parameter S_f remains more important than that of E_p.

The variability (uncertainty) of subgrade reaction modulus K_s and flexibility parameter S_f is evaluated through their coefficients of variation. Subsequently, it is worth studying their effects on the buckling behavior of buried pipes as discussed in the following section.

7. Uncertainty estimation of critical hoop force of buried pipes

Four variables have an influence on the uncertainty on critical buckling hoop force of pipes buried in Winkler foundation: the values of the subgrade reaction modulus K_S and pipe flexibility S_f as well as their corresponding coefficients of variation CV_{K_S} and CV_{Sf}, respectively. The FOSM method is applied on the expression of critical hoop force (Eq. (3)) to study the influence of the uncertainties of subgrade reaction modulus K_s and pipe flexibility parameter S_f. Critical hoop force can be decomposed into two parts as shown in Eq. (15). The first part is a function of the uncertainty of K_s while the other part is a function of the uncertainty of pipe flexibility EI/R^3. Therefore, these uncertainties can be written in terms of coefficients of variation as given in Eqs. (16)-(17)

$$CV_N^2 = CV_{N/K_s}^2 + CV_{N/S_f}^2 \tag{15}$$

Where

$$CV_{N/K_s}^2 = \left(\frac{\partial N}{\partial K_s} \cdot \frac{\overline{K}_s}{N_{cr}} \right)^2 CV_{K_s}^2 \tag{16}$$

$$CV_{N/S_f}^2 = \left(\frac{\partial N}{\partial S_f} \cdot \frac{\overline{S}_f}{N_{cr}} \right)^2 CV_{S_f}^2 \tag{17}$$

In Eq. (16), CV_{N/K_S} is the coefficient of variation of critical buckling force in respect of K_S; CV_{K_S} is the coefficient of variation of K_S; N is the hoop force which is a function of K_S and θ coordinate; $\overline{K_S}$ is the mean value of K_S; and N_{cr} is the critical hoop force for a given value of K_S corresponding to an abscissa θ. For this abscissa, the partial derivative of N with respect to K_S is then calculated. Eq. (17) is defined in the same manner as Eq. (16), except the parameters and the partial derivative are in respect of S_f.

The uncertainty effects of soil and structural properties on the buckling behavior of buried pipes is presented herein by mean of the uncertainties of both subgrade reaction modulus K_s and flexibility parameter S_f on the critical hoop force by considering different soil stiffnesses. The range of soil parameter E_s is considered between 2 MPa and 80 MPa to take into account of different soil morphologies (low to high stiffness). The results are presented in the following sections for buried pipes of different thicknesses (from 1 to 10 mm) with a radius of 0.50 m and E_P=2.1×10^5 MPa.

7.1 Influence of the uncertainty of K_S on the uncertainty of critical buckling force

It is possible to calculate the ratio between the coefficient of variation of critical buckling force with respect to K_S (CV_{N/K_S}), and the coefficient of variation of subgrade reaction modulus CV_{K_S} using Eq. (16). Fig. 7 depicts the evolution of this ratio $CV_{N/K_S}/CV_{K_S}$ as a function of subgrade reaction modulus for different pipe wall thicknesses (t=1, 2, 4, 6, 8 and 10 mm).

As illustrated in Fig. 7, the high influence of subgrade reaction modulus K_s on the ratio of $CV_{N/K_S}/CV_{K_S}$ is shown especially for the case of low soil stiffness ($K_s \leq 40$ MN/m^3) surrounding pipes with wall thickness $t \geq 6$ mm. Indeed, the higher the subgrade reaction modulus K_s, the higher the ratio $CV_{N/K_S}/CV_{K_S}$ is. This can be practically explained by the significant effect of soil support on the uncertainty of critical buckling load. However, it is clearly observed that the influence of subgrade reaction modulus K_s can be negligible in the case of low thickness of pipe ($t \leq 4$ mm). In

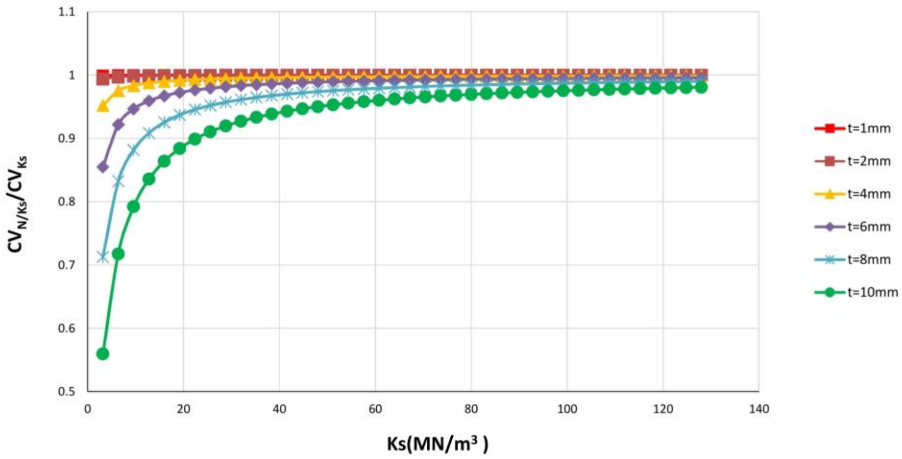


Fig. 7 Influence of uncertainty of K_s on the uncertainty of critical buckling force

fact, the coefficient of variation of the critical buckling with respect to K_s reaches its maximum values when it is close to the coefficient of variation of K_s ($CV_{N/K_s} \approx CV_{K_s}$). On the other hand, when the value of subgrade reaction modulus is larger than 40 MN/m^3 (K_s > 40 MN/m^3) and regardless of pipe thickness, the value of the ratio $CV_{N/K_s}/CV_{K_s}$ remains almost unchanged ($CV_{N/K_s}/CV_{K_s} \approx 1$) which indicates that the influence of stiff soil support is negligible.

7.2 Influence of the uncertainty of S_f on the uncertainty of critical buckling force

The ratio between the coefficient of variation of critical buckling load with respect to pipe flexibility S_f ($CV_{N/sf}$) buried in Winkler foundation, and the coefficient of variation of S_f (CV_{sf}) can be calculated based on Eq. (17). Fig. 8 illustrates the evolution of such ratio $CV_{N/S_f}/CV_{S_f}$ as a function of subgrade reaction modulus K_s. As shown in Fig. 8, considering the case of very low soil stiffness surrounding pipes with wall thickness t≥4 mm, the highest values of the ratio $CV_{N/S_f}/CV_{S_f}$ can be found. In this case, the uncertainty of pipe geometry has a significant effect on the uncertainty of critical buckling load. However, when subgrade reaction modulus K_s increases to 50 MN/mm^3, the ratio $CV_{N/S_f}/CV_{S_f}$ decreases especially for the case of pipes with wall thickness t≥6 mm. This shows that the ratio $CV_{N/S_f}/CV_{S_f}$ is more affected by subgrade reaction modulus K_s rather than pipe flexibility S_f. For pipes with low wall thickness (t≤4 mm), the influence of subgrade reaction modulus K_s on the ratio $CV_{N/S_f}/CV_{S_f}$ can be negligible wherein the influence of the uncertainty of flexibility parameter S_f on the uncertainty of critical buckling load with respect to S_f is insignificant ($CV_{N/S_f} \approx 0$). Moreover, for high values of K_s (stiff soil) and regardless of pipe thickness, the influence of soil support on the ratio $CV_{N/S_f}/CV_{S_f}$ is negligible and the coefficient of variation of critical buckling with respect to K_s reaches its minimum values ($CV_{N/S_f} \approx 0$). Therefore, in the studied case of soil stiffness, the uncertainty of flexibility parameter S_f on the uncertainty of critical buckling load with respect to S_f is still insignificant.

Based on Figs. 7-8, it is worth noting that the uncertainties of both subgrade reaction modulus

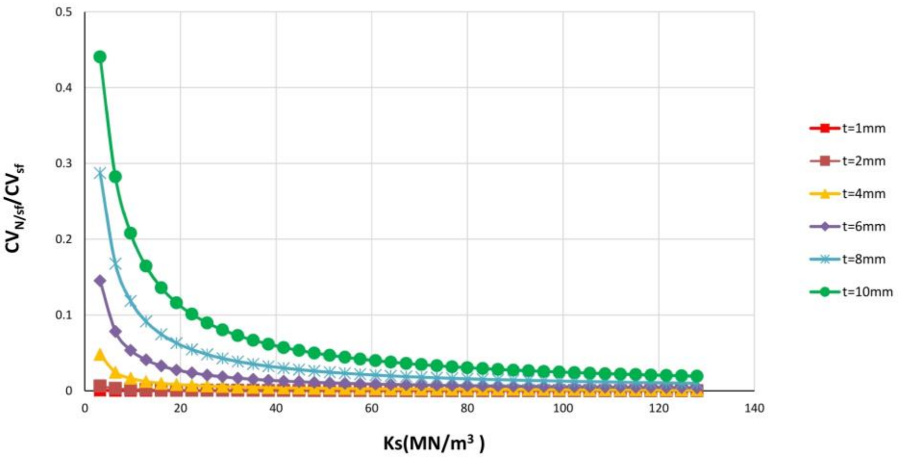


Fig. 8 Influence of uncertainty of S_f on the uncertainty of critical buckling force

K_s and flexibility parameter S_f have contradictory but complementary effect on the uncertainty of critical buckling load. For instance, when pipe's wall thickness t=10 mm, it is observed from the first point of both curves that the uncertainty of critical buckling load is affected by 55% of the subgrade reaction modulus K_s and 45% of the uncertainty of flexibility parameter S_f. When soil stiffness increases, the uncertainty effect of parameters K_s increases to reach its maximum values $(CV_{N/K_s} \approx 100\%)$ which represents the totality effect on the uncertainty of critical buckling load whereas the uncertainty effect of parameter S_f will be insignificant $(CV_{N/S_f} \approx 0\%)$.

7.3 Influence of the uncertainties of K_s and S_f on critical buckling load with respect to burial depth h

The effect of shallow cover on buckling stress of pipe wall has been examined by Luscher (1966) through an approximate approach in which subgrade reaction modulus K_s is modified to account for shallow depth of soil. By using Eqs. (16)-(17), it is then possible to calculate the ratio between the coefficient of variation of critical buckling load with respect to either K_s or S_f ($CV_{N/Ks}$ and $CV_{N/Sf}$, respectively) and their corresponding coefficients of variation (CV_{Ks} and CV_{Sf}, respectively). In this study, in addition to the same geometrical parameters previously deemed, a value of soil stiffness E_s = 10 MPa is taken herein to highlight the implicitly influence of burial depth h on critical hoop force N_{cr}.

As shown in Fig. 9, when burial depth h increases to 0.5 m, the ratio $CV_{N/K_s}/CV_{K_s}$ increases gradually for pipe thickness $t \geq 6$ mm. However, for values of burial depth h>0.5 m, the uncertainty of subgrade reaction modulus with respect to burial depth h does not influence the uncertainty of critical buckling load, since its maximum effect has been reached.

Therefore, it can be concluded that the effect of the uncertainty of soil support on the uncertainty of critical buckling load increases with burial depth especially when h value is less than pipe radius R. Beyond h>R, this uncertainty reaches its major effect which is between 0.9 and 1. For the case of low pipe thickness ($t \leq 4$ mm), the uncertainty of subgrade reaction modulus K_s on the uncertainty of critical buckling load reaches its maximum effect $(CV_{N/K_s}/CV_{K_s} \approx 1)$ regardless of the values of burial depth h. Overall, the soil support strongly affects the uncertainty of critical buckling load of low stiff pipes shallowly buried in Winkler foundation.

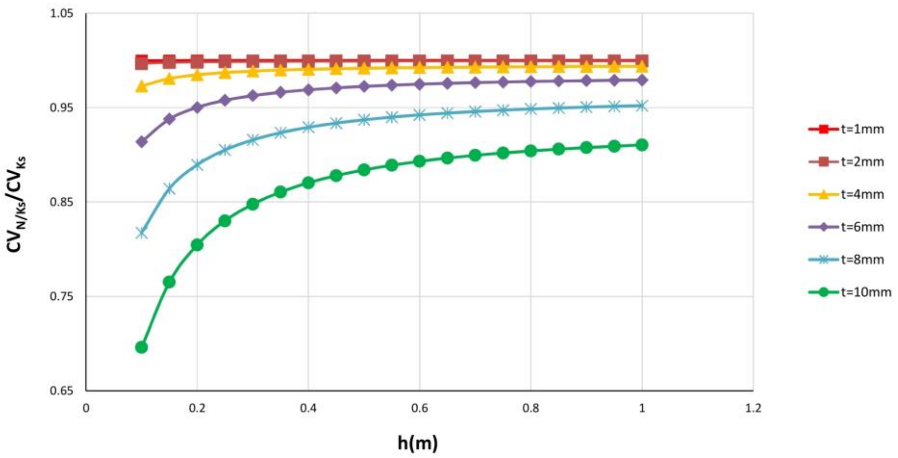


Fig. 9 Influence of the uncertainty of K_s as a function of *h* on the uncertainty of critical hoop force

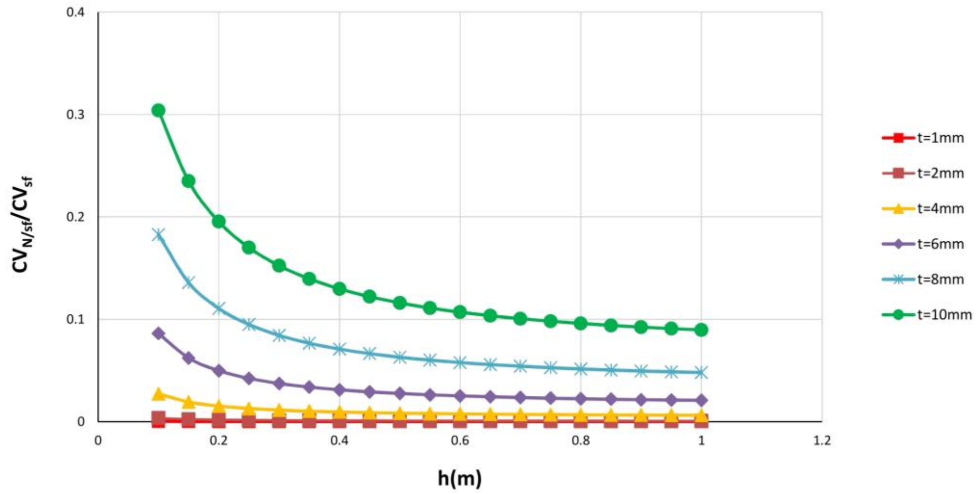


Fig. 10 Influence of the uncertainty of S_f as a function of *h* on the uncertainty of critical hoop force

It is clearly shown in Fig. 10 that for low values of burial depth *h*, the uncertainty of pipe flexibility S_f on the uncertainty of critical buckling load is in the maximum effect for all studied pipes. However, when burial depth increases to 0.5 m, the ratio $CV_{N/S_f}/CV_{S_f}$ decreases especially for the case of pipes thickness $t \geq 6$ mm. Moreover, for the values of *h* greater than 0.5 m, the ratio $CV_{N/S_f}/CV_{S_f}$ remains unchanged and the effect of the uncertainty of flexibility parameter S_f reaches to the minimum values (0 to 0.1). Therefore, the uncertainty of pipe geometry on the uncertainty of critical buckling load is important when the pipe is buried in a depth less than pipe radius *R*. Beyond $h>R$, this uncertainty will be insignificant. For the case of low pipe thickness ($t \leq 4$ mm), the effect of the uncertainty of pipe flexibility S_f on the uncertainty of critical buckling load is negligible ($CV_{N/S_f}/CV_{S_f} \approx 0$) regardless of the values of burial depth *h*. Overall, the uncertainty of pipe geometry has insignificant effect on the uncertainty of critical buckling load in the case of low stiff pipes shallowly buried in Winkler foundation.

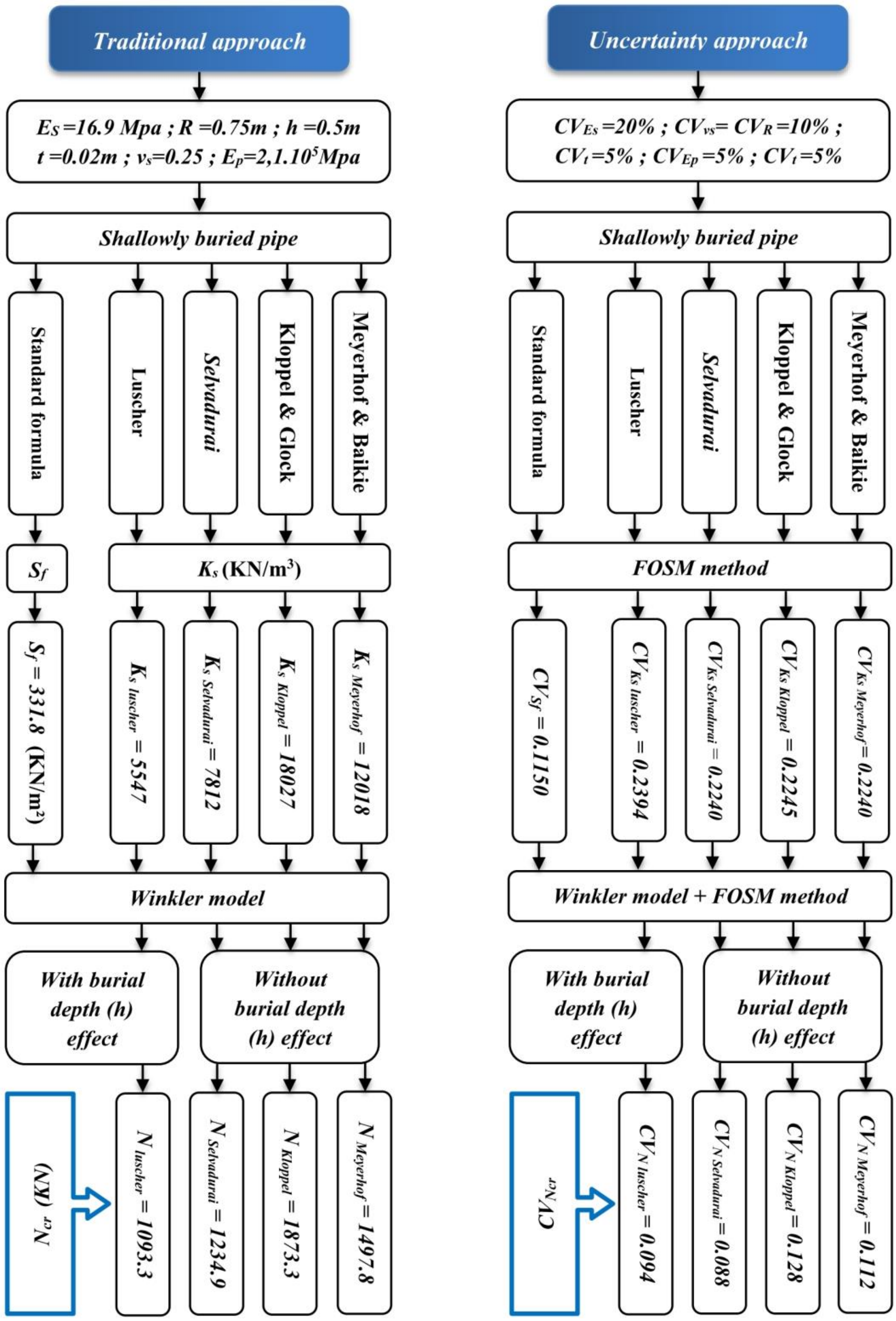


Fig. 11 Flow chart to use both traditional and uncertainty approches

8. Application

To highlight the proposed approach, it is worth presenting a parametric study in order to show in one hand, how the input and uncertainty parameters can be used for soil-buried pipe interaction analysis of the transversal behavior; on the other hand, to illustrate the significant effect of the uncertainties in the obtained outcomes. These investigations would generally enable the determination of more realistic design and confidence limits of prediction the failure of the studied phenomena. In this context, Figs. 11-12 present a methodology of analysis based on both traditional and uncertainty approaches for studying the transversal buckling behavior of shallowly buried flexible pipe. To perform the above task, , a steel pipe buried in soil is taken as an example with a modulus (E_s) of 16,9 MPa and v_s=0.25 with coefficient of variation 20% and 10% respectively which are taken from published values in the literatures and expert judgment (Table 1).

The mechanical property and the geometrical dimensions of this buried pipe are E_p=2.10^5 MPa, d=1.5 m and t = 0.02 m. This pipe is considered buried in a shallow depth of 0.5 m (h=0.5 m).

Firstly, starting from the values of the characterizing parameters (E_s, v_s, d, E_p, h), the different values of the subgrade reaction modulus (K_s) can be obtained from each semi-empirical model. Subsequently, through the application of the traditional approach of the Winkler model, the deterministic values of the critical buckling force are carried out (Fig. 11).

Secondly, regarding the uncertainty approach, knowing the coefficient of variation of each characterizing parameter (CV_{Es}, CV_{vs}, CV_d, CV_{Ep}, CV_h), the coefficient of variation of K_s from each considered model can be determined through the application of the FOSM method. This latter, is applied again on the analytical equation of the critical buckling hoop force to determine its

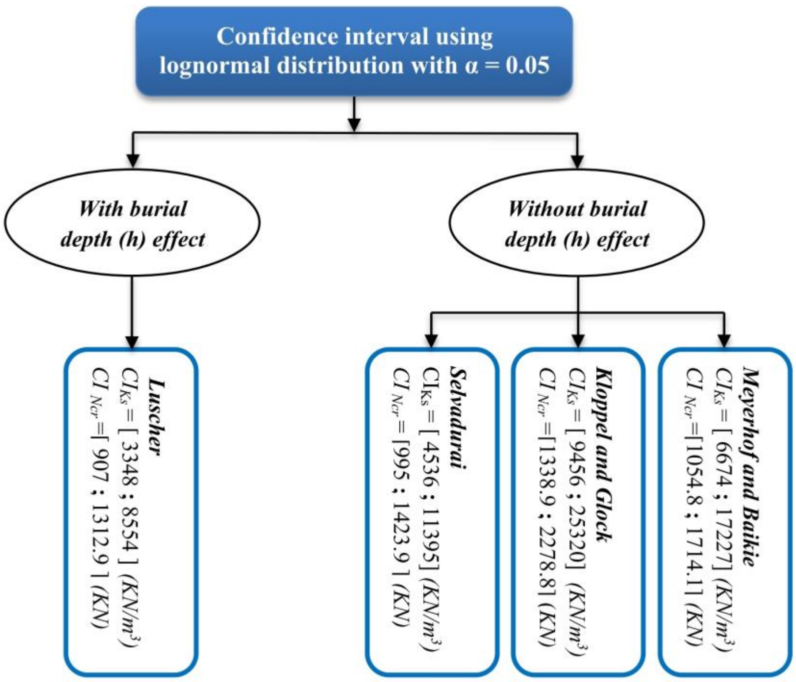
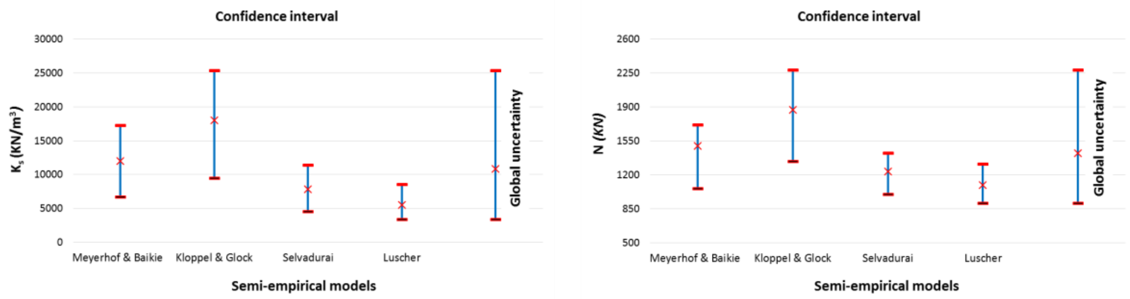


Fig. 12 Confidence interval results from the applied methodology



(a) (b)

Fig. 13 Global uncertainties for (a) the subgrade reaction modulus K_s, (b) the critical buckling force N_{cr}

coefficient of variation for each model (Fig. 11).

Finally, by assuming a lognormal distribution (which is a fairly common assumption) for the subgrade reaction modulus K_s and the critical buckling force N_{cr}, both approaches (traditional approach and uncertainty approach) are combined to calculate the confidence intervals for each semi-empirical model. In this context, the obtained results of 95% confidence bounds for the four considered models are presented in Fig. 12.

More representative results are shown in Fig. 13 wherein a global uncertainty is proposed. This includes the uncertainties from each semi-empirical model and corresponds to the range between the maximum of the four upper bounds and the minimum of the four lower bounds. Therefore, the global uncertainties for K_s and N_{cr} for the study example, are [3348; 25320] (KN/m^3) and [907; 2278.8] (KN) respectively (Fig. 13).

From the exposed above (Figs. 11-13) two cases have been carried out: the first case where the effect of burial depth was not taken into account in the uncertainty analysis (Mayerhof, Kllopel and Selvadurai models), whereas the second case, where it is considered (Luscher model). It is expected from the performed analysis that the obtained outcomes are different for each semi-empirical model. Additionally, the deterministic values of soil subgrade reaction modulus K_s and the critical buckling force N_{cr} and their associated confidence intervals are the highest for the first case (without burial depth "*h*" effect) however, the smallest and the most important are found in the second case (with burial depth "*h*" effect).

9. Conclusions

This paper presents a study of buckling behavior in the transversal direction of flexible pipes shallowly buried in Winkler foundation. This behavior is characterized by critical hoop force which is the main parameter analyzed in terms of uncertainty effect based on soil-pipe interaction properties. Based on the proposed uncertainty approach, the main aim is to analyze the uncertainty effects of the properties of soil support in terms of subgrade reaction modulus K_s and those of the pipe in terms of flexibility parameter S_f on the uncertainty of the critical hoop force. In this context, the obtained outcomes have revealed three main issues:

1. The uncertainty of subgrade reaction K_s is mainly affected by the inherent soil modulus E_s whereas the uncertainty of pipe wall thickness t and radius R have equal and significant effect on the uncertainty of the pipe flexibility S_f.

2. Regarding the buckling behavior of flexible pipes, the uncertainty of critical hoop force is mainly affected by the uncertainty of soil properties rather than that of structural features.
3. The effect of both uncertainties of soil support and pipe geometry on the uncertainty of critical buckling hoop force reduces when burial depth is taken into account in the model since its confidence interval is the smallest.

Therefore, it is worth mentioning that these uncertainties influence the buckling failure event of buried pipes which should be considered in the design in order to avoid unexpected failure due to buckling mode. Moreover, these results of uncertainty analysis could be used as a tool for a good reliability analysis against the transversal buckling failure mode wherein the precisely limit states could be defined and therefore target reliability indices can be ensured.

Finally, for the purpose of accuracy and convenience, it is practical and efficient to conduct a similar analysis introducing the coupling effect between the discrete Winkler springs which can be performed by assuming that the springs are connected by a shear layer membrane or beam. In this context, in order to enhance the obtained results, similar analytical models taking into account more parameters such as Vlassov & Pasternak model with two parameters, or Kerr's model with three parameters could be applied and compared.

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