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Thermo-mechanical vibration analysis of temperature-dependent porous FG beams based on Timoshenko beam theory

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Abstract. In this paper thermo-mechanical vibration analysis of a porous functionally graded (FG) Timoshenko beam in thermal environment with various boundary conditions are performed by employing a semi analytical differential transform method (DTM) and presenting a Navier type solution method for the first time. The temperature-dependent material properties of FG beam are supposed to vary through thickness direction of the constituents according to the power-law distribution which is modified to approximate the material properties with the porosity phases. Also the porous material properties vary through the thickness of the beam with even and uneven distribution. Two types of thermal loadings, namely, uniform and linear temperature rises through thickness direction are considered. Derivation of equations is based on the Timoshenko beam theory in order to consider the effect of both shear deformation and rotary inertia. Hamilton's principle is applied to obtain the governing differential equation of motion and boundary conditions. The detailed mathematical derivations are presented and numerical investigations are performed while the emphasis is placed on investigating the effect of several parameters such as porosity distributions, porosity volume fraction, thermal effect, boundary conditions and power-low exponent on the natural frequencies of the FG beams in detail. It is explicitly shown that the vibration behavior of porous FG beams is significantly influenced by these effects. Numerical results are presented to serve benchmarks for future analyses of FG beams with porosity phases.

Keywords: thermo-mechanical vibration; functionally graded beam; porous material; DTM; thermal effect

1. Introduction

Functionally graded materials (FGMs) are new type of composite materials formed of two or more phases which their both composition and structure gradually change over gradient directions smoothly and continuously. Therefore by changing the properties of the material it is possible to perform a certain function of material properties of mechanical strength and thermal conductivity.

The FGMs were introduced by Japanese scientists in mid-1980s as aerospace application for the first time. FGMs possess various advantages in comparison with traditional composites. For instance, multi functionality, ability to control deformation, corrosion resistance, dynamic

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response, minimization or remove stress concentrations, smoothing the transition of thermal stress and resistance to oxidation. Hence, FGMs have received wide engineering applications in modern industries including aerospace, nuclear energy, turbine components, rocket nozzles, critical furnace parts, batteries/fuel cells, critical furnace parts, etc. (Jha *et al*. 2013, Wattanasakulpong *et al*. 2012, Thai and Vo 2012, Şimşek and Kocatürk 2009). These wide engineering applications is cause that researchers attracted to FGMs, and study their vibration, static and dynamic’s behavior of the FG structures (Ebrahimi *et al*. 2009, Ebrahimi *et al*. 2009). Many investigation are reported in literature to study the dynamic and static behavior of functionally graded beams, here some of these disquisitions are mentioned briefly. Aydogdu and taskin (Aydogdu and Taskin) discussed free vibration analysis simply supported FG beam with power-law and exponential material graduation. They used different higher order shear deformation and classical beam theory (CBT) for deriving the differential equations of motion and solved them by Navier type solution method. Also they concluded that, increasing the mode number is couse increasing the difference between CBT and higher order theories. Simsek (201) investigated the vibration analysis of FGM beams based classical, the first-order and different higher order shear deformation beam theories by using Rayleigh-Ritz method. Also in another paper, non-linear vibration of FG Timoshenko beam subjected to a moving harmonic loading has been studied by him (Şimşek 2010). Sina *et al*. (2009) analyze the free vibration of FG beams by developing a new beam theory for laminated composite beams, which has a little different with first-order shear deformation beam theory. Pradhan and Chakraverty (2013) have presented free vibration characteristics FG beams based on Euler and Timoshenko beam theory with various boundary conditions by using Rayleigh-Ritz method. An analytical method has been presented by Wei *et al*. (201) for free vibration of cracked FGM beams with axial loading, shear deformation and rotary inertia. Akgoz and Civalek developed a shear deformation beam model and new shear correction factors for Bending and buckling analyses of simply supported FG micro beams (Akgöz and Civalek 2014). Civalek and Kiracioglu presented a numerical solution, the DSC method for free vibration of Timoshenko beams (Civalek and Kiracioglu 2010).

Due to huge application of beams in different fields such as civil, marine and aerospace engineering, and difference between the making temperatures and working temperatures of structures, for more efficient design, it is important to take into account the thermal effect when designing FGM structures. Xiang and Yang (2008) exploited free and forced vibration three-layer laminated Timoshenko beams of variable thickness with FGM in thermal environment. Differential quadrature method (DQM) has been used to solve equations of motions. Xiang and Yang (2008) presented an analytical solution for free vibration FG beams based on unified higher order shear deformation beam theory with general boundary conditions. Material properties were assumed to be temperature-dependent and the material graduation is taken to three type of power, exponential and sigmoid low. The important influence of temperature change on the vibration response of the FG beams is also taken into account. Using an analytical approach, bending and free vibration analysis of simply supported beam FG beams is investigated by Thai and Vo based on various higher-order shear deformation beam theories (Thai and Vo 2012). Recently Timoshenko beam theory was used to investigate thermo-mechanical vibration of pre/post-buckled FG beams resting on elastic foundation by Komijani *et al.* (2014). The material properties are assumed temperature and microstructure dependent and generalized differential quadrature method is used to solve motion equations. Analytical solutions are presented by Fourier series expansion. Akgoz and Civalek performed the thermo-mechanical buckling characteristics of embedded FG micro beams in elastic medium based on trigonometric shear deformation beam and modified couple stress theories (Akgöz and Civalek 2014). Ebrahimi and Salari (2015) propounded thermal buckling and vibrations characteristic of FG nano beams. They concluded reduction of natural frequency and buckling temperature is result of exist of nonlocality.

With the rapid progression in technology of structure elements, structures with graded porosity can be introduced as one of the latest development in FGMs. The structures consider pores into microstructures by taking the local density into account. Because of existence of technical problem in manufacturing of FGM, porosity or micro voids occurring inside FGM, thus it is necessary to consider the effect of porosity on static and dynamic's behavior of these materials in this investigation. The porous materials are combined of two elements as solid (body) and liquid or gas that wood, stone, sponge and bone are example of these materials in the nature. Studies on the vibration response of porous FG structures, especially for beams, are still limited in number. For porous plates, the linear and nonlinear dynamic stability of a circular porous plate has been investigated to determine the critical loads in two separate study by Magnucka-Blandzi (2009, 2010). In another study, she also presented the problem of axi-symmetrical deflection and buckling of circular porous plates (Magnucka-Blandzi 2008). Moreover, the wave propagation of an infinite FG plate having porosities by using various simple higher-order shear deformation theories has been studied by Ait Yahia *et al*. (2015). Yahia *et al*. have presented non-linear free vibrations analysis of FG porous annular plates resting on elastic foundations (Yahia *et al*. 2015). They concluded that porosity volume fraction and type of porosity distribution have a significant influence on the geometrically non-linear free vibration response of the FG annular plates at large amplitudes. Mechab *et al*. have developed a nonlocal elasticity model for free vibration of FG porous nanoplates resting on elastic foundations (Mechab 2016). They utilized exponential shear deformation plate theory. Ebrahimi and Mokhtari (2014) studied transverse vibration analysis of rotating FG beam with porosities based on Timoshenko beam theory. DTM has been presented to solve the equations of motions. Wattanasakulpong and Ungbhakorn (2014) used Euler-Bernoulli beam theory to investigate linear and nonlinear vibrations analysis of porous functionally graded beams with elastically restrained ends. Material properties of FG beam have been described by a modified rule of mixture. Translational and rotational springs have been used to simulate the non-classical boundary condition for FGM beams. Moreover, Wattanasakulpong and Chaikittiratana (2015) predicted flexural vibration of porous FG beams based on Timoshenko beam theory by using Chebyshev collection method. They find out the porosities caused decrease the mass, strength and density of FG beams, so should be considered especially for the beams with clamped-clamped boundary condition. Atmane *et al*. (2015) used an efficient beam theory to study bending, free vibration and buckling analysis of porous FG beams on elastic foundations. Literature search in the area of vibration analysis of FG porous beams indicated that there is no report considered the thermal environment effects on vibration characteristics of porous FG beams and the materials properties were assumed temperature independent. While one of the most important features of FGMs is thermal insulations so there is scientific need to be familiar with the thermo-mechanical behavior of FG porous structures subjected thermal loadings.Most recently Ebrahimi *et al*. (2015) studied the vibration of porous FG Euler beams subjected to thermal loadings. It should be noted that in the above-mentioned study, only one specific porosity distribution was considered and no detailed discussion concerning the effects of different porosity distributions on the thermo-mechanical of porous beams was given. Also they utilized EBT, it is well-known that, the EBT ignores the effect of shear deformation and rotary inertia of the thick beams. In other words, this theory is based on the assumption that plane sections of the cross-section remain plane and perpendicular to the beam axis. The EBT is only suitable for vibration of thin beams, when a beam is moderately deep, or made of high-strength composite materials with a high anisotropy ratio, the theory needs some modifications to include the effect of transverse shear. Literature search in the area of vibration analyses of FG porous beams indicate that there is not any report considered the thermal environment effects on vibration characteristics of porous FG beams based on Timoshenko beam theory. As stated before the most important feature of FGMs is their behavior in severe and high temperature environments, hence it is necessary to consider the changing material properties due to thermal environments, for instance, under a high temperature environment the materials become softer and the Young's modulus and thermal expansion coefficients usually decrease with rising temperature.

In this paper, vibration characteristics of temperature dependent FGM beams with two types of porosity volume fractions namely even and uneven porosity volume fractions and considering the effect of both uniform and linear temperature rising is presented. FGM with even porosity volume fraction has porosity evenly through cross-section of the beam, while in FGM beam with unevenly porosities, volume fraction centralized mostly around the middle zone of the cross-section and decrease linearly to zero at the top and bottom surfaces as shown in Fig. 2. The material properties are assumed to vary continuously through the thickness direction according to modified power-low form and are temperature dependent. Timoshenko beam theory is used to determined displacement and strain's field and consider the effects of shear deformation and rotatoriy inertia. Equations of motion and boundary conditions have been derived by Hamilton's principle. These equations are solved by using Navier type method and DTM. The influence of several parameters such as thermal effects, gradient indexes, porosity volume fraction, type of porosity distributions, aspect ratio and boundary conditions on vibration behavior of FG Timoshenko beams with porosities is investigated. Comparisons with analytical solutions and the result from the existing literature are provided for two-constituents metal-ceramic beams and good agreement between the result of this article and those available in literature validated the presented approach. New numerical results can also be useful as valuable sources for validating other approaches and approximate methods.

2.Theory and formulation

2.1 Power-low functionally graded beams with porosities

Consider a uniform FG beam with porosities of length *L*, width *b*and thickness *h*, according to Fig. 1 Cartesian coordinate system *O*(*x*,*y*,*z*) is shown on the central axis of the beam, as *x*-axis is matched with neutral axis of the beam in the undeflected position, the *y*-axis in the width direction,

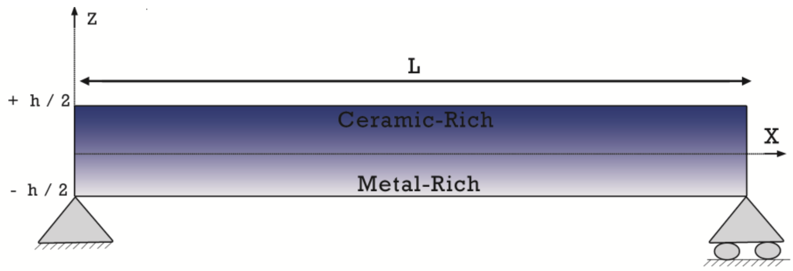


Fig. 1 Geometry and cordinates of functionally graded material beam

and the z-axis in the depth direction. The beam is made of homogeneous and isotropic functionally graded materials which properties of it's varying continuously only in the depth direction. Functionally graded materials (FGMs) are composite materials formed of two or multi phases that mechanical properties of them is gradually changed in one or more directions. A FG beam typically composed of two different materials as ceramics and metal, which at the top surface is full of ceramic (materials with good resistance to heat) and the bottom surface is metal rich (materials with good toughness property) and properties of ceramic varying to the metal propertie from top surface to the bottom surface smoothly. In this paper imperfect FGM with two type of porosities distribution (even and uneven) across the beam thickness has been studied, that porosities appear during manufacturing due to technical problem. Material properties of FG beam are supposed to vary through thickness direction of the constitutes according to modified power-low distribution. P-FGM is one of the most favorable models for FGMs. Effective material properties such as Young's modulus (E), shear modulus (G), mass density (ρ) and thermal expansions (α) are assumed to vary continuously in the depth direction according to power-low. Poissons'ratio is assumed to be constant in the z-axis direction. The effective material properties of FG beam with two kind of porosities that distributed identical in two phases of ceramic and metal can be expressed by using the modified rule of mixture as Wattanasakulpong and Ungbhakorn (2014)

$$P = P_c(V_c - \frac{\alpha}{2}) + P_m(V_m - \frac{\alpha}{2}) \tag{1}$$

Where P_c and P_m are the material properties of ceramic and metal and α is the volume fraction of porosities $(\alpha \prec\prec 1)$, for perfect FGM α is set to zero, V_c and V_m are the volume fraction of ceramic and metal that are attached as (Şimşek 2010)

$$v_c + v_m = 1 \tag{2}$$

The power-low volume fraction of the ceramics constituents of the beam is assumed to be given by Şimşek (2010)

$$v_c = (\frac{z}{h} + \frac{1}{2})^p \tag{3}$$

Here Z is the distance from the mid-plane of the FGM beam and p is the non-negative variable parameter (power-low exponent) which determine the material distribution through the thickness of the beam, according to this distribution we have a fully metal beam for large value of p and when p equal to zero a fully ceramic beam remain.For the even distribution of porosities (FGM-I), the effective material properties are obtained as Wattanasakulpong and Ungbhakorn (2014)

$$P(z) = \left(P_c - P_m\right)\left(\frac{z}{h} + \frac{1}{2}\right)^p + P_m - \frac{\alpha}{2}\left(P_c + P_m\right) \tag{4a}$$

$$E(z) = \left(E_c - E_m\right)\left(\frac{z}{h} + \frac{1}{2}\right)^p + E_m - \frac{\alpha}{2}\left(E_c + E_m\right) \tag{4b}$$

$$\rho(z)=\left(\rho_c-\rho_m\right)\left(\frac{z}{h}+\frac{1}{2}\right)^p+\rho_m-\frac{\alpha}{2}\left(\rho_c+\rho_m\right) \tag{4c}$$

$$G(z)=\left(G_c-G_m\right)\left(\frac{z}{h}+\frac{1}{2}\right)^p+G_m-\frac{\alpha}{2}\left(G_c+G_m\right) \tag{4d}$$

$$\alpha(z)=\left(\alpha_c-\alpha_m\right)\left(\frac{z}{h}+\frac{1}{2}\right)^p+\alpha_m-\frac{\alpha}{2}\left(\alpha_c+\alpha_m\right) \tag{4e}$$

For second type, uneven distribution of porosities (FGM-II),the effective material properties are replaced by following form Wattanasakulpong and Ungbhakorn (2014)

$$P(z)=\left(P_c-P_m\right)\left(\frac{z}{h}+\frac{1}{2}\right)^p+P_m-\frac{\alpha}{2}\left(P_c+P_m\right)\left(1-\frac{2|z|}{h}\right) \tag{5.a}$$

$$G(z)=\left(G_c-G_m\right)\left(\frac{z}{h}+\frac{1}{2}\right)^p+G_m-\frac{\alpha}{2}\left(G_c+G_m\right)\left(1-\frac{2|z|}{h}\right) \tag{5.b}$$

$$E(z)=\left(E_c-E_m\right)\left(\frac{z}{h}+\frac{1}{2}\right)^p+E_m-\frac{\alpha}{2}\left(E_c+E_m\right)\left(1-\frac{2|z|}{h}\right) \tag{5.c}$$

$$\rho(z)=\left(\rho_c-\rho_m\right)\left(\frac{z}{h}+\frac{1}{2}\right)^p+\rho_m-\frac{\alpha}{2}\left(\rho_c+\rho_m\right)\left(1-\frac{2|z|}{h}\right) \tag{5.d}$$

$$\alpha(z)=\left(\alpha_c-\alpha_m\right)\left(\frac{z}{h}+\frac{1}{2}\right)^p+\alpha_m-\frac{\alpha}{2}\left(\alpha_c+\alpha_m\right)\left(1-\frac{2|z|}{h}\right) \tag{5.e}$$

To predict the behavior of FGMs under high temperature more accurately, it is necessary to consider the temperature dependency on material properties. The nonlinear equation of thermo-elastic material properties in function of temperature $T(K)$ can be expressed as (Touloukian 1966)

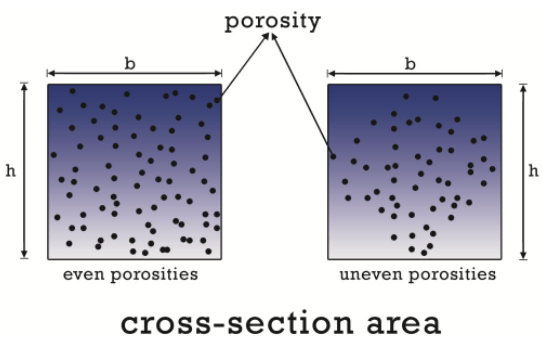


Fig. 2 Cross-section area of FGM beam with even and uneven porosities

$$P = P_0(P_{-1}\,T^{-1} + 1 + P_1\,T + P_2\,T^2 + P_3\,T^3) \tag{6}$$

Where P_0, P_{-1}, P_1, P_2 and P_3 are the temperature dependent coefficients which can be seen in the table materials properties (Table 3) for Si_3N_4 and SUS304. The bottom surface (z=-h/2) of FG porous beam is pure metal (SUS304), whereas the top surface (z=h/2) is pure ceramics (Si_3N_4).

2.2 Formulation of motion of FG porous beam using Timoshenko beam theory

The equations of motion is derived by using the Timoshenko beam theory following to Timoshenko beam theory the displacement field at any point of the beam can be written as (Ebrahimi and Salari 2015)

$$u_x\left(x,z,t\right) = u\left(x,t\right) + z\varphi(x,t) \tag{7}$$

$$u_y\left(x,z,t\right) = 0 \tag{8}$$

$$u_z(x,z,t) = w(x,t) \tag{9}$$

Where $u$is the axial displacement along x-axis, w is the transverse displacement along z-axis, φ is the rotational angle due to bending and t is the time. Then the strains field can be expressed as:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} + z\,\frac{\partial \varphi}{\partial x} \tag{10}$$

$$\gamma_{xz} = \frac{\partial w}{\partial x} + \varphi \tag{11}$$

ε_{xx}, γ_{xz} are normal and shear strain. The Euler Lagrange equations has been used to derive the equation of motion by using a Hamilton's principle, which can be stated as (Tauchert 1974)

$$\int_0^t \delta(U - T + V)\,dt = 0 \tag{12}$$

Where t_1, t_2 are the initial and end time δu is the virtual variation of strain energy, δV is the virtual variation of work done by external loads and δT is the virtual variation of kinetic energy. Here strain energy, kinetic energy and potential energy (external loading) can be calculated step by step and the equation of motion has been obtained by using rules of calculus of variations and Hamilton's principle.

a) In the first step we define strain energy as

$$U_t = U_b + U_s \Rightarrow \delta U_t = \delta U_b + \delta U_s \tag{13}$$

$$\delta U = \int_v \sigma_{ij}\delta\,\varepsilon_{ij}\,dV = \int_v (\sigma_{xx}\delta\,\varepsilon_{xx} + \sigma_{xz}\delta\,\gamma_{xz})\,dV \tag{14}$$

Where total strain energy has been obtained by summation of bending strain energy U_b and shear strain energy U_s, then by substituting the quantity of strains from Eqs. (10)- (11) into Eq. (14) as

$$\delta U=\int_0^L\int_A\sigma_{xx}\delta(\frac{\partial U}{\partial x}+z\frac{\partial\varphi}{\partial x})dAdx+\int_0^L\int_A\sigma_{xz}\delta(\frac{\partial W}{\partial x}+\varphi)dAdx \tag{15}$$

By defining N, M, Q as axial force, bending moment and shear force components as following and replacing these resultants into Eq. (15) , get to Eq. (18).

$$(N,M)=\int_A\sigma_{xx}(1,Z)\,dA,\quad Q=\int_A K_s\sigma_{xz}\,dA \qquad (16),\ (17)$$

$$\delta U=\int_0^L(N(\delta\frac{\partial u}{\partial x})+M(\delta\frac{\partial\varphi}{\partial x})+Q(\delta\frac{\partial w}{\partial x}+\delta\varphi))\,dx \tag{18}$$

Where coefficient K_s is called the Timoshenko shear correction factor and the exact value of it depends on cross section parameters and material properties of the beam. Here, K_s for rectangular beams has been assumed equal to 5/6 approximately.

b) in the second step the kinetic energy expression for Timoshenko beam theory can be expressed as

$$T=\frac{1}{2}\int_0^L\int_A\rho(z,T)\left(\left(\frac{\partial u_x}{\partial t}\right)^2+\left(\frac{\partial u_y}{\partial t}\right)^2+\left(\frac{\partial u_z}{\partial t}\right)^2\right)dAdx \tag{19}$$

$\frac{\partial u}{\partial t}$ the velocity along x, y and z-axises has been obtained by derivative of the coordinates with respect to time (t) as

$$T=\frac{1}{2}\int_0^L\int_A\rho(z,T)\left[\left(\frac{\partial u}{\partial t}\right)^2+Z^2\left(\frac{\partial\varphi}{\partial t}\right)^2+2z\frac{\partial\varphi}{\partial t}\frac{\partial u}{\partial t}+\left(\frac{\partial w}{\partial t}\right)^2\right]dAdx \tag{20}$$

$$T=\frac{1}{2}\int_0^L\left[I_0\left(\frac{\partial u}{\partial t}\right)^2+I_2\left(\frac{\partial\varphi}{\partial t}\right)^2+2I_1\frac{\partial\varphi}{\partial t}\frac{\partial u}{\partial t}+I_0\left(\frac{\partial w}{\partial t}\right)^2\right]dx \tag{21}$$

Where (I_0, I_1, I_2) are the mass moment of inertias that can be define as

$$(I_0,I_1,I_2)=\int_A\rho(z,T)(1,z,z^2)\,dA \tag{22}$$

Also the virtual variation of kinetic energy can be derived as

$$\delta T=\int_0^L\left[I_0(\frac{\partial u}{\partial t}\frac{\partial\delta u}{\partial t}+\frac{\partial w}{\partial t}\frac{\partial\delta w}{\partial t})+I_1(\frac{\partial\varphi}{\partial t}\frac{\partial\delta u}{\partial t}+\frac{\partial u}{\partial t}\frac{\partial\delta\varphi}{\partial t})+I_2\frac{\partial\varphi}{\partial t}\frac{\partial\delta\varphi}{\partial t}\right]dx \tag{23}$$

C) In the last step the variation of potential energy can be obtained as

$$\delta V=\int_0^L[f(x)\delta U+q(x)\delta W+\bar{N}\frac{\partial W}{\partial x}\frac{\partial(\delta\mathrm{W})}{\partial x}]\mathrm{d}x \tag{24}$$

Where $f(x)$, $q(x)$ are the axial and transverse loading that in this investigate equal to zero and $\bar{N}$ is the external loading due to thermal environment or elastic foundation.

In this study for analyzing vibration of porous FG beam in thermal environment, material properties are taken to be temperature-dependent and the temperature distribution is considered vary through thickness direction in two cases as: uniform and linear temperature distribution.

The first variation of external loadings due to temperature change can be written in the form as

$$\delta V = \int_0^L N^T \frac{\partial W}{\partial x}\frac{\partial(\delta \mathrm{W})}{\partial x}\mathrm{dx} \tag{25}$$

Which N^T is obtained as (Ebrahimi and Salari 2015)

$$N^T = \int E(z,T)\alpha(z,T)\Delta T dz \tag{26}$$

In which α is the coefficient of thermal dilatation that is typically positive and very small $(0 \prec \alpha, \alpha \ll 1)$. At last by substituting Eqs. (18)- (23) and (25) into Eq. (12) as

$$\begin{gathered}\int_{t_1}^{t_2}\int_0^l \left(-\frac{\partial N}{\partial X} + I_0\frac{\partial^2 U}{\partial t^2} + I_1\frac{\partial^2 \varphi}{\partial t^2}\right)\delta U + \left(Q - \frac{\partial M}{\partial X} + I_1\frac{\partial^2 U}{\partial t^2} + I_2\frac{\partial^2 \varphi}{\partial t^2}\right)\delta\varphi + \\ \left(-\frac{\partial Q}{\partial X} + I_0\frac{\partial^2 W}{\partial t^2} + \frac{\partial}{\partial X}N^T\frac{\partial W}{\partial X}\right)\delta W = 0\end{gathered} \tag{27}$$

and embedded equal to zero the coefficients of ∂u, $\partial\varphi$ and ∂W, the governing equations of motion of porous FG Timoshenko beam in thermal environment can be obtained as

$$(\delta U:0),\quad \frac{\partial N}{\partial x} - I_0\frac{\partial^2 U}{\partial t^2} - I_1\frac{\partial^2 \varphi}{\partial t^2} = 0 \tag{28}$$

$$(\delta\varphi:0),\quad \frac{\partial M}{\partial x} - Q - I_1\frac{\partial^2 U}{\partial t^2} - I_2\frac{\partial^2 \varphi}{\partial t^2} = 0 \tag{29}$$

$$(\delta W:0),\quad \frac{\partial Q}{\partial x} - \frac{\partial}{\partial x}(N^T\frac{\partial W}{\partial x}) - I_0\frac{\partial^2 W}{\partial t^2} = 0 \tag{30}$$

For a material that is linearly elastic and obeys the 1D Hooke's law, the relation between stress-strain can be described as

$$\sigma_{xx} = E(z)\varepsilon_{xx} \tag{31}$$

$$\sigma_{xz} = G(Z)\gamma_{XZ} \tag{32}$$

Where G is the shear modulus and E is the Young's modulus, by substituting the Eqs. (10)-(11) into (16)-(17) and integrating over the beam's cross-section, axial force, bending moment and shear force can be derived as following

$$N = A_{xx}\frac{\partial u}{\partial x} + B_{xx}\frac{\partial \varphi}{\partial x},\quad M = B_{xx}\frac{\partial u}{\partial x} + D_{xx}\frac{\partial \varphi}{\partial x},\quad Q = C_{xx}(\frac{\partial w}{\partial x} + \varphi) \tag{33}$$

In which the cross-section stiffness are defined as

$$(A_{xx}, B_{xx}, D_{xx}) = \int_A (1, z, z^2) E(z) dA, \quad C_{xx} = \int_A k_s G(z) dA \tag{34),(35}$$

And the last form of Euler-Lagrange equations for FG Timoshenko beam with porosities in thermal environment in terms of displacement can be derived as

$$A_{xx}\frac{\partial^2 u}{\partial x^2}+B_{xx}\frac{\partial^2 \varphi}{\partial x^2}-I_0\frac{\partial^2 u}{\partial t^2}-I_1\frac{\partial^2 \varphi}{\partial t^2}=0 \tag{36}$$

$$B_{xx}\frac{\partial^2 u}{\partial x^2}+D_{xx}\frac{\partial^2 \varphi}{\partial x^2}-C_{xx}(\frac{\partial w}{\partial x}+\varphi)-I_1\frac{\partial^2 U}{\partial t^2}-I_2\frac{\partial^2 \varphi}{\partial t^2}=0 \tag{37}$$

$$C_{xx}(\frac{\partial^2 w}{\partial x^2}+\frac{\partial \varphi}{\partial x})-N^T\frac{\partial^2 W}{\partial x^2}-I_0\frac{\partial^2 W}{\partial t^2}=0 \tag{38}$$

3.Sulution method

3.1 Analytical solution

In this section, by using Navier’s method (an analytical solution) Euler-Lagrange equations for free vibration of simply-supported porous FG beam has been solved. The displacement functions are expressed as combinations of non significant coefficients and known trigonometric functions to satisfy Lagrange equations and boundary conditions at x=0, x=L the following displacements functions are assumed to be of the formed

$$U = \sum_{m=1}^{\infty} U_m \cos(\frac{m\pi}{L}x)e^{iw_m t} \tag{39}$$

$$W = \sum_{m=1}^{\infty} W_m \cos(\frac{m\pi}{L}x)e^{iw_m t} \tag{40}$$

$$\theta = \sum_{m=1}^{\infty} \theta_m \cos(\frac{m\pi}{L}x)e^{iw_m t} \tag{41}$$

In which (U_m, W_m, θ_m) are the unknown Fourier coefficient that will be calculated for each value of m.

Boundary conditions for a simply-supported beam are as Eqs. (42)-(43) (Ebrahimi and Salari 2015).

$$u(0) = 0 \ , \ \frac{\partial u}{\partial x}(L) = 0 \tag{42}$$

$$w(0)=w(L)=0\ ,\ \frac{\partial\varphi}{\partial x}(0)=\frac{\partial\varphi}{\partial x}(L)=0 \tag{43}$$

By substituting Eqs. (40)-(41) into Eqs. (36)-(37)-(38) respectively, leads to Eqs. (44)-(45),

$$\left(-A_{xx}(\frac{m\pi}{L})^2+I_0{\omega_m}^2\right)U_m-\left(B_{xx}(\frac{m\pi}{L})^2-I_1{\omega_m}^2\right)\varphi_m=0 \tag{44}$$

$$\left(-C_{xx}(\frac{m\pi}{L})\right)\varphi_m+\left(-C_{xx}(\frac{m\pi}{L})^2+N^T(\frac{m\pi}{L})^2+I_0{\omega_m}^2\right)W_m=0 \tag{45}$$

$$\left(-B_{xx}(\frac{m\pi}{L})^2+I_1{\omega_m}^2\right)U_m+\left(-D_{xx}(\frac{m\pi}{L})^2-C_{xx}+I_2{\omega_m}^2\right)\varphi_m-\left(C_{xx}(\frac{m\pi}{L})\right)W_m=0 \tag{46}$$

By finding determinant of the coefficient matrix of the above equations and setting this multinomial to zero, we can find natural frequencies ω_n.

3.2 Application of differential transform method to free vibration problems

In this section, DTM is performed to solving equations of motions, which is a semi-analytic transformation technique based on Taylor series expansion equations and is a useful tool to obtain analytical solutions of these differential equations. Certain transformations rules are applied to governing equations and the boundary conditions of the system in order to transform them into a set of algebraic equations in terms of the differential transforms of the original functions. This method construct an analytical solution in the form of polynomials. It is different from the high-order Taylor series method, which requires symbolic computation of the necessary derivative of the data functions and is expensive for large orders. The Taylor series method is computationally expansive for large orders. DTM is an iterative procedure for obtaining analytic Taylor series solutions of differential equations, in fact this method tries to find coefficients of series expansions of unknown function with using the initial data on the problem.

Differential transformation of the nth derivative function $y(x)$ and differential inverse transformation of $Y(k)$ are respectively defined as follow (Hassan 2002)

$$Y(k)=\sum_{k=0}^{\infty}\frac{1}{k!}\left[\frac{d^k}{dx^k}y(x)\right]_{x=0} \tag{47}$$

$$y(x)=\sum_{0}^{\infty}X^kY(k) \tag{48}$$

In which $y(x)$ is the original function and $Y(k)$ is the transformed function. Consequently from Eqs. (47)-(48) can obtain

$$Y(k)=\sum_{k=0}^{\infty}\frac{x^k}{k!}\left[\frac{d^k}{dx^k}y(x)\right]_{x=0} \tag{49}$$

$$y(x)=\sum_{k=0}^{\infty}X^{k}Y(k) \tag{50}$$

In this calculations $y(x)=\sum_{n+1}^{\infty}X^{k}Y(k)$ is small enough to be neglected, and N is determined by the convergence of the eigenvalues. From definitions of DTM in Eqs. (47)-(48)-(49), the fundamental theorems of differential transforms method can be performed that are listed in Tables 1-2. present the differential transformation of conventional boundary conditions. Assuming a sinusoidal variation of $w(x,t)$ and $\theta(x,t)$, which the functions are approximated as

$$w(x,t)=\bar{w}e^{i\omega t} \quad \text{and} \quad \theta(x,t)=\bar{\theta}e^{i\omega t} \tag{51}$$

By substituting Eq. (51) into Eqs. (37)-(38) equations of motions has been turned to

$$(D_{xx}-\frac{B_{xx}{}^{2}}{A_{xx}})\frac{\partial^{2}\bar{\varphi}}{\partial x^{2}}-C_{xx}\frac{\partial\bar{w}}{\partial x}-C_{xx}\bar{\varphi}-I_{1}\omega^{2}\frac{B_{xx}}{A_{xx}}\bar{\varphi}+I_{2}\omega^{2}\bar{\varphi}=0 \tag{52}$$

Table 1 Some of the transformation rules of the one-dimensional DTM (Ju 2004)

Original function	Transformed function
$f(x)=g(x)\pm h(x)$	$F(K)=G(K)\pm H(K)$
$f(x)=\lambda g(x)$	$F(K)=\lambda G(K)$
$f(x)=g(x)h(x)$	$F(K)=\sum_{l=0}^{K}G(K-l)H(l)$
$f(x)=\frac{d^{n}g(x)}{dx^{n}}$	$F(K)=\frac{(k+n)!}{k!}G(K+n)$
$f(x)=x^{n}$	$F(K)=\delta(K-n)=\begin{cases}1 & k=n\\ 0 & k\neq n\end{cases}$

Table 2 Transformed boundary conditions (B.C.) based on DTM (Ju 2004)

x=0		*x*=*L*	
Original B.C.	Transformed B.C.	Original B.C.	Transformed B.C.
$\mathrm{f}(0)=0$	$F[0]=0$	$\mathrm{f}(L)=0$	$\sum_{k=0}^{\infty}F[k]=0$
$\frac{\mathrm{df}(0)}{\mathrm{dx}}=0$	$F[1]=0$	$\frac{\mathrm{df}(L)}{\mathrm{dx}}=0$	$\sum_{k=0}^{\infty}kF[k]=0$
$\frac{d^{2}f(0)}{\mathrm{dx}^{2}}=0$	$F[2]=0$	$\frac{d^{2}f(L)}{\mathrm{dx}^{2}}=0$	$\sum_{k=0}^{\infty}k(k-1)F[k]=0$
$\frac{d^{3}f(0)}{\mathrm{dx}^{3}}=0$	$F[3]=0$	$\frac{d^{3}f(L)}{\mathrm{dx}^{3}}=0$	$\sum_{k=0}^{\infty}k(k-1)(k-2)F[k]=0$

$$(C_{xx} - N^T)\frac{\partial^2 \overline{w}}{\partial x^2} + C_{xx}\frac{\partial \overline{\varphi}}{\partial x} + I_0\omega^2 \overline{w} = 0 \tag{53}$$

According to the basic transformation operations in Table 2, the transformed form of the governing Eqs. (52) and (53) around x_0=0 may be obtained as

$$(D_{xx} - \frac{B_{xx}{}^2}{A_{xx}})(k+1)(k+2)\varphi\left[k+2\right] + (I_2\omega^2 - C_{xx} - I_1\omega^2\frac{B_{xx}}{A_{xx}})\varphi\left[k\right] - C_{xx}(k+1)w\left[k+1\right] = 0 \tag{54}$$

$$(C_{xx} - N^T)(k+1)(k+2)w\left[k+2\right] + I_0\omega^2 w\left[k\right] + C_{xx}(k+1)\varphi\left[k+1\right] = 0 \tag{55}$$

Transformed functions of $w(x)$, $\varphi(x)$ are $w[k]$, $\varphi[k]$, by using the theorems introduced in Table 2, transformed various boundary conditions can be expressed as follow:

Simply supported-Simply supported:

$$w\left[0\right] = 0,\ \ \varphi\left[1\right] = 0$$
$$\sum_{k=0}^{\infty} w\left[k\right] = 0,\ \ \sum_{k=0}^{\infty} k\ \varphi\left[k\right] = 0 \tag{56a}$$

Clamped-Simply supported:

$$w\left[0\right] = 0,\ \ \varphi\left[0\right] = 0$$
$$\sum_{k=0}^{\infty} w\left[k\right] = 0,\ \ \sum_{k=0}^{\infty} k\ \varphi\left[k\right] = 0 \tag{56b}$$

Clamped-Clamped:

$$w\left[0\right] = 0,\ \ \varphi\left[0\right] = 0$$
$$\sum_{k=0}^{\infty} w\left[k\right] = 0,\ \ \sum_{k=0}^{\infty} \varphi\left[k\right] = 0 \tag{56c}$$

By using Eqs. (54)-(55) together with the transformed boundary conditions one arrives at the following eigenvalue problem

$$\begin{bmatrix} M_{11}{}^{(n)}(\omega) & M_{12}{}^{(n)}(\omega) \\ M_{21}{}^{(n)}(\omega)_{21} & M_{22}{}^{(n)}(\omega) \end{bmatrix}\left[C\right] = 0 \tag{57}$$

Where $[C]$ correspond to the missing boundary conditions at x=0 and M_{ij} are polynomials in terms of (ω) corresponding to the nth term. for the non-trivial solutions of Eq. (57) , it is necessary that the determinant of the coefficient matrix set equal to zero

$$\begin{bmatrix} M_{11}{}^{(n)}(\omega) & M_{12}{}^{(n)}(\omega) \\ M_{21}{}^{(n)}(\omega)_{21} & M_{22}{}^{(n)}(\omega) \end{bmatrix} = 0 \tag{58}$$

The ith estimated eigenvalue may be obtained by for the nth iteration, by solving Eq. (58). The total number of iterations is related to the accuracy of calculations which can be determined by following equations

$$\left|\omega_i^{(n)} - \omega_i^{(n-1)}\right| < \varepsilon \tag{59}$$

In this study ε=0.0001 in procedure of finding eigenvalues which results in four-digit precision in estimated eigenvalues. Furtherthe computer package Mathematica has been developed according to the DTM rules as stated before to find eigenvalues. As mentioned before, DTM implies an iterative procedure to obtain the high-order Taylor series solution of differential equations. The Taylor series method requires a long computational time for large orders, whereas one advantage of employing DTM in solving differential equations is a fast convergence rate and small calculation error.

4. Thermal environment and temperature distributions

For a porous FG beam in thermal environment, temperature is assumed vary along the thickness directions at two ways as:

4.1 uniform temperature rise (UTR)

Consider a porous FG that is at initial temperature equal to T_0=300 and beam is free of stresses at initial temperature and temperature of beam change to final temperature with the difference of ΔT as

$$\Delta T = T - T_0 \tag{60}$$

4.2 Linear temperature rise (LTR)

Consider the temperature of the top surface of the porous FG beam is T_t and vary linearly from T_t to T_b, the bottom surface temperature finallythe temperature rise is given as (Kiani and Eslami 2013)

$$T = T_m + \Delta T \left(\frac{1}{2} + \frac{z}{h}\right) \tag{61}$$

And ΔT should be defined as

$$\Delta T = T_t - T_b \tag{62}$$

5. Numerical result and discussions

Through this section, after validation for S-S porous FG beam the influence of different types of porous distributions, porosity volume fraction, power-law exponent, temperature rises, boundary conditions on the natural frequencies of the porous FG beam will be perceive. The functionally graded porous beam is combined of Steel (SUS304) and Silicon nitride (Si_3N_4) where

Table 3 Temperature dependent coefficients of Young's modulus, thermal expansion coefficient, mass density and Poisson's ratio for Si_3N_4 and SUS304

Material	Properties	P_0	P_{-1}	P_1	P_2	P_3
Si_3N_4	E (Pa)	348.43e+9	0	-3.070e-4	2.160e-7	-8.946e-11
	α(K^{-1})	5.8723e-6	0	9.095e-4	0	0
	ρ (Kg/m^3)	2370	0	0	0	0
	ν	0.24	0	0	0	0
SUS304	E (Pa)	201.04e+9	0	3.079e-4	-6.534e-7	0
	α(K^{-1})	12.330e-6	0	8.086e-4	0	0
	ρ (Kg/m^3)	8166	0	0	0	0
	ν	0.3262	0	-2.002e-4	3.797e-7	0

Table 4 Convergence study for the first dimensionless natural frequency of even porous FG beam under linear temperature rise with L/h=20, α=0.1, ΔT=40 [K], p=0.5

Method	Iteration	C-C		C-S		S-S	
		λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
DTM	11	10.7467	14.4549	7.0142	13.8114	4.3251	12.6788
	12	10.1801	109.828	6.9473	16.0877	4.3137	1244.48
	13	9.9226	103.094	6.8887	103.216	4.3143	129.986
	14	10.1257	19.0002	6.9117	18.6422	4.3150	16.1182
	15	10.0998	22.2776	6.9116	19.6668	4.3149	16.8066
	16	10.0944	28.0001	6.9110	20.9890	4.3149	18.5615
	17	10.0922	124.042	6.9107	124.205	4.3149	18.0688
	18	10.0937	25.4995	6.9108	22.1407	4.3149	17.7426
	19	10.0936	26.6848	6.9108	22.2208	4.3149	17.7748
	20	10.0936	27.4266	6.9108	22.3566	4.3149	17.8086
	21	10.0936	27.8241	6.9108	22.4705	4.3149	17.8054
	22	10.0936	27.3745	6.9108	22.4121	4.3149	17.8022
	23	10.0936	27.4184	6.9108	22.4118	4.3149	17.8025
	24	10.0936	27.4347	6.9108	22.4138	4.3149	17.8027
	25	10.0936	27.4383	6.9108	22.4148	4.3149	17.8027
	26	10.0936	27.4344	6.9108	22.4144	4.3149	17.8027
	27	10.0936	27.4347	6.9108	22.4144	4.3149	17.8027
	28	10.0936	27.4349	6.9108	22.4144	4.3149	17.8027
	29	10.0936	27.4349	6.9108	22.4144	4.3149	17.8027
	30	10.0936	27.4349	6.9108	22.4144	4.3149	17.8027
Analytical		-	-	-	-	4.3147	17.7994

its properties are given in Table 3. It is assumed that the temperature increase in metal surface to reference temperature T_0 of the FG beam is $Tm-T_0$=5K (Kiani and Eslami 2013).

The non-dimensional natural frequencies (λ) can be calculated by relations in Eq. (63).

$$\lambda = \omega \frac{L^2}{h}\sqrt{\frac{\rho_m}{e_m}} \tag{63}$$

Table 4 shows the convergence study of DTM for first two frequencies. It is observed that after a certain number of iterations, the eigenvalues converged to a value with good precision, hence the number of iterations is important in DTM method. From the result of Table 4, high convergence rate of the method can be easily observed, as seen that the first and second natural frequency of S-S FG beam with even porosity and p=0.5, ΔT=40 [K], α=0.1, L/h=20 converged after 15 and 24 iterations with four digit precision while for the C-S boundary conditions first and second natural frequencies converged after 18 and 26 iterations at last for C-C supports converged after 19, 28 iterations respectively.

In Table 5, numerical result are compared with Simsek (2010) for validating of the present research, hereupon natural frequencies of FG beams combined of alumina and aluminum with following material and beam properties (E_{Al}=70 GPa, ρ_{Al}=2702 kg/m^3, v_{Al}=0.3, $E_{Al_2o_3} = 380GPa$,, $\rho_{Al_2o_3} = 3960kg/m^3$, $v_{Al_2o_3} = 0.3$) for L/h=5, 20 and various gradient indexes with simply-simply boundary conditions are obtained by two solution method as DTM and analytical. The present frequencies are in good agreement with results of Simsek (201). It is observed that the fundamental frequency parameters obtained in the present research are in approximately enough to the results provided in the study that is used for comparison and validate the proposed method of solutions. The fundamental frequency parameters obtained in the present investigation are in approximately close enough to the results provided in these literatures and thus validates the proposed method of solution.

In the next tables effects of different parameters such as temperature change, porosity parameter, types of temperature rising, porosities distribution, gradient indexes and boundary conditions on free vibration behavior of porous FG beam are represented. Three different boundary conditions are considered in the following: simply-supported/simply-supported (S-S) , simply supported/clamped (C-S) and clamped/clamped (C-C) that predicate the edge conditions at x=0, x=L of the beam.

Table 5 Comparison of the nondimensional fundamental fequency for a S-S FG beam with various gradient indexes

Power-law Exponent	L/h	present		Simsek (2010)
		Analytical	DTM	Lagrange’s equations
p=0	5	5.15247847	5.15247850	5.1524
	20	5.46031881	5.4603185	5.4603
p=0.2	5	4.80540725	4.80605463	4.8065
	20	5.08133551	5.08138658	5.0826
p=0.5	5	4.40789132	4.41066441	4.4083
	20	4.65090965	4.65112669	4.6513
p=1	5	3.99024091	3.99659975	3.9902
	20	4.20505401	4.20554478	4.2050
p=2	5	3.63438622	3.64478827	3.6343
	20	3.83667615	3.83756665	3.8367

The first dimensionless natural frequency of the porous beam with the simply supported has been presented at Table 6. For (L/h=20) and different values of the gradient index (p=0.1, 0.2, 0.5, 1), volume fraction of porosity (α=0, 0.1, 0.2) and temperature changes (ΔT=20, 40, 80K) based on both DTM and analytical solution method. Two types of porosities distribution is considered as even (FGM *I*) and uneven (FGM *II*), as well as temperature rising is inclusive uniform and linear distributions at this table.

By studying the results of Table 6. It is obsereved that fundamental frequency will be growth by increasing the porosity parameter for every temperature rising and porous distributions, and the

Table 6 Temperature and material graduation effect on first dimensionless natural frequency of a S-S FG porous beam with different FG type and porosity parameter and thermal loading (L/h=20)

ΔT=20 [K]										
FGM type	a	Load type	n=0.1		n=0.2		n=0.5		n=1	
			DTM	Analytical	DTM	Analytical	DTM	Analytical	DTM	Analytical
FGM (I)	0	UTR	5.5581	5.5581	5.0726	5.0727	4.2788	4.2786	3.7278	3.7276
		LTR	5.6159	5.6158	5.1324	5.1323	4.3405	4.3403	3.7893	3.7891
	0.1	UTR	5.9110	5.9109	5.3085	5.3084	4.3692	4.3690	3.7449	3.7446
		LTR	5.9624	5.9623	5.3619	5.3618	4.4245	4.4243	3.7999	3.7991
	0.2	UTR	6.4184	6.4183	5.6278	5.6277	4.4790	4.4786	3.7590	3.7586
		LTR	6.4636	6.4635	5.6750	5.6749	4.5280	4.5277	3.8078	3.8074
FGM (II)	0	UTR	5.5581	5.5580	5.0727	5.0726	4.2788	4.2786	3.7278	3.7276
		LTR	5.6159	5.6158	5.1324	5.1323	4.3405	4.3403	3.7893	3.7891
	0.1	UTR	5.7884	7.7884	5.2469	5.2468	4.3807	4.3806	3.7916	3.7913
		LTR	5.8424	5.8423	5.3028	5.3027	4.4386	4.4384	3.8491	3.8488
	0.2	UTR	6.0557	6.0557	5.4444	5.4443	4.4920	4.4919	3.8592	3.8589
		LTR	6.1061	6.1061	5.4968	5.4968	4.5462	4.5460	3.9129	3.9127
ΔT=40 [K]										
FGM type	a	Load type	n=0.1		n=0.2		n=0.5		n=1	
			DTM	Analytical	DTM	Analytical	DTM	Analytical	DTM	Analytical
FGM (I)	0	UTR	5.2929	5.2928	4.8109	4.8108	4.0226	4.0225	3.4765	3.4763
		LTR	5.4806	5.4805	5.0024	5.0023	4.2182	4.2180	3.6717	3.6715
	0.1	UTR	5.6709	5.6709	5.0731	5.0730	4.1407	4.1405	3.5218	3.5215
		LTR	5.8380	5.8380	5.2437	5.2437	4.3149	4.3147	3.6953	3.6951
	0.2	UTR	6.2020	6.2019	5.4180	5.4179	4.2777	4.2774	3.5636	3.5633
		LTR	6.3492	6.3491	5.5683	5.5681	4.4311	4.4308	3.7160	3.7157
FGM (II)	0	UTR	5.2929	5.2928	4.8109	4.8108	4.0227	4.0225	3.4765	3.4763
		LTR	5.4806	5.4806	5.0024	5.0023	4.2182	4.2180	3.6717	3.6715
	0.1	UTR	5.5386	5.5386	5.0012	5.0012	4.1415	4.1414	3.5578	3.5575
		LTR	5.7139	5.7139	5.1801	5.1800	4.3240	4.3239	3.7395	3.7392
	0.2	UTR	5.8203	5.8202	5.2138	5.2137	4.2689	4.2687	3.6420	3.6417
		LTR	5.9839	5.9838	5.3808	5.3807	4.4391	4.4390	3.8109	3.8107

Table 6 Continued

			ΔT=80 [K]							
FGM type	a	Load type	n=0.1		n=0.2		n=0.5		n=1	
			DTM	Analytical	DTM	Analytical	DTM	Analytical	DTM	Analytical
FGM (I)	0	UTR	4.7006	4.7006	4.2210	4.2210	3.4341	3.4340	2.8885	2.8884
		LTR	5.1937	5.1937	4.7266	4.7265	3.9584	3.9583	3.4216	3.4214
	0.1	UTR	5.1432	5.1432	4.5511	4.5510	3.6246	3.6244	3.0088	3.0086
		LTR	5.5752	5.5752	4.9941	4.9940	4.0833	4.0831	3.4739	3.4737
	0.2	UTR	5.7335	5.7334	4.9596	4.9595	3.8301	3.8300	3.1217	3.1215
		LTR	6.1079	6.1079	5.3434	5.3433	4.2270	4.2267	3.5229	3.5226
FGM (II)	0	UTR	4.7006	4.7006	4.2210	4.2210	3.4341	3.4340	2.8885	2.8884
		LTR	5.1937	5.1937	4.7266	4.7266	3.9584	3.9583	3.4216	3.4214
	0.1	UTR	4.9864	4.9864	4.4534	4.4533	3.5984	3.5982	3.0179	3.0177
		LTR	5.4422	5.4422	4.9205	4.9204	4.0816	4.0814	3.5073	3.5071
	0.2	UTR	5.3044	5.3043	4.7044	4.7043	3.7675	3.7673	3.1464	3.1462
		LTR	5.7260	5.7259	5.1362	5.1361	4.2130	4.2128	3.5957	3.5954

Table 7 Temperature and material graduation effect on first dimensionless natural frequency of a C-S FG beam with different FG type, porosity and thermal loading (L/h=20)

FGM type	a	Load type	ΔT=20[K]				ΔT=40[K]				ΔT=20			
			Power-low exponent											
			n=0.1	n=0.2	n=0.5	n=1	n=0.1	n=0.2	n=0.5	n=1	n=0.1	n=0.2	n=0.5	n=1
			DTM	DTM	DTM	DTM	DTM	DTM	DTM	DTM	DTM	DTM	DTM	DTM
FGM (I)	0	UTR	8.8331	8.0764	6.8385	5.9779	8.6374	7.8843	6.6526	5.7972	8.2237	7.4760	6.2530	5.4050
		LTR	8.8699	8.1155	6.8803	6.0205	8.7618	8.0128	6.7859	5.9314	8.5378	7.8000	6.5904	5.7471
	0.1	UTR	9.3630	8.4237	6.9591	5.9844	9.1842	8.2496	6.7922	5.8230	8.8100	7.8826	6.4360	5.4750
		LTR	9.3941	8.4580	6.9964	6.0225	9.2936	8.3631	6.9109	5.9427	9.0841	8.1670	6.7342	5.7780
	0.2	UTR	10.1338	8.9012	7.1100	5.9865	9.9710	8.7444	6.9619	5.8443	9.6337	8.4174	6.6484	5.5398
		LTR	10.1605	8.9305	7.1427	6.0202	10.0651	8.8431	7.0661	5.9497	9.8681	8.6627	6.9079	5.8046
FGM (II)	0	UTR	8.8331	8.0765	6.8385	5.9779	8.6374	7.8844	6.6526	5.7972	8.2237	7.4760	6.2530	5.4050
		LTR	8.8699	8.1155	6.8803	6.0205	8.7618	8.0128	6.7859	5.9314	8.5378	7.8000	6.5904	5.7471
	0.1	UTR	9.1786	8.3345	6.9839	6.0637	8.9933	8.1533	6.8095	5.8947	8.6038	7.7702	6.4365	5.5301
		LTR	9.2124	8.3707	7.0230	6.1035	9.1087	8.2728	6.9339	6.0200	8.8941	8.0704	6.7498	5.8476
	0.2	UTR	9.5821	8.6293	7.1443	6.1559	9.4064	8.4583	6.9808	5.9983	9.0393	8.0987	6.6330	5.6597
		LTR	9.6131	8.6627	7.1807	6.1930	9.5133	8.5693	7.0968	6.1149	9.3071	8.3764	6.9235	5.9538

growthing of the frequency for FGM (I) is more tangible than FGM (II).

This growthing in frequency value emphasis on the importance of porosity effect. Also it is obvious from this table that increasing temperature change yields decreasing of natural frequencies in two types of temperature risings, thus temperature change has a significant effect on the dimensionless natural frequencies. As we know, increasing of the power indexes lead to rise the percentage of metal phase thereupon FG beames will be more flexible and fundamental frequency values reduce. In addition, for a certain of values of temperature change, gradient indexes, temperature rise and porous parameter, dimensionles natural frequency of the even distribution is more than uneven one. It can also be seen that the dimensionless natural frequencies perdicated by DTM are in close agreement with those evaluated by analytical solution.

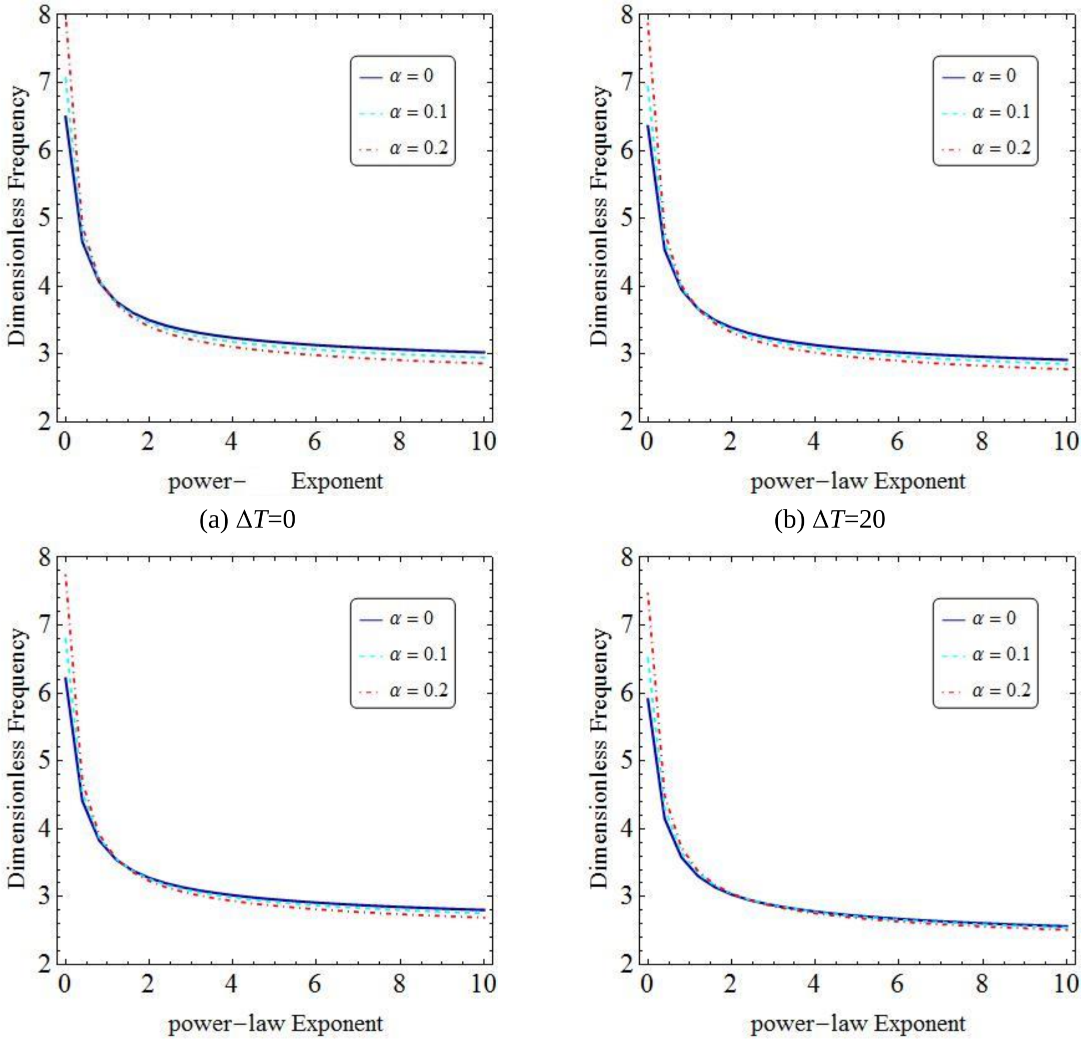
Also, Table 7, contains the effect of temperature change, porosity parameter, material graduation on natural frequencies of the porous FG beams of Clamped-Simply boundary condition with different volume fraction of porosity parameters and two cases of temperature rise. It is observable from this table that fundamental frequency increase with increasing porous parameter, also increasing gradient index and temperature change yields to comes down of natural frequency values, furthermore fundamental frequencies of FGM beams with uneven porosity distribution are less than even distribution one.

Table 8 Temperature and material garduation effect on first dimensionless natural frequency of a C-C FG beam with different FG type, porosity and thermal loading (L/h=20)

FGM type	a	Load type	ΔT=20[K]				ΔT=40[K]				ΔT=80[K]			
			Power-low exponent											
			n=0.1 DTM	n=0.2 DTM	n=0.5 DTM	n=1 DTM	n=0.1 DTM	n=0.2 DTM	n=0.5 DTM	n=1 DTM	n=0.1 DTM	n=0.2 DTM	n=0.5 DTM	n=1 DTM
FGM (I)	0	UTR	12.8585	11.7664	9.9789	8.7346	12.7112	11.6227	9.8415	8.6022	12.4113	11.3282	9.5563	8.3248
		LTR	12.8781	11.7887	10.0051	8.7626	12.7868	11.7033	9.9291	8.6928	12.6008	11.5293	9.7743	8.5508
	0.1	UTR	13.6108	12.2551	10.1405	8.7316	13.4751	12.1238	10.0164	8.6128	13.2013	11.8570	9.7605	8.3652
		LTR	13.6260	12.2734	10.1633	8.7564	13.5387	12.1930	10.0936	8.6936	13.3607	12.0293	9.9516	8.5655
	0.2	UTR	14.7111	12.9317	10.3458	8.7222	14.5859	12.8124	10.2350	8.6172	14.3367	12.5725	10.0080	8.3992
		LTR	14.7215	12.9459	10.3652	8.7441	14.6368	12.8699	10.3017	8.6881	14.4644	12.7153	10.1725	8.5743
FGM (II)	0	UTR	12.8585	11.7664	9.9789	8.7346	12.7112	11.6227	9.8415	8.6022	12.4113	11.3282	9.5562	8.3247
		LTR	12.8781	11.7887	10.0051	8.7626	12.7868	11.7033	9.9291	8.6928	12.6008	11.5293	9.7743	8.5508
	0.1	UTR	13.3475	12.1293	10.1793	8.8489	13.2073	11.9931	10.0499	8.7248	12.9233	11.7154	9.7824	8.4656
		LTR	13.3645	12.1493	10.2034	8.8750	13.2759	12.0671	10.1312	8.8092	13.0954	11.8995	9.9840	8.6754
	0.2	UTR	13.9202	12.5452	10.4015	8.9727	13.7866	12.4161	10.2797	8.8566	13.5175	12.1541	10.0291	8.6148
		LTR	13.9348	12.5629	10.4236	8.9968	13.8484	12.4835	10.3549	8.9350	13.6725	12.3218	10.2150	8.8091

Table 9 The effects of porosity on dimensionless frequency for higher modes of porous FG beam under linear temperature rise with L/h=20, ΔT=80[K], p=0.2

Type of FGM	a	**C-C** λ_1	**C-C** λ_2	**C-C** λ_3	**C-S** λ_1	**C-S** λ_2	**C-S** λ_3	**S-S** λ_1	**S-S** λ_2	**S-S** λ_3
FGM (I)	0	11.5293	31.5200	65.7397	7.8000	25.6924	53.5747	4.7266	20.3281	45.4899
	0.1	12.0293	32.8314	67.5212	8.16698	26.7788	55.7663	4.9940	21.2110	47.3732
	0.2	12.7153	34.6451	69.1749	8.66256	28.2762	58.8064	5.3434	22.4215	49.9788
FGM (II)	0	11.5293	31.5200	65.7397	7.8000	25.6924	53.5774	4.7266	20.3281	45.4899
	0.1	11.8569	32.6811	66.8532	8.0704	26.4296	61.0442	4.9205	20.9809	46.9587
	0.2	12.5724	34.5408	68.3520	8.37641	62.5695	62.5695	5.1362	21.7251	48.5382



(a) Δ*T*=0 (b) Δ*T*=20

(c) Δ*T*=40 (d) Δ*T*=80

Fig. 3 The variation of the first dimensionless frequency of S-S FGM(I) beam with material graduation and porosity for different linear temperature rises (*L/h*=20)

The effects of porosity on dimensionless frequency for higher modes of porous FG beam under linear temperature rise with different boundary condition and constant values of *L/h*=20, Δ*T*=80[K], *p*=0.2 presented at Table 9. It is concluded that porosity effect is more tangible for higher modes of porous FG beam. So it is necessary to consider porosity effect for vibration of FG beams. It is found that increasing of volume fraction of porosity is couse of increasing of dimensionless frequencies for both porosity distribution.

Atlast, the fundamental frequency parameter is presented at Table 8. for FG beam subjected to uiform and linear temperature rising with different power-low indexes, porous distributions with Clamped-Clamped boundary conditions. Also, the conclusions that derived from this table for the effect of the porosity and power index parameters on the natural frequency are similar to two

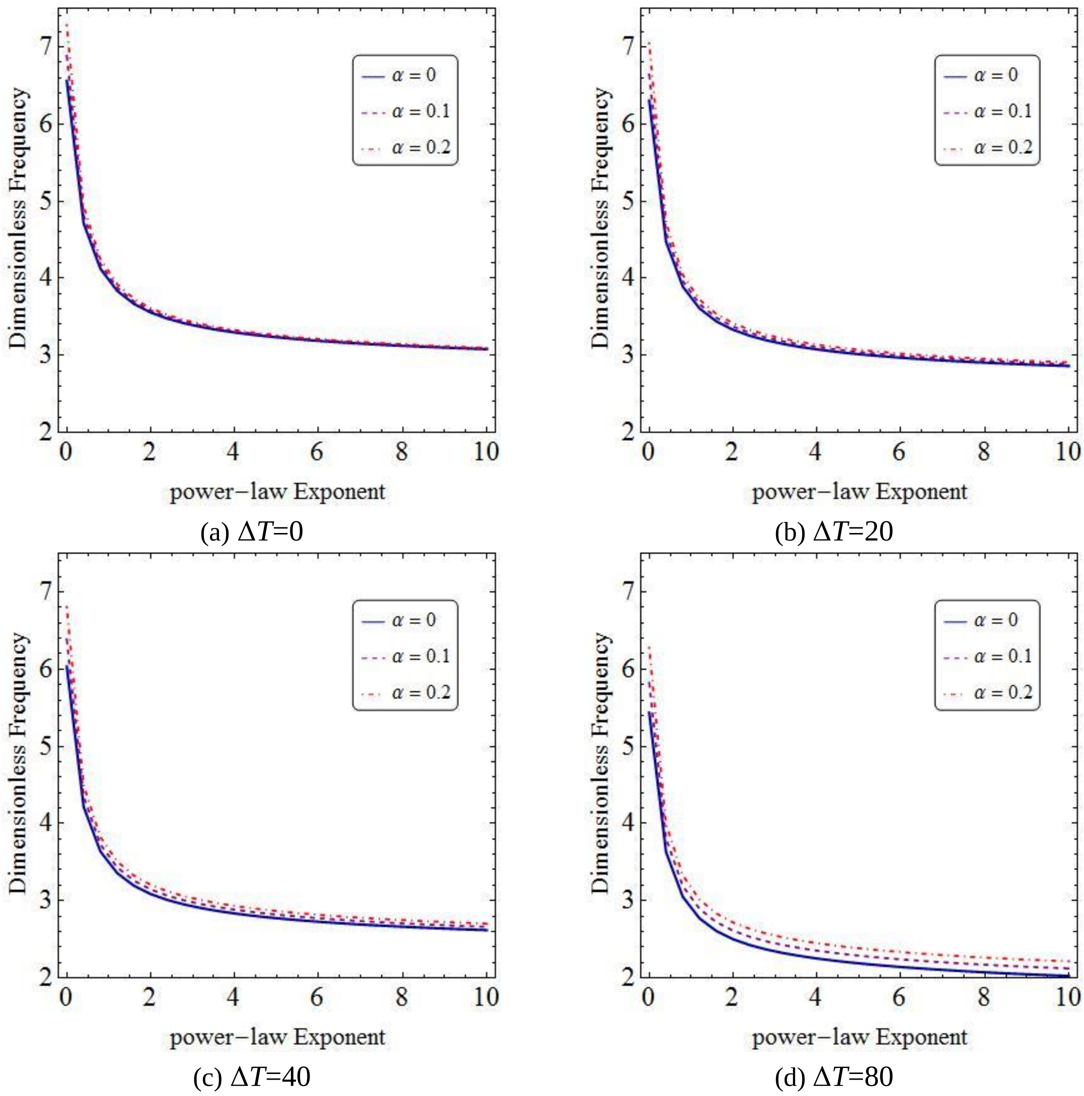
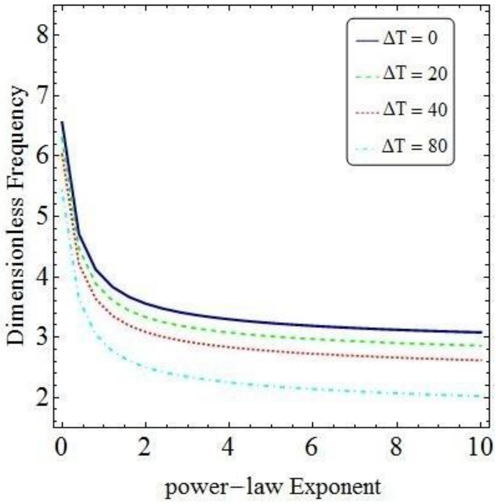


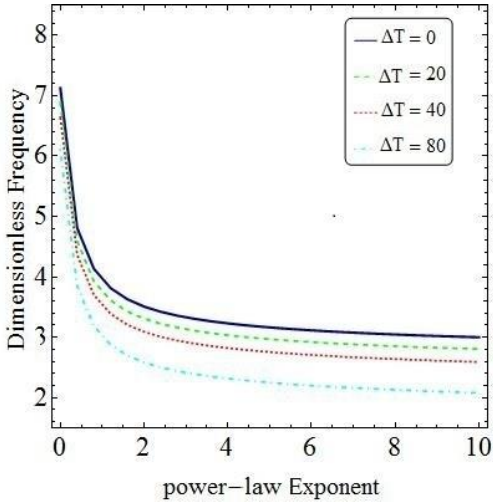
Fig. 4 The variation of the first dimensionless frequency of S-S FGM(II) beam with material graduation and porosity for different uniform temperature rises (L/h=20)

previously discussed edge conditions except that for even porosity with n=1 and ΔT=20, it is observed that with increasing of porosity, the fundamental frequencies will go come down. Indeed the effect of porosity on natural frequency depends on the values of power low exponent. By increasing values of power-low exponent from a certain value of n, increment of the porosity leads to decreasing of the frequencies and increasing of temperature changings leads that the certain value of the n (which from then on increasing of the porosity leads to decreasing of the frequency) has gone up.

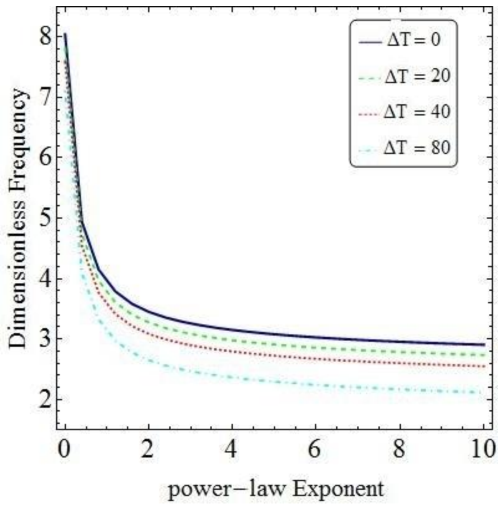
Therefore by comparing the frequency values for porous FG beams for a prescribed porous distribution, temperature rise and gradient indexes in Tables 6-8, can observe the influence of boundary conditions on frequencies. The greatest frequency at pre-buckling region, is obtained for



(a) α=0



(b) α=0.1

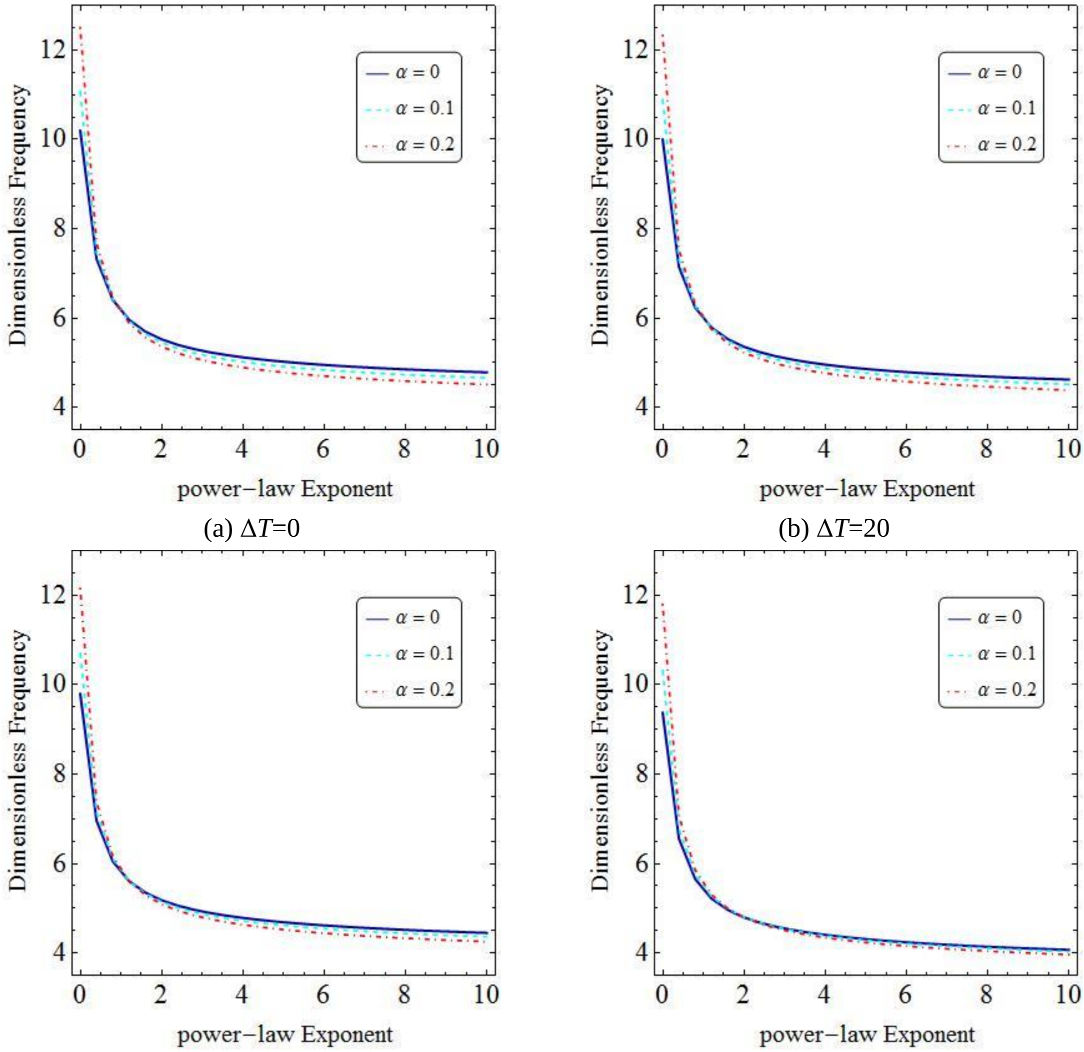


(c) α=0.2

Fig. 5 The variation of the first dimensionless frequency of S-S FGM (*I*) beam with material graduation and uniform temperature rises for different porosities (L/h=20)

the FG beam with C-C boundary conditions followed with C-S and S-S, respectively.

Comparison of the first dimensionless natural frequencies of S-S FG (*I*) beam subjected to UTR and FG (*II*) beam subjected to LTR with changing of porosity volume fraction and power exponent are presented in Figs. 3-4 at constant slenderness ratio (L/h=20). Four type of temperature changing are considered as 0, 20, 40 and 80. It is observed from the results of Figs. 3 and 4 that the dimensionless natural frequencies of porous FG beam decrease with the increase of power indexes. When the power exponent is in the range of 0 to 2, reducing is higher than where power exponent is in range between 2 to 10. Also the effect of temperature changing is obvious, the dimensionless natural frequencies will be decreased by increasing of temperature changing for all gradient indexes, thus both temperature rises have a significant effect on the fundamental



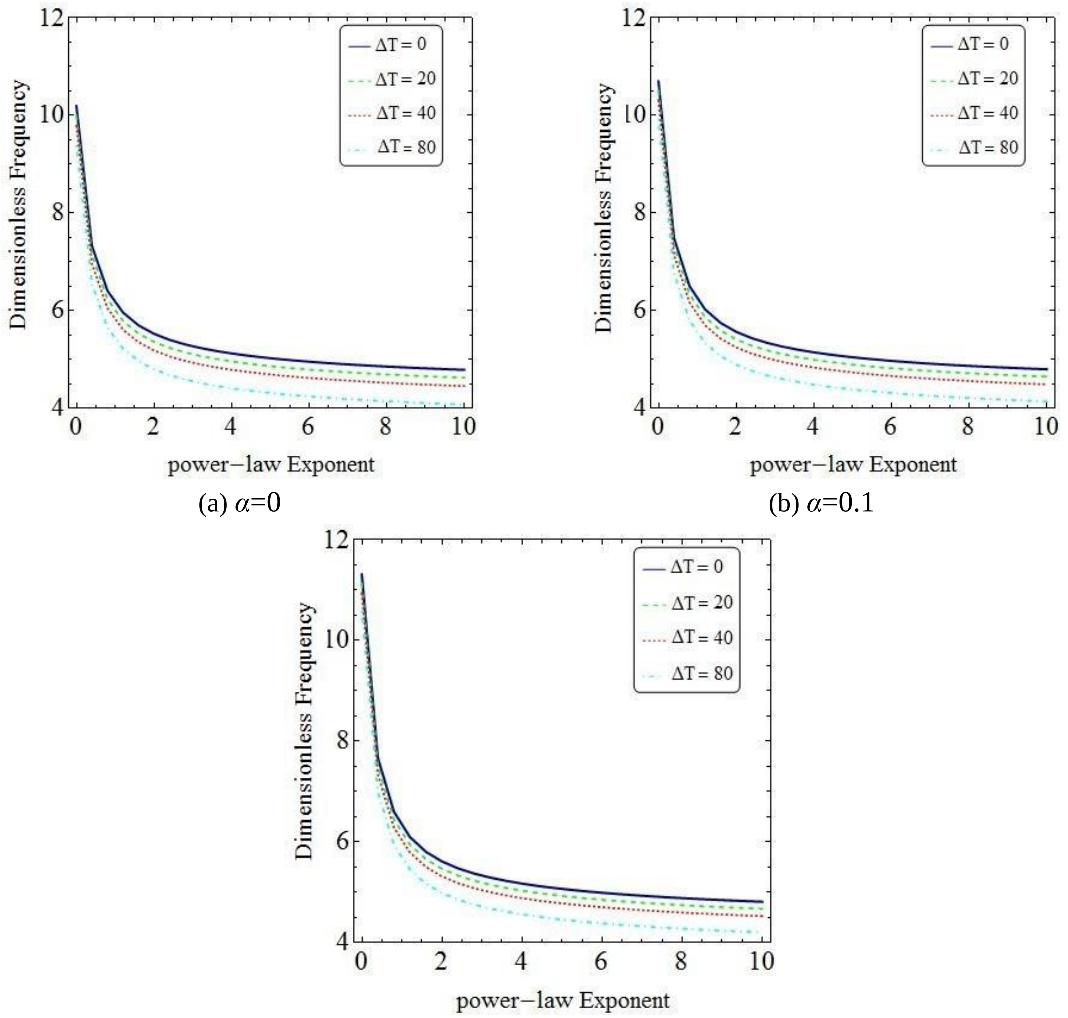
(a) ΔT=0 (b) ΔT=20

(c) ΔT=40 (d) ΔT=80

Fig. 6 The variation of the first dimensionless frequency of C-S FGM (*I*) beam with material graduation and porosity for different uniform temperature rises (*L/h*=20)

frequency of the porous FG beam. Also it is concluded that even distribution porosity effect depends on power indexes, for low values of n increasing of porosity leads to increasing of fundamental frequencies and from a certain value of the n frequencies will be decreased by increasing of porosity. The certain value of the n has gone up by increment of temperature different in accordance with Fig. 3.

The natural frequency parameter as a function of power law indexes for uniform temperature rise and porosity parameters is presented in Fig. 5 for FGM (*I*) with S-S boundary conditions. Different porosity parameter has been considered as (α=0, α=0.1, α=0.2). It is easily deduced that an increase in temperature change gives rise to decrease in the first dimensionless natural frequency for all gradient indexes. Also it is revealed that increasing of porosity parameter yields



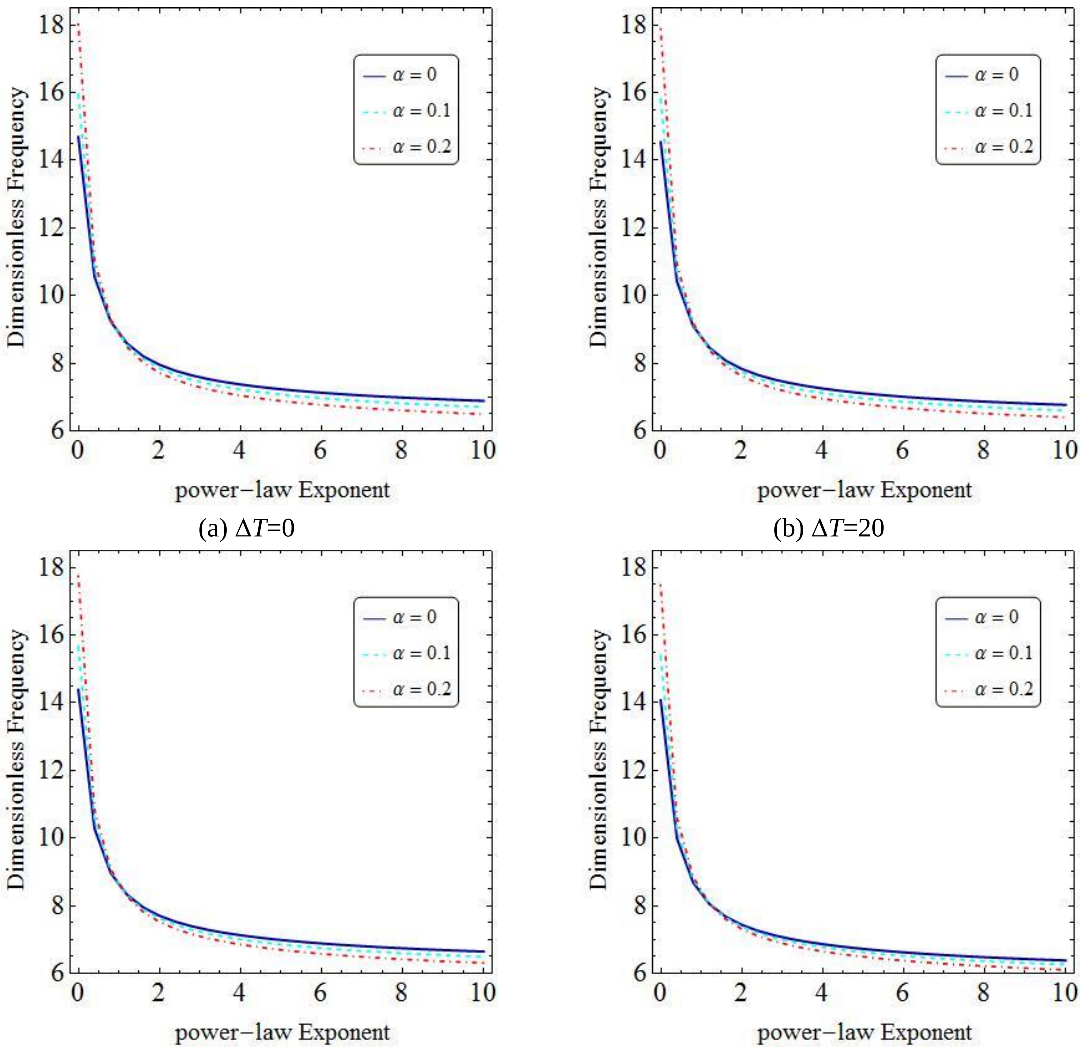
(a) α=0 (b) α=0.1 (c) α=0.2

Fig. 7 The variation of the first dimensionless frequency of C-S FGM (*II*) beam with material graduation and uniform temperature rises for different porosities (*L/h*=20)

the growth in dimensionless frequencies, thus it is necessary to consider porosity effect.

Also the variation of the first dimensionless frequency of C-S FG (*I*) asubjected to uniform temperature rises is depicted in Fig. 6 with different porosity parameter and power exponents. Four temperature changing has been considered as 0, 20, 40 and 80. It is seen that increasing of power indexes and growing of temperature changing are couse of decreasing of frequencies. As well as by perceiving Fig. 8 similar results has been obtained for C-C porous FG beam.

Fig. 7 display the variations of the first dimensionless natural frequency of the C-S FG beam with uneven porosity distributions respect to UTR for different values of gradient indexes and porosity parameters. Three porosity values has been considered as 0, 0.1 and 0.2. A comparison between these figure show that reducing of the fundamental frequencies is due to increasing of the temperature changes, and it is more palpable for high power indexes. Similarly, fundamental



(a) ΔT=0 (b) ΔT=20

(c) ΔT=40 (d) ΔT=80

Fig. 8 The variation of the first dimensionless frequency of C-C FGM (*I*) beam with material graduation and porosity for different uniform temperature rises (*L/h*=20)

frequency of C-C FG beam with uneven porosity distributions subjected to UTR is presented in Fig. 9. It is found that increasing of porosity parameter yields to increasing of nondimensional frequency.

Comparison of the first dimensionless natural frequencies of S-S FG beam subjected to both cases of thermal loading (UTR and LTR) with changing of porosity volume fraction and power exponent at constant slenderness ratio *L/h*=20 are presented in Fig. 10. It is concluded that frequency of the beam subjected to uniform is less than linear temperature rises, And the difference will be hight by increasing of the temperature changes. By comparing the three Figs with different porosity parameter we can found that increasing of porosity parameter yields to increasing of fundamental frequency.

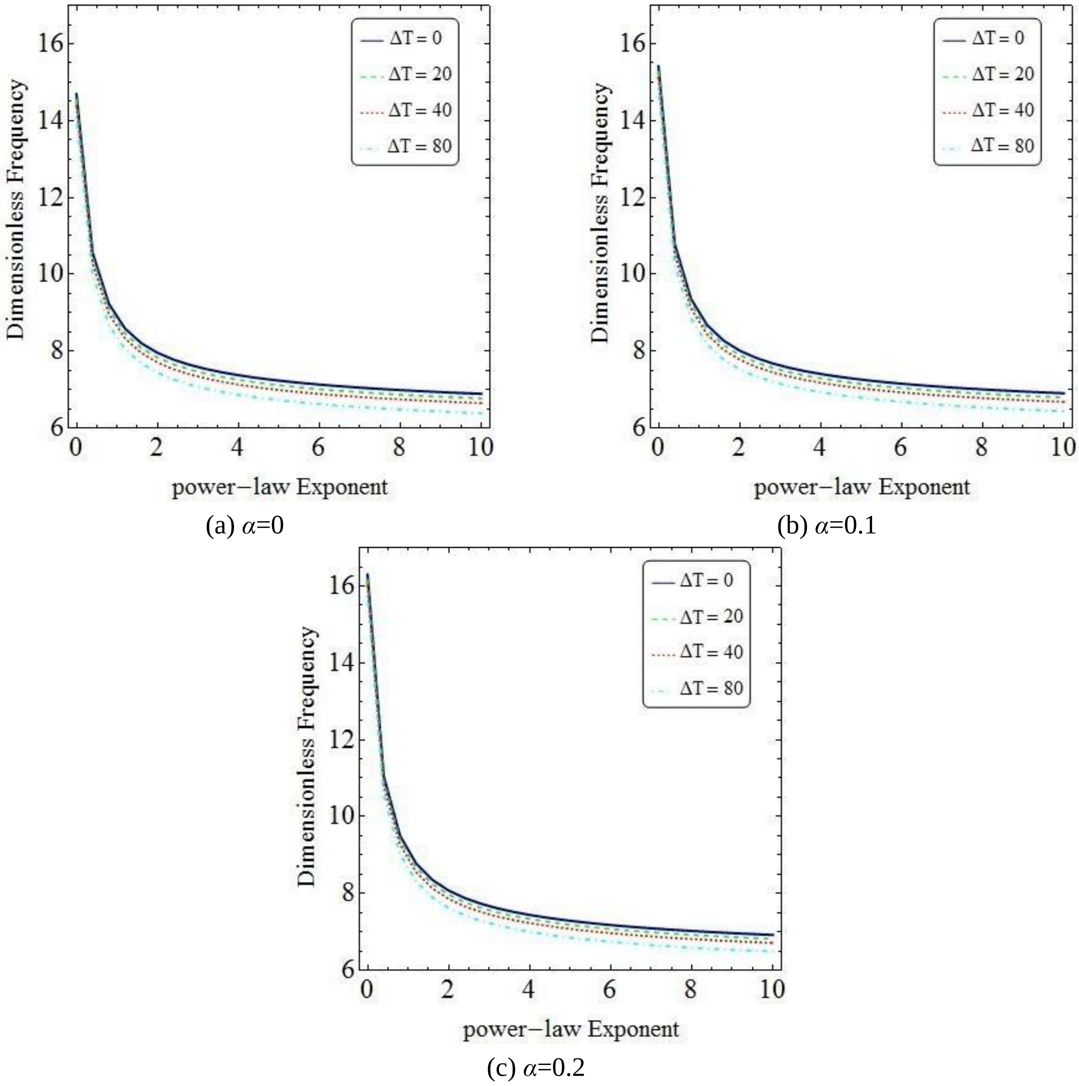
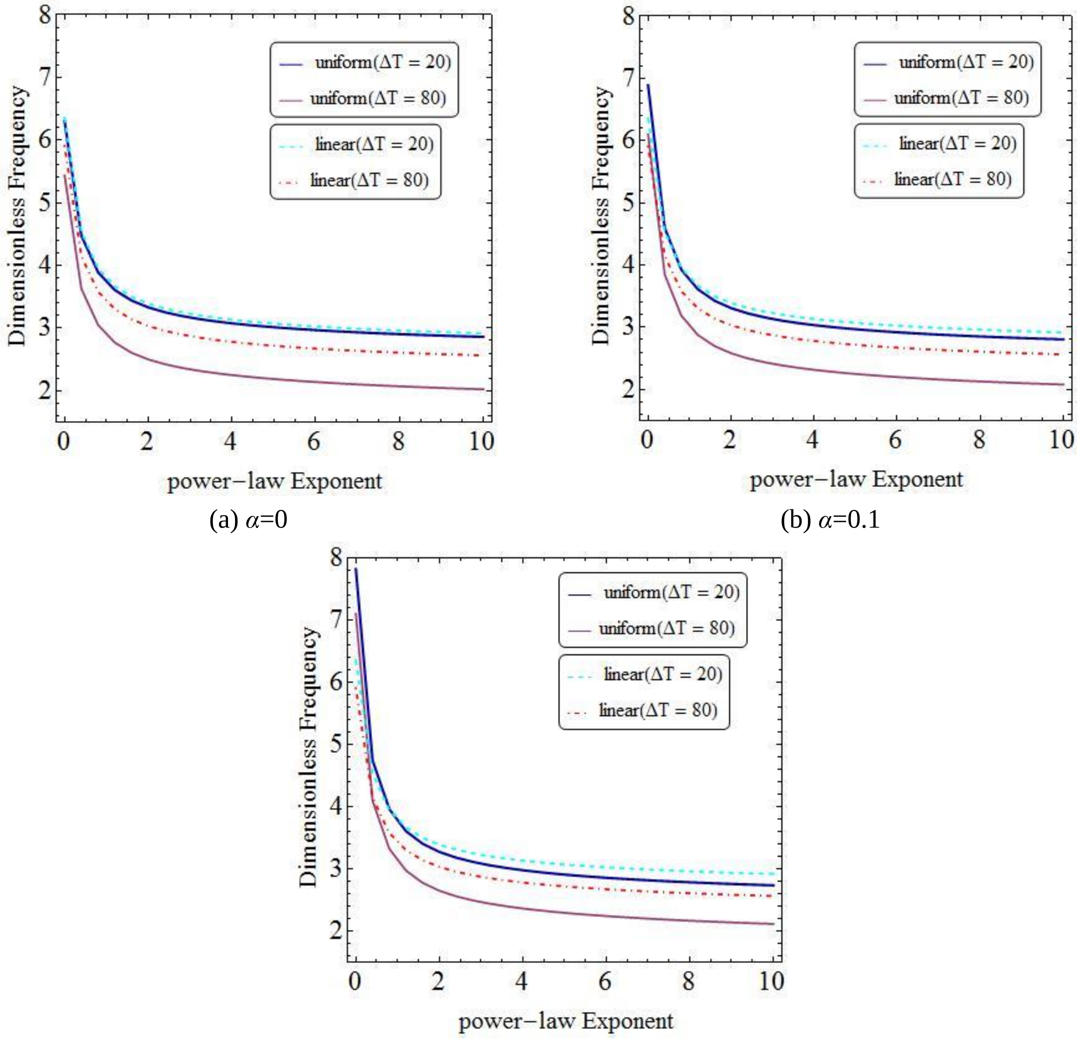


Fig. 9 Variations of the first dimensionless natural frequency of the C-C FGM (*II*) beam with respect to uniform temperature change for different values of gradient indexes and porosities (*L*/*h*=20)

6. Conclusions

In this research thermo-mechanical vibrational characteristic of the temperature-dependent FGM beams with porosities volume fraction based on Timoshenko beam theory is presented. Material's properties of the FG beams are assumed to be dependent to temperature and thickness based on modified rule of mixture. Hamilton's principle is used to derive governing differential equations and boundry conditions. The Navier-based analytical model and a semi analytical differential transformation method are used to solve governing partial differential equations.



(a) α=0

(b) α=0.1

(c) α=0.2

Fig. 10 Comparisonof the first dimensionless frequency of S-S FG (*I*) beam with material graduation and uniform and linear temperature rises for different porosities (*L/h*=20)

According to the numerical results, it is revealed that the proposed modeling and semi analytical approach can provide accurate frequency results of the FG beams as compared to analytical results and also some cases in the literature.

Finally, the effect of different parameters are investigated, the effect of volume fraction of porosity, two cases of thermal loadings (UTR and LTR), material property gradient index and boundry conditions on fundamental frequencies of porous FGM are investigated.

- It is concluded that increasing in temperature is cause of decreasing of fundamental frequency
- Also it is revealed that for even distributions of porosity increment of the volume fraction of porosity yields the increase in fundamental frequencies for low values of power indexes and from a certain value of the n, yields the decrease in frequencies. And certain value of the

ndepends on boundary conditions and temperature effects.

• For uneven distributions of porosity fundamental, natural frequencies will be increased by increment of the volume fraction of porosity for all gradient indexes. Which emphasizes on the importance of inspected porosity volume fraction effect.

• Dimensionless frequencies decrease by increasing in the gradient index value.

• It is concluded that the dimensionles natural frequencies with the uniform temperature rise is less than linear temperature one.

• Moreover it is revealed that under temperature rise, the greatest frequency at pre-buckling region is obtained for the porous FG beam with C-C boundry condition followed with S-S and C-S conditions, respectively.

• It is concluded that various factor such as porosity parameter, temperature rises, boundary conditions play important roles in dynamic behavior of FG beams with porosities. Therefore, the porosity and thermal effects should be considered in the analysis of vibration bahaviour of structures.

• Porosity effect is more tangible for higher modes of porous FG beam.

References

Akgöz, B. and Civalek, Ö. (2014), "Shear deformation beam models for functionally graded microbeams with new shear correction factors", *Compos. Struct*, **112**, 214-225.

Akgöz, B. and Civalek, Ö. (2014), "Thermo-mechanical buckling behavior of functionally graded microbeams embedded in elastic medium", *Int. J. Mech. Sci*., **85**, 90-104.

Atmane, H.A., Tounsi, A. and Bernard, F. (2015), "Effect of thickness stretching and porosity on mechanical response of a functionally graded beams resting on elastic foundations", *Int. J. Mech. Mater. Des*., 1-14.

Aydogdu, M. and Taskin, V. (2007), "Free vibration analysis of functionally graded beams with simply supported edges", *Mater. Des*., **28**(5), 1651-1656.

Civalek, Ö. and Kiracioglu, O. (2010), "Free vibration analysis of Timoshenko beams by DSC method", *Int. J. Numer. Meth. Biomed. Eng*., **26**(12), 1890-1898.

Ebrahimi, F. and Mokhtari, M. (2014), "Transverse vibration analysis of rotating porous beam with functionally graded microstructure using the differential transform method", *J. Braz. Soc. Mech. Sci. Eng*., 1-10.

Ebrahimi, F. and Salari, E. (2015), "Thermal buckling and free vibration analysis of size dependent Timoshenko FG nanobeams in thermal environments", *Compos. Struct*., **128**, 363-380.

Ebrahimi, F., Ghasemi, F. and Salari, E. (2015), "Investigating thermal effects on vibration behavior of temperature-dependent compositionally graded Euler beams with porosities", *Meccanica*, **51**(1), 223-249..

Ebrahimi, F., Naei, M.H. and Rastgoo, A. (2009), "Geometrically nonlinear vibration analysis of piezoelectrically actuated FGM plate with an initial large deformation", *J. Mech. Sci. Technol*, **23**(8), 2107-2124.

Ebrahimi, F., Rastgoo, A. and Atai, A. (2009), "A theoretical analysis of smart moderately thick shear deformable annular functionally graded plate", *Eur. J. Mech-A/Solid*., **28**(5), 962-973.

Hassan, I.A.H. (2002), "On solving some eigenvalue problems by using a differential transformation", *Appl. Math. Comput*., **127**(1), 1-22.

Jha, D., Kant, T. and Singh, R. (2013), "A critical review of recent research on functionally graded plates", *Compos. Struct*., **96**, 833-849.

Ju, S.P. (2004), "Application of differential transformation to transient advective-dispersive transport equation", *Appl. Math. Comput*., **155**(1), 25-38.

Kiani, Y. and Eslami, M. (2013), "An exact solution for thermal buckling of annular FGM plates on an

elastic medium", *Compos. Part B: Eng*., **45**(1), 101-110.
Komijani, M., Esfahani, S.E., Reddy, J.N., Liu, Y.P. and Eslami, M.R. (2014), "Nonlinear thermal stability and vibration of pre/post-buckled temperature-and microstructure-dependent functionally graded beams resting on elastic foundation", *Compos. Struct.*, **112**, 292-307.
Magnucka-Blandzi, E. (2008), "Axi-symmetrical deflection and buckling of circular porous-cellular plate", *Thin Wall. Struct*., **46**(3), 333-337.
Magnucka-Blandzi, E. (2009), "Dynamic stability of a metal foam circular plate", *J. Theor. Appl. Mech*., **47**, 421-433.
Magnucka-Blandzi, E. (2010), "Non-linear analysis of dynamic stability of metal foam circular plate", *J. Theor. Appl. Mech*., **48**(1), 207-217.
Mechab, I., Mechab, B., Benaissa, S., Serier, B. and Bouiadjra, B.B. (2016), "Free vibration analysis of FGM nanoplate with porosities resting on Winkler Pasternak elastic foundations based on two-variable refined plate theories", *J. Braz. Soc. Mech. Sci. Eng*., 1-19.
Pradhan, K. and Chakraverty, S. (2013), "Free vibration of Euler and Timoshenko functionally graded beams by Rayleigh-Ritz method", *Compos. Part B: Eng*., **51**, 175-184.
Şimşek, M. (2010), "Fundamental frequency analysis of functionally graded beams by using different higher-order beam theories", *Nucl. Eng. Des*., **240**(4), 697-705.
Şimşek, M. (2010), "Non-linear vibration analysis of a functionally graded Timoshenko beam under action of a moving harmonic load", *Compos. Struct*., **92**(10), 2532-2546.
Şimşek, M. and T. Kocatürk, (2009), "Free and forced vibration of a functionally graded beam subjected to a concentrated moving harmonic load", *Compos. Struct*., **90**(4), 465-473.
Sina, S., Navazi, H. and Haddadpour, H. (2009), "An analytical method for free vibration analysis of functionally graded beams", *Mater. Des*., **30**(3), 741-747.
Tauchert, T.R. (1974), *Energy Principles in Structural Mechanics*, McGraw-Hill Co.
Thai, H.T. and Vo, T.P. (2012), "Bending and free vibration of functionally graded beams using various higher-order shear deformation beam theories", *Int. J. Mech. Sci*., **62**(1), 57-66.
Touloukian, Y.S. (1966), "Thermophysical properties of high temperature solid materials", **4**, Oxides and Their Solutions and Mixtures, Part I. Simple Oxyg. Compd. Mix., DTIC Document.
Wattanasakulpong, N. and Chaikittiratana, A. (2015), "Flexural vibration of imperfect functionally graded beams based on Timoshenko beam theory: Chebyshev collocation method", *Meccanica*, **50**(5), 1-12.
Wattanasakulpong, N. and Ungbhakorn, V. (2014), "Linear and nonlinear vibration analysis of elastically restrained ends FGM beams with porosities", *Aerosp. Sci. Technol*., **32**(1), 111-120.
Wattanasakulpong, N., Prusty, B.G., Kelly, D.W. and Hoffman, M. (2012), "Free vibration analysis of layered functionally graded beams with experimental validation", *Mater. Des*., **36**, 182-190.
Wei, D., Liu, Y. and Xiang, Z. (2012), "An analytical method for free vibration analysis of functionally graded beams with edge cracks", *J. Sound Vib*., **331**(7), 1686-1700.
Xiang, H. and Yang, J. (2008), "Free and forced vibration of a laminated FGM Timoshenko beam of variable thickness under heat conduction", *Compos. Part B: Eng*., **39**(2), 292-303.
Yahia, S.A., Atmane, H.A., Houari, M.S.A. and Tounsi, A. (2015), "Wave propagation in functionally graded plates with porosities using various higher-order shear deformation plate theories", *Struct. Eng. Mech*., **53**(6), 1143.

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