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Topology optimization of bracing systems using a truss-like material model

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Abstract. To minimize the compliance of frame, a method to optimize the topology of bracing system in a frame is presented. The frame is first filled uniformly with a truss-like continuum, in which there are an infinite number of members. The frame and truss-like continuum are analysed by the finite element method altogether. By optimizing the distribution of members in the truss-like continuum over the whole design domain, the optimal bracing pattern is determined. As a result, the frame's lateral stiffness is enforced. Structural compliance and displacement are decreased greatly with a smaller increase in material volume. Since optimal bracing systems are described by the distribution field of members, rather than by elements, fewer elements are needed to establish the detailed structure. Furthermore, no numerical instability exists. Therefore it has high calculation effectiveness.

Keywords: topology optimization; compliance; bracing systems; truss-like continuum; lateral stiffness

1. Introduction

Bracing systems in steel frames (Majid *et al*. 2012, Tabeshpour *et al*. 2012) and reinforced concrete frames (Faella *et al*. 2014, Massumi and Absalan 2013) can significantly increase structural stiffness to resist lateral loads, such as wind and earthquake. To increase the structural lateral stiffness, in practical engineering, different kinds of bracing systems are used (Hsiao *et al*. 2012, Lui and Zhang 2013, Kutuk *et al*. 2014, Nouri *et al*. 2015, Yu *et al*. 2015). Furthermore, "X" bracing systems are widely used, although they are not the best type (Zhou 2003). In fact there are infinite number of potential positions in which various kinds of braces can be arranged. Mijar and Swan (1998), Qing *et al* (2000) adopted topology optimization methods to design the optimal bracing system. In the topology optimization procedure, the frame is filled by a uniform isotropic continuum generally, which is divided into finite elements. The structural topology is optimized by deleting parts of elements (Bendsoe and Kikuchi 1988, Xie and Steven 1993). The structural topology is expressed by the "existence" and "nonexistence" of elements. The frame members are indicated by a series of connected elements. To obtain the detailed frame structure, more elements are required in continuum structural topology optimization methods. Furthermore, a group of numerical instability problems, such as checkerboard patterns and one-node connected hinges,

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needs to be overcome (Sigmund and Petersson 1998), for which some additional techniques are required.

In fact, a topology optimization structure is generally a truss-like continuum. In this paper, the truss-like material model is used. Based on the optimal truss-like material, the bracing systems of frame are suggested. To improve the calculation efficiency, a fully stressed optimality criteria method is applied to solve the problem.

In a truss-like continuum, there are finite numbers of members with infinitesimal spaces along different directions at any point. To establish the truss-like continuum, a truss-like material model was constructed (Zhou and Li 2006, 2008). Based on the truss-like material model, a topology-optimization method to minimize the natural frequencies of the braced frame was presented (Zhou and Chen 2014). In the topology-optimization procedure, the space between members of the frame is the design domain, which is completely filled by the truss-like continuum. The structure with the truss-like continuum is analysed by the finite element method. The densities and orientations of members at nodes in the truss-like continuum are taken as design variables. The densities and orientations of members in an element are interpolated by these values at nodes belonging to this element. Therefore the densities and orientations of members vary continuously in the design domain. Because no intermediate densities are suppressed in optimization iteration, there is no numerical instability.

2. Truss-like material model

In the truss-like continuum, members are laid in a weak matrix at any point. By omitting the stiffness of the weak matrix, the linear elastic relation between stress σ and strain ε of the member can be assumed to be

$$\sigma = Et\varepsilon \tag{1}$$

where E is Young's modulus and t is the density of the member. The material properties in tension and compression are assumed to be identical. If two families of members are arranged along the orthotropic orientations (which are defined as the principal axes of the material) with densities of t_1 and t_2 at any point, the stress–strain relation is expressed as

$$\boldsymbol{\sigma} = \boldsymbol{D}(t_1, t_2, 0)\boldsymbol{\varepsilon} \tag{2}$$

where

$$\boldsymbol{D}(t_1, t_2, 0) = E \cdot \mathrm{diag}[t_1 \quad t_2 \quad (t_1 + t_2)/4] \tag{3}$$

is the elastic matrix along the principal axes of the material at any point. $\boldsymbol{\sigma}=[\sigma_1\ \sigma_2\ \tau_{12}]^T$ and $\boldsymbol{\varepsilon}=[\varepsilon_1\ \varepsilon_2\ \gamma_{12}]^T$ are the stress and strain along the principal axes of the material, respectively. With the assumption of shear stiffness, the third term of the matrix in Eq. (3), Eq. (3) can describe isotropic material in the case of $t_1=t_2$. If the principal axes of the material lie along the direction with angles of α from the global coordinate x-axis, the elastic matrix in the global coordinates system can be calculated with the aid of the coordinates transformation matrix $\boldsymbol{T}(\alpha)$

$$\boldsymbol{D}(t_1, t_2, \alpha) = \boldsymbol{T}^{\mathrm{T}}(\alpha)\boldsymbol{D}(t_1, t_2, 0)\boldsymbol{T}(\alpha) = E\sum_{b=1}^{2} t_b \sum_{r=1}^{3} s_{br} g_r(\alpha)\boldsymbol{A}_r \tag{4}$$

where s_{br} and $g_r(\alpha)$ are the components of the matrices

$$\boldsymbol{s}=\begin{bmatrix}1 & 1 & 1\\ -1 & -1 & 1\end{bmatrix},\quad \boldsymbol{g}(\alpha)=\begin{bmatrix}\cos 2\alpha & \sin 2\alpha & 1\end{bmatrix} \tag{5}$$

$\boldsymbol{A}_r$ are the constant matrices

$$\boldsymbol{A}_1=\frac{1}{2}\mathrm{diag}[1 \quad -1 \quad 0],\quad \boldsymbol{A}_2=\frac{1}{4}\begin{bmatrix}0 & 0 & 1\\ 0 & 0 & 1\\ 1 & 1 & 0\end{bmatrix},\quad \boldsymbol{A}_3=\frac{1}{2}\mathrm{diag}[1 \quad 1 \quad \frac{1}{2}] \tag{6}$$

$\boldsymbol{T}(\alpha)$ is the coordinates transformation matrix

$$\boldsymbol{T}(\alpha)=\begin{bmatrix}\cos^2\alpha & \sin^2\alpha & 0.5\sin 2\alpha\\ \sin^2\alpha & \cos^2\alpha & -0.5\sin 2\alpha\\ -\sin 2\alpha & \sin 2\alpha & \cos 2\alpha\end{bmatrix} \tag{7}$$

If the densities t_{bj} and orientations α_j of member b at node j are taken as design variables, the elastic matrix at node j is expressed as

$$\boldsymbol{D}(t_{1j},t_{2j},\alpha_j)=E\sum_{b=1}^{2}t_{bj}\sum_{r=1}^{3}s_{br}g_r(\alpha_j)\boldsymbol{A}_r,\quad j=1,2,...,J \tag{8}$$

where J is the total number of nodes. Since the truss-like material is not distributed uniformly, the elastic matrix at any point in an element e would be calculated by the interpolation of the elastic matrices at the nodes belonging to the element

$$\boldsymbol{D}_e(\xi,\eta)=\sum_{j\in S_e}N_j(\xi,\eta)\boldsymbol{D}(t_{1j},t_{2j},\alpha_j) \tag{9}$$

where $N_j(\xi, \eta)$ is the shape function. ξ and η are local coordinates in an element. S_e is the set of nodes belonging to the element e. Introducing Eq. (8) into Eq. (9) leads to the elastic matrix in the element e

$$\boldsymbol{D}_e=E\sum_{j\in S_e}N_j\sum_{b=1}^{2}t_{bj}\sum_{r=1}^{3}s_{br}g_r(\alpha_j)\boldsymbol{A}_r \tag{10}$$

Introducing Eq. (10) into the definition of the elementary stiffness matrix leads to

$$\boldsymbol{k}_e=E\sum_{j\in S_e}\sum_{b=1}^{2}t_{bj}\sum_{r=1}^{3}s_{br}g_r(\alpha_j)\int_{V_e}N_j\boldsymbol{B}^{\mathrm{T}}\boldsymbol{A}_r\boldsymbol{B}\mathrm{d}V=\sum_{j\in S_e}\sum_{b=1}^{2}t_{bj}\sum_{r=1}^{3}s_{br}g_r(\alpha_j)\boldsymbol{H}_{ejr} \tag{11}$$

where

$$\boldsymbol{H}_{ejr}=E\int_{V_e}N_j\boldsymbol{B}^{\mathrm{T}}\boldsymbol{A}_r\boldsymbol{B}\mathrm{d}V \tag{12}$$

is a constant matrix that is independent of the elements if regular rectangular elements are used.

The volume of the truss-like continuum is obtained by the integration of densities of members over all elements

$$V = \sum_{e} \sum_{b=1}^{2} \int_{V_e} \sum_{j \in S_e} N_j t_{bj} \mathrm{d}V \tag{13}$$

If regular rectangular elements are used, the volume can be calculated by

$$V = \frac{1}{4} V_e \sum_{j=1}^{J} z_j \sum_{b=1}^{2} t_{bj} \tag{14}$$

where z_j is the number of elements around the node j. V_e is the area of the element.

3. Optimization method

The structure is analysed by the finite element method

$$\boldsymbol{KU} = \boldsymbol{F} \tag{15}$$

where $\boldsymbol{K}$, $\boldsymbol{U}$, and $\boldsymbol{F}$ are the stiffness matrix, nodal displacement vector, and nodal force vector, respectively. The objective function of the optimization problem is the compliance (external force work)

$$c = \boldsymbol{F}^{\mathrm{T}} \boldsymbol{U} \tag{16}$$

The optimization problem is stated as

$$\begin{array}{ll} \text{find} & t_{bj}, \alpha_j, \quad j = 1, 2, \ldots, J; \; b = 1, 2 \\ \min & c = \boldsymbol{F}^{\mathrm{T}} \boldsymbol{U} \\ \text{s.t.} & V \le \overline{V} \end{array} . \tag{17}$$

Introducing Eq. (14) into Eq. (15) leads to

$$c = \boldsymbol{U}^{\mathrm{T}} \boldsymbol{K} \boldsymbol{U} = \sum_{e} \boldsymbol{U}_e^{\mathrm{T}} \boldsymbol{k}_e \boldsymbol{U}_e \tag{18}$$

where $\boldsymbol{U}_e$ is the nodal displacement vector of one element. Further introducing Eq. (11) into Eq. (18) leads to

$$c = \sum_{e} \sum_{j \in S_e} \sum_{b=1}^{2} t_{bj} \sum_{r=1}^{3} s_{br} g_r(\alpha_j) \boldsymbol{U}_e^{\mathrm{T}} \boldsymbol{H}_{ejr} \boldsymbol{U}_e \tag{19}$$

The frame is filled with a truss-like continuum material that is meshed by plane rectangular elements. The frame is divided into beam elements accordingly. These nodes, which are sheared by beam elements and continuum elements, share identical displacement along two coordinate axes. The members of the frame are prescribed. The members in the truss-like continuum are to be optimized. The truss-like continuum is distributed non-uniformly and continuously over the whole design domain. The densities and orientations of members in the truss-like continuum at nodes are taken as the design variables. The distributions of members in elements are interpolated by the values at nodes.

Cox (1965) proved that structures with minimum volume under stress constraint and structures with minimum compliance under volume constraint are identical structures. For this reason, the fully stressed criterion was applied to minimize the structural compliance. To get a symmetrical structure, loads act on the structure symmetrically. Although there are infinite numbers of members in the truss-like continuum, only a finite number of important members (linking loads and supporters) remain in the final optimal structure to avoid the difficulties of fabricating a truss-like continuum. Members in the truss-like continuum are generally curves. When finite numbers of members remain, curve members have to be replaced by straight members. To make the structure easier to build, nodes that are close to each other are combined in the final optimal structure. The optimization procedure is as follows.

(1) The frame is filled by the truss-like continuum. The frame and truss-like continuum are divided into beam elements and four-node plane rectangular elements, respectively. The densities and orientations of members in the truss-like continuum at nodes are initialized as

$$t_{bj}^{0}=0.025\,,\quad \alpha_{j}^{0}=0\,,\quad b=1,2;\; j=1,2,\ldots,J$$

where the superscript is the iteration index.

(2) The structure is analysed by the finite element method. The stresses σ_{bj}^{i} of the truss-like continuum along member b at node j are calculated. The densities of members in the truss-like continuum at node j are updated by the fully stressed criterion

$$t_{bj}^{i+1}=\max(t_{c}^{i},\, t_{bj}^{i}\sigma_{bj}^{i}/\sigma_{p})$$

where σ_p is the permitted stress. t_c^i is a small positive value

$$t_{c}^{i}=\max_{b,j}(t_{bj}^{i})\times 10^{-7}$$

to avoid the singularity of the stiffness matrix. The orientations of the members in the truss-like continuum are aligned with the orientations of principal stress at all nodes.

(3) The procedure returns to step (2) if the relative change of design variables is greater than 1%.

(4) The truss-like continuum is replaced with finite numbers of straight members, and nodes that are close to each other are merged.

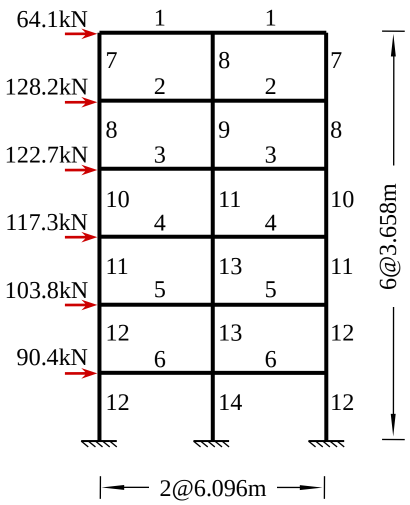
4. Numerical examples

The first example is a two-bay six-storey plane frame (Qing *et al*. 2000) as shown in Fig. 1(a). Members are divided into 14 groups (marked in Fig. 1(a)) with wide flange sections, listed in Table 1. Young's modulus is E=200 GPa.

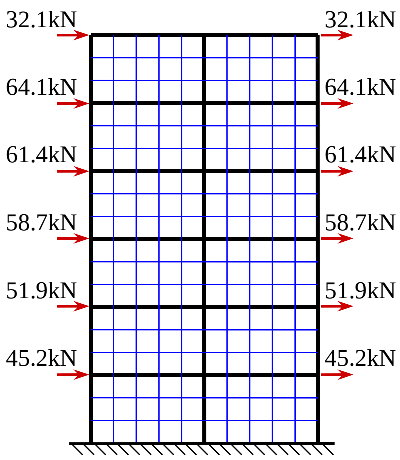
The truss-like continuum filled in the frame is meshed by 180 plane rectangular elements; the frame is divided into 114 beam elements as shown in Fig. 1(b). To obtain a symmetric structure, loads are applied symmetrically. The truss-like continuum is optimized after 25 iterations. The iteration history of compliance of the frame braced with the truss-like continuum is shown in Fig. 2, which displays the stable convergence of the objective function. The densities and orientations of the distributed members in the optimal truss-like continuum are illustrated in Fig. 3(a). The lengths and orientations of lines represent the densities and orientations of the members in the truss-like continuum. To make the figure legible, some lines that are too long are cut shorter. According to the densities and orientations of members in Fig. 3(a), the bracing system of the frame is established in Fig. 3(b). Furthermore, curve members are replaced with straight members

Table 1 Sectional shape of example 1

Group Number	Sectional model	Sectional area (cm^2)	Sectional inertical moment (cm^4)
1	W8×21	39.74	3134
2	W8×28	53.23	4079
3	W10×26	49.10	5994
4	W12×26	49.35	8491
5	W14×26	49.61	10198
6	W10×19	36.26	4008
7	W10×17	32.20	3409
8	W8×10	19.10	1282
9	W12×19	35.94	5411
10	W12×14	26.84	3688
11	W14×22	41.97	8283
12	W16×26	49.68	12529
13	W16×31	58.71	15609
14	W24×62	117.4	64516



(a) Structure and the number of member groups



(b) Truss-like continuum elements filled in frame

Fig. 1 Mechanics model of example 1

and nodes that are close to one another are merged, which leads to the structure shown in Fig. 3(c).

The volume of the original frame is V_0=0.62 m^3. The volume of truss-like material filled in the frame is controlled so that 0.2V_0=0.124 m^3. With exactly the same amount of bracing material, the compliances of frames with different bracing patterns are compared in Table 2. It can be found from Table 2 that the compliance of the frame with a truss-like continuous bracing system, as

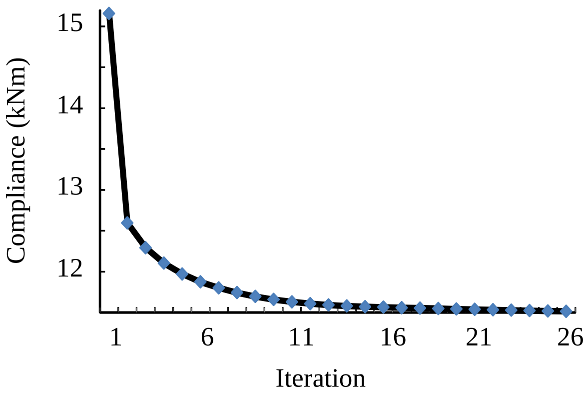
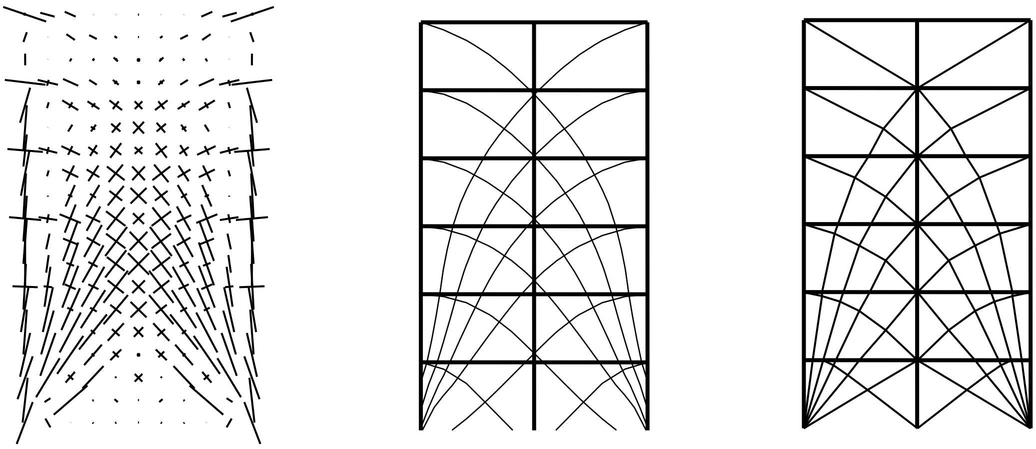
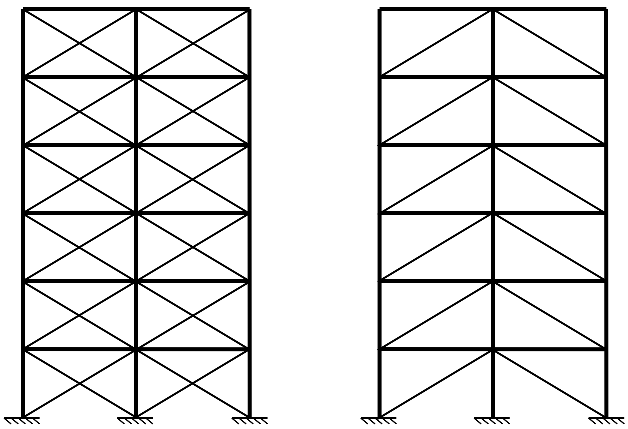


Fig. 2 History of iteration of example 1



(a) Optimal truss-like continuum (b) Optimal bracing systems (c) Adjusted bracing systems

Fig. 3 Optimal structure of example 1



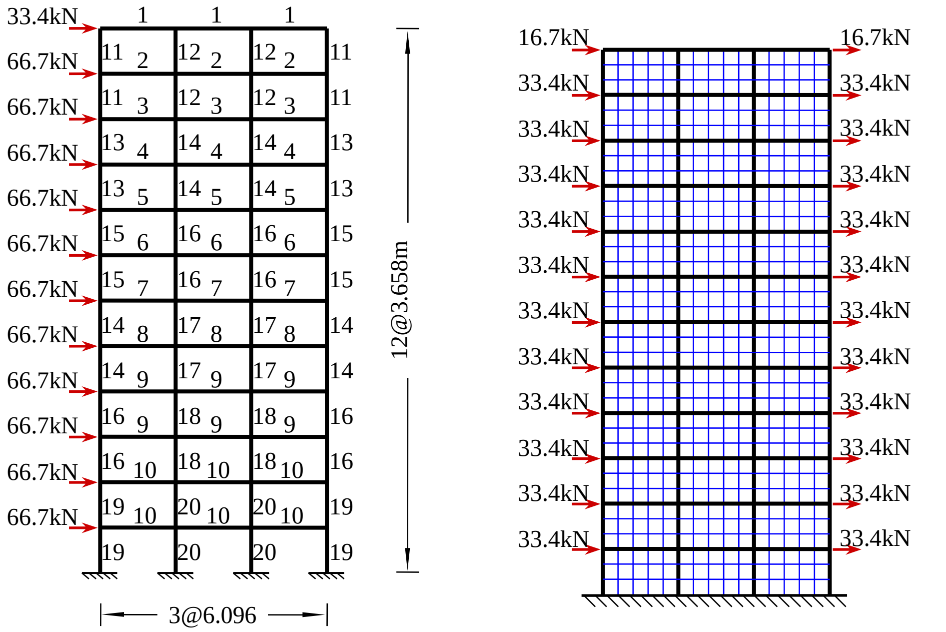
(a) X-bracing systems (b) A-bracing systems

Fig. 4 Frame with bracing systems of example 1

Table 2 Calculation results

Example	Structures	Figure	Compliance (kNm)	Volume of bracing structure (m^3)
Example 1 2-bay 6-story frame	Original frame	Fig. 1(a)	22.46	(0.620)*
	Truss-like continuous brace	Fig. 3(a)	11.51	0.124
	Truss-like discrete brace	Fig. 3(c)	16.67	0.124
	X-brace	Fig. 4(a)	18.82	0.124
	A-brace	Fig. 4(b)	20.62	0.124
Example 2 3-bay 12-story frame	Original frame	Fig. 5(a)	253.3	(2.990)*
	Truss-like continuous brace	Fig. 7(a)	4.190	0.598
	Truss-like discrete brace	Fig. 7(c)	17.90	0.598
	X-brace	Fig. 8	23.39	0.598

* The volume of original frame without brace.



(a) Structure and the number of member groups (b) Truss-like continuum elements filled in frame

Fig. 5 Mechanics model of example 2

shown in Fig. 2(a), decreased to 52% of that of the original frame shown in Fig. 1(a), while the volume of the material of the frame increased by only 20%.

In practical engineering, simpler bracing systems, as shown in Fig. 4, are most likely to be adopted. For comparison, these structures are analysed under loads and boundaries identical to those mentioned above. The calculation results are given in Table 2. It can be seen that the truss-

Table 3 Sectional shape of example 2

Group Number	Sectional model	Sectional area(cm^2)	Sectional inertical moment (cm^4)
1	150 UB 18.0	23	905
2	180 UB 18.1	23	1210
3	200 UB 29.8	38.2	2910
4	250 UB 37.3	47.5	5570
5	310 UB 40.4	52.1	8640
6	360 UB 50.7	64.7	1420
7	360 UB 56.7	72.4	1610
8	410 UB 53.7	68.9	1880
9	460 UB 67.1	85.8	2960
10	460 UB 74.6	95.2	3350
11	150 UC 23.4	29.8	1260
12	150 UC 37.2	47.3	2220
13	200 UC 46.2	59	4590
14	200 UC 59.5	76.2	6130
15	200 UC 52.2	66.6	5280
16	250 UC 72.9	93.2	11400
17	250 UC 89.5	114	14300
18	310 UC 96.8	124	22300
19	310 UC 118	150	27700
20	310 UC 137	175	32900

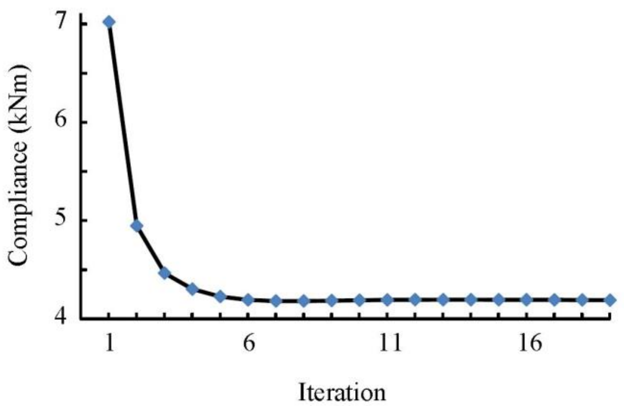
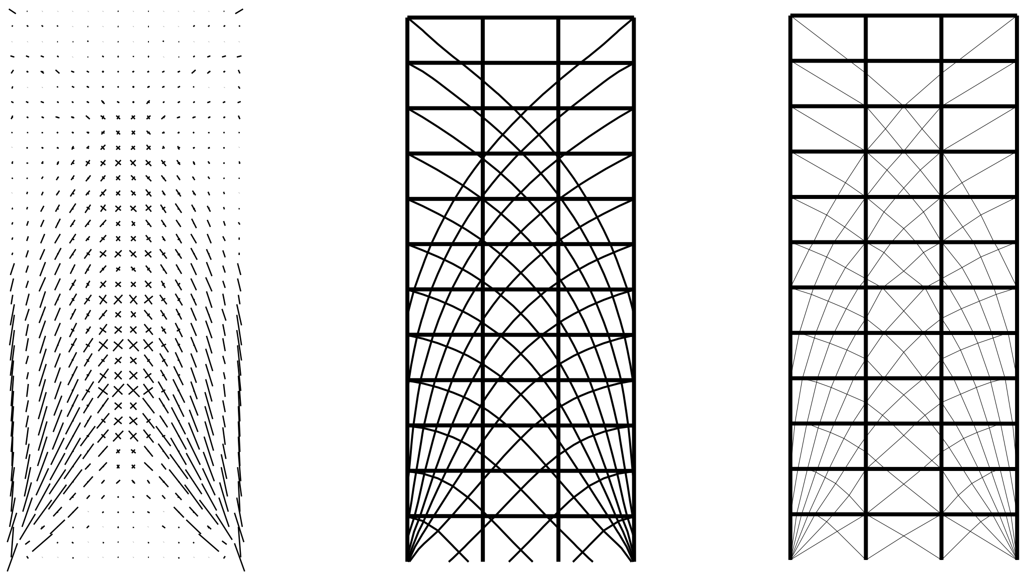


Fig. 6 History of iteration of example 2

like bracing systems have less compliance than the frames with the bracing systems shown in Fig. 4.

The second example is a three-bay twelve-story plane frame, as shown in Fig. 5(a). Members are divided into 20 groups with universal columns, and beam sections are shown in Table 3. Young’s modulus is E=200 GPa.



(a) Optimal truss-like continuum (b) Optimal bracing systems (c) Improved bracing systems

Fig. 7 Optimal bracing structure of example 2

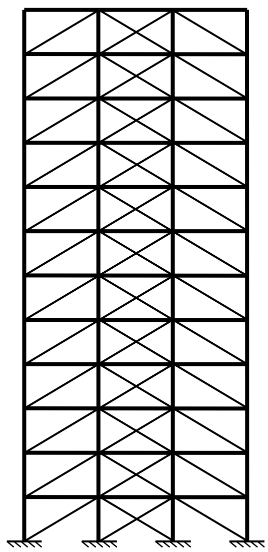


Fig. 8 Frame with bracing systems of example 2

The truss-like continuum is meshed by 540 plane rectangular elements; the frame is divided into 466 beam elements as shown in Fig. 5(b). The truss-like continuum is optimized after 19 iterations. The iteration history of compliance of the frame braced by truss-like material is shown in Fig. 6. The densities and orientations of members at nodes in the optimal truss-like continuum are shown in Fig. 7(a). From Fig. 7(a), the bracing system of the frame is established in Fig. 7(b). Replacing curve members with straight members and merging nodes that are close to one another leads to the structure shown in Fig. 7(c).

These structures are analysed and the results are shown in Table 2. The volume of the original frame is V_0=2.99 m^3. The volume of truss-like material filled in the frame is controlled so that 0.2V_0=0.598 m^3. With exactly the same amount of bracing material, the compliances of frames with different bracing patterns are compared in Table 2. It can be found from Table 2 that the compliance of the frame with the truss-like continuous bracing system, as shown in Fig. 7(a), decreased dramatically to 1.7% of that of the original frame, while the volume of the material of the frame increased by only 20%. The compliance of the structure derived from the truss-like continuum brace shown in Fig. 7(c) is 77% of that of the X-braced frame shown in Fig. 8.

5. Conclusions

By using the truss-like continuum material model to optimize the bracing systems of the frame, the compliance of frame is decreased greatly with a smaller increase in material volume. Since the optimal bracing systems of the frame are expressed by the distribution field of the truss-like continuum rather than by elements, fewer elements are needed and more detailed structures are obtained. The optimal discrete bracing systems are suggested by the distribution field of the truss-like continuum. The frames with the optimal bracing frame have less compliance than others. The effectiveness to increase structural lateral stiffness is remarkable, especially for high-rise buildings.

Here buckling is not taken into account, which need further research.

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