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Reliability evaluation of water distribution network considering mechanical characteristics using informational entropy

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Abstract. Many studies have been carried out to investigate the important factors in calculating the realistic entropy amount of water distribution networks, but none of them have considered both mechanical and hydraulic characteristics of the networks. Also, the entropy difference in various networks has not been calculated exactly. Therefore, this study suggested a modified entropy function to calculate the informational entropy of water distribution networks so that the order of demand nodes and entropy difference among various networks could be calculated by taking into account both mechanical and hydraulic characteristics of the network. This modification was performed through defining a coefficient in the entropy function as the amount of outflow at each node to all dissipated power in the network. Hence, a more realistic method for calculating entropy was presented by considering both mechanical and hydraulic characteristics of network while keeping simplicity. The efficiency of the suggested method was evaluated by calculating the entropy of some sample water networks using the modified function.

Keywords: network reliability; water distribution network; mechanical characteristics; informational entropy; energy dissipation

1. Introduction

A water distribution system is a network of source nodes, links, demand nodes, and other hydraulic components such as pumps, valves, and tanks. Water distribution systems are mainly meant to supply water at a sufficient pressure level and quantity to all their users and provide water for firefighting. Quantification of water distribution networks' reliability, as a lifeline system (Moghtaderizadeh and Kiureghian 1983, Selçuk and Yücemen 1999), as well as the water system maintenance (Quimpo and Shamsi 1991) has drawn much attention as important research topics in risk management in the past decades. Reliability of a water distribution network can be defined as the probability of a demand node in the system receiving sufficient supply with satisfactory pressure head (Tanyimboh *et al.* 1999). Several measures of reliability for water distribution networks have been proposed by various researchers, some of which include a surrogate measure

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of reliability based on informational entropy (Cullinane 1985, Wagner *et al.* 1988, Tanyimboh, Tabesh and Burrows 2001, Shinstine, Ahmed and Lansey 2002). However, one of the reasons that reliability assessment has not become a common phase in design practice is its complexity (Ostfeld 2001). Some researchers have proposed that it might be possible to use entropy as a general performance indicator for water distribution systems (Awumah *et al.* 1990). This method has several advantages over other performance and reliability indices, for example, it is extremely rapid and far easier to calculate than other measures, has minimal data requirements, could be directly incorporated into design optimization frameworks (Tanyimboh and Templeman 2000), and can be a measure of system redundancy (Awumah *et al.* 1991).

Redundancy, on the other hand, in a water distribution network implies the reserve capacity of the network and alternative supply paths of the demand nodes when links fail in performing the desired service (Awumah *et al.* 1991).Redundancy, which is closely related to reliability, is an aspect of the overall system performance that is often neglected. A redundant network is inherently very reliable. Considering the seismic performance of lifeline networks during the past earthquakes shows that a single redundancy index can remarkably increase the system reliability. In other words, networks with some redundancy have much higher capacity to respond to partial failure in the network (Javanbarg *et al.* 2006). Thus, redundancy can be considered as a surrogate measure in calculating the reliability of water distribution networks.

Awumah and his colleagues (Awumah *et al.* 1990, Awumah *et al.* 1991) seem to be the earliest researchers to propose the use of Shannon’s entropy function (Shannon 1948) as a surrogate measure to calculate the reliability of water distribution networks. Later, Tanyimboh and Templeman (1993a) offered a more suitable definition of the entropy function for water distribution networks by using a multiple probability space model and the conditional probability of Khinchin (1953). They also developed a non-iterative algorithm to find the maximum-entropy flow distribution in single-source networks. In their study, network topology, flow directions in every link, and supplies and demands at every node were taken into account whereas some other parameters such as link length, diameter, and roughness were neglected. There are an almost infinite number of possible flow distributions unless the network is of the branching-tree type. This non-iterative algorithm was formulated using the path entropy concept and Laplace’s principle of insufficient reason (Tanyimboh and Templeman 1993b). Using the super-source concept, they also tried to extend the single source algorithm to cover multiple-source networks, which has proved to be inconsistent with Walter’s research (1995). In another study, Yassin-Kassab *et al*. (1999) presented a non-iterative algorithm based on the single-source algorithm for calculating the maximum entropy flow distribution in multiple source networks. Later, the relationship between the entropy and reliability of water distribution networks was investigated by Tanyimboh and Templeman (2000) whose study supports the hypothesis that water distribution networks which are designed to carry the maximum entropy flows prove to be more reliable.

A further study by Tanyimboh and Sheahan (2002) explored the possibility of optimizing the layout of water distribution systems by using the minimum-cost/maximum-entropy design concept. In their research, they attempted to advance the entropy flows in water distribution networks to the stage where applications were possible; however, they could not properly interpret the meaning of network entropy.

Ang and Jowitt (2003) investigated the meaning of network entropy by using a simple water distribution network. Their research explored the relationship between the total power dissipated by the water distribution network and the numerical value of the network’s entropy. They used a simple loop network whose link diameters varied from zero to infinity and calculated the corresponding network entropy and energy loss for the water distribution network. In two other articles (2005a, 2005b), they presented an alternative method to calculate the network entropy of water distribution systems, which offered new insights into the concept of network entropy. Their alternative method, termed as the Path Entropy Method (PEM), provides a simpler explanation for the entropy of branching-tree networks and the maximum entropy of water distribution networks. The formulation of the PEM was based on the different paths available to a water molecule for moving from a super-source to a super-sink. More explanation about PEM will be given in this paper later.

In their definition of entropy function, Tanyimboh and Templeman failed to take into account the differences between branching-tree networks having different layouts and the same number of supply and demand nodes which all have the same PEM diagram. To overcome this problem, Hosseini and Emamjomeh (2010) suggested a penalty number for each link, which is equal to the amount of loss in case of its failure, based on which a new weighting ratio was proposed. The order of demand nodes in the network was considered in entropy calculations by calculating a new coefficient in the entropy function.

Previously defined redundancy indices for water distribution networks in the literature are generally based on either hydraulic or mechanical characteristics of the network, i.e., none of them take into account both types of characteristics in their calculations. However, the network risk is highly affected by both of these characteristics. Hence, the aim of the present study was to explore deficiencies of the previous definitions of water distribution networks entropy and to suggest a new weighted entropy-based measure for assessing the reliability of water distribution networks in terms of both mechanical and hydraulic characteristics of the system.

2. Entropy function for water distribution networks

The formulation of the entropy function mainly relies on Shannon's measure of uncertainty (Shannon 1948), which is an underlying principle of information theory. Tanyimboh and Templeman (1993a) were the first to develop a proper entropy function by using the multiple probability space model and the conditional probability of Khinchin (1953). The available network data are the topological layout, the supply and demand at all nodes, and the flow direction in each link. It is noteworthy that the flow direction in each link is a key assumption as there will be a flow distribution maximum entropy for each set of flow directions. Moreover, length, diameter, and roughness of the links are not used directly in their formulation. There will be a very large number of feasible flow patterns except for branching-tree networks.

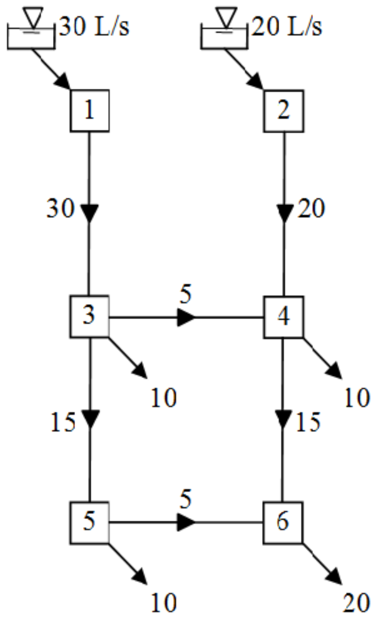
The network entropy function developed by Tanyimboh and Templeman is

$$\frac{s}{K} = S = S_0 + \sum_{n=1}^{N} P_n S_n \tag{1}$$

Where S is the entropy defined by Shannon, N is the total number of nodes, and K is the Boltzman constant which is usually set to unity. The entropy of the external inflows (S_0) is represented by

$$S_0 = -\sum_{i \in I} P_{0i} \ln P_{0i} \tag{2}$$

Where I is the set of all source nodes



$$S=-\left[\frac{30}{50}\ln(\frac{30}{50})+\frac{20}{50}\ln(\frac{20}{50})\right]$$
$$-\frac{30}{50}\times\left[\frac{15}{30}\ln(\frac{15}{30})+\frac{10}{30}\ln(\frac{10}{30})+\frac{5}{30}\ln(\frac{5}{30})\right]$$
$$-\frac{25}{50}\times\left[\frac{15}{25}\ln(\frac{15}{25})+\frac{10}{25}\ln(\frac{10}{25})\right]$$
$$-\frac{15}{50}\times\left[\frac{10}{15}\ln(\frac{10}{15})+\frac{5}{15}\ln(\frac{5}{15})\right]=1.80731$$

Fig. 1 A sample network with two sources and four demand nodes as well as details of entropy calculations (Ang and Jowitt 2005b)

$$P_{0i}=\frac{q_{0i}}{T_0} \tag{3}$$

Where q_{0i} is the external inflow at source node i, and T_0 is the total supply or demand.

The second term in the entropy function consists of the outflow entropy at each node (S_n) weighted by P_n ratio of the total outflow of each node to the total inflow of the whole network.

$$P_n=\frac{T_n}{T_0} \tag{4}$$

Where T_n is the total outflow at node n. An important point in the definition of outflow is that it is inclusive of any demand at the node. In Eq. (1), the outflow entropy at each node (S_n) is given by

$$S_n=-\sum_{n\in ND_n}P_{nj}\ln P_{nj} \tag{5}$$

Where ND_n is the set of all outflows from node n, and

$$P_{nj}=\frac{q_{nj}}{T_n} \tag{6}$$

Where q_{nj} is the flow from node n to node j.

The entropy function in Eq. (1) indicates that the entropy of a water distribution network has two components. The entropy of the external inflows (S_0), as the first component, is the uncertainty

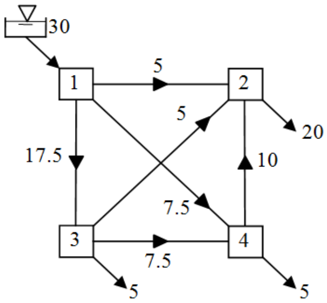
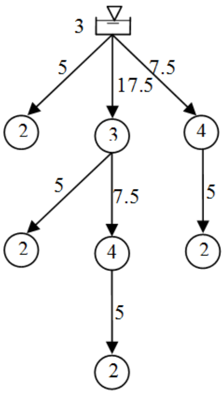


Fig. 2 Sample fully-connected network with maximum network entropy (Hosseini and Emamjomeh 2014)



$$
\begin{aligned}
S = &-\frac{30}{30}\times\left[\frac{5}{30}\ln(\frac{5}{30})+\frac{17.5}{30}\ln(\frac{17.5}{30})+\frac{7.5}{30}\ln(\frac{7.5}{30})\right] \\
&-\frac{17.5}{30}\times\left[2\times\frac{5}{17.5}\ln(\frac{5}{17.5})+\frac{7.5}{17.5}\ln(\frac{7.5}{17.5})\right] \\
&-\frac{15}{30}\times\left[\frac{5}{15}\ln(\frac{5}{15})+\frac{10}{15}\ln(\frac{10}{15})\right]=1.9073
\end{aligned}
$$

Fig. 3 Tree diagram of sample network, shown in Fig. 2, with entropy calculations (Hosseini and Emamjomeh 2014)

faced by a water molecule moving from the super-source to the individual supply nodes. This term for multiple-source networks and single-source networks is non-zero and zero, respectively. Moreover, the second part consists of the weighted entropy values at every demand node. Informational entropy measures the amount of uncertainty in a situation or system. For a water distribution network, the uncertainty can be imagined from the viewpoint of a water molecule.

Fig. 1 shows a sample network with two sources, four demand nodes, and details of entropy calculations using the entropy function proposed by Tanyimboh and Templeman.

The term in the first bracket is related to the first part of the entropy function or S_0 term. In addition, other calculations are related to the second part of entropy function or weighed entropy in each demand node.

3. Path entropy method (PEM) for calculating the entropy of water distribution networks

Fig. 2 demonstrates the water distribution network used in this section to describe the PEM method. The network includes one source and three demand nodes, and the maximum entropy flow is allocated to the links. Entropy calculation details of Fig. 2 and its tree diagram are shown in Fig. 3. Since the network includes one source, the S_0 term equals zero, and only the second part of the entropy function, weighed entropy, exists in each demand node. Considering the stated definition of entropy in the last part and Fig. 3, the entropy of a water distribution network can be presented by the number of paths available for a water molecule when it moves from the source to the demand node. Based on this observation, an alternative method for calculating network entropy is the path entropy method (PEM) (Ang and Jowitt 2005a).

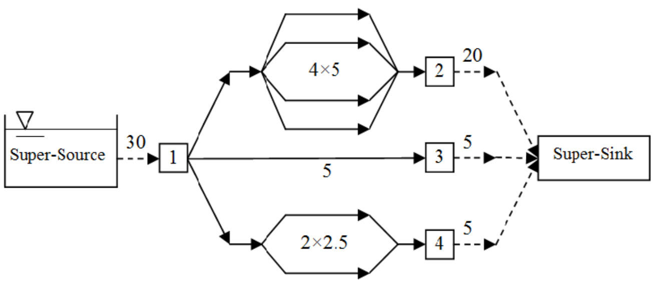
The PEM diagram shows the number of paths from the super-source to the super-sink and the amount of flow in each path. Development of the PEM diagram includes two main steps. The first step involves determining the number of paths from the source nodes to every demand node and drawing the PEM diagram with all the nodes and links. In the second step, the flow carried by each link is determined by examining the flow rates in all network links.

Once the PEM diagram is developed, the calculation of the network entropy is relatively straightforward, as compared to the network entropy equation suggested by Tanyimboh and Templeman (1993b). However, it must be noted that the less complicated entropy calculation is the result of the efforts spent in organizing the data into the PEM diagram. The true strength of the PEM lies in its ability to offer new insights into the meanings of the network entropy such as the entropy of branching-tree networks and the maximum-entropy flows of a single-source network with certain flow directions. Fig. 4. Demonstrates the PEM diagram of the sample network shown in Fig. 2 and its entropy calculations.

4. Discussion on the previous definitions of entropy function

As it was maintained by Walters (1995), all branching-tree networks have the same minimum entropy value. Furthermore, Ang and Jowitt (2005a) showed this fact by using the path entropy method. Fig. 5 shows all different layouts of branching-tree networks related to the sample network shown in Fig. 2.

According to Fig. 5, there is only one path from the source node to each demand node in all layouts; therefore, from the informational point of view, all of them have essentially the same entropy. The PEM diagram for the branching-tree sample network is shown in Fig. 6, which can be used for representing any of the different layouts. but, in Tanyimboh and Templeman’s definition of entropy function, no differences among branching-tree networks having different layouts and



$$S = -4\times\frac{5}{30}\ln(\frac{5}{30}) - 1\times\frac{5}{30}\ln(\frac{5}{30}) - 2\times\frac{2.5}{30}\ln(\frac{2.5}{30}) = 1.9073$$

Fig. 4 PEM diagram of the sample network, shown in Fig. 2, with entropy calculations (Hosseini and Emamjomeh 2014)

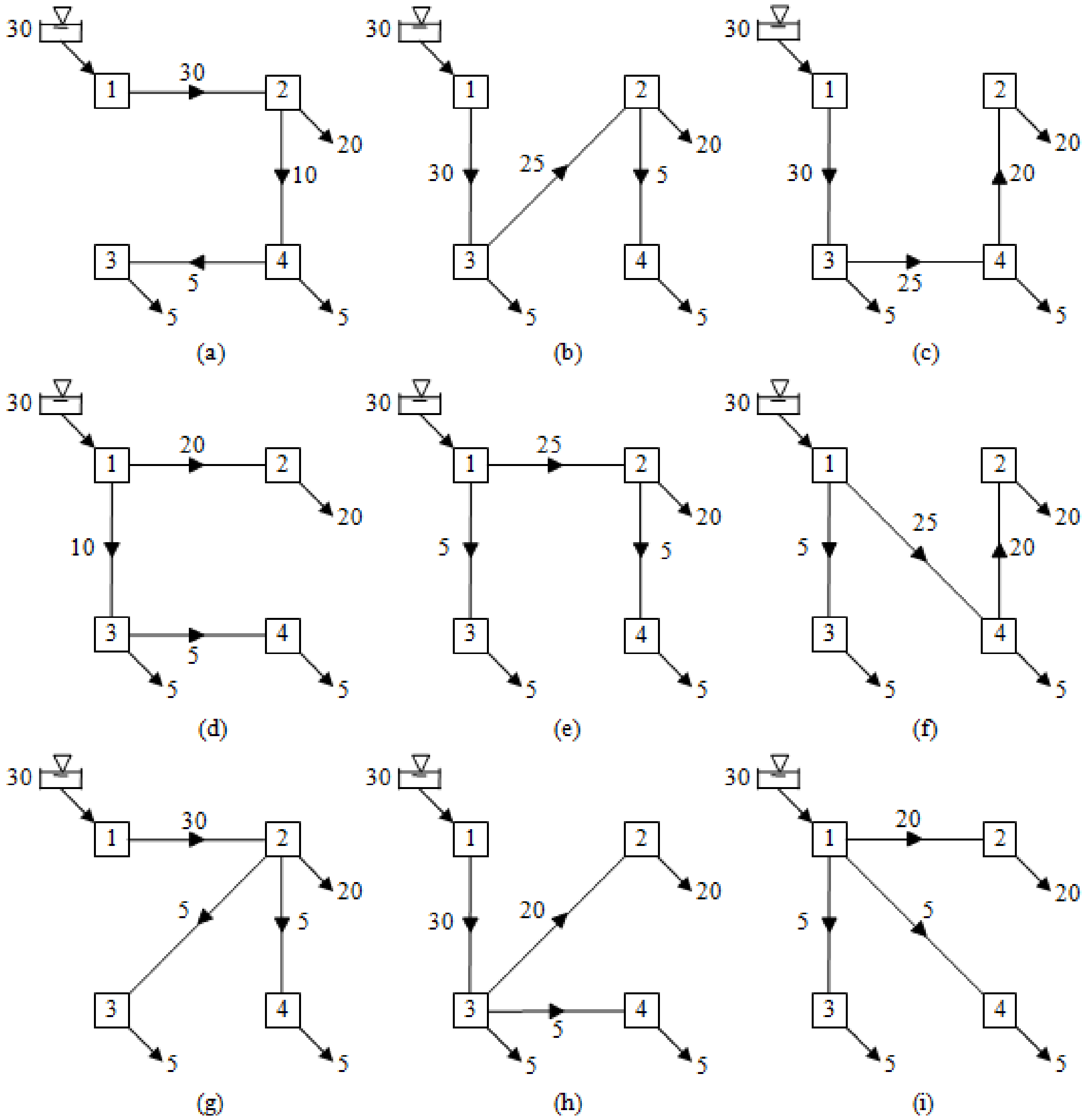
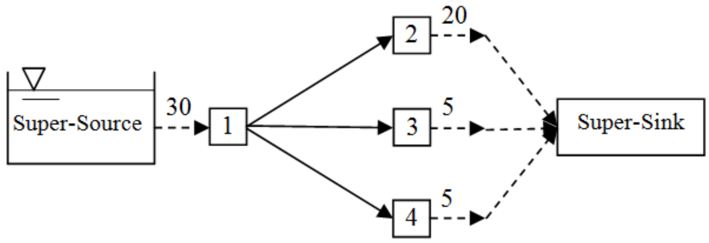


Fig. 5 Various branching-tree networks of the sample fully-connected network shown in Fig. 2 (Hosseini and Emamjomeh 2010)

the same number of supply and demand nodes (like networks shown in Fig. 5) could be considered. All these branching-tree networks have the same PEM diagram as shown in Fig. 6.

With a cursory look at networks shown in Fig. 5, it is obvious that some of them are more sensitive than others to damage in one of their links. For example, if the link 1-3 in networks (c), (d), and (e) in Fig. 5 gets damaged due to any hazards like earthquake, the amount of loss would be 30, 10, and 5 L/s, respectively. Therefore, the amount of service loss in a network depends not only on its main configuration as parallel or series but also on the connectivity order of different demand nodes to the supply node.



$$S = -\frac{20}{30}\ln(\frac{20}{30}) - 2\times\frac{5}{30}\ln(\frac{5}{30}) = 0.8676$$

Fig. 6 PEM diagram of the tree-branching networks

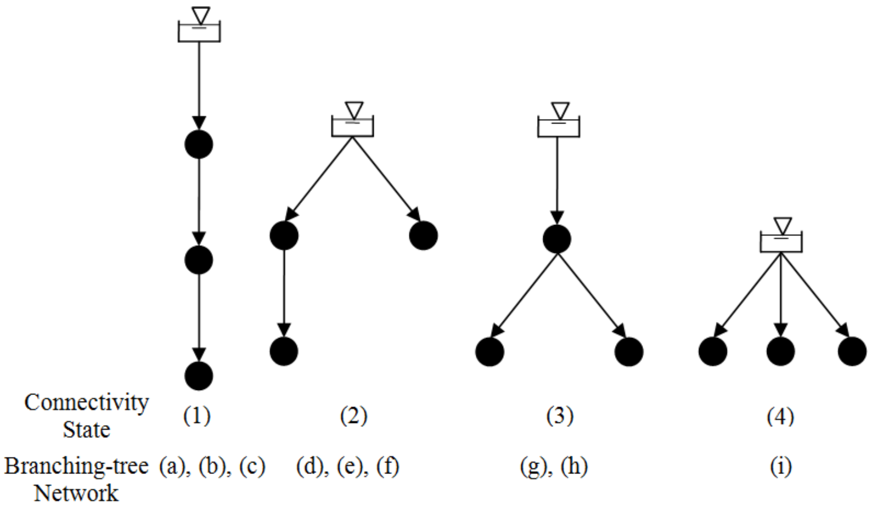


Fig. 7 Different supply-demand connectivity states of the branching-tree networks shown in Fig. 5 (Hosseini and Emamjomeh 2010)

For a better understanding of the main difference between the sensitivity states of various branching-tree networks, the networks shown in Fig. 6 were examined more meticulously by Hosseini and Emamjomeh (2010, 2014). These networks could be divided into four major categories based on their connectivity state as shown in Fig. 7. It is obvious that the fourth pattern has the greatest redundancy because each demand node has a separate path to the source, and failure of any link does not affect the serviceability of other demand nodes. Conversely, the first pattern is the most vulnerable water distribution network because serviceability levels of its different nodes are highly correlated.

To add the effects of connectivity state and order which, in fact, determine the sensitivity of a network, Hosseini and Emamjomeh (2010) defined a penalty number (T_P) for each link, which is equal to the amount of loss in case of its failure. Based on these penalty numbers, they introduced a new weighting ratio (P'_n) as follows

$$P_n^{'} = \frac{T_n}{T_p} \tag{7}$$

Table 1 The amount of entropy (S') for branching-tree networks in Fig. 5, based on the modifications proposed by Hosseini and Emamjomeh (2010)

Network	*a*	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*
S'	0.5784	0.4338	0.3470	0.7436	0.7436	0.5206	0.6507	0.4732	0.8676

Where T_P is the summation of penalty factors for all links in the network. They used this weighting ratio instead of the previous one, P'_n in Eq. (1), in the calculation of networks' entropy values.

The amount of entropy (S') for all branching-tree networks shown in Fig. 5, based on the modifications proposed by Hosseini and Emamjomeh, is presented in Table 1.

According to Table 1, only network (i) has a similar entropy amount to the one suggested by Tanyimboh and Templeman since each demand node in this network has a separate link to the source.

The amount of entropy in networks with more connections to the source significantly reduces with the new weighing ratio. Therefore, Hosseini and Emamjomeh could manage to modify the effect of connectivity order in demand nodes of the network and the difference between various branching-tree networks by defining a new weighing index (P'_n) in the entropy function.

Considering the results shown in Table 1, when the modified entropy function by Hosseini and Emamjomeh is used, the same amount of entropy is calculated for branching-tree networks (d) and (e). However, with a more careful analysis of these two networks, it could be observed that if link 1-2 breaks in network (d), the amount of dissipated water will be 20 L/s; therefore, the second consumer's demands would not be met. If the same link breaks in network (e), the amount of dissipated water will be 25 L/s, so the second and fourth consumer's needs would not be met. Therefore, network (d), in comparison to network (e), has a higher level of reliability, and if we consider entropy as a reliability measure, it should be higher in network (d) than in network (e).

As a result, Hosseini and Emamjomeh's modification cannot determine the real difference between the reliability of different networks having the same mechanical characteristics, namely, length, diameter, and roughness coefficient. Also, the reliability of different networks with various mechanical characteristics could not be measured by this modified version. For example, if the length, diameter, or material changes in one of the branching-tree networks, the effect of this change on the amount of entropy could not be measured by this entropy function.

In the next section, a new ratio in entropy function will be presented so that both mechanical and hydraulic characteristics of the network could be considered in calculating the entropy amount. In addition, the connectivity order of the network demand nodes and differences in various networks will be examined more carefully.

5. Identifying the deficiencies of the previous definitions of entropy function and presenting a new function

As it concluded in the previous part, the entropy function presented by Tanyimboh and Templeman with all modifications made on it, has two major deficiencies. First, both the mechanical and hydraulic characteristics of the network were not considered in this function; more specifically, only the flow rate of the links was present in entropy calculations. Also, mechanical parameters like length, diameter, and roughness of the link were not considered in entropy calculation, while it is clear that the aforementioned parameters highly influence the entropy amount as a network reliability measure. Secondly, with all modifications made on the entropy function, they could not show the numerical difference between various branching-tree networks having the same number of sources and demand nodes.

In the present study, dissipated power was used to consider the mechanical characteristics of the network like length, diameter, and roughness of the links along with hydraulic characteristics. For this purpose, based on the amount of the dissipated power calculated in each link, a new P''_n ratio was defined for each node of the network. The equation is as follows

$$P_n^{''} = \frac{T_n}{\sum_{i=1}^{np} P_{wi}} \tag{8}$$

Where, similar to the function proposed by Tanyimboh and Templeman, T_n is the total outflow at node n, $\sum_{i=1}^{np} P_{wi}$ is the total dissipated power by the water distribution network (W), and np is the total number of the links in the network.

The dissipated power by link i (P_{wi}) can be calculated as follows

$$P_{wi} = \rho g h_i Q_i \tag{9}$$

Where ρ is the density of water (kg/m^3), g is the gravity acceleration (m/s^2), h_i is the head loss for link i (m), and Q_i is the flow rate of link i (m^3/s).

In this paper, the link friction head loss is computed using the Hazen-Williams equation, because this equation is the most appropriate one for water distribution networks, as follows

$$h_i = \frac{10.6 L_i}{D_i^{4.865} C_i^{1.85}} Q_i^{1.85} = K_i Q_i^{1.85} \tag{10}$$

Where L_i is the length of the link i (m), D_i is the diameter of link i (m), C_i is the coefficient of roughness for link i, and K_i is the resistance coefficient for link i (s/m^2).

Based on the above equations, the dissipated power by link i can be calculated as follows

$$P_{wi} = \rho g K_i Q_i^{2.85} \tag{11}$$

In calculating the amount of network entropy, P''_n was used instead of P_n while the rest of the calculations are the same as Tanyimboh and Templeman's calculations. The entropy amount in branching-tree networks, shown in Fig. 5, is presented in Table 2. These numbers were calculated using the modified function in this study; furthermore, their counter amounts were computed using the modified method of Hosseini and Emamjomeh. In calculation of the dissipated power, the length of the peripheral links in the network, the diameter of all links, and the coefficient of roughness were 1000 m, 400 mm, and 130.

For example, the new entropy of branching-tree network (b) in the Fig. 5 could be calculated as follows:

$$L_{1-3}, L_{2-4} = 1000m, L_{3-2} = 1414.21m$$

Table 2 The amount of entropy for branching tree networks in Fig. 5, based on the modifications proposed by Hosseini and Emamjomeh (S') and the modification proposed in this study (S'')

Network	*a*	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*
S'	0.5784	0.4338	0.3470	0.7436	0.7436	0.5206	0.6507	0.4732	0.8676
S''	0.4957	0.2799	0.2707	1.4181	0.8519	0.4449	0.5096	0.3562	1.5812

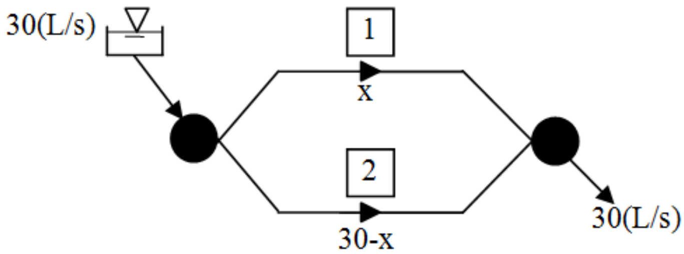


Fig. 8 A parallel network with two links

$$P_{w(1-3)} = 50.34W, P_{w(2-3)} = 42.34W, P_{w(2-4)} = 0.305W$$

$$\sum_{i=1}^{3} P_{wi} = 92.985W$$

$$S_b^{''} = -\frac{30}{92.985} \times \left[\frac{5}{30}\ln(\frac{5}{30}) + \frac{25}{30}\ln(\frac{25}{30})\right] - \frac{25}{92.985} \times \left[\frac{20}{25}\ln(\frac{20}{25}) + \frac{5}{25}\ln(\frac{5}{25})\right] = 0.2799$$

With due consideration to Table 2, it can be seen that with modifications made in this study, the entropy amount in network (d) is more than the entropy amount in network (e), which is in line with our expectation.

6. Examining the behavior of the proposed entropy function in the parallel network

A parallel network with two links as shown in Fig. 8 was used to examine the behavior of the proposed function in parallel networks. In this network, the inflow rate, the length of the links, the diameter of the links, and the roughness coefficient of the links were 30 L/s, 1000 m, 400 mm, and130, respectively. If the flow rate of the first link is taken as x, the flow rate of the second link would be 30-x.

Based on the proposed function in this study, entropy variations of the parallel network versus x is shown in Fig. 9.

As can be seen in Fig. 9, the closer is the flow amount in the two links; the higher is the entropy amount. More specifically the maximum entropy occurs when flow amount in two links are exactly the same. In other words, the maximum entropy value corresponds to x=15, and the graph of the entropy function is symmetric with respect to this value. These variations are exactly based on our expectation regarding the entropy behavior.

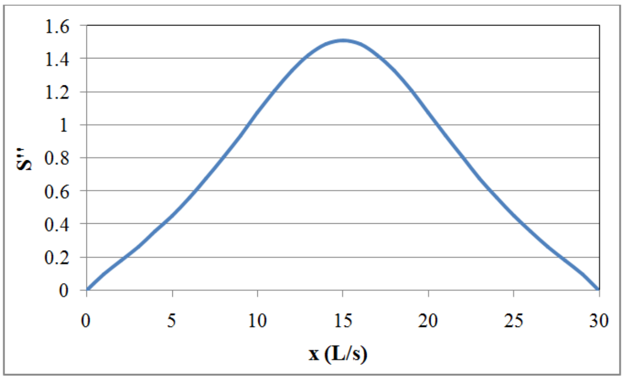


Fig. 9 Entropy variations in the parallel network versus different flow amounts in the first link (x), using the proposed entropy function in this study

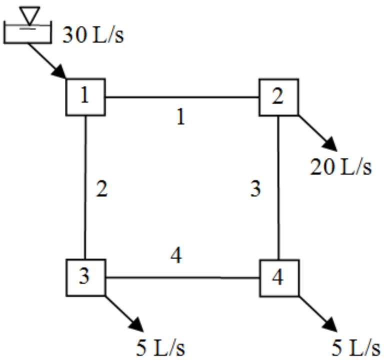


Fig. 10 Sample loop network with one source node and three demand nodes (Ang and Jowitt 2003)

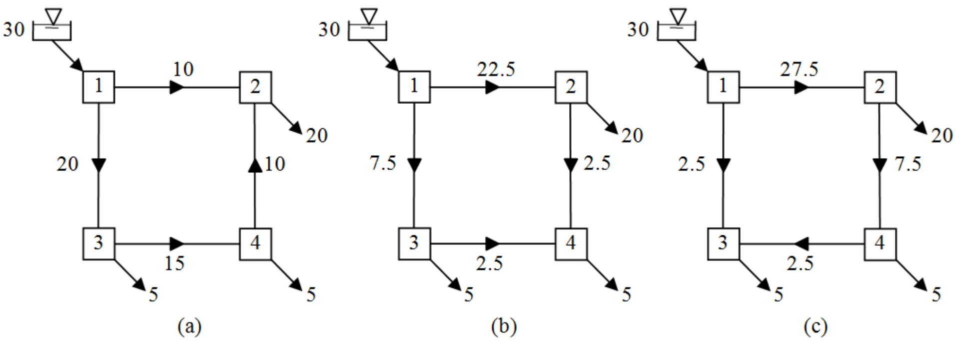


Fig. 11 Three different possible flow directions for the network shown in Fig. 10 (Ang and Jowitt 2003)

7. Examining the behavior of the proposed entropy function in the loop network

A network with one supply node and three demand nodes, as shown in Fig. 10, was utilized to assess the proposed entropy function in loop networks. This network was used by Ang and Jowitt (2003) to examine Tanyimboh and Templeman’s entropy function.

Table 3 The entropy amount for three flow directions shown in Fig. 11, using Tanyimboh and Templeman's function (S) as well as the modification proposed by Hosseini and Emamjomeh (S') and the one suggested in the present study (S'') to this function

Network	*a*	*b*	*c*
S	1.3296	0.9831	0.9831
S'	0.7253	0.8426	0.7373
S''	1.6459	1.4266	0.8215

The elevation of the source node and the demand nodes were assumed to be 100m and 99m, respectively. The length, diameter, roughness coefficient of all links were considered the same, i.e., 890m, 400mm, and 130, respectively. Fig. 11 illustrates three various states of flow direction. Table 3 shows the entropy amounts calculated using Tanyimboh and Templeman's method (S), the modified method of Hoseini and Emamjomeh (S'), and the one proposed by entropy in this study (S'').

According to Fig. 11, it could be concluded that the reliability of network (a) is more than that of (b), and the reliability of network (b) is more than that of (c). In fact, the breakage of link 1-2 in networks (a), (b), and (c) causes 10, 22.5, and 27.5 L/s water to be wasted, respectively. Furthermore, the maximum difference between the flow rates in the network's links in state (a) is 10 L/s, while this difference is 20 L/s and 25 L/s in states (b) and (c) respectively. Based on these two facts it is expected that the network in state (a) has more entropy value than state (b), and in state (b) more than state (c). However, Table 3 demonstrates that although the entropy amount in network (a) is more than the entropy amounts in networks (b) and (c), the entropy amount in network (b) and (c) are the same, using the proposed entropy function by Tanyimboh and Templeman.

Based on the modified function proposed by Hosseini and Emamjomeh, the entropy amount in network (b) is more than the entropy amount in (c), and the entropy amount in network (c) is more than the entropy amount in network (a).

Finally, it should be noted the entropy amounts calculated via the proposed method in this study, as shown in Table 3, are in agreement with our expectation. In other words, the entropy amount in network (a) is more than the entropy amount in network (b), and the amount of entropy in network (b) is more than the entropy amount in network (c).

8. Examining the relationship between the proposed entropy function and water distribution network reliability

One of the most important issues about the entropy of water distribution networks is the relationship between the entropy amount and the network reliability. In a study conducted by Tanyimboh and Templeman (2000), various water networks with different number of links and loops were used, and this relationship was proved to be strong. As a result, they concluded that networks with higher entropy have higher reliability.

In this section, the relationship between the proposed entropy function and the reliability of the water network is examined. Im this study the network was utilized for this purpose had been used by Tanyimboh and Templeman (2000) to compare their own entropy function with the network

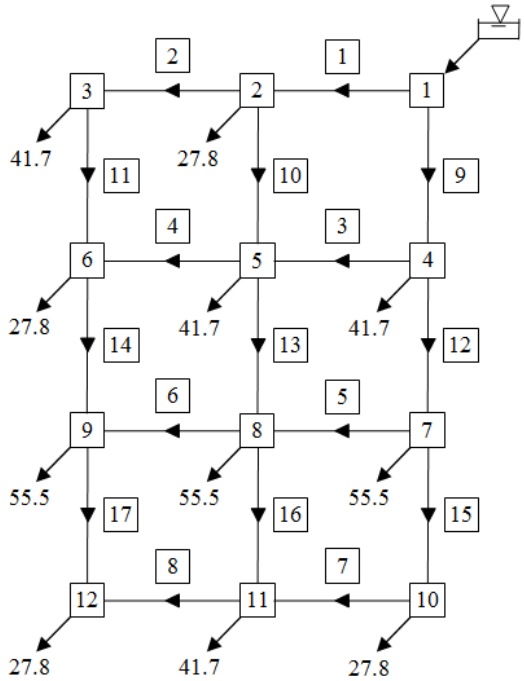


Fig. 12 Sample water distribution network with one source and 11 demand nodes (Tanyimboh and Templeman 2000)

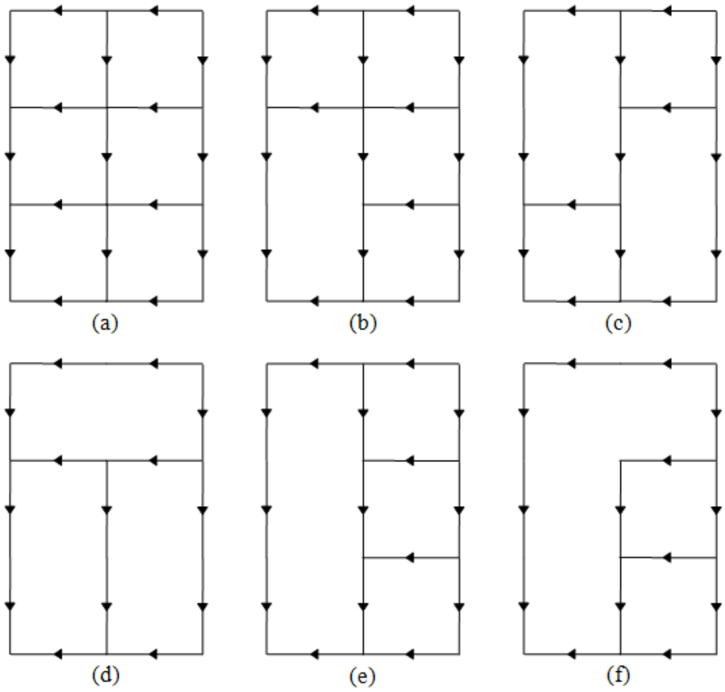


Fig. 13 Different layouts examined for the network shown in Fig. 12 (Tanyimboh and Templeman 2000)

Table 4 Diameter and flow rate of links related to different layouts shown in Fig. 13 (Tanyimboh and Templeman 2000)

Link No	*D* (mm)						Flow Rate (L/s)					
	a	*b*	*c*	*d*	*E*	*F*	*a*	*b*	*c*	*d*	*e*	*f*
1	302	310	325	251	326	284	175.6	186.2	210.4	117.5	212.9	158.0
2	192	206	227	232	265	268	61.6	71.6	90.6	89.7	129.4	130.2
3	226	226	232	302	186	225	87.6	87.5	92.1	174.4	56.3	90.3
4	175	209	0.1	182	0.1	0.1	42.9	65.3	0.0	49.2	0.0	0.0
5	179	157	0.1	0.1	161	184	44.2	32.8	0.0	0.0	35.2	51.0
6	178	0.1	194	0.1	0.1	0.1	37.5	0.0	48.4	0.0	0.0	0.0
7	119	123	139	163	130	143	12.0	12.9	17.0	27.6	15.0	20.2
8	135	157	147	148	180	176	10.8	15.9	13.7	13.9	23.4	22.6
9	361	354	337	390	336	368	268.9	258.3	234.1	327.0	231.6	286.5
10	228	227	231	0.1	185	0.1	86.3	86.9	92.0	0.0	55.7	0.0
11	138	160	190	193	237	240	19.9	29.9	48.9	48.0	87.7	88.5
12	275	265	234	244	270	286	139.5	129.0	100.3	110.9	133.5	154.4
13	239	209	293	223	221	188	89.3	67.4	142.4	83.5	70.4	48.6
14	182	231	149	233	212	215	35.0	67.4	21.2	69.4	59.9	60.7
15	169	172	185	199	177	184	39.8	40.7	44.8	55.4	42.8	48.0
16	184	200	178	163	213	200	40.5	44.6	38.5	28.0	50.1	44.1
17	162	139	149	146	100	105	17.0	11.9	14.1	13.9	4.4	5.2

reliability. The general structure of the network is shown in Fig. 12. As can be seen in the Fig. 12, this network has one source node and eleven demand nodes with various amounts of demand. The length of all links and their coefficient of roughness are 1000m and 130, respectively. The elevation of the source node and the demand nodes are assumed to be as 100m and zero, respectively.

A variety of layouts could be suggested for this network. Six different layouts were used in this study as it is shown in Fig. 13.

Using Tanyimboh and Templeman's proposed entropy function, these networks were designed in such a way that the amount of flow rate in different links has the maximum entropy with the minimum cost. Table 4 shows the related diameter and the flow rate of links for patterns shown in Fig. 13 (Tanyimboh and Templeman 2000).

One of the formulations for calculating network reliability has been the function proposed by Tanyimboh and Templeman (1998) in which reliability is defined as the ratio of the mean value of the flow delivered to the required flow. Therefore, the reliability formulation of a single-source water distribution network can be stated as follows

$$\begin{aligned} R = \frac{1}{T}\left[P(0)T(0) + \sum_{m=1}^{M} P(m)T(M) + \sum_{\substack{m=1 \\ \forall n \neq m}}^{M} P(m,n)T(m,n) + \ldots\right] \\ + \frac{1}{2}\left[1 - P(0) - \sum_{m=1}^{M} P(m) - \sum_{\substack{m=1 \\ \forall n \neq m}}^{M} P(m,n) - \ldots\right] \end{aligned} \tag{12}$$

Table 5 The amount of reliability and entropy based on Tanyimboh and Templeman's function (S) and the proposed function in this study (S'') related to layouts shown in Fig. 13

Network	*a*	*b*	*c*	*d*	*e*	*f*
R	0.999027	0.998933	0.998872	0.998777	0.998555	0.998485
S	3.1291	3.0201	2.8640	2.6424	2.7932	2.6362
S''	0.00692	0.00666	0.00630	0.00585	0.00612	0.00578

In the above formulation, R is network reliability, $P(0)$ is the probability that no link is available, $P(m)$ is the probability that only link m is unavailable, and $P(m,n)$ is the probability that only links m and n are unavailable. Similarly, $T(0)$, $T(m)$, and $T(m,n)$ are the total flows supplied at adequate pressure with no links available, only link m unavailable, and only links m and n unavailable, respectively. Finally, M stands for the number of links while T represents the total demand (Tanyimboh and Templeman 2000).

The reliability of layouts shown in Fig. 13 along with the amount of their entropy, based on Tanyimboh and Templeman's entropy function (S) and the proposed function in this study (S''), are presented in Table 5.

As can be seen in Table 5, as the reliability of different layouts reduces, the amount of network entropy generally reduces, using Tanyimboh and Templeman's entropy function and the one proposed in this study. Therefore, the entropy function proposed in this study has a direct relationship with network reliability, just like Tanyimboh and Templeman's entropy function, but in addition to the advantages of Tanyimboh and Templeman's function, our proposed entropy function can take into account network's both mechanical and hydraulic parameters. In addition, the suggested entropy function, compared with other reliability methods, is less complicated in terms of calculations.

9. Conclusions

Based on various water distribution networks discussed and numerical results presented in this study, it could be concluded that the amount of entropy in water distribution networks can be a good criterion for evaluating the reliability of networks exposed to either natural or man-made hazards. The more the number of network characteristics is used in entropy calculation, the more realistic criterion it will be in calculation of network reliability.

After examining the available methods proposed by several researchers for calculating the entropy in water distribution networks, it became evident that the entropy function presented by Tanyimboh and Templeman and its modification suggested by various researchers have two basic deficiencies. First, all of them have considered only the flow rate of the links while they have failed to take into account mechanical characteristics of the network such as length, diameter, and roughness coefficient of the links. Second, despite all modifications recommended for these functions, they cannot indicate the numerical difference between various branching-tree networks having the same number of source and demand nodes as well as different kinds of flow directions in a specific network. In order to overcome these two deficiencies and improve reliability evaluation in these networks, in this study suggested a modification on the entropy function on the entropy function proposed by Tanyimboh and Templeman. This modification was carried out through defining a coefficient for each node as the amount of outflow at the node to all dissipated power in the network.

After using the modified function for a number of networks, it was revealed that the suggested function, despite its simplicity, can take into account the order of source and demand nodes' linkage, the difference among various water distribution networks, and the effects of both mechanical and hydraulic characteristics of the network. Therefore, the proposed entropy function can be used to evaluate the reliability of existing water distribution networks and choose the optimum layout for designing the new networks.

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