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Lateral stability analysis of multistory buildings using the differential transform method

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Abstract. The determination of the critical buckling load of multistory structures is important since this load is used in second order analysis. It is more realistic to determine the critical buckling load of multistory structures using the whole system instead of independent elements. In this study, a method is proposed for designating the system critical buckling load of torsion-free structures of which the load-bearing system consists of frames and shear walls. In the method presented, the multistory structure is modeled in accordance with the continuous system calculation model and the differential equation governing the stability case is solved using the differential transform method (DTM). At the end of the study, an example problem is solved to show the conformity of the presented method with the finite elements method (FEM).

Keywords: stability; differential transform method; continuous system; multistory structure; wall-frame

1. Introduction

It is known that the critical buckling load of multistory structures is important to be designated since this load is an auxiliary parameter in second order analysis (Kollar 2008). Existing design codes take the subsystem approach as a basis instead of the analysis of the whole structural system in the determination of the critical buckling load (Kollar 2008, Girgin and Özmen 2008, Zalka 2013). In fact, more accurate results can be obtained by determining the critical buckling load for the whole structural system.

There are a number of studies (Wood 1974a, Wood 1974b, Wood 1974c, Rosman 1974, Rosman 1981, Rutenberg *et al.* 1988, Wang *et al.* 1991, Syngellakis and Kameshki 1994, Aristizabal-Ochoa 1997, Wang 1997, Zalka 1999, Li 2001, Aristizabal-Ochoa 2002, Hoenderkamp 2002, Zalka 2002a, Zalka 2002b, Aristizabal-Ochoa 2003, Potzta and Kollar 2003, Zalka 2003, Xenidis and Makarios 2004, Gantes and Mageirou 2005, Gustafsson and Hehir 2005, Ozmen and Girgin 2005, Girgin *et al.* 2006, Kaveh and Salimbahrami 2006, Mageirou and Gantes 2006, Tong and Ji 2007, Gomes *et al.* 2007, Girgin and Ozmen 2007, Xu and Wang 2007, Gengshu *et al.* 2008, Gengshu and Yun 2008, Bozdogan and Ozturk 2010, Orumu 2013, Ellwanger 2013, Kaveh 2013, Lal and Ahlawat 2015, Zhang *et al.* 2015) in literature for the designation of critical buckling load

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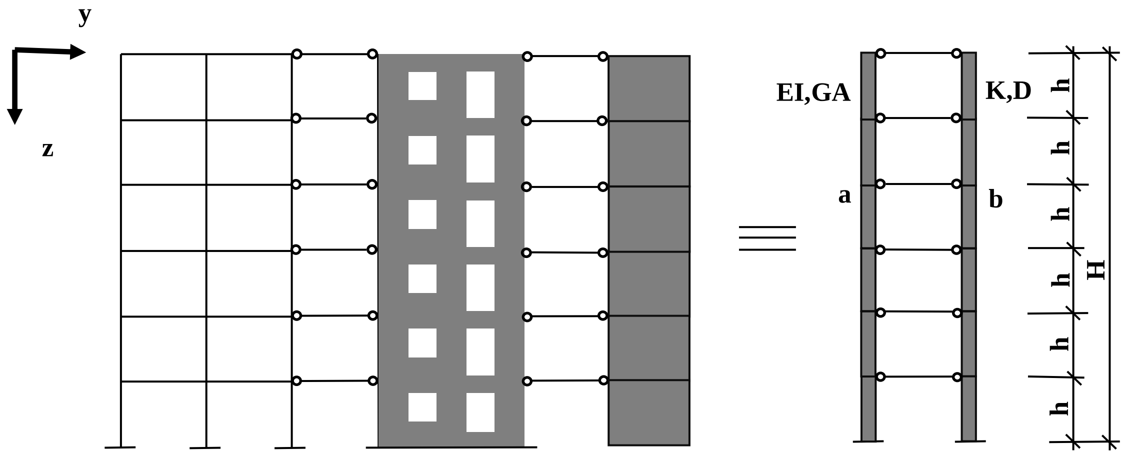

Fig. 1 Equivalent model of multistoy buildings

in multistory structures.

In this study, an approach based on the differential transform method (DTM) which has been commonly used in mechanics in recent years is proposed for the determination of critical buckling load of multistory structures. The following assumptions were made for developing the approach proposed:

(a) The materials are linearly elastic.
(b) The structure is torsion-free.
(c) The material and geometric characteristics of the structure are uniform throughout the height of the structure.

2. Stability equations of multistory structures

The behavior of the multistory structures of which the load-carrying system consists of frames, coupled shear walls and shear walls can be idealized as the behavior of an equivalent sandwich beam according to the continuous system calculation model.(Potzta and Kollar 2003) The model of a multistory structure example modeled as an equivalent sandwich beam is shown in Fig. 1. As can be seen, the structure is formed by connecting the two rods such as *a* and *b* with elements jointed at two ends at floor levels.

The stability case differential equation system for the distributed load of the sandwich beam given is written as

$$GA\left(\frac{d^2y}{dz^2}-\frac{d^2y_{ae}}{dz^2}\right)+K\left(\frac{d^2y}{dz^2}-\frac{d^2y_{be}}{dz^2}\right)-\left(qz\frac{dy}{dz}\right)'=0 \tag{1}$$

$$GA\left(\frac{dy}{dz}-\frac{dy_{ae}}{dz}\right)=-EI\frac{d^3y_{ae}}{dz^3} \tag{2}$$

$$K\left(\frac{dy}{dz}-\frac{dy_{be}}{dz}\right)=-D\frac{d^3y_{be}}{dz^3} \tag{3}$$

where y is the total displacement function, y_{ae} is the flexural displacement in the equivalent beam a, y_{be} is the flexural displacement in the equivalent beam b, $q(z)$ is the axial distributed load along the height of the structure and z is the axis along the height of the structural elements. GA indicates the total shear rigidity of the column and shear wall elements and can be calculated using Eq. (4).

$$GA=\frac{E}{2*(1+\mu)}*\frac{A}{k} \tag{4}$$

Where μ is Poisson's ratio, E is the modulus of elasticity, A is the cross-sectional area of the column or the shear wall and k is the shape factor for shear.

EI refers to the total flexural rigidity of the column and shear walls and can be calculated using Eq. (5).

$$EI=\sum_{i=1}^{n}(EI_i) \tag{5}$$

where n is the total number of columns and shear walls, I_i is the moments of inertia of the column and shear wall elements.

D refers to the global rigidity and can be calculated using Eq. (6) (Zalka 2000, Potzta and Kollar 2003).

$$D=E\sum_{i=1}^{m}(A_it_i^2) \tag{6}$$

where A_i is the cross-sectional area of the elements and t_i is the distance between ith arm of the elements and the center of gravity. K refers to the equivalent shear rigidity and is calculated as follows for the frame element. (Fig. 2)

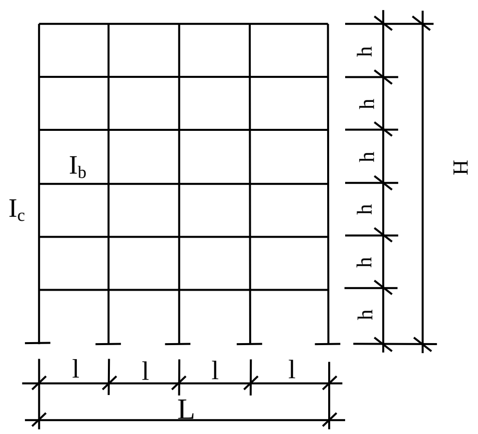

Fig. 2 Planar frame

$$K = (\frac{1}{K_b} + \frac{1}{K_c})^{-1} \tag{7}$$

where K_b is the equivalent shear rigidity of the beams and K_c is the equivalent shear rigidity of the columns. These values are calculated using the following formulas (Zalka 2000, Potzta and Kollar 2003, Zalka 2013)

$$K_b = \sum_{i=1}^{n-1} (\frac{12EI_{bi}}{l_i h}) \tag{8}$$

where I_{bi} is the moments of inertia of the beams, l_i is the beam length and h is the floor height.

$$K_c = \sum_{i=1}^{n} (\frac{\pi^2 EI_{ci}}{h^2}) \tag{9}$$

where I_{ci} is the moments of inertia of the columns.

The equivalent shear rigidity for coupled shear walls is calculated using the following formula

$$K^c = (\frac{1}{K_b^c} + \frac{1}{K_c^c})^{-1} \tag{10}$$

where K_c^c is the contribution of the wall in the coupled shear wall shear wall to the equivalent shear resistance and K_b^c is the contribution of the coupling beams to the equivalent shear resistance. These values are calculated as follows (Zalka 2000, Potzta and Kollar 2003)

$$K_b^c = \sum_{i=1}^{m-1} \frac{6EI_{bi}[(d_i + s_i)^2 + (d_i + s_{i+1})^2]}{d_i^3 h(1 + \frac{12kEI_{bi}}{GA_{bi} d_i^2})} \tag{11}$$

$$K_c^c = \sum_{i=1}^{n} (\frac{\pi^2 EI_{wi}}{h^2}) \tag{12}$$

As shown in Fig. 3, d_i is the width of the ith wall, s_i is the distance between the shear wall axes in between the two edges of the coupled shear wall, EI_{bi} and GA_{bi} are the flexural and shear rigidities, respectively, of the coupling beams and EI_{wi} is the flexural rigidity of the walls.

The boundary conditions can be written as follows considering that the bending moments and total shear force at the top (z=0) and the displacement and rotations at the bottom (z=H) are zero

$$z = 0 \qquad EI \frac{d^2 y_{ae}}{dz^2} = 0 \tag{13}$$

$$z = 0 \qquad D \frac{d^2 y_{be}}{dz^2} = 0 \tag{14}$$

$$z = 0 \qquad GA\left(\frac{dy}{dz} - \frac{dy_{ae}}{dz}\right) + K\left(\frac{dy}{dz} - \frac{dy_{be}}{dz}\right) - \left(qz \frac{dy}{dz}\right) = 0 \tag{15}$$

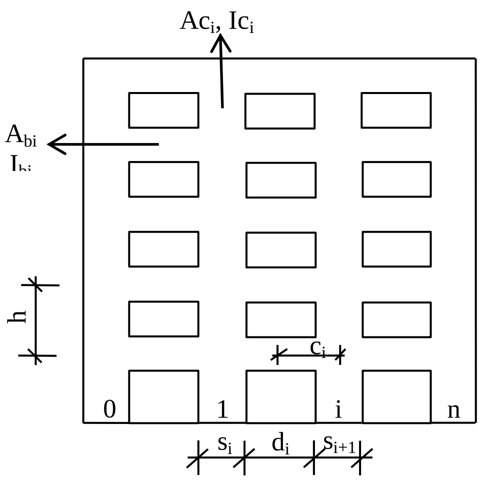

Fig. 3 Coupled shear wall

$$z = H \qquad y = 0 \tag{16}$$

$$z = H \qquad \frac{dy_{ae}}{dz} = 0 \tag{17}$$

$$z = H \qquad \frac{dy_{be}}{dz} = 0 \tag{18}$$

Integrating Eq. (1) once gives Eq. (19)

$$GA\left(\frac{dy}{dz} - \frac{dy_{ae}}{dz}\right) + K\left(\frac{dy}{dz} - \frac{dy_{be}}{dz}\right) - \left(qz\frac{dy}{dz}\right) = c \tag{19}$$

Considering the boundary condition given in Eq. (15) in Eq. (19), c is found to be zero. In that case, Eq. (19) takes the following form

$$GA\left(\frac{dy}{dz} - \frac{dy_{ae}}{dz}\right) + K\left(\frac{dy}{dz} - \frac{dy_{be}}{dz}\right) - \left(qz\frac{dy}{dz}\right) = 0 \tag{20}$$

Applying the transform $\varepsilon = \frac{z}{H}$ on the Eqs. (20), (2) and (3) in order to make these equations dimensionless, the following set of equations are obtained

$$\frac{GA}{H}\left(\frac{dy}{d\varepsilon} - \frac{dy_{ae}}{d\varepsilon}\right) + \frac{K}{H}\left(\frac{dy}{d\varepsilon} - \frac{dy_{be}}{d\varepsilon}\right) - \left(\frac{q\varepsilon H}{H}\frac{dy}{d\varepsilon}\right) = 0 \tag{21}$$

$$\frac{GA}{H}\left(\frac{dy}{d\varepsilon} - \frac{dy_{ae}}{d\varepsilon}\right) = -\frac{EI}{H^3}\frac{d^3 y_{ae}}{d\varepsilon^3} \tag{22}$$

$$\frac{K}{H}\left(\frac{dy}{d\varepsilon} - \frac{dy_{be}}{d\varepsilon}\right) = -\frac{D}{H^3}\frac{d^3 y_{be}}{d\varepsilon^3} \tag{23}$$

Multiplying Eq. (21) by $\frac{H^3}{EI}$, Eq. (24) is obtained.

$$\frac{GAH^2}{EI}\left(\frac{dy}{d\varepsilon}-\frac{dy_{ae}}{d\varepsilon}\right)+\frac{KH^2}{EI}\left(\frac{dy}{d\varepsilon}-\frac{dy_{be}}{d\varepsilon}\right)-\left(\frac{q\varepsilon H^3}{EI}\frac{dy}{d\varepsilon}\right)=0 \tag{24}$$

Multiplying Eq. (22) by $\frac{H^3}{EI}$, Eq. (25) is obtained.

$$\frac{GAH^2}{EI}\left(\frac{dy}{d\varepsilon}-\frac{dy_{ae}}{d\varepsilon}\right)=-\frac{d^3y_{ae}}{d\varepsilon^3} \tag{25}$$

Multiplying Eq. (23) by $\frac{H^3}{D}$, Eq. (26) is obtained.

$$\frac{KH^2}{D}\left(\frac{dy}{d\varepsilon}-\frac{dy_{be}}{d\varepsilon}\right)=-\frac{d^3y_{be}}{d\varepsilon^3} \tag{26}$$

If we equate the coefficients on the left hand side of Eqs. (24), (25) and (26) to dimensionless parameters in order to shorten the differential equations, the following equation can be written

$$t\left(\frac{dy}{d\varepsilon}-\frac{dy_{ae}}{d\varepsilon}\right)+m\left(\frac{dy}{d\varepsilon}-\frac{dy_{be}}{d\varepsilon}\right)-\left(\alpha\varepsilon\frac{dy}{d\varepsilon}\right)=0 \tag{27}$$

$$t\left(\frac{dy}{d\varepsilon}-\frac{dy_{ae}}{d\varepsilon}\right)=-\frac{d^3y_{ae}}{d\varepsilon^3} \tag{28}$$

$$r\left(\frac{dy}{d\varepsilon}-\frac{dy_{be}}{d\varepsilon}\right)=-\frac{d^3y_{be}}{d\varepsilon^3} \tag{29}$$

The dimensionless parameters *m*, *r*, *t* and α in Eqs. (27), (28) and (29) are defined as follows

$$\frac{KH^2}{EI}=m \quad (30a) \qquad \frac{KH^2}{D}=r \quad (30b) \qquad \frac{GAH^2}{EI}=t \quad (30c) \qquad \frac{qH^3}{EI}=\alpha \quad (30d)$$

If the transformation $\varepsilon=\frac{z}{H}$ is also applied on Eqs. (13)-(18), the boundary conditions can be rewritten as

$$\varepsilon=0 \qquad \frac{\mathrm{EI}}{H^2}\frac{d^2y_{ae}}{d\varepsilon^2}=0 \tag{31}$$

$$\varepsilon=0 \qquad \frac{\mathrm{D}}{H^2}\frac{d^2y_{be}}{d\varepsilon^2}=0 \tag{32}$$

$$\varepsilon=1 \qquad y=0 \tag{33}$$

$$\varepsilon = 1 \qquad \frac{1}{H}\frac{dy_{ae}}{d\varepsilon} = 0 \tag{34}$$

$$\varepsilon = 1 \qquad \frac{1}{H}\frac{dy_{be}}{d\varepsilon} = 0 \tag{35}$$

3. Application of the differential transform method

The DTM which was firstly proposed by Pukhov (1981) for solving Ordinary differential equations has been developed and used in the solution of many mechanical problems (Chen and Liu 1998, Ozgumus and Kaya 2006, Keskin *et al*. 2007, Rajasekaran 2008, Chai and Chen 2009, Liu *et al*. 2013). The DTM involves calculating the coefficients of the Taylor series. Below is given the main equations of DTM in brief (Rajasekaran 2009).

$$Y[k] = \frac{1}{k!}[\frac{d^k y(x)}{dx^k}]_{x=0}; \qquad 0 \le \mathrm{x} \le 1 \tag{36}$$

$$y(x) = \sum_{k=0}^{\infty} x^k Y[k] \tag{37}$$

$$y(x) = \sum_{k=0}^{\infty} \frac{x^k}{k!}[\frac{d^k y(x)}{dx^k}]_{x=0} \tag{38}$$

$$D\mathrm{T}(\mathrm{x}^{\alpha}\mathrm{y}^{\beta}) = [\prod_{i=1}^{\beta}(k-\alpha+i)]Y(k-\alpha+\beta) \tag{39}$$

$$y^{\beta} = \frac{d^{\beta}y}{dx^{\beta}} \tag{40}$$

The differential expressions necessary to apply the DTM in the main equations (Eqs. (27), (28) and (29)) are written as follows in accordance with the DTM method

$$\frac{dy}{d\varepsilon} = (k+1)Y[k+1] \tag{41}$$

$$\frac{dy_{ae}}{d\varepsilon} = (k+1)Y_{ae}[k+1] \tag{42}$$

$$\frac{dy_{be}}{d\varepsilon} = (k+1)Y_{be}[k+1] \tag{43}$$

$$\frac{d^3 y_{ae}}{d\varepsilon^3} = (k+3)(k+2)(k+1)Y_{ae}[k+3] \tag{44}$$

$$\frac{d^3 y_{be}}{d\varepsilon^3} = (k+3)(k+2)(k+1)Y_{be}[k+3] \tag{45}$$

Substituting the differential transform expressions in Eqs. (27), (28) and (29), the equations are written as follows according to the DTM

$$Y[k+1]=\frac{tY_{ae}[k+1]+mY_{be}[k+1]}{(t+m)}+\frac{\alpha \mathrm{kY[k]}}{(\mathrm{k}+1)(\mathrm{t}+\mathrm{m})} \tag{46}$$

$$Y_{ae}[k+3]=\frac{-t\{Y[k+1]-Y_{ae}[k+1]\}}{(k+3)(k+2)} \tag{47}$$

$$Y_{be}[k+3]=\frac{-r\{Y[k+1]-Y_{be}[k+1]\}}{(k+3)(k+2)} \tag{48}$$

Similarly, applying the DTM in the boundary conditions given in (31) to (35), the following formulas are found

$$\varepsilon=0 \qquad Y_{ae}[2]=0 \tag{49}$$

$$\varepsilon=0 \qquad Y_{be}[2]=0 \tag{50}$$

$$\varepsilon=1 \qquad \sum_{\mathrm{i}=0}^{\mathrm{n}}(Y[i])=0 \tag{51}$$

$$\varepsilon=1 \qquad \sum_{\mathrm{i}=1}^{\mathrm{n}}(iY_{ae}[i])=0 \tag{52}$$

$$\varepsilon=1 \qquad \sum_{\mathrm{i}=1}^{\mathrm{n}}(iY_{be}[i])=0 \tag{53}$$

To start with, we choose the unknowns $Y[0]$, $Y_{ae}[1]$ and $Y_{be}[1]$ from the boundary conditions written in accordance with DTM. The other unknown coefficients are written in terms of these unknowns with help from the recurrence relationships (46), (47) and (48) and substituted in the Eqs. (51),(52) and (53). In this way, the following matrix equation is obtained

$$\begin{bmatrix} s(1,1) & s(1,2) & s(1,3) \\ s(2,1) & s(2,2) & s(2,3) \\ s(3,1) & s(3,2) & s(3,3) \end{bmatrix} \begin{Bmatrix} Y[0] \\ Y_{ae}[1] \\ Y_{be}[1] \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{54}$$

For Eq. (54) to have a non-zero solution, the determinant of the matrix s in the above equation must be zero. The minimum value of α which makes the value of the determinant zero is found. After α is found, the critical buckling load is found with the help of Eq. (30d).

4. Lateral stability of multistory structures resting on elastic foundation

Especially in structures with shear walls, which rest on soft soil, the rotations below the shear walls increase the displacements, prolong the structural periods at the level of dynamic analysis and decrease the critical buckling load at the level of stability analysis. For this reason, for the critical buckling load calculation in structures resting on elastic foundations, the rotations below

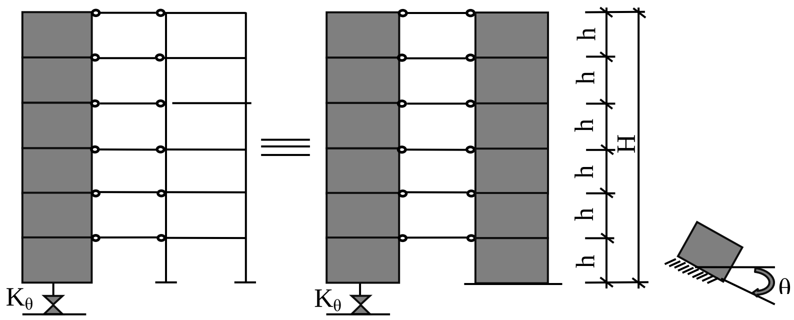

Fig. 4 Equivalent model of Multistory building resting on elastic

the shear walls should necessarily be taken into account.

The behavior of multistory structures resting on elastic foundation, of which the load carrying system consists of frames and shear walls, can be idealized as the behavior of an equivalent sandwich beam as shown in Fig. 4 according to the continuous system calculation model.

In case the rotations below shear walls are taken into account, only the boundary condition (17) differs. In this case, the boundary condition (17) becomes

$$z = H \qquad K_\theta \frac{dy_{ae}}{dz} - EI\frac{d^2 y_{ae}}{dz^2} = 0 \tag{55}$$

To make the given boundary condition dimensionless, it is written as

$$\varepsilon = 1 \qquad \frac{HK_\theta}{EI}\frac{dy_{ae}}{d\varepsilon} - \frac{d^2 y_{ae}}{d\varepsilon^2} = 0 \tag{56}$$

In Eq. (50), if we define

$$\frac{HK_\theta}{EI} = p \tag{57}$$

then

$$p\frac{dy_{ae}}{d\varepsilon} - \frac{d^2 y_{ae}}{d\varepsilon^2} = 0 \tag{58}$$

5. The procedural steps of the method:

The implementation phase of the DTM method presented in this study are briefly provided below:

(1) The rigidities *K*, *D*, *GA* and *EI* are calculated using the relevant equations,

(2) *m*, *t* and *r* values are calculated using formula (30),

(3) The matrix Eq. (49) is formed,

(4) With the help of α, the critical buckling load *qH* is found from Eq. (30d).

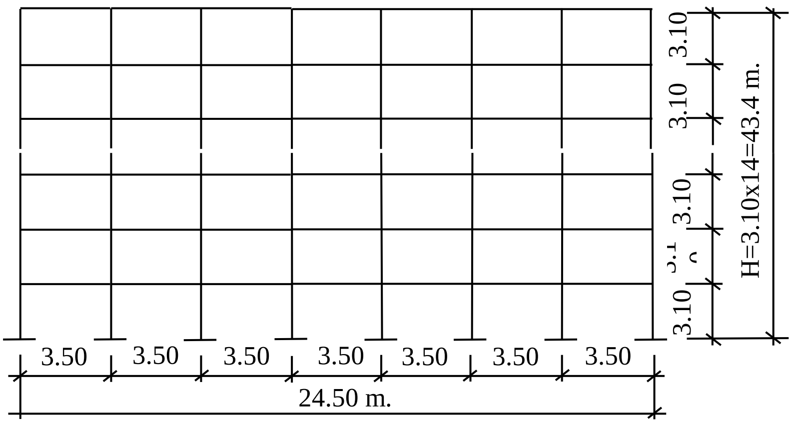

Fig. 5 Multistory planar frame (example 1)

Table 1 The parameters used in the calculation with DTM

EI	1,250,000 kNm^2
GA	20,833,333.33 kN
K	411,135.0851 kN
D	3,858,750,000 kNm^2
m	619.518
r	0.201
t	31,392.667
α	817.7004

6. Numerical examples

In this part, three examples of different features are solved using DTM and compared using FEM. The solution using DTM is carried out with the help of a program prepared in MATLAB. In the analysis with SAP2000 shear walls are modelled with simplified model (frame) and the axial loads are considered at floor levels.

6.1 Example 1

A 14-story structure of which the load carrying system consists of a plane frame, which rests on a rigid foundation and whose cross-section is shown in Fig. 5 is considered. In the plane frame considered, the axle separation in x-direction is 3.50 m, story height at all floors is 3.10 m, the dimension of all columns is 0.50 m/0.50 m and the dimension of all beams is 0.25 m/0.50 m, modulus of elasticity is 3×10^7 kN/m^2, Poisson ratio is 0.2 and the shape factor is 1.2.

In practice, the critical buckling load of the structure under its own weight is calculated both with the DTM method using the package software and also using SAP2000 structural analysis software which finds the solution using the FEM. In the analysis with SAP2000, the axial loads are considered at floor levels. The parameters used in the calculation of critical buckling load with the DTM are given in Table 1 and the comparative results for the critical buckling load obtained using the DTM and FEM are given in Table 2.

Table 2 Comparison of the critical buckling loads calculated with the DTM and FEM (Example 1)

Critical buckling load	DTM	SAP2000	[(DTM-FEM)/FEM].%
	542.656 MN	497.589 MN	9.05

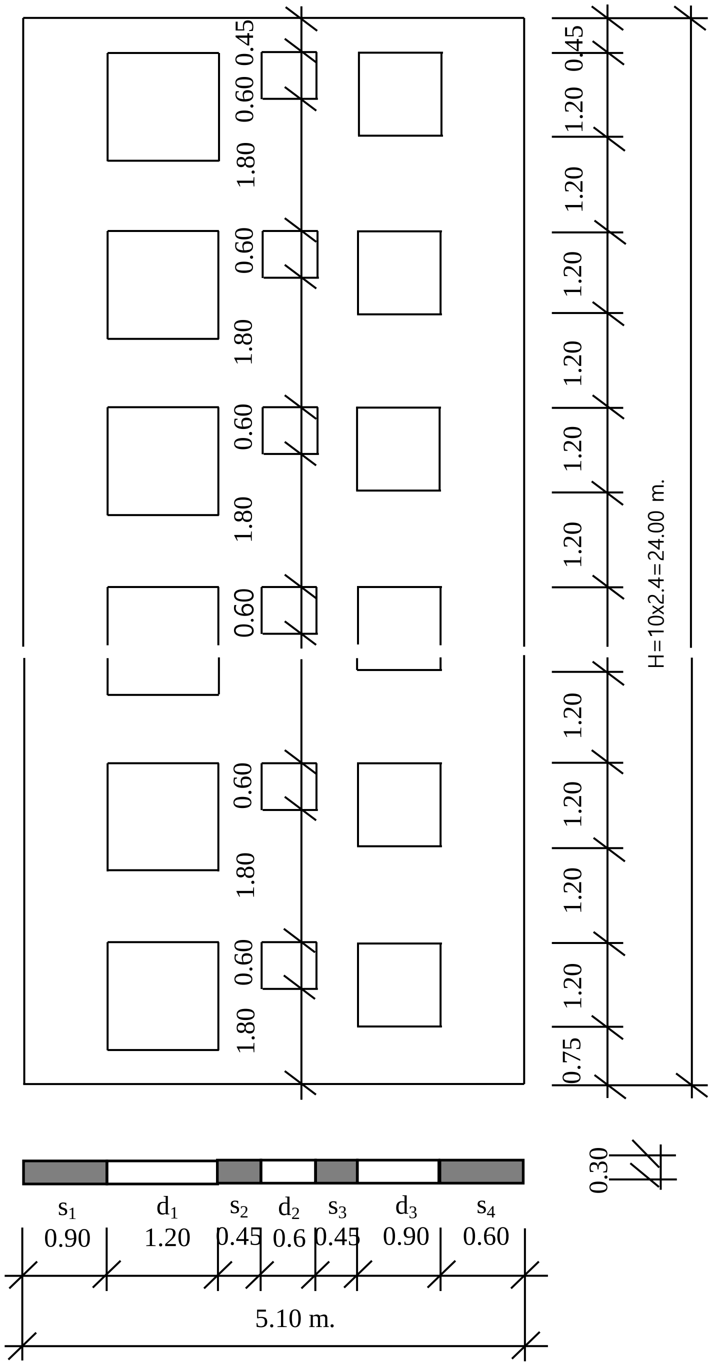

Fig. 6 (example 2)

Table 3 Parameters used in the calculation with the DTM (Example 2)

EI	845,437.500 kNm^2
GA	7,500,000.000 kN
K	1,252,622.829 kN
D	65,746,687.500 kNm^2
m	853.417
r	10.974
t	5,109.780
α	451.825

Table 4 Comparison of the critical buckling loads calculated with the DTM and FEM (Example 2)

	DTM	SAP2000	[(DTM-FEM)/FEM].%
Critical buckling load	663.177 MN	617.747 MN	7.35

6.2 Example 2

For the coupled shear wall which was formed with inspiration from Colunga and Hernandez study (Colunga and Hernandez 2015), whose cross-section and shape are given in Fig. 6 and which has irregular openings and rests on a rigid foundation, the thickness of the coupled shear wall is given as 0.30 m, story height at floors as 2.40 m, the modulus of elasticity as 3×10^7 kN/m^2, Poisson's ratio as 0.2 and the shape factor as 1.2.

The critical buckling load of the coupled shear wall given under its own weight is calculated both with the DTM and with the FEM using SAP2000. In the analysis with SAP2000, the coupled shear wall is modelled with frame elements.

The parameters used in the calculation of the critical buckling load with the DTM are collectively shown in Table 3.

The comparative results for the critical buckling load obtained using the DTM and FEM are given in Table 4.

As can be seen in Table 4, the results obtained from DTM are sufficiently convergent with the results obtained from FEM.

6.3 Example 3

The 20-story building whose load-carrying system consists of solid shear wall-frames and whose cross-section, appearance and dimensions are shown in Fig. 7 is considered. In the structure considered, the axle separation in the x-direction is 4 m in the frame and 6.50 m in between the shear wall and the frame, the story height at all floors is 3.00 m, the dimension of all columns is 0.30 m/0.60 m, the dimension of all beams is 0.25 m/0.50 m, the dimension of the shear wall is 0.50 m/4.00 m, Poisson's ratio is 0.2, the shape factor is 1.2 and the modulus of elasticity is 3×10^7 kN/m^2.

The critical buckling load of this system whose load-carrying system consists of shear wall-frames under its own weight is calculated for different soil classes both with the DTM and with the FEM. In the analysis with SAP2000, the shear wall is modelled with shell elements.

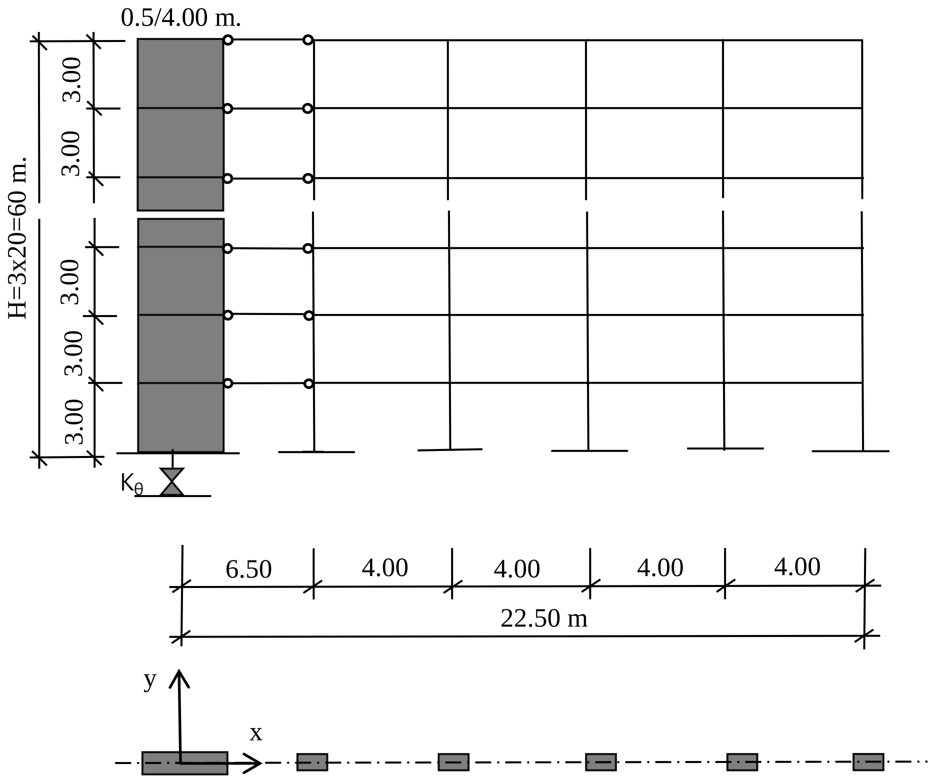

Fig. 7 Wall frame sytem (example 3)

Table 5 Rotation spring constants for various soils (Nadjai and Johnson 1998)

Soil Type	K_θ (kN.m/rad)
Rigid foundation	∞
Dense sand	1.56×10^7
Sand with 60% density	8.9×10^6
Sand with 15% density	2.2×10^6

Table 6 The parameters used in the calculation with the DTM

	Rigid foundation	Dense sand	Sand with 60% density	Sand with 15% density
EI	80,810,000 kNm^2	80,810,000 kNm^2	80,810,000 kNm^2	80,810,000 kNm^2
GA	30,208,333.330 kN	30,208,333.330 kN	30,208,333.330 kN	30,208,333.330 kN
K	231,171.598 kN	231,171.598 kN	231,171.598 kN	231,171.598 kN
D	864×10^6 kNm^2	864×10^6 kNm^2	864×106 kNm^2	864×106 kNm^2
m	10.298	10.298	10.298	10.298
r	0.963	0.963	0.963	0.963
t	1345.749	1345.749	1345.749	1345.749
p	∞	11.583	6.608	1.633
α	27.493	24.288	22.591	17.848

Table 7 Comparison of the critical buckling loads calculated with the DTM and FEM (Example 3)

	Soil Type	DTM	SAP2000	[(DTM-FEM)/FEM].%
Critical buckling load	Rigid foundation	617. 141 MN	563.859 MN	9.45
	Dense sand	545.198 MN	502.908 MN	8.41
	Sand with 60% density	507.105 MN	471.721 MN	7.50
	Sand with 15% density	400.638 MN	385.373 MN	3.97

The spring coefficients for the soil classes considered are given in Table 5. The values given in the table are taken from the Reference (Nadjai and Johnson 1998).

The dimensionless coefficients necessary for the DTM calculation of the system considered is given in Table 6.

In Table 7, the comparative results obtained from the DTM and from the FEM are collectively given.

As can be seen in the table, as the soil loosens, the critical buckling load decreases, as expected.

7. Conclusions

In this article, the DTM method is proposed for the calculation of the critical buckling load in multistory structures whose structural characteristics are uniform throughout the structure height. The rotations below the foundation in shear wall elements are also taken into account in the study. The examples solved shows that the results obtained from the DTM are in sufficient conformity with the results obtained from the FEM. The solutions of DTM are always larger than FEM ones The calculations of stiffness and rigidities (equivalent shear rigidity, global rigidity) of the governing differential equations generate the errors. The DTM method provides easy and fast solutions. Through considering the structure as a continuous system for the solution, the structural behavior is represented with minimal number of parameters. As a conclusion, the method presented can be relied upon to both easily understand the behavior of the structure and check the results obtained using the FEM.

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