Structural Engineering and Mechanics, *Vol. 57, No. 4 (2016) 617-639*
DOI: http://dx.doi.org/10.12989/sem.2016.57.4.617

Thermo-mechanical postbuckling of symmetric S-FGM plates resting on Pasternak elastic foundations using hyperbolic shear deformation theory

Abdelbaki Chikh[1], Ahmed Bakora[1], Houari Heireche[2], Mohammed Sid Ahmed Houari[3,4], Abdelouahed Tounsi*[1,2,3,4] and E.A. Adda Bedia[1,4]

[1]Material and Hydrology Laboratory, Faculty of Technology, Civil Engineering Department, University of Sidi Bel Abbes, Algeria
[2]Laboratoire de Modélisation et Simulation Multi-échelle, Université de Sidi Bel Abbés, Algeria
[3]Laboratoire des Structures et Matériaux Avancés dans le Génie Civil et Travaux Publics, Université de Sidi Bel Abbes, Faculté de Technologie, Département de Génie Civil, Algeria
[4]Algerian National Thematic Agency of Research in Science and Technology (ATRST), Algeria

(Received January 21, 2015, Revised November 30, 2015, Accepted January 11, 2016)

Abstract. In this work, an analytical formulation based on both hyperbolic shear deformation theory and stress function, is presented to study the nonlinear post-buckling response of symmetric functionally graded plates supported by elastic foundations and subjected to in-plane compressive, thermal and thermo-mechanical loads. Elastic properties of material are based on sigmoid power law and varying across the thickness of the plate (S-FGM). In the present formulation, Von Karman nonlinearity and initial geometrical imperfection of plate are also taken into account. By utilizing Galerkin procedure, closed-form expressions of buckling loads and post-buckling equilibrium paths for simply supported plates are obtained. The effects of different parameters such as material and geometrical characteristics, temperature, boundary conditions, foundation stiffness and imperfection on the mechanical and thermal buckling and post-buckling loading capacity of the S-FGM plates are investigated.

Keywords: functionally graded materials; post-buckling; hyperbolic shear deformation theory; elastic foundation; imperfection

1. Introduction

Functionally Graded Material (FGM) is relatively novel technology employed in components subjected to high temperature. Laminated composite materials allow design flexibility to obtain a desirable stiffness and strength through the choice of lamination system. Laminated composite structures generally subjected to stress concentrations and because of discontinuities in material characteristics failures seen in laminated composites in the form of debonding, matrix cracking, and adhesive bond separation. FGM can support such problems because of continuous change of material characteristics from one surface to the other, especial in thick direction. Thus, these new

*Corresponding author, Professor, E-mail: tou_abdel@yahoo.com

http://www.techno-press.org/?journal=sem&subpage=8 ISSN: 1225-4568 (Print), 1598-6217 (Online)

materials are selected to utilize in structural members of aircraft, aerospace vehicles, nuclear plants as well as various temperature shielding structures often employed in industries (Shahrjerdi *et al.* 2011, Benachour *et al.* 2011, Tounsi *et al*. 2013a, Ould Larbi *et al.* 2013, Bouderba *et al*. 2013, Golmakani 2013, Chakraverty and Pradhan 2014a, b, Hadji *et al.* 2014, Zidi *et al*. 2014, Mantari and Granados 2015, Chakraverty and Pradhan 2015, Sallai *et al.* 2015, Pradhan and Chakraverty 2015a, b, c, Rad 2015, Mahi *et al.* 2015, Ait Atmane *et al.* 2015, Ait Yahia *et al.* 2015, Attia *et al.* 2015, Ebrahimi and Dashti 2015, Kar and Panda 2015, Bouchafa *et al.* 2015, Akbaş 2015, Arefi 2015, Darılmaz 2015, Bellifa *et al.* 2016).

The buckling and post-buckling of rectangular functionally graded plates has been a topic of investigation in solid mechanics for more than a century. Birman (1995) studied the buckling behavior of FGM hybrid composite plates based on the multi-cell model. Yang and Shen (2003) used a perturbation technique along with one-dimensional differential quadrature approximation and Galerkin procedure to examine the post-buckling response of fully clamped FGM rectangular plates based on the classical plate theory under the transverse and in-plane loads. The first order shear deformation theory is employed by Wu (2004) to determine the analytical expressions of critical buckling temperatures for simply supported FGM plates. Woo *et al*. (2005) utilized a mixed Fourier series solution to obtain analytical solutions to investigate the post-buckling response of moderately thick FG plates and shallow shells under edge compressive load and specified temperature field. Prakash *et al.* (2008) employed an eight-nodded C0 shear flexible quadrilateral plate element to examine the nonlinear bending/pseudo-post-buckling response of FGM plates based on the first order shear deformation theory under thermo-mechanical load. Matsunaga (2009) proposed a 2 D global higher-order deformation theory for thermal buckling of FG plates. He computed the critical buckling temperatures of a simply supported FG plate under uniformly and linearly distributed temperatures. Moradi and Mansouri (2012) studied the thermal buckling behavior of rectangular composite laminated plates by using the Differential Quadrature method. Using different shear deformation theories, Daneshmehra *et al.* (2013) studied the post-buckling behavior of FG beams. Ahmed (2014) discussed the post-buckling of FG sandwich beams using a consistent higher order theory. A novel refined hyperbolic shear deformation theory was developed by El Meiche *et al.* (2011) utilizing the Navier's solution method for the buckling and free vibration responses of FG sandwich plates. Bourada *et al.* (2012) presented a new four-variable refined plate theory for thermal buckling analysis of FG sandwich plates. Duc and Cong (2013) investigated the post-buckling behaviors of sigmoid FG plates subjected in-plane compressive, thermal and thermo-mechanical loads using Reddy's third order shear deformation plate theory. A refined and simple nth-order shear deformation theory is developed by Yaghoobi and Fereidoon (2014) for the buckling analysis of FG plates resting on elastic foundation. Swaminathan and Naveenkumar (2014) proposed an analytical formulation for the stability analysis of simply supported FG sandwich plates based on two higher-order refined computational models. Ait Amar Meziane *et al*. (2014) developed an efficient and simple refined shear deformation theory for the vibration and buckling of exponentially graded material sandwich plate resting on elastic foundations under various boundary conditions. Yaghoobi *et al*. (2014) studied the nonlinear vibration and post-buckling behaviors of FG beams resting on nonlinear elastic foundation and subjected to thermo-mechanical loading. Khalfi *et al*. (2014) proposed a refined and simple shear deformation theory for thermal buckling of solar FG plates on elastic foundation. Bakora and Tounsi (2015) studied the thermo-mechanical post-buckling response of thick P-FGM plates resting on elastic foundations. Nguyen *et al.* (2015) developed a refined higher-order shear deformation theory for bending, vibration and buckling analysis of FG sandwich plates. Tagrara *et*

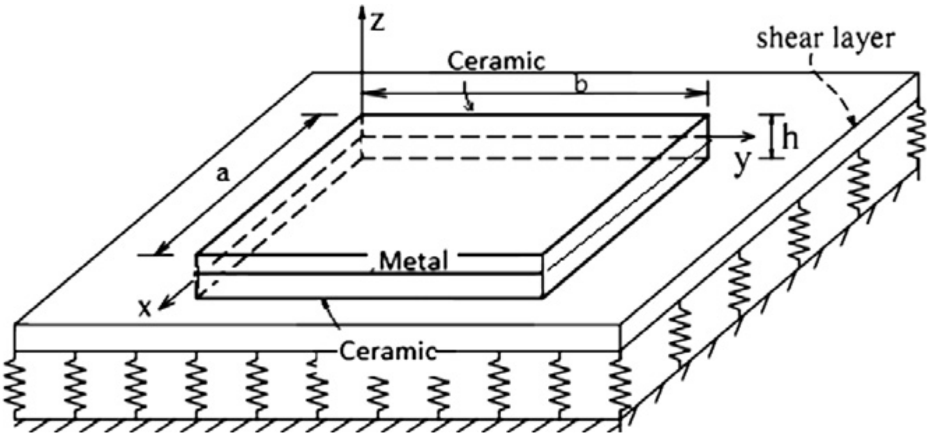

Fig. 1 Symmetrical S-FGM plate on elastic foundation

al. (2015) studied the bending, buckling and vibration responses of FG carbon nanotube-reinforced composite beams. Tebboune *et al.* (2015) investigated the thermal buckling analysis of FG plates resting on elastic foundation based on an efficient and simple trigonometric shear deformation theory. Bouguenina *et al*. (2015) studied the thermal buckling of FGM plates with variable thickness using a finite differential method.

In this work we research the post-buckling responses of thick FG plates resting on elastic foundations and subjected to axial compressive, thermal and thermo-mechanical loads using a new hyperbolic shear deformation plate theory, stress function for FGM plate with Sigmoid power law distribution of the volume of constituents (S-FGM), considering into account geometrical nonlinearity, initial geometrical imperfection, temperature and the plate-foundation interaction is represented by Pasternak model. Analytical expressions of buckling loads and post-buckling load-deflection curves for simply supported FG plates are determined by Galerkin technique. A parametric study is considered to assess the influences of geometrical and material properties, temperature, boundary conditions, foundation stiffness and imperfection on the buckling and post-buckling of the symmetric S-FG plates.

2. Material properties of symmetric S-FGM plate

In this work, a symmetrical rectangular S-FGM plate that consists of three layers fabricated with functionally graded ceramic and metal materials and is midplane-symmetric. The outer surface layers of the plate are ceramic-rich, but the midplane layer is fully metallic. A coordinate system (x, y, z) is considered in which (x, y) plane is the midplane of the plate and z is thickness direction ($-h/2 \leq z \leq h/2$) as indicated in Fig. 1.

The material properties P of S-FG plate such as the modulus of elasticity E and the coefficient of thermal expansion α, vary in the thickness direction z according to a linear rule of mixture as

$$P(z) = P_m V_m(z) + P_c V_c(z) \tag{1}$$

where P_m and P_c are the corresponding properties of the metal and ceramic, respectively. The volume fractions of metal and ceramic, V_m and V_c, are assumed as

$$V_m(z)=\begin{cases}\left(\dfrac{2z+h}{h}\right)^N & \textbf{for } -h/2\le z\le 0\\ \left(\dfrac{-2z+h}{h}\right)^N & \textbf{for } 0\le z\le h/2\end{cases}\quad ;\quad V_c(z)=1-V_m(z) \tag{2}$$

where $\boldsymbol{N}$ is the power law index which takes the value greater or equal to zero.

The reaction-deflection relation of Pasternak foundation is expressed by (Besseghier *et al.* 2015, Ait Atmane *et al.* 2016, Bounouara *et al.* 2016, Salima *et al.* 2016)

$$f_e=k_w w-k_g\nabla^2 w \tag{3}$$

where $\nabla^2=\partial^2/\partial x^2+\partial^2/\partial y^2$, w is the transverse displacement of the plate, k_w is Winkler foundation modulus and k_g is the shear layer foundation stiffness of Pasternak model.

3. Theoretical formulations

Based on a new hyperbolic shear deformation theory, the following displacement field is assumed

$$u(x,y,z)=u_0(x,y)-z\frac{\partial w(x,y)}{\partial x}+\Psi(z)\ \phi_x(x,y) \tag{4a}$$

$$v(x,y,z)=v_0(x,y)-z\frac{\partial w(x,y)}{\partial y}+\Psi(z)\ \phi_y(x,y) \tag{4b}$$

$$w(x,y,z)=w_0(x,z) \tag{4c}$$

with

$$\Psi(z)=z-\frac{1}{10}\left(\frac{h\sinh\left(10\dfrac{z}{h}\right)}{\cosh(5)}\right)+\frac{h}{100} \tag{4d}$$

here u_0, v_0, w_0, ϕ_x, ϕ_y are five unknown displacements of the midplane of the plate.

The non-linear von Karman strain-displacement equations are as follows

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}+z\begin{Bmatrix}k_x\\ k_y\\ k_{xy}\end{Bmatrix}+\Psi(z)\begin{Bmatrix}\eta_x\\ \eta_y\\ \eta_{xy}\end{Bmatrix},\quad \begin{Bmatrix}\gamma_{xz}\\ \gamma_{yz}\end{Bmatrix}=\Psi'(z)\begin{Bmatrix}\gamma_{xz}^0\\ \gamma_{yz}^0\end{Bmatrix} \tag{5}$$

where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} u_{0,x} + (w_{0,x})^2/2 \\ v_{0,x} + (w_{0,y})^2/2 \\ u_{0,y} + v_{0,x} + w_{0,x} w_{0,y} \end{Bmatrix}, \quad \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} = \begin{Bmatrix} -w_{0,xx} \\ -w_{0,yy} \\ -2w_{0,xy} \end{Bmatrix}, \quad \begin{Bmatrix} \eta_x \\ \eta_y \\ \eta_{xy} \end{Bmatrix} = \begin{Bmatrix} \phi_{x,x} \\ \phi_{y,y} \\ \phi_{x,y} + \phi_{y,x} \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{xz}^0 \\ \gamma_{yz}^0 \end{Bmatrix} = \begin{Bmatrix} \phi_x \\ \phi_y \end{Bmatrix}, \tag{6}$$

The linear constitutive relations of an S-FG plate can be expressed as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 & 0 & 0 \\ \nu & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1-\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1-\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_x - \alpha\,\Delta T \\ \varepsilon_y - \alpha\,\Delta T \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \tag{7}$$

where ΔT is temperature rise from stress free initial state or temperature difference between two surfaces of the S-FG plate.

By utilizing the virtual work principle to minimize the functional of total potential energy function result in the expressions for the nonlinear equilibrium equations of a perfect plate resting on two parameters elastic foundation as

$$N_{x,x} + N_{xy,y} = 0 \tag{8a}$$

$$N_{xy,x} + N_{y,y} = 0 \tag{8b}$$

$$(M_{x,xx} + 2M_{xy,xy} + M_{y,yy}) + N_x w_{,xx} + 2\, N_{xy} w_{,xy} + N_y w_{,yy} - k_w w + k_g \nabla^2 w = 0 \tag{8c}$$

$$S_{x,x} + S_{xy,y} - Q_x = 0 \tag{8d}$$

$$S_{xy,x} + S_{y,y} - Q_y = 0 \tag{8e}$$

where the force and moment resultants (N, Q, S and M) of the S-FG plate are determined by

$$(N_i,\, M_i,\, S_i) = \int_{-h/2}^{h/2} \sigma_i\,(1, z, \Psi(z))\, dz, \quad (i = x, y, xy) \tag{9a}$$

$$Q_i = \int_{-h/2}^{h/2} \sigma_j\; \Psi'(z)\, dz, \qquad (i = x, y); \;\; (j = xz, yz) \tag{9b}$$

Substitution of Eqs. (5) and (7) into Eq. (9) yields the constitutive relations as

$$\begin{aligned}(N_x,\, M_x, S_x) = \frac{1}{1-\nu^2}&[(E_1, E_2, E_3)(\varepsilon_x^0 + \nu\,\varepsilon_y^0) + (E_2, E_4, E_5)(k_x + \nu\, k_y) \\ &+ (E_3, E_5, E_7)(\eta_x + \nu\,\eta_y) - (1+\nu)(\Phi_1, \Phi_2, \Phi_3)]\end{aligned} \tag{10a}$$

$$(N_y, M_y, S_y) = \frac{1}{1-\nu^2}[(E_1, E_2, E_3)(\varepsilon_y^o + \nu\,\varepsilon_x^0) + (E_2, E_4, E_5)(k_y + \nu\,k_x) + (E_3, E_5, E_7)(\eta_y + \nu\,\eta_x) - (1+\nu)(\Phi_1, \Phi_2, \Phi_3)] \tag{10b}$$

$$(N_{xy}, M_{xy}, S_{xy}) = \frac{1}{2(1+\nu)}[(E_1, E_2, E_3)\gamma_{xy}^0 + (E_2, E_4, E_5)k_{xy} + (E_3, E_5, E_7)\eta_{xy}] \tag{10c}$$

$$(Q_x, Q_y) = \frac{1}{2(1+\nu)} E_8\left(\gamma_{xz}^0, \gamma_{yz}^0\right) \tag{10d}$$

where

$$(E_1, E_3, E_4, E_5, E_7) = \int_{-h/2}^{h/2} (1, \Psi, z^2, z\Psi, \Psi^2)E(z)dz,\; E_2 = \int_{-h/2}^{h/2} z\,E(z)dz = 0,$$
$$E_8 = \int_{-h/2}^{h/2} (\Psi'(z))^2\, E(z)\,dz \tag{11a}$$

$$(\Phi_1, \Phi_2, \Phi_3) = \int_{-h/2}^{h/2} (1, z, \Psi)E(z)\,\alpha(z)\,\Delta T(z)\,dz \tag{11b}$$

The last three equations of Eq. (8) may be expressed into two equations in terms of variables w_0 and $\phi_{x,x}+\phi_{y,y}$ by substituting Eqs. (6) and (10) into Eqs. (8c)-(8e). Subsequently, elimination of the variable $\phi_{x,x}+\phi_{y,y}$ from two the resulting equations, conducts to the following system of equilibrium equations

$$N_{x,x} + N_{xy,y} = 0 \tag{12a}$$

$$N_{xy,x} + N_{y,y} = 0 \tag{12b}$$

$$(D_1 D_3 - D_2^2)\nabla^6 w - D_1 D_4\; \nabla^4 w - D_3 \nabla^2 (N_x w_{,xx} + 2N_{xy} w_{,xy} + N_y w_{,yy} - k_w w + k_g \nabla^2 w) + D_4\,(N_x w_{,xx} + 2N_{xy} w_{,xy} + N_y w_{,yy} - k_w w + k_g \nabla^2 w) = 0 \tag{12c}$$

where

$$D_1 = \frac{E_4}{(1-\nu^2)}, \quad D_2 = \frac{E_5}{(1-\nu^2)}, \quad D_3 = \frac{E_1\,E_7 - E_3^2}{E_1(1-\nu^2)}, \quad D_4 = \frac{E_8}{2(1+\nu)} \tag{13}$$

For an imperfect S-FG plate, Eqs. (12) are modified into form as

$$(D_1 D_3 - D_2^2)\nabla^6 w - D_1 D_4 \nabla^4 w - D_3 \nabla^2 [\,f_{,yy}(w_{,xx} + w^*_{,xx}) - 2\,f_{,xy}(w_{,xy} + w^*_{xy}) + f_{,xx}(w_{,yy} + w^*_{,yy}) - k_w w + k_g \nabla^2 w] + D_4\,[\,f_{,yy}(w_{,xx} + w^*_{,xx}) - 2\,f_{,xy}(w_{,xy} + w^*_{xy}) + f_{,xx}(w_{,yy} + w^*_{,yy}) - k_w w + k_g \nabla^2 w] = 0 \tag{14}$$

in which $w^*(x,y)$ is a known function denoting initial small imperfection of the plate. Note that Eq. (14) gets a complicated form under the hyperbolic shear deformation theory which includes the 6th-order partial differential term $\nabla^6 w_0$. Also, $f\,(x,y)$ is stress function defined by

$$N_x = f_{,yy}\ ,\ N_y = f_{,xx}\ ,\ N_{xy} = -f_{,xy} \tag{15}$$

The geometrical compatibility equation for an imperfect plate is expressed as

$$\varepsilon^0_{x,yy} + \varepsilon^0_{y,xx} - \gamma^0_{xy,xy} = w^2_{0,xy} - w_{0,xx}w_{0,yy} + 2\,w_{0,xy}w^*_{0,xy} - w_{0,xx}w^*_{0,yy} - w_{0,yy}w^*_{0,xx} \tag{16}$$

From the constitutive relations (10) and Eq. (15) one can write

$$\begin{aligned}(\varepsilon^0_x, \varepsilon^0_y) &= \frac{1}{E_1}[(f_{,yy}, f_{,xx}) - \nu\,(f_{,xx}, f_{,yy}) - E_3\,(\eta_x, \eta_y) + \Phi_1\,(1,1)]\\ \gamma^0_{xy} &= -\frac{1}{E_1}[2\,(1+\nu)\,f_{,xy} + E_3\,\eta_{xy}]\end{aligned} \tag{17}$$

Substituting Eq. (17) into Eq. (16), the compatibility equation of an imperfect S-FG plate becomes

$$\nabla^4 f - E_1\,(w^2_{0,xy} - w_{0,xx}\,w_{0,yy} + 2w_{0,xy}\,w^*_{0,xy} - w_{0,xx}\,w^*_{0,yy} - w_{0,yy}\,w^*_{0,xx}) = 0 \tag{18}$$

It is noted that Eqs. (14) and (18) are nonlinear equations employed to study the stability of thick S-FG plates resting on elastic foundations subjected to mechanical, thermal and thermo-mechanical loads.

Three cases of boundary conditions are considered in this work, referred to as Cases 1, 2 and 3 (Librescu and Lin 1997, Lin and Librescu 1998).

- Case 1: Four edges of the plate are simply supported and freely movable (FM). The associated boundary conditions are

$$w_0 = N_{xy} = \phi_y = M_x = S_x = 0, \quad N_x = N_{x0}\ \textbf{at}\ x = 0, a \tag{19a}$$

$$w_0 = N_{xy} = \phi_x = M_y = S_y = 0, \quad N_y = N_{y0}\ \textbf{at}\ y = 0, b \tag{19b}$$

- Case 2: Four edges of the plate are simply supported and immovable (IM). In this case, boundary conditions are

$$w_0 = u_0 = \phi_y = M_x = S_x = 0, \quad N_x = N_{x0}\ \textbf{at}\ x = 0, a \tag{20a}$$

$$w_0 = v_0 = \phi_x = M_y = S_y = 0, \quad N_y = N_{y0}\ \textbf{at}\ y = 0, b \tag{20b}$$

- Case 3: All edges are simply supported. Two edges x=0, a are freely movable and subjected to compressive load in the x direction, whereas the remaining two edges y=0, b are unloaded and immovable. For this case, the boundary conditions are defined as

$$w_0 = N_{xy} = \phi_y = M_x = S_x = 0, \quad N_x = N_{x0}\ \textbf{at}\ x = 0, a \tag{21a}$$

$$w = v = \phi_x = M_y = S_y = 0, \quad N_y = N_{y0}\ \textbf{at}\ y = 0, b \tag{21b}$$

N_{y0} are axial compressive loads at movable edges (i.e., Case 1 and the first of Case 3) or are fictitious compressive edge loads at immovable edges (i.e., Case 2 and the second of Case 3).

The proposed solutions of w and f respecting boundary conditions (17)-(19) are considered to be (Librescu and Lin 1997, Lin and Librescu 1998)

$$\left(w, w^*\right) = (W, \mu\, h)\sin(\lambda_m x)\sin(\delta_n y) \tag{22a}$$

$$\begin{aligned} f &= A_1\cos(2\lambda_m x) + A_2\cos(2\delta_n y) + A_3\sin(\lambda_m x)\sin(\delta_n y) \\ &\quad + \frac{1}{2}N_{x0}y^2 + \frac{1}{2}N_{y0}x^2 \end{aligned} \tag{22b}$$

$$\phi_x = B_1\cos(\lambda_m x)\sin(\delta_n y), \quad \phi_y = B_2\sin(\lambda_m x)\cos(\delta_n y) \tag{22c}$$

where $\lambda_m = m\pi/a$, $\delta_n = n\pi/b$, m, n are odd numbers, W is amplitude of the deflection and μ is imperfection parameter. The coefficients A_i (i=1,2,3) are determined by substitution of Eqs. (22a, b) into Eq. (18) as

$$A_1 = \frac{E_1\delta_n^2}{32\lambda_m^2}W(W + 2\mu\, h), \quad A_2 = \frac{E_1\lambda_m^2}{32\delta_n^2}W(W + 2\mu\, h), \quad \mathrm{A}_3 = 0 \tag{23}$$

Using Eqs. (6) and (10) in Eqs. (8d, e) and substituting Eqs. (22a, c) into the resulting equations, the coefficients B_1 and B_2 are determined as

$$B_1 = \frac{a_{12}a_{23} - a_{22}a_{13}}{a_{12}^2 - a_{11}a_{22}}W, \quad B_2 = \frac{a_{12}a_{13} - a_{11}a_{23}}{a_{12}^2 - a_{11}a_{22}}W \tag{24}$$

in which

$$(a_{11}, a_{22}, a_{12}) = D_3(\lambda_m^2, \delta_m^2, \nu\lambda_m\delta_n) + \frac{1-\nu}{2}D_3(\lambda_m^2, \delta_m^2, \lambda_m\delta_n) + D_4(1,1,0) \tag{25a}$$

$$(a_{13}, a_{23}) = -\,D_2(\lambda_m^3 + \lambda_m\delta_n^2,\, \delta_n^3 + \delta_n\lambda_m^2) \tag{25b}$$

Then, setting Eqs. (22a, b) into Eq. (14) and employing the Galerkin method for the resulting equation yield

$$\begin{aligned} &(-(D_1D_3 - D_2^2)(\lambda_m^2 + \delta_n^2)^3 - D_1D_4(\lambda_m^2 + \delta_n^2)^2 - [k_w + k_g(\lambda_m^2 + \delta_n^2)]\,[(D_3(\lambda_m^2 + \delta_n^2) + D_4])\,W \\ &- \frac{E_1}{16}(D_3(\lambda_m^4\delta_n^2 + \lambda_m^2\delta_n^4 + \lambda_m^6 + \delta_n^6) + D_4(\lambda_m^4 + \delta_n^4)) \times W(W + \mu h)\,(W + 2\mu h) \\ &- (D_3(\lambda_m^2 + \delta_n^2) + D_4) \times (N_{x0}\,\lambda_m^2 + N_{y0}\,\delta_n^2)\,(W + \mu h) = 0 \end{aligned} \tag{26}$$

This equation will be employed to investigate the buckling and post-buckling responses of thick S-FG plates under mechanical, thermal and thermo-mechanical loads.

3.1 Mechanical post-buckling analysis

A simply supported symmetric S-FG plate with all movable edges is considered and this plate is supported by elastic foundations and subjected to axial edge compressive loads (F_x, F_y) uniformly distributed on edges x=0, a and y=0, b, respectively. In this case, pre-bucking force resultants are (Samsam Shariat and Eslami 2007)

$$N_{x0} = -F_x h,\ N_{y0} = -F_y h \tag{27}$$

and Eq. (26) leads to

$$F_x = e_1^1 \frac{W}{h(W+\mu h)} + e_2^1 \frac{W}{h}(W + 2\mu h) \tag{28}$$

where

$$e_1^1 = \frac{(D_1 D_3 - D_2^2)(\lambda_m^2 + \delta_n^2)^3 + D_1 D_4 (\lambda_m^2 + \delta_n^2)^2}{(\lambda_m^2 + \beta\ \delta_n^2)[D_3(\lambda_m^2 + \delta_n^2) + D_4]} + \frac{[K_w + K_g a^2 (\lambda_m^2 + \delta_n^2)]\, D_1}{a^4 (\lambda_m^2 + \beta\ \delta_n^2)} \tag{29a}$$

$$e_2^1 = \frac{E_1}{16(\lambda_m^2 + \beta\ \delta_n^2)[D_3(\lambda_m^2 + \delta_n^2) + D_4]} \times [D_3(\lambda_m^4 \delta_n^2 + \lambda_m^2 \delta_n^4 + \lambda_m^6 + \delta_n^6) + D_4(\lambda_m^4 + \delta_n^4)] \tag{29b}$$

in which

$$\beta = F_y / F_x\ ,\ K_w = \frac{k_w a^4}{D_1}\ ,\ K_g = \frac{k_g a^2}{D_1} \tag{30}$$

For a perfect FG plate, Eq. (28) reduces to an equation from which buckling compressive load may be determined as $F_{xb} = e_1^1$.

3.2 Thermal post-buckling analysis

A simply supported symmetric S-FG plate with all immovable edges is assumed here. The plate is also supported by an elastic foundation and under temperature environments or subjected to through the thickness temperature gradient. The in-plane condition on immovability at all edges, i.e., u_0=0 at x=0, a and v_0=0 at y=0, b, is given in an average sense as (Tung and Duc 2010)

$$\int_0^b \int_0^a \frac{\partial u_0}{\partial x} dx dy = 0\ ,\ \int_0^a \int_0^b \frac{\partial v_0}{\partial x} dy dx = 0 \tag{31}$$

From Eqs. (6) and (10) one can determine the following expressions in which Eq. (15) and imperfection have been introduced

$$\frac{\partial u_0}{\partial x} = \frac{1}{E_1}(f_{,yy} - \nu f_{,xx}) - \frac{E_3}{E_1}\phi_{x,x} - \frac{1}{2} w_{,x}^2 - w_{,x} w_{,x}^* + \frac{\Phi_1}{E_1} \tag{32a}$$

$$\frac{\partial v_0}{\partial y} = \frac{1}{E_1}(f_{,xx} - \nu f_{,yy}) - \frac{E_3}{E_1}\phi_{y,y} - \frac{1}{2} w_{,y}^2 - w_{,y} w_{,y}^* + \frac{\Phi_1}{E_1} \tag{32b}$$

Introduction of Eq. (22) into Eq. (32) and then the result into Eq. (31) give

$$N_{x0} = -\frac{\Phi_1}{1-\nu} + \frac{4E_3}{mn\pi^2(1-\nu^2)}(\lambda_m B_1 + \nu\delta_n B_2)W + \frac{E_1}{8(1-\nu^2)}(\lambda_m^2 + \nu\delta_n^2)W(W+2\mu h) \tag{33a}$$

$$N_{y0} = -\frac{\Phi_1}{1-\nu} + \frac{4E_3}{mn\pi^2(1-\nu^2)}(\nu\lambda_m B_1 + \delta_n B_2)W + \frac{E_1}{8(1-\nu^2)}(\nu\lambda_m^2 + \delta_n^2)W(W+2\mu h) \tag{33b}$$

When the deflection dependence of fictitious edge loads is ignored, i.e., W=0, Eq. (33) becomes

$$N_{x0} = N_{y0} = -\frac{\Phi_1}{1-\nu} \tag{34}$$

Substituting Eq. (33) into Eq. (26) yields the expression of thermal parameter as

$$\begin{aligned}\frac{\Phi_1}{1-\nu} &= \left[\frac{(D_1D_3 - D_2^2)(\lambda_m^2+\delta_n^2)^2 + D_1D_4(\lambda_m^2+\delta_n^2)}{D_3(\lambda_m^2+\delta_n^2)+D_4} + \frac{k_w + k_g(\lambda_m^2+\delta_n^2)}{(\lambda_m^2+\delta_n^2)}\right]\frac{W}{W+\mu h} \\ &+ \frac{4E_3}{mn\pi^2(1-\nu^2)(\lambda_m^2+\delta_n^2)} \times (\lambda_m^3 B_1 + \nu\lambda_m^2\delta_n B_2 + \nu\lambda_m\delta_n^2 B_1 + \delta_n^3 B_2)W \\ &+ \left[\frac{E_1\left[D_3(\lambda_m^4\delta_n^2 + \lambda_m^2\delta_n^4 + \lambda_m^6 + \delta_n^6) + D_4(\lambda_m^4+\delta_n^4)\right]}{16\left[D_3(\lambda_m^2+\delta_n^2)+D_4\right](\lambda_m^2+\delta_n^2)} + \frac{E_1\left[(\lambda_m^4 + 2\nu\lambda_m^2\delta_n^2 + \delta_n^4)\right]}{8(1-\nu^2)(\lambda_m^2+\delta_n^2)}\right]W(W+2\mu h)\end{aligned} \tag{35}$$

3.3 Uniform temperature rise

The symmetric S-FG plate is exposed to temperature environments uniformly raised from stress free initial state T_i to final value T_f, and temperature change $\Delta T = T_f - T_i$ is assumed to be independent from thickness variable. The thermal parameter Φ_1 is obtained from Eq. (11b), and substitution of the result into Eq. (35) yields

$$\Delta T = e_1^2 \frac{W}{W+\mu h} + e_2^2 W + e_3^2 W(W+2\mu h) \tag{36}$$

where

$$e_1^2 = \frac{(1-\nu)}{L\left[D_3(\lambda_m^2+\delta_n^2)+D_4\right]} \times \left[(D_1D_3 - D_2^2)(\lambda_m^2+\delta_n^2)^2 + D_1D_4(\lambda_m^2+\delta_n^2)\right] + \frac{\left[K_w + K_g a^2(\lambda_m^2+\delta_n^2)\right](1-\nu)D_1}{a^4 L(\lambda_m^2+\delta_n^2)}, \tag{37a}$$

$$e_2^2 = \frac{4E_3}{mnL\pi^2(1+\nu)(\lambda_m^2+\delta_n^2)} \times (\lambda_m^3 B_1 + \nu\lambda_m^2 \delta_n B_2 + \nu\lambda_m \delta_n^2 B_1 + \delta_n^3 B_2) \tag{37b}$$

$$\begin{aligned} e_3^2 = {} & \frac{E_1(1-\nu)}{16L(\lambda_m^2+\delta_n^2)[D_3(\lambda_m^2+\delta_n^2)+D_4]} \times [D_3(\lambda_m^4\delta_n^2+\lambda_m^2\delta_n^4+\lambda_m^6+\delta_n^6)+D_4(\lambda_m^4+\delta_n^4)] \\ & + \frac{E_1(\lambda_m^4+2\nu\lambda_m^2\delta_n^2+\delta_n^4)}{8L(1+\nu)(\lambda_m^2+\delta_n^2)} \end{aligned} \tag{37c}$$

in which

$$L = E_m\alpha_m + \frac{E_m\alpha_{cm}+E_{cm}\alpha_m}{N+1} + \frac{E_{cm}\alpha_{cm}}{2N+1} \tag{38}$$

By Setting μ=0, Eq. (36) leads to an equation from which buckling temperature change of the perfect symmetric S-FG plates may be obtained as $\Delta T_b = e_1^2$.

3.4 Thermo-mechanical post-buckling analysis

The symmetric S-FG plate resting on elastic foundation is uniformly compressed by F_x on two movable edges x=0, a and simultaneously exposed to elevated temperature environments or subjected to nonlinear temperature distribution. The two edges y=0, b are assumed to be immovable. In this case, $N_{x0}=-F_x h$ and fictitious compressive load on immovable edges is obtained by setting the second of Eq. (32) in the second of Eq. (31) as

$$N_{y0} = \nu N_{x0} - \Phi_1 + \frac{4\delta_n}{mn\pi^2}\left[E_3 B_2\right]W + \frac{E_1\delta_n^2}{8}W(W+2\mu h) \tag{39}$$

Then, N_{x0} and N_{y0} are placed in Eq. (26) to give

$$F_x = e_1^3\frac{W}{h(W+\mu h)} + e_2^3\frac{W}{h} + e_3^3 W\frac{(W+2\mu h)}{h} + \frac{L\delta_n^2\Delta T}{h(\lambda_m^2+\nu\delta_n^2)} \tag{40}$$

where the coefficients e_1^3; e_2^3; e_3^3 are defined as follows

$$e_1^3 = \frac{(D_1D_3-D_2^2)(\lambda_m^2+\delta_n^2)^3 + D_1D_4(\lambda_m^2+\delta_n^2)^2}{(\lambda_m^2+\nu\delta_n^2)[D_3(\lambda_m^2+\delta_n^2)+D_4]} + \frac{[K_w+K_g a^2(\lambda_m^2+\delta_n^2)]D_1}{a^4(\lambda_m^2+\nu\delta_n^2)}, \tag{41a}$$

$$e_2^3 = \frac{4\delta_n^3}{mn\pi^2(\lambda_m^2+\nu\delta_n^2)} \times \left[E_3 B_2\right] \tag{41b}$$

$$\begin{aligned} e_3^3 = {} & \frac{E_1}{16(\lambda_m^2+\nu\delta_n^2)[D_3(\lambda_m^2+\delta_n^2)+D_4]} \times [D_3(\lambda_m^4\delta_n^2+\lambda_m^2\delta_n^4+\lambda_m^6+\delta_n^6)+D_4(\lambda_m^4+\delta_n^4)] \\ & + \frac{E_1\delta_n^4}{8(\lambda_m^2+\nu\delta_n^2)} \end{aligned} \tag{41c}$$

Eqs. (28), (36) and (40) are explicit expressions of load-deflection curves for thick symmetric

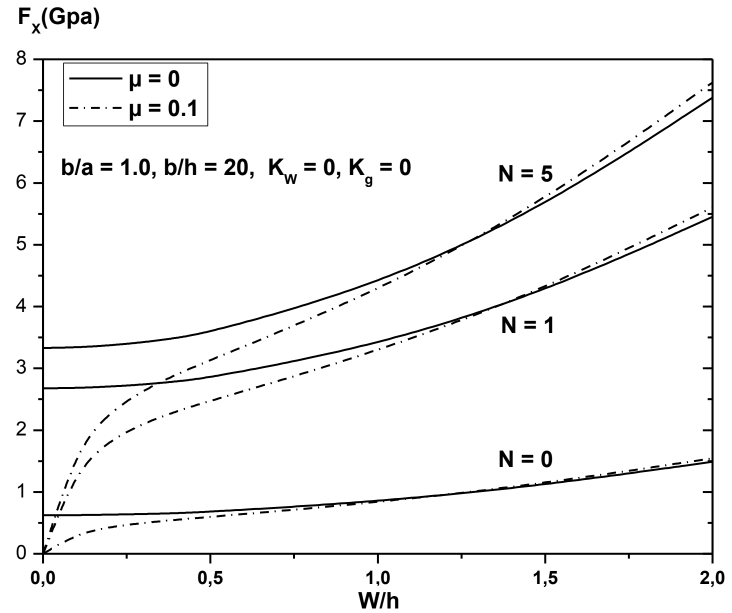

Fig. 2 Effects of the power law index *N* on the post-buckling of symmetrical S-FG plates under in-plane compressive load (all FM edges)

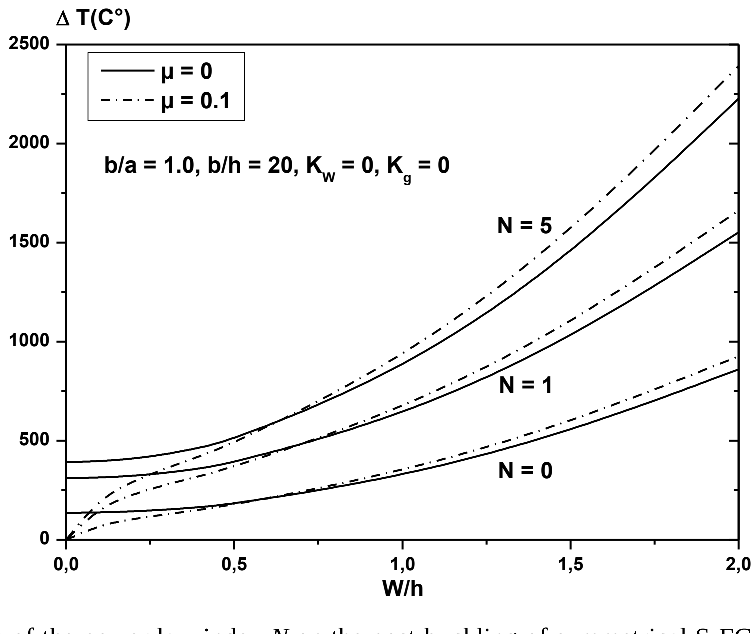

Fig. 3 Effects of the power law index *N* on the post-buckling of symmetrical S-FG plates under uniform temperature rise (all IM edges)

S-FG plates supported by Pasternak elastic foundations and subjected to axial compressive, thermal and thermo-mechanical loads, respectively.

4. Results and discussion

In this part of the work, numerical results are analyzed for checking the accuracy of the present formulation in predicting the buckling and post-buckling responses of thick symmetric S-FG plates supported by elastic foundations. A square ceramic-metal plate with the following properties is considered:

- $E_m = 70\,\text{GPa}$, $\nu_m = 0.3$, $\alpha_m = 23\times10^{-6}\,°\text{C}^{-1}$
- $E_c = 380\,\text{GPa}$, $\nu_c = 0.3$, $\alpha_c = 7.4\times10^{-6}\,°\text{C}^{-1}$

In this case, the buckling of perfect plates will be initiated for m=n=1, and these values of half waves are also utilized to present graphically load-deflection equilibrium paths for both perfect and imperfect plates. In figures, *W/h* indicates the dimensionless maximum deflection and the S-FG plate-foundation interaction is ignored, unless otherwise stated.

Influences of the power law index *N* on the post-buckling of S-FG plates subjected to in-plane compressive load and uniform temperature rise are demonstrated in Figs. 2 and 3. It is shown that the mechanical load and the thermal resistance become considerably important if the volume *N* increases or the percentage of ceramic increases. Decreasing the power law index *N* leads to a strong drop of both critical buckling loads and post-buckling carrying capacity.

Figs. 4 and 5 show a comparison between the mechanical and thermal post-buckling load-deflection curves determined by the present formulation and the third order shear deformation theory presented by Duc and Cong (2013) with various volume fractions of the S-FG plate. The results demonstrate an excellent agreement between the present formulation and the third order shear deformation theory used by Duc and Cong (2013). The results also show us that the imperfect plate has a better mechanical and thermal loading capacity than those of the perfect plate. The obtained results in Fig. 4 are presented numerically in Table 1. It is noted that these

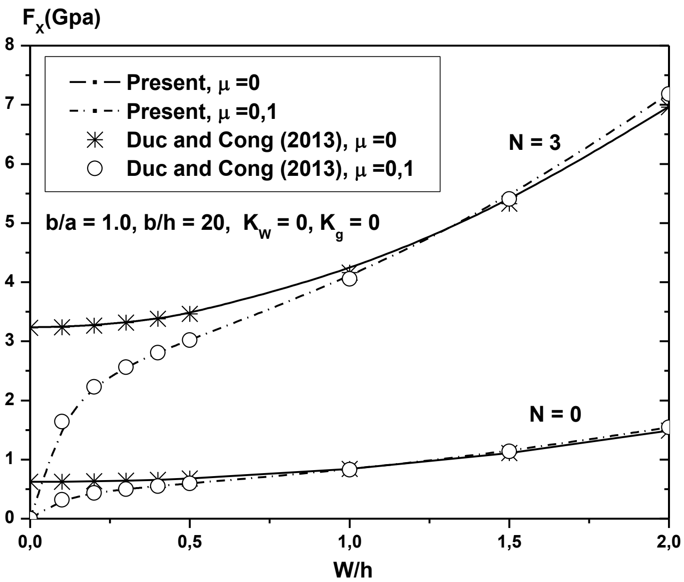

Fig. 4 Comparisons of mechanical post-buckling load-deflection curves for S-FG plates with various of volume fractions *N*

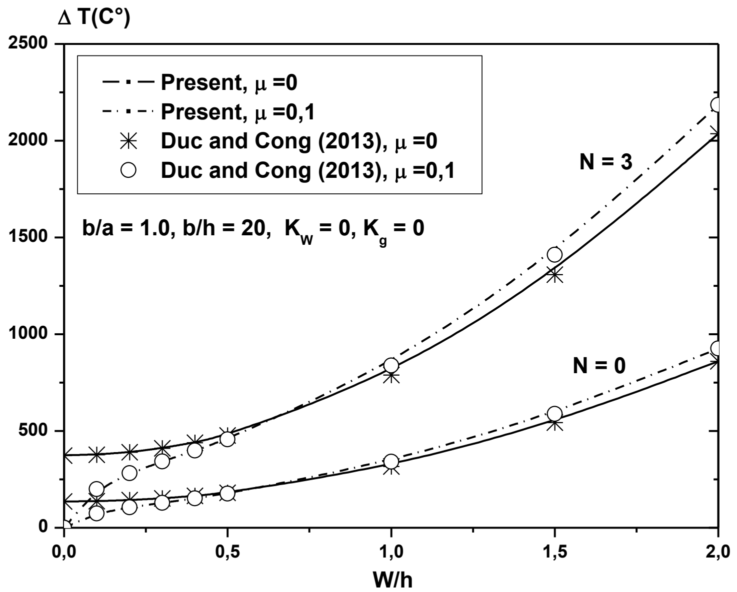

Fig. 5 Comparisons of thermal post-buckling load-deflection curves for S-FG plates with various of volume fractions *N*

Table 1 Comparisons of mechanical post-buckling loads for S-FG plates with various of volume fractions *N*

$\frac{W}{h}$	Theory	N=0		N=3	
		μ=0	μ=0,1	μ=0	μ=0,1
0	Present	0,62411	0	3,23613	0
	Duc and Cong (2013)(*)	0,62387	0	3,22944	0
0.1	Present	0,62627	0,31853	3,24546	1,64605
	Duc and Cong (2013)(*)	0,62603	0,31841	3,23876	1,64271
0.2	Present	0,63274	0,43334	3,27345	2,23206
	Duc and Cong (2013)(*)	0,63251	0,43319	3,26675	2,2276
0.3	Present	0,64354	0,50047	3,32009	2,56704
	Duc and Cong (2013)(*)	0,6433	0,50047	3,3134	2,56202
0.4	Present	0,65865	0,5511	3,3854	2,81282
	Duc and Cong (2013)(*)	0,65842	0,55091	3,37871	2,80746
0.5	Present	0,67808	0,59565	3,46937	3,02332
	Duc and Cong (2013)(*)	0,67785	0,59546	3,46268	3,01774
1	Present	0,84001	0,82645	4,16911	4,06152
	Duc and Cong (2013)(*)	0,83977	0,82623	4,16242	4,05543
1.5	Present	1,10988	1,13564	5,33534	5,41298
	Duc and Cong (2013)(*)	1,10964	1,13542	5,32865	5,40671
2	Present	1,4877	1,54434	6,96807	7,18716
	Duc and Cong (2013)(*)	1,48746	1,54411	6,96138	7,18079

(*) Values computed by authors

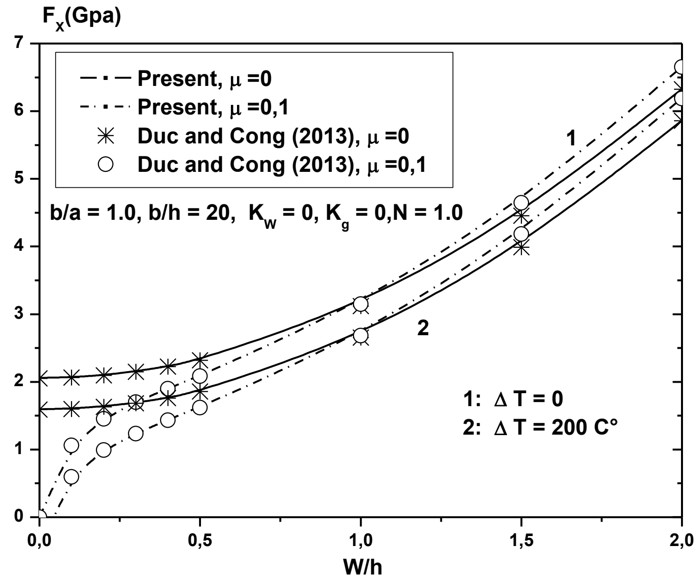

Fig. 6 Comparisons of mechanical post-buckling load-deflection curves for S-FG plates with different temperature ΔT

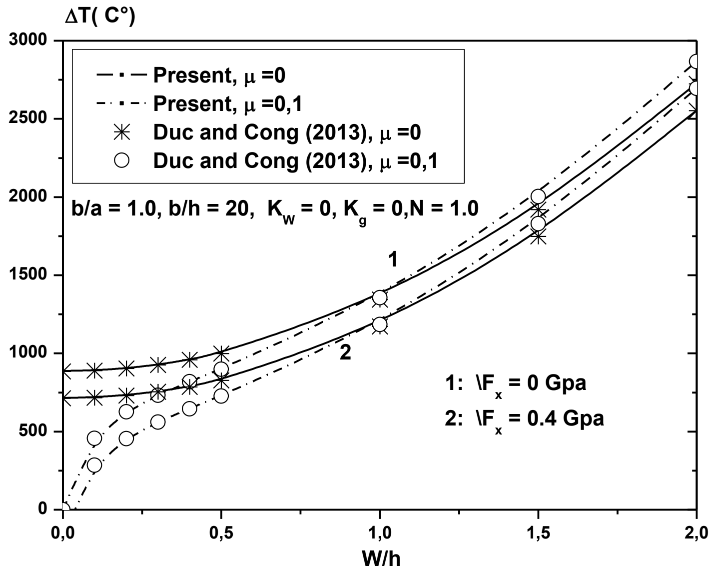

Fig. 7 Comparisons of mechanical post-buckling load-deflection curves for S-FG plates with different mechanical loads F_x

results are computed by both the present theory and the method proposed by Duc and Cong (2013). One can conclude that an excellent agreement between two methods is confirmed.

A comparison study between the mechanical and thermal post-buckling load-deflection curves determined by the present formulation and the third order shear deformation theory presented by

Duc and Cong (2013) is carried out by considering various of thermal and mechanical loads and the results are shown in Figs. 6 and 7. Obviously, with the same power law index, the critical loadings of post-buckling of the S-FG plate are different. Also, similar to above two figures, the critical mechanical and thermal loadings for the present theory are identical to those of the third order shear deformation.

The effect of initial imperfections on post-buckling of S-FG plate under uniaxial compressive force (all FM edges) and under uniform temperature (all IM edges) is indicated in Figs. 8 and 9. Fig. 8 proves us that the critical compressive forces diminish with μ in the region of the small bending. By against, it increases with μ in the other region of the large bending, meaning the higher bending-load curve (i.e., the better loading ability). Fig. 9 indicates us that an initial imperfection has a significant effect on the thermal resistance of S-FG at the threshold value of the bending.

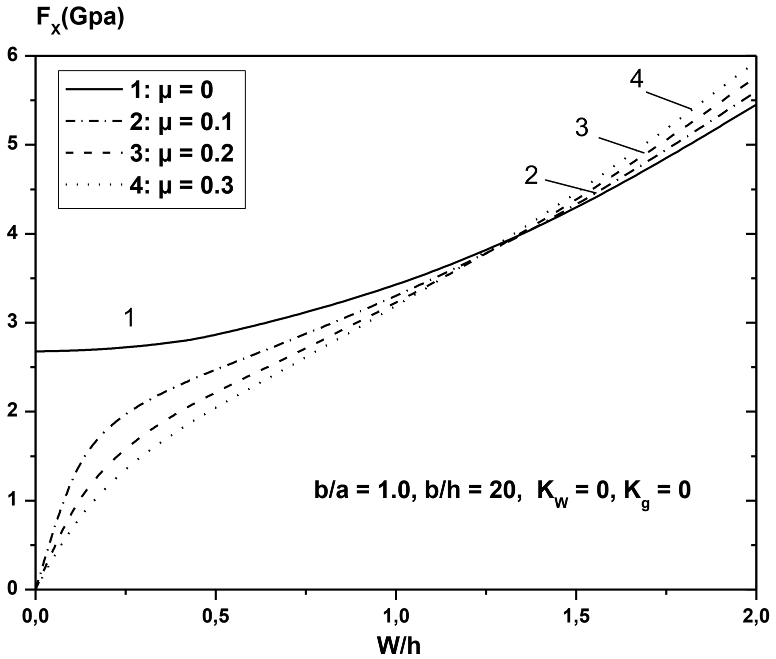

Fig. 8 The effect of imperfections on the post-buckling of symmetrical S-FG plates under uniaxial compressive force (all FM edges)

Figs. 10 and 11 illustrate a considerable effect of elastic foundations on the post-buckling of S-FG plate under uniaxial compressive load (all FM edges) and uniform temperature (all IM edges). The influence of Pasternak foundation K_g on the critical compressive forces and the thermal resistance of S-FG is important than the Winkler foundation K_w.

The thermo-mechanical stability investigation has been carried out by using Eq. (40). Figs. 12 and 13 have been prepared under the supposition of the third boundary conditions (Case 3) for the FM edges x=0, a and IM edges y=0, b which are simultaneously under the compressive uniform loading on the edge x=0, a. Fig. 12 proves the influence of the temperature gradient of the

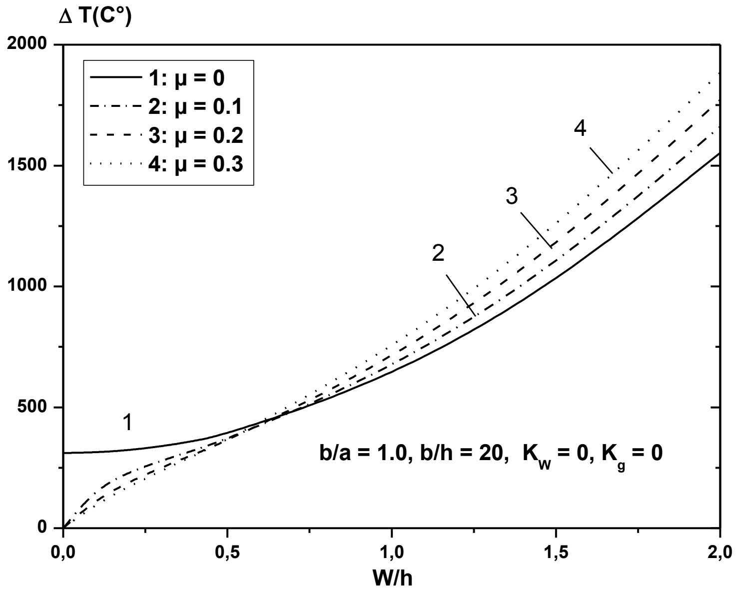

Fig. 9 The effect of imperfections on the post-buckling of symmetrical S-FG plates under uniform temperature rise (all IM edges)

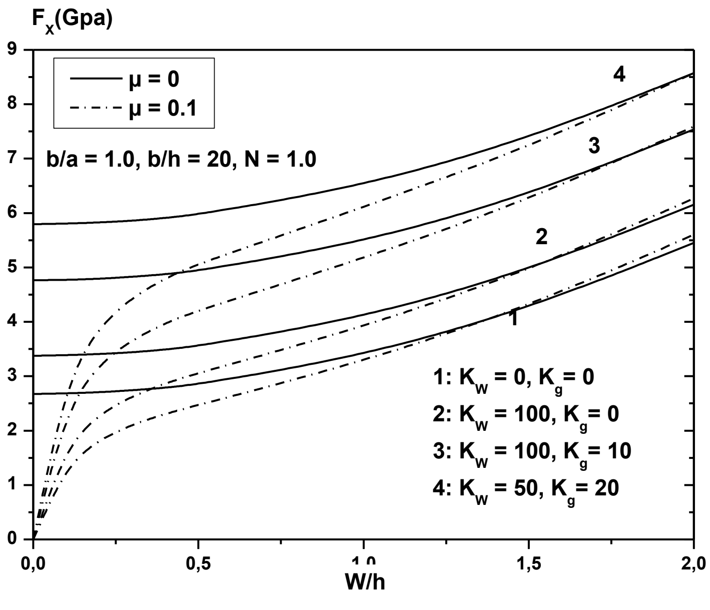

Fig. 10 Effects of the elastic foundations on the post-buckling of symmetrical S-FG plates under uniaxial compressive load (all FM edges)

surrounding environment on the response of uniaxial compressive force. The inclusion of temperature reduces the loading capacity (for both perfect and imperfect plates) and the imperfect plate obtains a curvature immediately even if there is no mechanical compressive load.

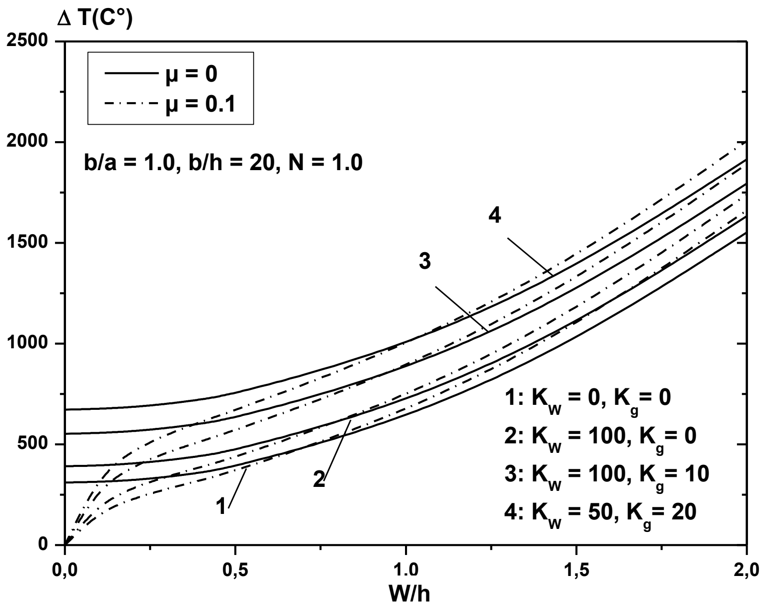

Fig. 11 Effects of the elastic foundations on the post-buckling of symmetrical S-FG plates under uniform temperature rise (all IM edges)

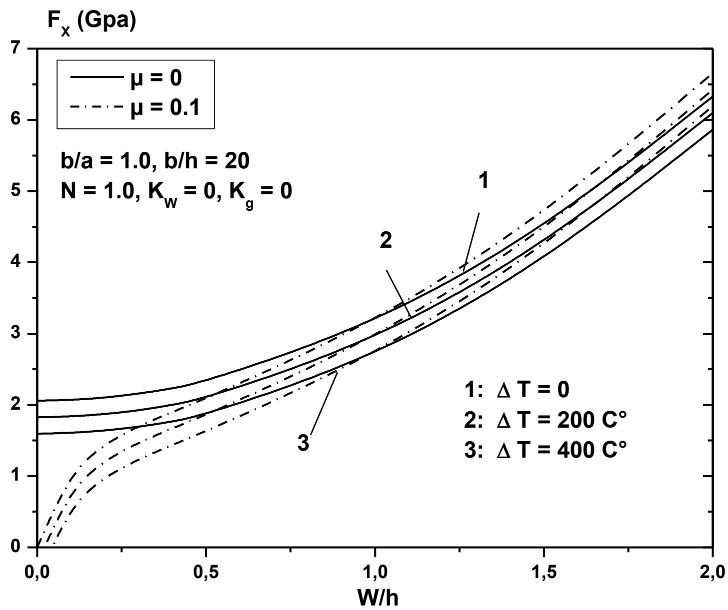

Fig. 12 Effect of temperature field and uniaxial compression on the post-buckling of symmetric S-FG plate under uniform temperature rise (FM on y=0, b; IM on x=0, a)

Fig. 13 shows the buckling and post-buckling response of the S-FG plate under the increased uniform temperature gradient field ΔT and the different values of the uniaxial compressive load

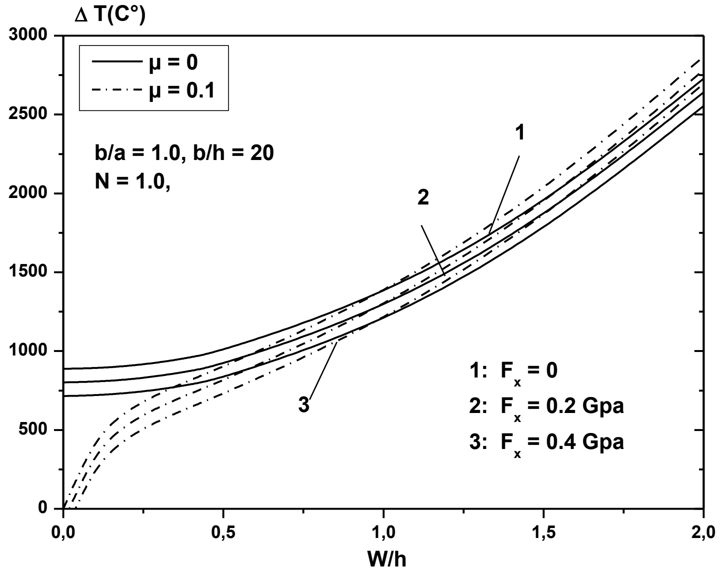

Fig. 13 Effect of temperature gradient and uniaxial compression on the post-buckling of symmetric S-FG. (FM on y=0, b; IM on x=0, a)

F_x. The inclusion of the mechanical loading diminishes the thermal loading capacity of the perfect and imperfect plates.

5. Conclusions

This work presents an analytical formulation to study the postbuckling responses of thick symmetric FG plates supported by Pasternak elastic foundations and subjected to in-plane compressive, thermal and thermomechanical loads. Both a new hyperbolic shear deformation plate theory and stress function are used in the present formulation by taking into consideration Von Karman nonlinearity, initial geometrical imperfection, temperature and Pasternak type elastic foundation. The effects of power law index and geometrical characteristics, temperature, boundary conditions, foundation stiffness and imperfection on the postbuckling loading capacity of the S-FG plates are investigated and discussed. It is concluded that the critical mechanical and thermal loadings for the proposed hyperbolic shear deformation theory are almost identical to those for the third order shear deformation theory and for the postbuclking period of the S-FGM plate, comparing with a perfect plate, an imperfect plate has a better mechanical and thermal loading capacity. An improvement of present formulation will be considered in the future work to account for the thickness stretching effect by employing quasi-3D shear deformation models (Saidi *et al.* 2013, Bousahla *et al.* 2014, Fekrar *et al.* 2014, Hebali *et al.* 2014, Belabed *et al.* 2014, Bourada *et al.* 2015, Meradjah *et al.* 2015, Hamidi *et al.* 2015, Bourada *et al.* 2015, Bennai *et al.* 2015, Bennoun *et al.* 2016), laminated composite plates (Chattibi *et al.* 2015, Draiche *et al.* 2014, Sadoune *et al.* 2014, Ozturk 2015, Kirkland and Uy 2015, Kar *et al.* 2015) and nanostructures (Tounsi *et al.* 2013b, Al-Basyouni *et al.* 2015, Belkorissat *et al.* 2015, Ould Youcef *et al.* 2015, Larbi Chaht *et al.* 2015, Chemi *et al.* 2015).

Acknowledgments

This research was supported by the Algerian National Thematic Agency of Research in Science and Technology (ATRST) and university of Sidi Bel Abbes (UDL SBA) in Algeria.

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