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C^0-type Reddy's theory for composite beams using FEM under thermal loads

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Abstract. To analyze laminated composite and sandwich beams under temperature loads, a C^0-type Reddy's beam theory considering transverse normal strain is proposed in this paper. Although transverse normal strain is taken into account, the number of unknowns is not increased. Moreover, the first derivatives of transverse displacement have been taken out from the in-plane displacement fields, so that the C^0 interpolation functions are only required for the finite element implementation. Based on the proposed model, a three-node beam element is presented for analysis of thermal responses. Numerical results show that the proposed model can accurately and efficiently analyze the thermoelastic problems of laminated composites.

Keywords: C^0-type Reddy's beam theory; thermal responses; laminated composite and sandwich beams; three-node beam element; thermal stresses

1. Introduction

Laminated composite and sandwich structures have been found extensive applications in aerospace, aeronautic, automotive, naval and building structures due to their high strength and specific stiffness as well as low-density. However, composite laminated structures are usually applied in high temperature situations. Rising temperature will induce significant thermal stresses because of different thermal properties of adjacent layers (Shokrieh *et al.* 2013), which can cause the failure of laminated composite and sandwich beams. Consequently, the thermal deformation and stresses become significant parameters for predicting the thermal responses of the laminated composite and sandwich beams.

Available approaches (Rolfs *et al.* 1998, Khdeir and Reddy 1991, Matsunaga 2004, Wu *et al.* 2010) have been used to analyze the thermal responses of composite laminates. A natural choice of model for the analysis of such structures is the three-dimensional models (Savoia and Reddy 1995), which require huge computational cost. In order to overcome the above problem, Wu and Tauchert (1980a, b) studied the thermal deformation and stress results in symmetric and antisymmetric laminates using the classical laminated plate theory (CLPT) based on the Kirchhoff hypothesis. However, the classical theory is inadequate for accurate prediction of thermal

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responses for multilayered plates owing to neglecting the effects of transverse shear and normal strains (Naganarayana *et al.* 1997). To consider the effects of transverse shear strains, the first-order shear deformation theory (FSDT) has been developed to analyze the thermal behaviors of laminated structures (Reddy and Hsu 1980). However, transverse shear strains in FSDT are assumed to be constant across the thickness direction and the shear correction factors have to be used to adjust the transverse shear stiffness for laminated composites. As a result, the accuracy of solutions of FSDT depends on the shear correction factor. To overcome the drawbacks of the first-order shear deformation theory, Reddy (1984) developed the third-order theory (Reddy's theory) which can satisfy the free conditions of the transverse shear stresses on the upper and lower surfaces. Reddy's theory accounts not only for transverse shear strains, but also for a parabolic distribution of the transverse shear strains along the thickness direction. Thus, there is no need to use shear correction factors. Subsequently, Reddy's theory is widely used in the study of composite laminates because of its efficiency and simplicity (Aydogdu 2006, Li and Zhu 2009, Xiang *et al.* 2011). For example, finite element models (Nayak *et al.* 2002, Sheikh and Chakrabarty 2003) based on Reddy's theory have been proposed for bending, vibration and thermal expansion analysis of composite laminates. Nayak *et al.* (2002) used a finite element model based on Reddy's theory to calculate the natural frequencies and loss factors of laminated composite and sandwich plates. He and Yang (2014) built a finite element model to analyze buckling response of two-layer composite beams using Reddy's theory. Based on Reddy's theory, Kadoli *et al.* (2008) studied the static behavior of functionally graded metal-ceramic (FGM) beams under ambient temperature. Simsek (2010) analyzed fundamental frequency of functionally graded beams by using different higher-order beam theories which include Reddy's theory.

However, the displacement fields of Reddy's theory involve the first derivative of transverse displacement, so the C^1 interpolation functions are required during finite element implementation. Therefore, it is difficult to construct higher-order elements. To avoid using C^1 interpolation functions, Bhar and Satsangi (2011) developed a C^0-type Reddy's theory and analyzed the bending problem of the laminated composite and sandwich structures. The first derivatives of transverse displacement in the displacement field have been removed, so that the C^0 interpolation functions are only required. In order to extend C^0-type Reddy's theory for the analysis of thermoelastic problems, a C^0-type Reddy's beam theory considering the transverse normal thermal strain has been proposed in this paper. Subsequently, to verify the accuracy and efficiency of the present model, a three-node beam element based on the proposed model is presented for thermal expansion and bending analysis of laminated composite and sandwich beams. In the finite element implementation, it is found that although transverse normal strain is considered, the displacement variables are not increased since thermal loads could be included in the generalized force vector. Moreover, if temperature field does not vary (T=0), the proposed theory can automatically return to the C^0-type Reddy's beam theory (Bhar and Satsangi 2011). However, the accuracy of the present approach for analysis of the thermal expansion and bending problems has been verified in comparison with the models which neglect transverse normal strains.

2. Theoretical formulations

In order to consider the transverse normal strain without increasing additional displacement variables, the transverse normal thermal strain induced by the temperature variation is introduced in transverse displacement field. The transverse normal thermal strain caused by temperature is given by Wu *et al.* (2013)

$$\varepsilon_{zT}^{k} = \alpha_{3}^{k} \Delta T\left(x,z\right) \tag{1}$$

where ε_{zT}^{k} represents transverse normal thermal strain due to thermal loading in which z denotes transverse coordinate through the thickness and the superscript k represents the layer number of the laminated beam; α_3^k is the transverse thermal expansion coefficient at the kth layer; ΔT is the rise in temperature with respect to the reference temperature.

The temperature field is distributed through the thickness of laminates as follows

$$\Delta T\left(x,z\right) = f(z)T(x) \tag{2}$$

in which $f\,(z)$ describes the temperature profiles through the thickness direction, $T(x)$ is the in-plane temperature field.

Integrating ε_{zT}^{k} across the thickness direction, the transverse normal thermal deformation at kth ply can be obtained as follows

$$w_T^k(x,z) = \Omega^k T\left(x\right) \tag{3}$$

where $\Omega^k(z) = \int \alpha_3^k f\left(z\right) dz$.

2.1 The third-order theory (TOT)

The in-plane displacement components using Taylor's expansions are expanded as cubic functions of the thickness coordinate z, whereas transverse displacement is assumed constant through thickness (Kant and Swaminathan 2002). Displacement field of the third-order theory (TOT) can be expressed as

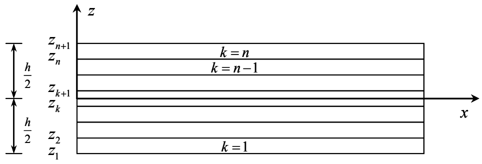
$$\begin{aligned} u &= u_0 + z u_1 + z^2 u_2 + z^3 u_3 \\ w &= w_0 \end{aligned} \tag{4}$$

2.2 Reddy's Theory (RT)

Based on the third-order theory (TOT), Reddy's theory (Reddy 1980) is proposed by using the free conditions of the transverse shear stresses on the upper and the lower surfaces. Reddy's theory can be written as

$$\begin{aligned} u &= u_0 + \Phi_1 u_1 + \Phi_2 \frac{\partial w_0}{\partial x} \\ w &= w_0 \end{aligned} \tag{5}$$

where $\Phi_1 = z - \dfrac{4z^3}{3h^2}, \Phi_2 = -\dfrac{4z^3}{3h^2}$, h is thickness of beam.



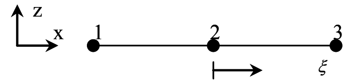


Fig. 1 Schematic figure of laminated beam segment and three-node beam element

2.3 C^0-type Reddy's beam theory (RT-C^0)

By satisfying the free conditions of the transverse shear stresses on the upper and the lower surfaces and removing the first derivatives of transverse displacement, C^0-type Reddy's beam theory (Bhar and Satsangi 2011) can be given by

$$\begin{aligned} u &= u_0 + \psi_1 u_1 + \psi_2 u_3 \\ w &= w_0 \end{aligned} \tag{6}$$

where $\psi_1=z$, $\psi_2=z^3$.

2.4 C^0-type Reddy's beam theory considering transverse normal thermal strain (TRTC)

Adding transverse normal thermal deformation w_T to transverse displacement field of RT-C^0, a C^0-type Reddy's beam theory considering transverse normal thermal strain is given by

$$\begin{aligned} u &= u_0 + z u_1 + z^2 u_2 + z^3 u_3 \\ w^k &= w_0 + w_T^k \end{aligned} \tag{7}$$

Transverse shear strain can be expressed as

$$\gamma_{xz}^k = \frac{\partial w^k}{\partial x} + \frac{\partial u}{\partial z} = \frac{\partial w_0}{\partial x} + \frac{\partial w_T^k}{\partial x} + u_1 + 2zu_2 + 3z^2 u_3 \tag{8}$$

Employing the free conditions of the transverse shear stresses on the upper and the lower surfaces and eliminating the first derivatives of transverse displacement, the final displacement fields in the present model can be given by

$$\begin{aligned} u &= u_0 + \phi_1 u_1 + \phi_2 u_3 + \phi_3 \frac{\partial T}{\partial x} \\ w^k &= w_0 + w_T^k \end{aligned} \tag{9}$$

where $\phi_1 = z$, $\phi_2 = z^3$, $\phi_3 = \frac{z^2}{h}\left(\frac{\Omega^1(z_1) - \Omega^n(z_{n+1})}{2}\right)$, z_i is shown in Fig. 1.

In Eq. (9), it is found that if temperature field does not vary (T=0), the proposed theory can automatically return to C^0-type Reddy's beam theory (Bhar and Satsangi 2011). From linear strain-displacement relationship, the strains of C^0-type Reddy's beam theory considering transverse normal thermal strain can be written as

$$\begin{aligned} \varepsilon_x &= \frac{\partial u_0}{\partial x} + \phi_1 \frac{\partial u_1}{\partial x} + \phi_2 \frac{\partial u_3}{\partial x} + \phi_3 \frac{\partial^2 T}{\partial x^2} \\ \gamma_{xz} &= \frac{\partial \phi_1}{\partial z} u_1 + \frac{\partial \phi_2}{\partial z} u_3 + \frac{\partial \phi_3}{\partial z} \frac{\partial T}{\partial x} + \frac{\partial w_0}{\partial x} + \frac{\partial {w_T}^k}{\partial x} \end{aligned} \tag{10}$$

In Eq. (10), it is found that in-plane stresses and transverse shear strains are affected by temperature component. As a result, C^0-type Reddy's beam theory considering transverse normal thermal strain can produce accurate results for analysis of the thermal expansion and bending problems of laminated composites.

3. Constitutive equations

In a common structural axis system, the stress-strain relationships accounting for transverse shear deformation and thermal effects for the kth layer can be given by

$$\begin{Bmatrix} \sigma_x \\ \tau_{xz} \end{Bmatrix}^k = \begin{bmatrix} Q_{11} & 0 \\ 0 & Q_{44} \end{bmatrix}^k \begin{Bmatrix} \varepsilon_x - \alpha_x \Delta T \\ \gamma_{xz} \end{Bmatrix}^k \tag{11}$$

in which ΔT is the rise of temperature; α_x is the linear thermal expansion coefficients in the direction of common structural axes; ${Q_{ij}}^k$ is the transformed material constants for the kth layer.

4. Finite element formulation

The first derivatives of transverse displacement have been taken from the in-plane displacement field in the proposed theory. Thus, the finite element counterparts only require the C^0 interpolation functions. According to Eq. (9), a three-node beam element is developed. Each node contains u_0, u_1, u_3 and w_0, as the nodal degrees of freedom, shown in Fig. 1.

4.1 Three-node beam element

Independent displacement variables involved in the present model are only discretized by using the Lagrangian quadratic shape functions of the three-node beam element. Using the nodal variables and the shape functions, the displacement variables within one element can be expressed as follows

$$u_0 = \sum_{i=1}^{3} N_i u_{0i}, u_1 = \sum_{i=1}^{3} N_i u_{1i}, u_3 = \sum_{i=1}^{3} N_i u_{3i}, w_0 = \sum_{i=1}^{3} N_i w_{0i} \tag{12}$$

where N_i (i=1~3) are the Lagrangian quadratic shape functions defined as:

$$N_1 = \frac{1}{2}\xi(\xi-1), N_2 = 1-\xi^2, N_3 = \frac{1}{2}\xi(\xi+1), \xi = 2\frac{x-x_c}{x_3-x_1}, x_c = \frac{x_1+x_3}{2}.$$

in which ξ is the natural coordinate, which can be found in Fig. 1.

4.2 Strain matrix and stiffness matrix

Using linear strain-displacement relations, the strains can be written as follows

$$\begin{Bmatrix} \varepsilon_x \\ \gamma_{xz} \end{Bmatrix} = \begin{bmatrix} \dfrac{\partial}{\partial x} & 0 \\ \dfrac{\partial}{\partial z} & \dfrac{\partial}{\partial x} \end{bmatrix} \begin{Bmatrix} u \\ w \end{Bmatrix} = B\delta^e + \bar{\varepsilon}_T \tag{13}$$

where the strain matrix B is the same as that of RT-C^0 (Bhar and Satsangi 2011),

$$B = \begin{bmatrix} B_1 & B_2 & B_3 \end{bmatrix},$$

$$\delta^e = \begin{bmatrix} \delta_1^e & \delta_2^e & \delta_3^e \end{bmatrix}^T,$$

$$\bar{\varepsilon}_T = \begin{bmatrix} \phi_3 \dfrac{\partial^2 T}{\partial x^2} \\ \left(\dfrac{\partial \phi_3}{\partial z} + \Omega(z) \right) \dfrac{\partial T}{\partial x} \end{bmatrix},$$

$$\delta_i^e = \begin{bmatrix} u_{0i} & w_{0i} & u_{1i} & u_{3i} \end{bmatrix}$$

$$B_i = \begin{bmatrix} \dfrac{\partial L_i}{\partial x} & 0 & \phi_1 \dfrac{\partial L_i}{\partial x} & \phi_2 \dfrac{\partial L_i}{\partial x} \\ 0 & \dfrac{\partial L_i}{\partial x} & L_i & \dfrac{\partial \phi_2}{\partial z} L_i \end{bmatrix}, (i = 1 \sim 3)$$

The potential energy of the laminated beams under thermal loads can be written as

$$V = \frac{1}{2} \int \varepsilon^T Q \varepsilon dv \tag{14}$$

where Q is the transformed material constant matrix, and $\varepsilon = \{\varepsilon_x - \alpha_x \Delta T \ \gamma_{xz}\}^T$.

Substituting Eqs. (11) and (13) into Eq. (14), the potential energy within one element can be given by

$$V = \frac{1}{2} \int_e \left[B\delta^e - \varepsilon_T \right]^T Q \left[B\delta^e - \varepsilon_T \right] dv \tag{15}$$

where $\varepsilon_T = \tilde{\varepsilon}_T - \bar{\varepsilon}_T, \tilde{\varepsilon}_T = \{\alpha_x \Delta T \quad 0\}^T$.

The equilibrium equation can be obtained by using the minimum potential principle

$$\left(\int_e B^T Q B dv \right) \delta^e - \int_e B^T Q \varepsilon_T dv = 0 \tag{16}$$

The above equation can be further simplified as follows

$$[K]^e \delta^e = \{P\}^e \tag{17}$$

in which $[K]^e = \int_e B^T QBdv$ is the element stiffness matrix, and $\{P\}^e = \int_e B^T Q\varepsilon_T dv$ is the nodal load.

From Eq. (17), it is found that in the finite element implementation although transverse normal deformation is considered, the displacement variables have not been increased since thermal loads could be absorbed in the generalized force vector.

For the whole structure

$$[K]\{\delta\} = \{P\} \tag{18}$$

where $[K] = \sum_{e=1}^{N} [K]^e$, $\{P\} = \sum_{e=1}^{N} \{P\}^e$, and N is the total number of elements.

5. Numerical results and assessment

In order to assess the performance and validity of the proposed model as well as the three-node element, simply-supported laminated composite and sandwich beams subjected to thermal loading are analyzed.

Material (1) laminated beams (Kapuria *et al.* 2003):

$$E_L = 181\text{GPa}\,, E_T = 10.3\text{GPa}\,, G_{LT} = 7.17\text{GPa}\,, G_{TT} = 2.87\text{GPa}\,,$$

$$\nu_{LT} = 0.28,\ \ \nu_{TT} = 0.33, \alpha_L = 0.02\times10^{-6}\,/\,\text{K}\,, \alpha_T = 22.5\times10^{-6}\,/\,\text{K}\,.$$

Material (2) sandwich beams (Matsunaga 2003):

Face sheets (h/10×2):

$$E_0 = 144.8\text{GPa}\,, E_L = E_0\,, E_T = 0.04E_0\,, G_{LT} = 0.008E_0\,, G_{TT} = 0.02E_0\,,$$

$$\nu_{LT} = 0.25\,, \alpha_0 = 10^{-6}\,/\,K\,, \alpha_L = 0.139\times10^{-6}\,/\,\text{K}\,, \alpha_T = 9\times10^{-6}\,/\,\text{K}\,.$$

Core material (4h/5):

$$E_0 = 144.8\text{GPa}\,, E_L^c = E_T^c = 0.0016E_0\,, G_{LT}^c = G_{TT}^c = 0.0024E_0\,, \nu_{LT}^c = 0.25\,,$$

$$\alpha_L = 0.139\times10^{-6}\,/\,\text{K}\,, \alpha_T = 9\times10^{-6}\,/\,\text{K}\,.$$

Material (3) laminated beams (Matsunaga 2003):

$$E_L = 144.8\text{GPa}\,,\ \ E_T = 9.65\text{GPa}\,,\ \ G_{LT} = 4.14\text{GPa}\,, G_{TT} = 3.45\text{GPa}\,, \nu_{LT} = 0.3\,,$$

$$\alpha_L = 0.139\times10^{-6}\,/\,\text{K}\,, \alpha_T = 9\times10^{-6}\,/\,\text{K}\,.$$

Where subscript L is the direction parallel to the fibers and subscript T denotes the transverse direction.

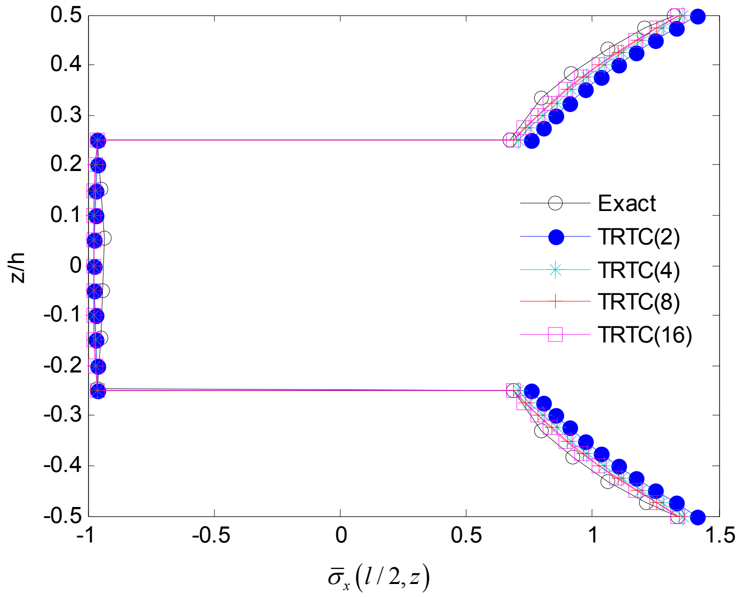


Fig. 2 Convergence study of in-plane stress (*l*/*h*=5)

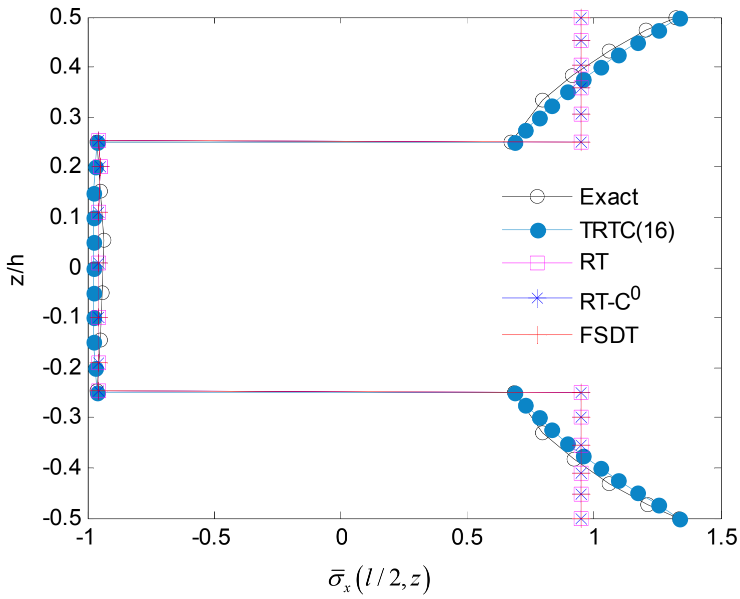


Fig. 3 In-plane stress through thickness of laminated beam (*l*/*h*=5)

Simply supported boundary conditions based on the proposed model TRTC is given by
$w_0=0$ at $x=0,l$.

Example 1 A three-layer [0°/90°/0°] simply-supported laminated beam subjected to thermal loads $\Delta T=T_0\sin(\pi x/l)$. l is the length of the beam along the x-axis.

The stresses are normalized as follows:

$$\bar{\sigma}_x(l/2,z)=\frac{\sigma_x(l/2,z)}{\alpha_T E_T T_0},\bar{\tau}_{xz}(0,z)=\frac{\tau_{xz}(0,z)l}{\alpha_T E_T T_0 h},T_0=1.$$

The mesh convergence study of in-plane stresses for laminated beam is shown in Fig. 2. Numerical results show that the present results with 16 elements have converged to the exact solution (Matsunaga 2003). In Fig. 3, distributions of in-plane stresses through the thickness of a three-layer beam with material (1) are presented. It may be readily seen from Fig. 3 that the results obtained from the model TRTC are in good agreement with the exact solution (Matsunaga 2003). However, FSDT, RT and RT-C^0 are less accurate for predicting the thermal expansion problems of laminated beams owing to neglecting transverse normal strain. A comparison of transverse shear stresses based on four different theories can be found in Fig. 4, and the similar accuracy is obtained.

Example 2 A simply-supported three-layer [0°/core/0°] sandwich beam subjected to thermal loads $\Delta T=T_0\sin(\pi x/l)$. l is the length of the beam along the x-axis.

The displacements and stresses are normalized as follows:

$$\bar{u}(0,z)=\frac{100u(0,z)h^2}{\alpha_0 T_0 l^3},\ \bar{\sigma}_x(l/2,z)=\frac{\sigma_x(l/2,z)}{\alpha_0 T_0 E_0},\ \bar{\tau}_{xz}(0,z)=\frac{\tau_{xz}(0,z)}{\alpha_0 T_0 E_0}.$$

To study the effects of transverse normal strain on thermal stresses, a three-layer sandwich beam with material (2) is considered. For the displacement and stresses distributions through the thickness of the sandwich beams are shown in Figs. 5, 6 and 7, respectively. It is noted that the results of TRTC agree well with the exact solutions (Matsunaga 2003) in comparison with the models FSDT, RT and RT-C$^{\mathbf{0}}$. Numerical results show that transverse normal strain can not be ignored for analysis of thermal expansion of sandwich beams.

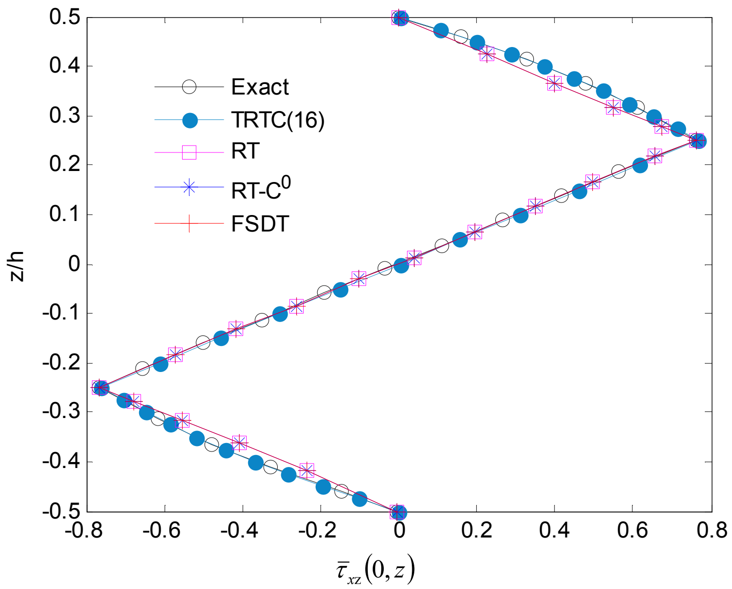


Fig. 4 Transverse shear stress through thickness of laminated beam (l/h=5)

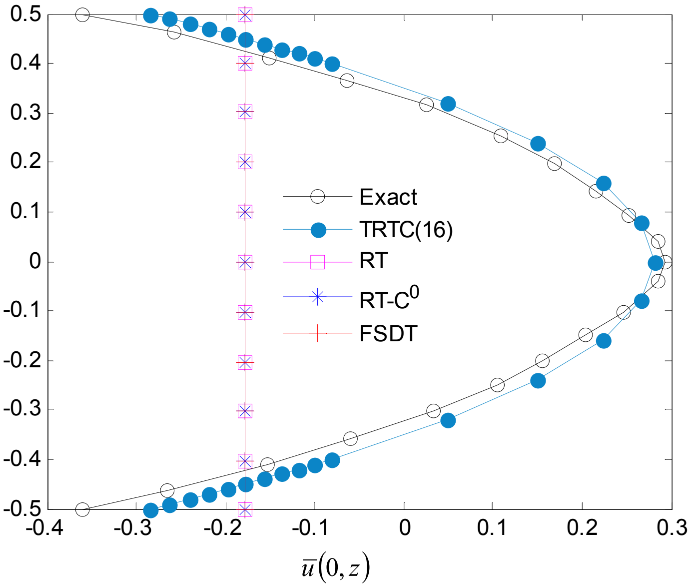


Fig. 5 In-plane displacement through thickness of sandwich beam (*l*/*h*=5)

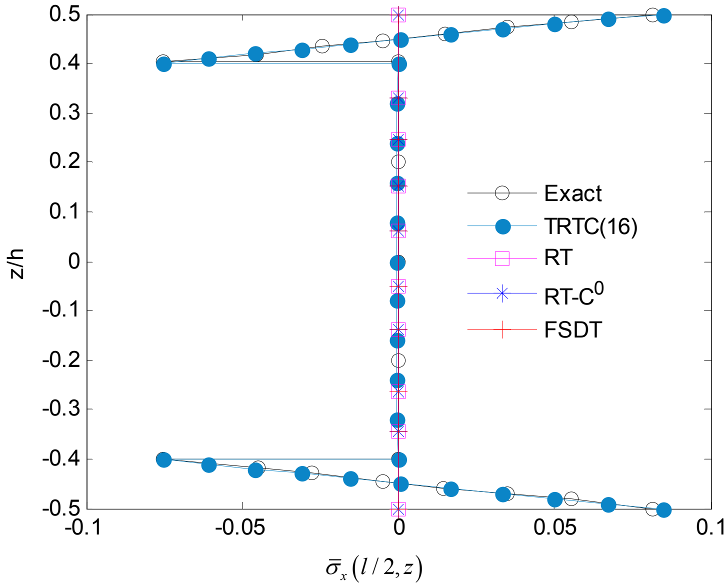


Fig. 6 In-plane stress through thickness of sandwich beam (*l*/*h*=5)

Example 3 A simply-supported 16-layer [0°/90°/.../0°/90°] laminated beam subjected to thermal loads $\Delta T = T_0 \sin(\pi x/l)$. l is the length of the beam along the x-axis.

The material (3) is used, and stresses are normalized as follows:

$$(\bar{\sigma}_x, \bar{\tau}_{xz}) = (\sigma_x, \tau_{xz}) / (\alpha_0 T_0 E_L), \alpha_0 = 10^{-6} / K .$$

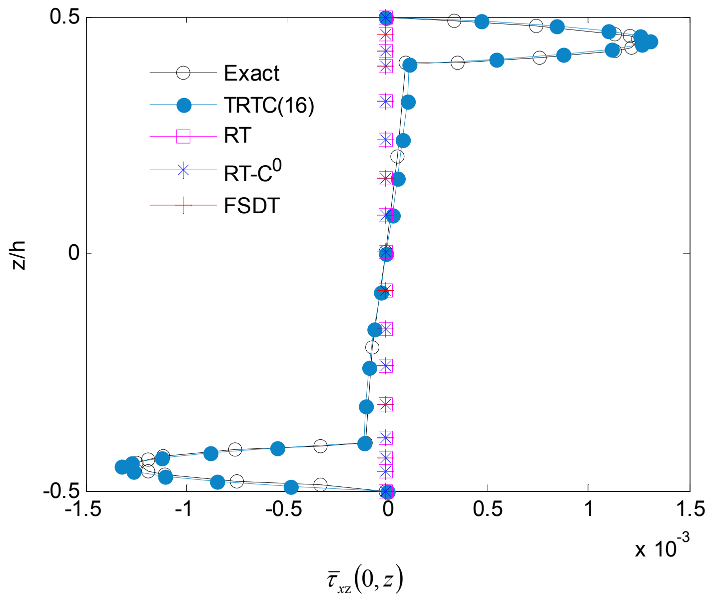


Fig. 7 Transverse shear stress through thickness of sandwich beam (*l*/*h*=5)

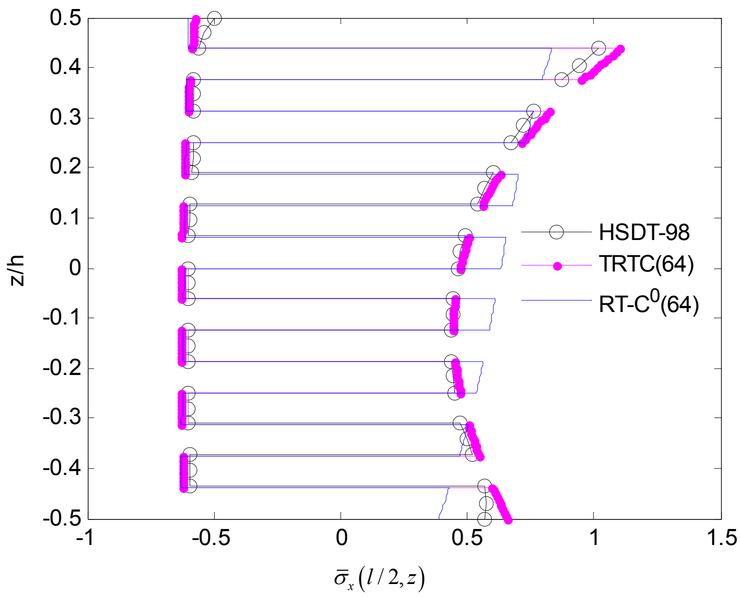


Fig. 8 In-plane stress for a 16-layer beam under thermal loads (*l*/*h*=5)

To evaluate the performance of the proposed model TRTC for multilayered beams, thermal expansion problems of unsymmetric 16-layer beams are studied in this example. Distributions of in-plane stresses and transverse shear stresses through the thickness of 16-layer beams with material (3) are respectively plotted in Figs. 8 and 9. It is observed that results obtained from the model TRTC agree well with HSDT-98 obtained by the authors based on the higher-order theory

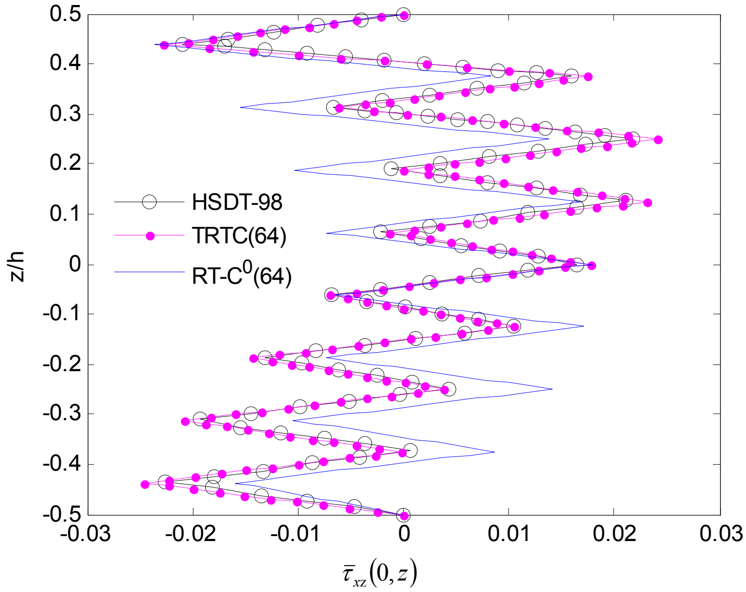


Fig. 9 Transverse shear stress for a 16-layer beam under thermal loads (l/h=5)

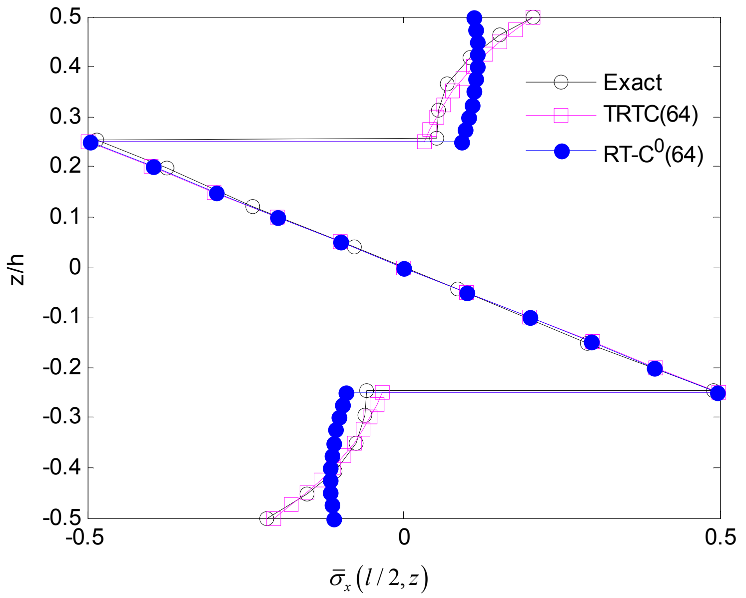


Fig. 10 In-plane stress for a three-layer beam under thermal loads (l/h=5)

proposed by Matsunaga (Matsunaga 2003). However, results of RT-C^0 seem to be less satisfactory owing to neglecting transverse normal strain.

Example 4 A simply-supported three-layer [0°/90°/0°] laminated beam subjected to thermal loads $\Delta T=(2zT_0/h)\sin(\pi x/l)$ is analyzed. l is the length of the beam along the x-axis.

The normalized stresses are the same as those of Example 1. To further verify the accuracy of

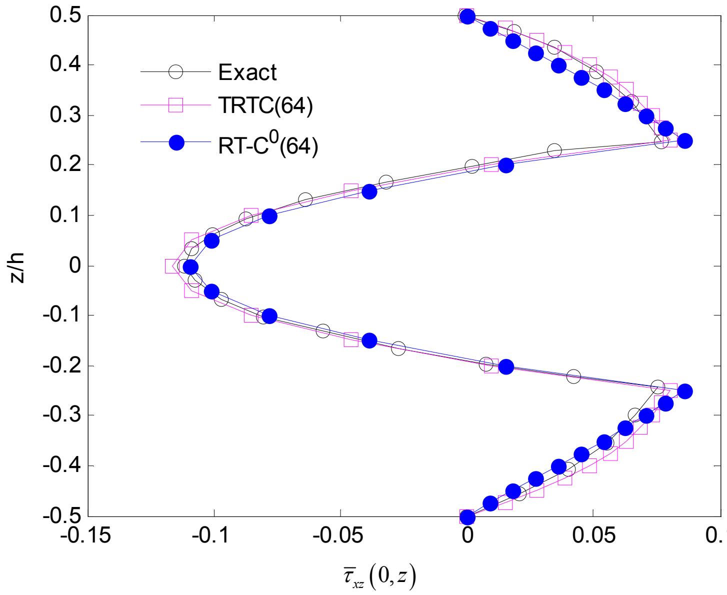


Fig. 11 Transverse shear stress for a three-layer beam under thermal loads (*l/h*=5)

the present model, thermal bending of a simply supported three-layer [0°/90°/0°] laminated beam of Material (1) is considered, and corresponding results are shown in Figs. 10 and 11. Numerical results show that present results of stresses are in good agreement with the exact solutions (Kapuria *et al.* 2003), whereas RT-C^0 is less accurate because of neglecting transverse normal strain.

5. Conclusions

Based on the C^0-type Reddy's beam theory considering transverse normal thermal strain, a three-node beam element is presented for analysis of thermal expansion and bending problems of simply-supported laminated composite and sandwich beams in this paper. On one hand, this paper draws the conclusions that the proposed model overcomes the C^1 requirement, so that its finite element counterparts only require C^0 interpolation functions. Owing to its C^0 requirement, the present model is adequate for implementations in commercial finite element codes to offer the more design practices. On the other hand, the proposed model can produce accurate stresses and displacements to predict the thermal response in contrast to FSDT, RT and RT-C^0, since transverse normal thermal deformation is introduced in the in-plane displacement field.

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